Multinomial logit estimation of a diameter growth matrix model of two Mediterranean pine species in Spain

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ABSTRACT

Introduction: Understanding diameter growth of the Mediterranean pine species is fundamental for evaluating and making appropriate strategic decisions in forest management. A matrix diameter growth model for two Mediterranean pine forests ecosystems in Spain has been developed.

Methods: Multinomial logistic (MNL) regression was employed to estimate the transition probabilities of a matrix growth model. The model combines individual-tree and stand attributes and explores the effects of independent variables and their relationships with tree size and the probabilities of stand diameter growth. The aim was to predict growth of individual trees by diameter class for a five-year period.

Results: MNL model results for diameter growth gave better predictions for Maritime pine (*Pinus pinaster* Aiton) than for Scots pine (*Pinus sylvestris* L.). Stand simulations showed that diameter growth probabilities depended on productivity and stand density. Stand simulations under fixed conditions showed that the probabilities of diameter growth increased as site productivity increased and decreased with increased stand density index.

Conclusions: This study demonstrates the usefulness of matrix growth models as tools to predict growth in Mediterranean pine forests. Stand density and site productivity are key factors in explaining Scots pine and Maritime pine forest growth in the study area.

Key words: Diameter class/ Growth/ Multinomial logit/ Stand simulations/ Transition probabilities

1. INTRODUCTION

Matrix population models are key tools in demographic studies based on the analysis of survival, growth and reproduction data. In operational forestry, assessing the long-term sustainability of forest practices and adequately estimating and projecting forest growth under different climate and management scenarios could allow foresters to make appropriate decisions with significant consequences at both economic and ecological levels. Matrix models have been applied in forestry (Usher, 1966; Usher, 1969; Buongiorno and Michie, 1980; Buongiorno, Peyron et al., 1995), because they are conceptually simple (Buongiorno and Michie, 1980) and useful for predicting the dynamics of the number of trees within individual diameter classes as a function of time. Such critical information is needed to simulate stands affected by different harvest scenarios (Volin and Buongiorno, 1996).

Matrix models have been widely used with temperate and tropical forests. López et al. (2008) showed a method for estimating the long-term sustainable harvest rates and the stable diameter distributions of uneven-aged managed Beech (*Fagus sylvatica* L.) stands in northwestern Navarra (Spain), using a projection matrix model. This methodology has also been applied in tropical and subtropical forests (Osho, 1991; Spathelf and Durlo, 2001) and in temperate forests (Roberts and Hruska, 1986; Monserud and Sterba, 1999; Hao, Meng et al., 2005), but not in Mediterranean forests. Matrix models normally require grouping trees into size classes that are usually based on diameter at breast height (Picard, Bar-Hen et al., 2003; Shimatani, Kubota et al., 2007). Diameter increment (Lowell and Mitchell, 1987) is a useful proxy for tree biomass growth and can be predicted by probabilistic models based on multinomial distribution.

Multinomial logit (MNL) models are frequently used in market decisions (Wang, Bennett et al., 2007), recreational activities (Mogas, Riera et al., 2006) and computational systems (Prinzie and Van den Poel, 2008). In forestry applications, they have been used in operational forestry (Kurttila, Hämäläinen et al., 2001), for valuing ancient forest ecosystems (Englin, McDonald et al., 2006), in tropical dry forests (Boltz and Carter, 2006), and to determinate of tree quality in natural uneven-aged pine stands (Prestemon and Buongiorno, 2000). However, this approach has not been used before to model tree-diameter growth in Mediterranean forests.

In this paper we attempt to develop a diameter growth model useful for decision-making in two Mediterranean pine ecosystems, using MNL estimation. Our goal was to develop a growth model using transition matrix elements based on probabilistic approach for diameter class which would adequately predict change in tree diameter growth. The MNL model enabled us to simulate stand development dynamics, with transitions influenced by changing characteristics.

2. MATERIALS AND METHODS

2.1. Study area and focus species

The study was conducted in Scots pine (*Pinus sylvestris* L.) and Maritime pine (*Pinus pinaster* Ait.) stands in the Central and Iberian Mountain Range Systems in Spain (0º37'–4º12'W; 39º48'–42º58'N). The altitude of Maritime pine stands ranges from 990 to 1200 m.a.s.l. and for Scots pine stands from 1270 to 2210 m.a.s.l. Scots pine is one of the most important species in Europe, dominating forest landscapes together with other species in boreal areas and Mediterranean mountains. In Spain, Scots pine stands occupy 840,000 ha as mono-specific stands and 370,000 ha as mixed-forest stands. Maritime pine is a conifer from the western Mediterranean Basin, covering more than 4 million ha over wide ranges of elevation, climate and soil. As the most extensive conifer in Spanish forests, it covers approximately 1,200,000 ha, including natural and artificial stands (Bravo-Oviedo, Río et al., 2004); over 700,000 ha are pure stands and 600,000 ha are mixed forest stands, most of them in the Mediterranean areas (Río, Bravo et al., 2004). Silviculture in the area is based on natural regeneration and silvicultural interventions are not frequent at early stages of development.

2.2. Data

Data from 101 permanent sample plots of Scots pine stands and 90 plots of Maritime pine stands were recorded (7316 trees tallied overall). These plots were installed between 2002 and 2004. The plots were located along the area of distribution of the species studied, and represent the existing range of ages, stand densities and sites. Plots consisted of three concentric subplots with radii of 5, 10 and 15 meters each. For these subplots, the minimum diameter at breast height (DBH) was recorded at 7.5, 12.5 and 22.5 cm, respectively. At plot establishment, the following data were recorded for every sample tree: species, DBH (trunk diameter outside bark at 1.3 m above the ground), total height (HT, m), height to crown base (HCB, m), height to the largest

crown width (HLCW, m) and horizontal distance from plot centre to the vertical axis of the tree in meters (D, m). Diameters were measured with a caliper in two perpendicular directions (to the nearest millimeter). Forest stands ranged in age from 20 to 188. A summary of the characteristics of the plots used in the study is presented in Table 1.

It was necessary to backdate all of the tree measurements to estimate their values at the start of the previous 5-year growth. Tree attribute backdating was obtained using methodology proposed by Hann and Hanus (2001). More information about the backdating process used can be found in Lizarralde (2008). No thinning treatments or harvesting operations were conducted in the sample plots in the previous 5-year period.

Table 1. Summary statistics of the independent variables and description of the stands.

Variable	Mean	Minimum	Maximum	Standard Deviation	Standard Error
Pinus sylvestris L. (4071 trees in 101 plots)					
DBH_t	30.119	6.130	64.280	9.889	0.15498
DBH_{t+5}	31.121	8.000	66.500	9.932	0.15567
SI	24.101	12.580	38.020	5.174	0.08109
A	89.182	30.250	188.250	35.264	0.55269
BA	1.229	0.000	4.590	0.622	0.00975
N	1027.868	212.210	4594.270	937.163	14.68807
SDI	1021.708	362.260	1546.710	230.919	3.61917
HSI	29.495	17.350	85.070	8.814	0.13814
BAL	28.747	0.000	77.620	16.484	0.25834
QMD	29.679	12.840	50.450	8.465	0.13266
D_{DBH}	0.499	0.000	1.000	0.289	0.00453
Pinus pinaster Ait. (3542 trees in 90 plots)					
DBH_t	25.724	5.980	59.100	7.831	0.13158
DBH_{t+5}	26.750	7.650	60.050	7.768	0.13052
SI	14.674	7.110	25.060	4.453	0.07482
A	71.146	20.750	127.500	29.287	0.49209
BA	0.978	0.000	3.880	0.373	0.00627
N	1055.871	159.150	4594.270	659.802	11.08637
SDI	834.841	377.690	1459.010	170.063	2.85749
HSI	44.039	18.880	95.120	10.657	0.17907
BAL	21.344	0.000	57.080	12.690	0.21323
QMD	24.760	10.280	43.510	6.682	0.11227
D_{DBH}	0.503	0.001	1.000	0.286	0.00481

DBH: Diameter at breast height over bark in cm, QMD: quadratic mean diameter in cm, N: number of trees per ha, BA: basal area in square meters per tree, BAL: basal area larger trees in square meters per ha, SI: site index (or productuvity), A: stand age in years, HSI: Hart-Becking Spacing Index, SDI: Reineke's Stand Density Index, D_{DBH}: distance from DBH of the individual-tree to the upper limit of its size class in cm.

2.3. Statistical analysis

Diameter classes were defined at 1 cm intervals and the growth interval was defined as a 5-year period. Data were classified in I diameter size-classes. Fifty-nine diameter classes for Scots pine stands and 54 diameter classes for Maritime pine stands were used respectively. MNL models were used to model relationships between a polytomous response variable and a set of regressor variables. A discrete random variable Y was then defined as the number of size-classes advanced by an individual tree during a period of time. The support of Y was defined by the set $\{0,1,...,K\}$ where K was the maximum number of size classes advanced. After observing our dataset, we consider $K = 5$ for Scots pine and $K = 4$ for Maritime pine. Let p_k with $k = 0,1,...,K$ represent the probability function for variable Y, i.e. $p_k = p(Y = k)$. Therefore, p_k is the probability for an individual tree to grow k-diameter classes in a time period, which we called, k-upgrowth probability. With this notation, MNL regression to estimate the probabilities p_k was employed using individual tree and stand attributes as explanatory variables.

Based on the data available for this study, individual-tree attributes such as diameter at breast height (DBH, cm), distance from DBH of the individual-tree to the upper limit of its size class (D_{DBH}, cm), basal area (BA, m^2), basal area in larger trees (BAL, m^2 ha^{-1}); and stand variables such as number of trees (N, ha^{-1}), Reineke stand density index (SDI), Hart-Becking index (HSI) or relative spacing index, stand age (A, years), site index (SI), quadratic mean diameter (QMD, m^2 ha^{-1}) were analyzed. These different candidate models were tested and stepwise method was used to select the independent variables. Finally, seven explanatory variables were selected for use in MNL regression (P>0.05). Goodness of fit for all models was examined (Nagelkerke, 1991) and adjusted-R^2 measures computed (McFadden, 1979). The best models were selected using Akaike's Information Criterion (AIC).

Two different MNL models were estimated and selected for each data set (by species). The first was a basic model that showed the importance of the choice of seven independent variables (henceforth, this model will be called Model 1). In the second model, the variable D_{DBH} was not included (henceforth, called Model 2). The models can be written as:

$$\ln\left(\frac{p_k}{p_0}\right) = Z_k = \beta_{i1}DBH + \beta_{i2}DBH^2 + \beta_{i3}BAL + \beta_{i4}SI + \beta_{i5}A + \beta_{i6}SDI + \beta_{i7}D_{DBH} \quad ..(1)$$

$$\ln\left(\frac{p_k}{p_0}\right) = Z_k = \beta_{i1}DBH + \beta_{i2}DBH^2 + \beta_{i3}BAL + \beta_{i4}SI + \beta_{i5}A + \beta_{i6}SDI \qquad (2)$$

Both equations above applied for $k = 1,2,...,K$; and $\beta_{i1},...,\beta_{i7}$ were parameters to be estimated. Note that, in both cases, a zero intercept was assumed to normalize the model, as proposed by Greene (2000). The maximum likelihood method was used to estimate the parameters of the models. The analysis was performed with the SAS v.9.1 program using PROC LOGISTIC procedure (SAS, 2004). Once the parameters were estimated, we obtained the estimated values $\hat{Z}_k$. Finally, the estimated probabilities $\hat{p}_k$ for each individual tree were obtained. The specific expression of MNL regression was given by:

$$\hat{p}_k = \frac{e^{\hat{Z}_k}}{1 + \sum_{l=1}^{K} e^{\hat{Z}_l}} \text{ for } k = 1,2,...,K \text{ and } \hat{p}_0 = \frac{1}{1 + \sum_{l=1}^{K} e^{\hat{Z}_l}} \text{...........................} (3)$$

The dependent variable was multinomial with four and five possible components (Maritime pine and Scots pine, respectively). We denoted each component as growth up diameter classes (K was 1, 2, 3, 4 or 5). Now, let suppose that we have a stand with fixed values SI, A, SDI and let DBH_i be the center of the diametric class i in cm and N_i is the number of trees per ha in this class for each $i = 1,2,...I$. We can then calculate basal area in square meters by ha, for class i as $BA = N_i \pi / 4 (DBH_i)^2$ and BAL in square meters by ha for class i as:

$$BAL_i = \sum_{l=i+1}^{I} BA_i \text{ for } i = 1,2,...,I-1 \text{ (evidently } BAL_I = 0 \text{)}.$$

Using MNL regression, these values allow estimating probabilities $\hat{p}_k(DBH_i, BAL_i, SI, A, SDI, 0.5)$ or $\hat{p}_k(DBH_i, BAL_i, SI, A, SDI)$, depending on the model used. Finally, if π_{ij} denotes the transition probability between diameter class i and j at the end of the growth period, in Model 1 we can estimate probability in the following way:

$$\hat{\pi}_{ij} = \hat{p}_{j-i}(DBH_i, BAL_i, SI, A, SDI, 0.5) \text{ for } j = i, i+1,...,i+K \qquad (4)$$

or, if appropriate, to implement Model 2, as follows:

$$\hat{\pi}_{ij} = \hat{p}_{j-i}(DBH_i, BAL_i, SI, A, SDI) \ \text{ for } \ j = i, i+1, ..., i+K \tag{5}$$

These values defined matrix transitions of probabilities. Thus, stand evolution could now be projected over several periods. No growth was used as a categorical reference. Therefore, the MNL estimates of transition probabilities were expressed in k events (depending on the species). The data were randomly divided into two subsets for model fitting and testing (80% and 20% of the total number of dataset, respectively). Once the model was selected, the entire dataset, for both species, was used for estimating parameters and transition probabilities of a matrix growth model.

3. RESULTS

3.1. Diameter growth model

Model response was multi-categorical. Our modeling approach using MNL regression was able to estimate four upgrowth diameter classes for Maritime pine stands and five events for Scots pine stands. Our results revealed that size (DBH) for Scots pine and age (A) for Maritime pine have influence on growth. For both species, site productivity (SI) showed a remarkable effect on diameter growth model (Table 3).

3.2. Parameters, selection and validation of the MNL models

Results of the adjusted-R^2 test fitted were acceptable (R^2=0.54 in Scots pine stands and R^2=0.57 in Maritime pine stands). Model 1 fits the data better than Model 2 as the low AIC values show (Table 2). However, Model 2 could also be considered useful. Although Model 2 shows lower adjusted-R^2 values than Model 1, the former uses only stand and diameter class information. The coefficient estimates of the model selected (Eq. 1 and 2) were 35 and 28 for Scots pine and Maritime pine, respectively. The number of estimate parameters on models differed due to k diameter class predicted and the number of independent variables used. In Model 2, DBH2 was not significant in Maritime pine stands so the number of parameters was consequently reduced to only 20 and 30, respectively.

The MNL regression results for both models are shown in Table 3. The estimated parameters of the selected growth models were consistent and they had reasonable values and signs. For both species, diameter class growth was higher on the better sites. It was observed that when DBH increased, the coefficient parameters of

β_{i1} decreased, whereas as stand age increased, parameters of β_{i5} increased. See Table 3 for Maritime pine (Model 1). Opposite effects were observed for Scots pine when the same variables were analyzed. The D_{DBH} variable had a negative effect on diameter growth prediction (k – diameter class) for both species.

Table 2. Statistical parameters of the models (Model 1 and 2).

Species	Model	Number of parameters	AIC	SC	-2 LOG L	Generalized R^2 (*)	Adjust-R^2 (**)
P. sylvestris	1	35	6766.82	6987.73	6696.82	0.8805	0.5361
P. pinaster	1	28	4915.75	5088.58	4859.75	0.8774	0.5687
P. sylvestris	2	30	7842.30	8031.65	7782.30	0.8353	0.4624
P. pinaster	2	20	6151.01	6274.46	6111.01	0.8077	0.4604

* Nagelkerke (1991); ** McFadden (1979)

Table 3. Maximum likelihood estimates (MLE) of transition model parameters (Model 1 and 2). Asymptotic standard errors are given in parentheses.

Variable	Upgrowth (k)	*Pinus sylvestris* L. Model 1	Model 2	*Pinus pinaster* Ait. Model 1	Model 2
DBH	1	0.20095 (0.01940)	0.07080 (0.01680)	0.13051 (0.03110)	-0.04150 (0.00961)
	2	0.23324 (0.02510)	0.06480 (0.02160)	0.01840 (0.04280) ns	-0.13940 (0.01430)
	3	0.19599 (0.04760)	0.00647 (0.04420) ns	-0.16177 (0.08090)	-0.22200 (0.03000)
	4	0.68821 (0.13340)	0.45350 (0.12830)	-0.01083 (0.32760) ns	-0.49180 (0.09910)
	5	2.26191 (1.28790) ns	1.79190 (1.03830) ns		
DBHxDBH	1	-0.00252 (0.00028)	-0.00078 (0.00024)	-0.00327 (0.00053)	
	2	-0.00290 (0.00036)	-0.00067 (0.00031)	-0.00325 (0.00073)	
	3	-0.00204 (0.00068)	0.00049 (0.00063) ns	-0.00185 (0.00147) ns	
	4	-0.01084 (0.00255)	-0.00767 (0.00250)	-0.01320 (0.00924) ns	
	5	-0.03698 (0.02490) ns	-0.02910 (0.01970) ns		
D_{DBH}	1	-4.62635 (0.20120)		-6.74732 (0.29640)	
	2	-6.70679 (0.26200)		-9.58148 (0.36650)	
	3	-8.05233 (0.52300)		-12.03700 (0.65910)	
	4	-8.46499 (1.00170)		-12.03259 (1.26950)	
	5	-6.85632 (2.08210)			
BAL	1	-0.02067 (0.00398)	-0.01600 (0.00348)	-0.05204 (0.00627)	-0.03480 (0.00496)
	2	-0.04190 (0.00533)	-0.03580 (0.00474)	-0.11505 (0.00900)	-0.08670 (0.00760)
	3	-0.03029 (0.01200)	-0.02220 (0.01170) ns	-0.19077 (0.01900)	-0.14200 (0.01730)
	4	-0.04151 (0.03560) ns	-0.03560 (0.03640) ns	-0.20892 (0.03980)	-0.17490 (0.03720)
	5	0.22798 (0.23370) ns	0.21820 (0.21200) ns		
SI	1	0.08966 (0.01310)	0.04610 (0.01170)	0.33824 (0.02400)	0.18150 (0.01660)
	2	0.18790 (0.01600)	0.12720 (0.01390)	0.64304 (0.03230)	0.40740 (0.02270)
	3	0.25343 (0.02610)	0.18360 (0.02410)	0.83418 (0.05920)	0.49200 (0.04320)
	4	0.13514 (0.05610)	0.07270 (0.05500) ns	0.67512 (0.12930)	0.46440 (0.08370)
	5	-0.47720 (0.45880) ns	-0.44010 (0.37150) ns		
A	1	-0.01206 (0.00210)	-0.00908 (0.00183)	0.01407 (0.00376)	0.00999 (0.00268)
	2	-0.02550 (0.00310)	-0.02120 (0.00279)	0.02354 (0.00546)	0.01370 (0.00416)
	3	-0.04097 (0.00814)	-0.03630 (0.00791)	0.04339 (0.01120)	0.02060 (0.00983)
	4	-0.08309 (0.02850)	-0.07580 (0.02830)	0.04067 (0.03050) ns	0.03950 (0.02380) ns
	5	-0.23899 (0.19650) ns	-0.22950 (0.19070) ns		
SDI	1	0.00001 (0.00023) ns	-0.00005 (0.00020) ns	0.00015 (0.00032) ns	-0.00012 (0.00025) ns
	2	-0.00170 (0.00029)	-0.00166 (0.00025)	-0.00176 (0.00044)	-0.00218 (0.00035)
	3	-0.00433 (0.00056)	-0.00424 (0.00053)	-0.00330 (0.00083)	-0.00373 (0.00074)
	4	-0.00692 (0.00150)	-0.00649 (0.00140)	0.00001 (0.00214) ns	0.00039 (0.00200) ns
	5	-0.01992 (0.01070) ns	-0.01730 (0.00916) ns		

ns=not significant; all other variables are significant at $p < 0.05$.

We validated models with a portion of the total dataset selected randomly (20%). The validation method indicated that the model could be applied across a wide range of growth conditions and forest structure. We compared and analyzed observed and

predicted diameter growth data. Table 4 shows that the models successfully predicted diameter growth. Figure 1 (a, b) illustrates the performances of Model 1 and 2 against some independent variables (A, SDI, BA and SI). The success rate in predicting no growth class change was 54.6% for Maritime pine stands and 46.7% for Scots pine stands. Although the overall average of diameter growth predictions was acceptable (k=1 predicted substantially more than 85%), it appears that the MNL models produce larger negative biases (sub-estimation) for larger growth diameter classes (e.g. k=2 and 3). However, Model 1 shows a better prediction for Maritime pine stands than for Scots pine stands, except within a particular range of site productivity (from 13 to 23 m). It was clear that Model 1 produced more consistent and accurate results than Model 2. In short, the behavior of the models was satisfactory, showing a high percentage of prediction accuracy.

Table 4. DBH upgrowth validation for *P. sylvestris* L. and *P. pinaster* Ait.

Species	Observed (K)	Predicted upgrowth (%)						Trees (n)
		0	1	2	3	4	5	
P. sylvestris (Mod. 1)	0	46.72	51.92	1.36	0.00	0.00	0.00	884
	1	9.68	85.91	4.41	0.00	0.00	0.00	2356
	2	2.79	74.93	22.28	0.00	0.00	0.00	718
	3	0.00	41.76	57.14	0.00	1.10	0.00	91
	4	0.00	26.32	57.89	0.00	15.79	0.00	19
	5	0.00	0.00	0.00	66.67	33.33	0.00	3
P. pinaster (Mod. 1)	0	54.57	45.27	0.16	0.00	0.00	----	645
	1	8.06	85.65	6.29	0.00	0.00	----	2195
	2	1.15	56.14	42.55	0.16	0.00	----	611
	3	0.00	29.33	70.67	0.00	0.00	----	75
	4	0.00	12.50	87.50	0.00	0.00	----	16
P. sylvestris (Mod. 2)	0	2.94	95.48	1.58	0.00	0.00	0.00	884
	1	1.36	94.57	4.07	0.00	0.00	0.00	2356
	2	0.28	81.89	17.83	0.00	0.00	0.00	718
	3	0.00	56.04	43.96	0.00	0.00	0.00	91
	4	0.00	31.58	68.42	0.00	0.00	0.00	19
	5	0.00	33.33	66.67	0.00	0.00	0.00	3
P. pinaster (Mod. 2)	0	1.24	97.98	0.78	0.00	0.00	----	645
	1	0.23	93.58	6.20	0.00	0.00	----	2195
	2	0.00	64.81	35.19	0.00	0.00	----	611
	3	0.00	44.00	56.00	0.00	0.00	----	75
	4	0.00	37.50	62.50	0.00	0.00	----	16

The highest prediction rate of the model for both species occurred for the growth of one diameter class (more than 85% successful predictions). The model showed balanced predictions, mainly for Maritime pine stands (no change in diameter class, growth by one or two diameter classes). The loss of prediction accuracy of the model occurred in larger upgrowth diameter classes ($k \geq 3$) due to the low stem number in

these diameter classes. Figure 1 (a, b) shows the success ratio of the model selected as it relates to some independent variables. As far as we can see, the success ratio of the diameter growth model with respect to some independent variables was quite similar for Models 1 and 2. The performance of the model selected in successful rate prediction was higher for Maritime pine than for Scots pine, although some exceptions have been observed (Figure 1a).

3.3. Transition probabilities of diameter growth

Multinomial equations allow the estimation of transition probabilities. Probabilities of tree diameter growth may be better explained in Scots pine stands than in Maritime pine stands. The width of diameter classes should be small enough to estimate transition probability in Mediterranean pine forests. In our study we used diameter growth classes at 1 cm width. This allowed us to observe five (Scots pine) and four (Maritime pine) maximum diameter growth classes. Boltz and Carter (2006) used multinomial formulation for estimating probabilities from one to three events (mortality, stability and upgrowth). Successful model predictions were very high for no growth (k=0) and one-step diameter growth (k=1). Differences were observed only in the larger upgrowth diameter class prediction ($k \geq 3$) due to the smaller number of individual trees in these classes. The growth transitions in simulated forest conditions were slightly different among species studied.

4. DISCUSSION

4.1. MNL growth model for Mediterranean pine forests

A matrix model to predict diameter upgrowth transition based on MNL regression for two Mediterranean pines in Spain has been developed. The independent variables included in the models represent the main factors affecting tree growth (size, productivity, competition, density and age). The MNL model predicts greater k– diameter growth when stand age ranged from 60 to 100 and SDI was lower for Scots pine stands (Figure 1a). In the present study, stand ingrowth and mortality were not considered in the matrix model. Buongiorno and Michie (1980) modeled recruitment of Sugar maple (*Acer saccharum* Marsh.) to predict long-term growth of undisturbed and managed stands in central and northern Wisconsin and the upper Michigan peninsula (United States). Buongiorno (1995) developed a deterministic model to predict stand evolution.

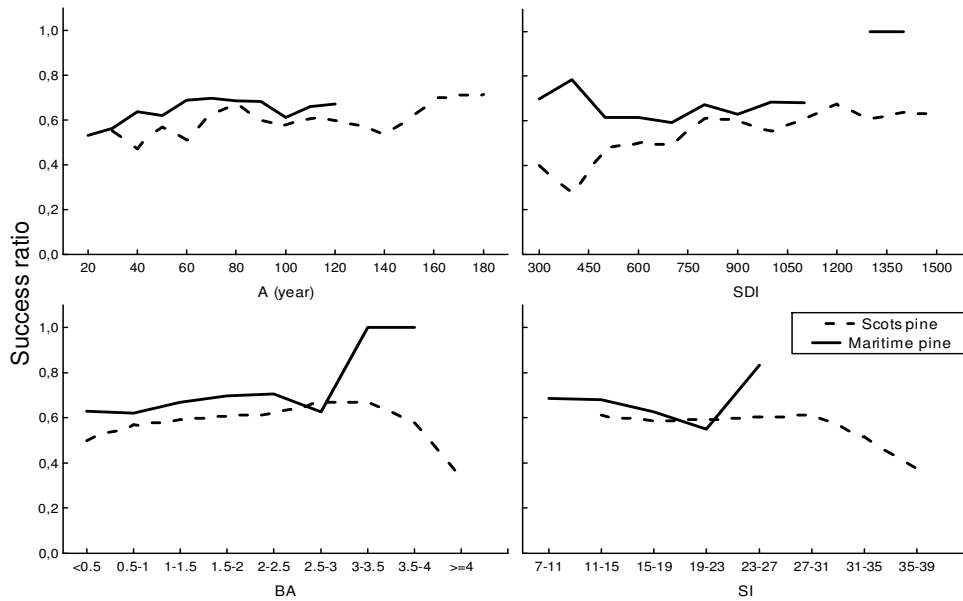*a) Model 1*
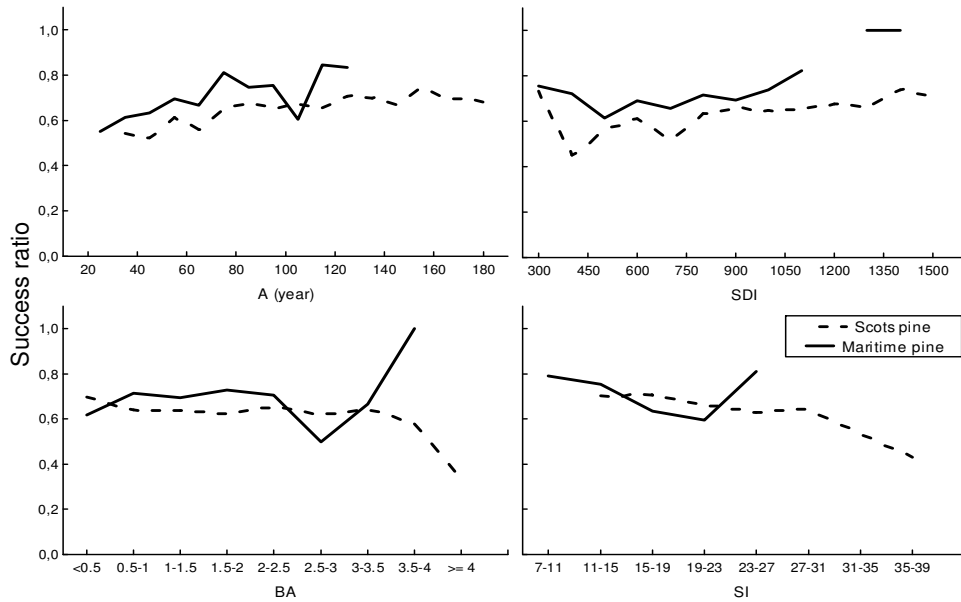
b) Model 2

Figure 1. Model validations are presented taking account of successful ratio of diameter growth prediction in relation to some independent variables. The success ratio was obtained between successful predictions by the models (*k*-upgrowth diameter class) and diameter growth observed in the period studied.

MNL has been used because this methodology allows multi-response analysis. In some cases, results are difficult to interpret given the large number of parameters estimated by the model. To avoid over-parameterization of the models, Shimatani

(2007) demonstrated effects of increasing intervals using 1-cm intervals, and 2-cm intervals on *Camellica japonica* and *Dendropanax trifidus*; the conclusion was that smaller interval classes were better, but as we will need more parameters an equilibrium point between the number of classes and parameters must be found. In our study, we have achieved a balance between number of diameter classes and parameters estimated by the models. The growth classes predicted from MNL show an alternative technique for forest growth modeling in Mediterranean forestry. Data in studies on topics such as differences among ecological guilds (Boltz and Carter, 2006), product distribution from regional inventories (Teeter and Zhou, 1998), and lumber grade and by-product yields in Scots pine (Lyhykäinen, Mäkinen et al., 2009) have been predicted by MNL regression. In Finland, strategic groups were identified by MNL, using forest-owner and forest-holding characteristics as explanatory variables (Kurttila, Hämäläinen et al., 2001). Studying *Pinus tadea* L. and *Pinus echinata* Mill. stands, Prestemon and Buongiorno (2000) used discrete multinomial logit and tobit models to predict the probability that a stem or log belonged to a certain quality class.

Another question is the choice of the diameter class to be used. Division into size classes inevitably involves some subjectivity. A diameter class width similar to ours was used by Shimatani et al. (2007) for forests on Tsushina Island. In other forest types, López et al. (2008) estimated population growth rates with respect to the width of the diameter classes using matrix models for beech (*Fagus sylvatica* L.) stands in Navarra (Spain). Wider diameter classes are usually chosen in tropical forests: e.g., six diameter classes were used in the construction of matrix models for tree population projection in a tropical rain forest of south-western Nigeria (Osho, 1991). The diameter distribution can be a reference to choose the width of the diameter class (López Torres, Fullana Belda et al., 2008) although narrow size classes are recommended in demographic studies using integral projection models (Zuidema, Jongejans et al., 2010). In other studies, various widths of the diameter class have been used (Lin, Buongiorno et al., 1998; Boltz and Carter, 2006), including 5.1 cm for Loblolly pine stands, 5 cm in dry forest or 4 cm for uneven-aged mixed-species forests (Hao, Meng et al., 2005).

On the other hand, diameter growth matrix model performance is affected by diameter class width (Picard, Bar-Hen et al., 2003). In this study, we used one-centimeter diameter class width because it allows better grouping of all individual trees.

This width is smaller than the width used in other previous matrix models (Buongiorno, Dahir et al., 1994), but is similar to the width used in other models (Picard, Bar-Hen et al., 2003). As we can see, different width of size-classes were used in previous studies, while our proposal used very fine classes and calculated a transition probability for every pair of classes. In population dynamics of tree species in Japanese forests, matrix models constructed based on Bayesian non-parametric estimation used also fine diameter classes (Shimatani, Kubota et al., 2007). In summary, many authors argue that narrow categories (1.3-3 cm DBH) should be used in tree matrix models to obtain the best estimations of tree age, population growth rate and elasticity.

By comparing model behavior against different variables (Figure 2a, b), we can evaluate the model performance and gain insight into forest dynamics, as well as obtain information useful for developing adequate management paths. Vanclay (1995) assumed, for transition matrices, that a tree in one of a finite number of size classes has a known probability of moving to another class, dependent only upon its current size. However, in our study the upgrowth probability depended not only on tree size but also on stand characteristics such as density, competition, age and productivity. The transition probabilities that the model assigned for each individual tree were evaluated considering only the maximum probability predicted for that tree. Picard et al. (2003) found a high sensitive of diameter width when they modeled diameter class distribution using a second-order matrix model.

Age influenced diameter growth rates in both species. The upgrowth probability increased for Maritime pine stands and decreased for Scots pine stands. The effect of stand density on diameter growth (Buongiorno, Peyron et al., 1995) was negative in the species studied, as had been previously reported in Loblolly pine stands (Lin, Buongiorno et al., 1998). The model performance is adequate (success rate over 50%) along different variables (A, BA, SDI, DBH and SI). However, performance decreases for extreme values (low or high), in some cases to below 40%. More data would be needed to improve these predictions for extreme values. The MNL model was used to study differences in stand development using transitions influenced by changing stand characteristics (Boltz and Carter, 2006). Site index is a proxy of forest productivity that reflects environmental factors at the stand level. In growth simulations presented here for Scots pine and Maritime pine stands, large growth variations were shown for each *k*-probability in relation to site quality and density.
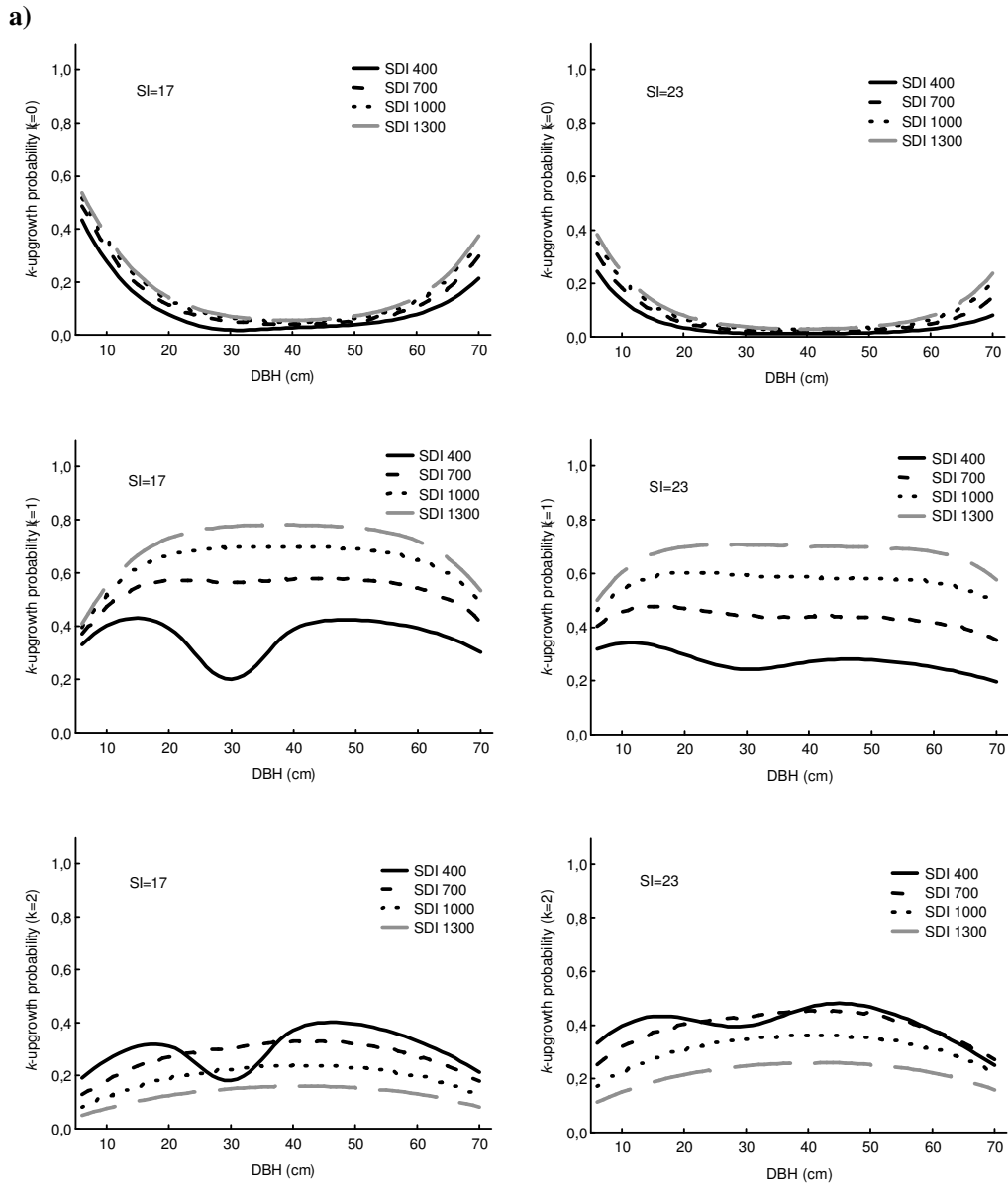
a)

Figure 2. Simulated upgrowth probabilities versus tree DBH for various stand conditions (DBH classes are 1-cm in width).

a) Simulation per upgrowth diameter class, various stand densities (SDI) and site index (SI) for Scots pine stands
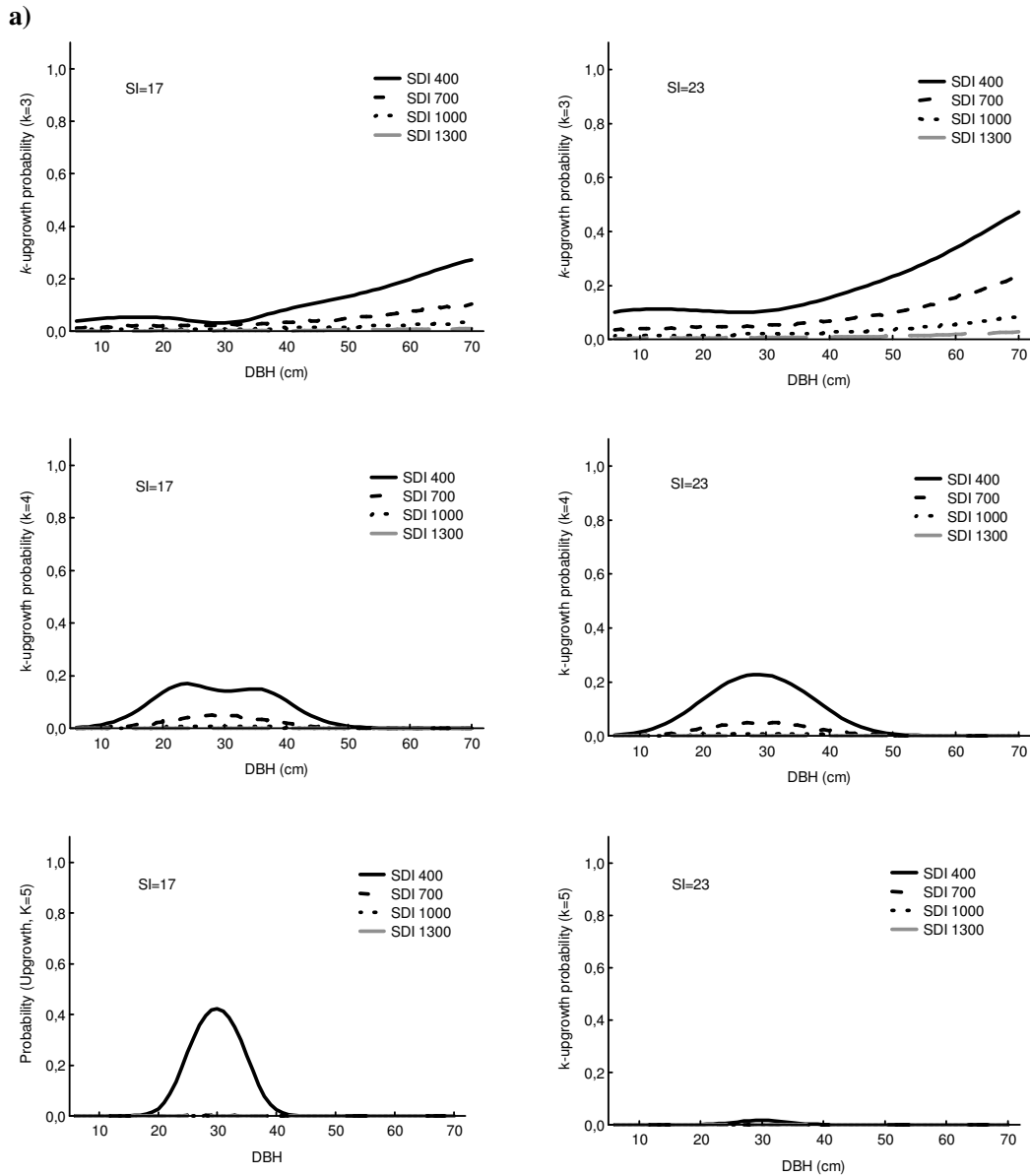Stand conditions: D_{DBH}=0.5, BAL=0, A=50, SI=17 and 23, SDI=400, 700, 1000, 1300

b) Simulation per upgrowth diameter class, various stand densities (SDI) and site index (SI) for Maritime pine stands
Stand conditions: D_{DBH}=0.5, BAL=0, A=50, SI= 9 and 14, SDI=400, 700, 1000, 1300

a)

Figure 2. Continued.

4.2. Growth simulations of Mediterranean pine forests

Simulations were widely used to evaluate differences in stand development (Buongiorno and Michie, 1980; Miina, 1993). To illustrate our growth models and its applicability, we performed simulations for both species in two different forest conditions. We varied initial stand densities (SDI=400, 700, 1000 and 1300) and site index: (a) Scots pine stands (SI=17 and 23 m) and (b) Maritime pine stands (SI=9 and 14 m). Stand age (A=50-year), basal area in larger trees (BAL=0) and D_{DBH} (0.5 cm) were fixed. Such simulations results are illustrated in detail in Figure 2.

15

b)

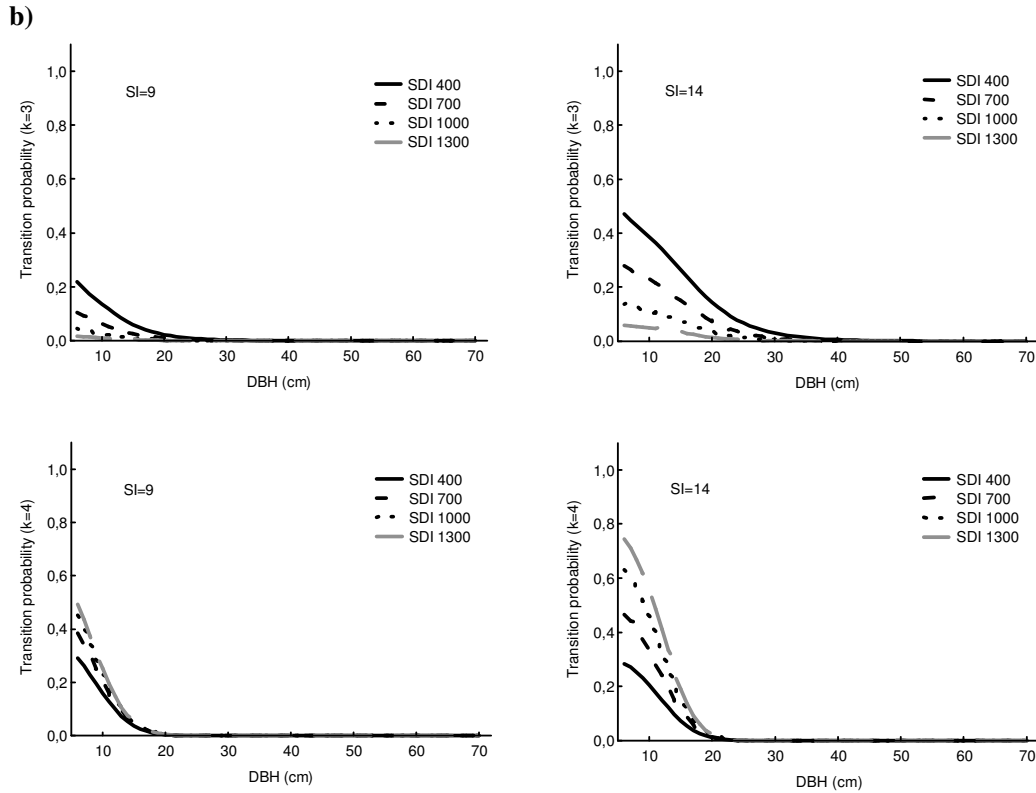
Figure 2. Continued.

b)
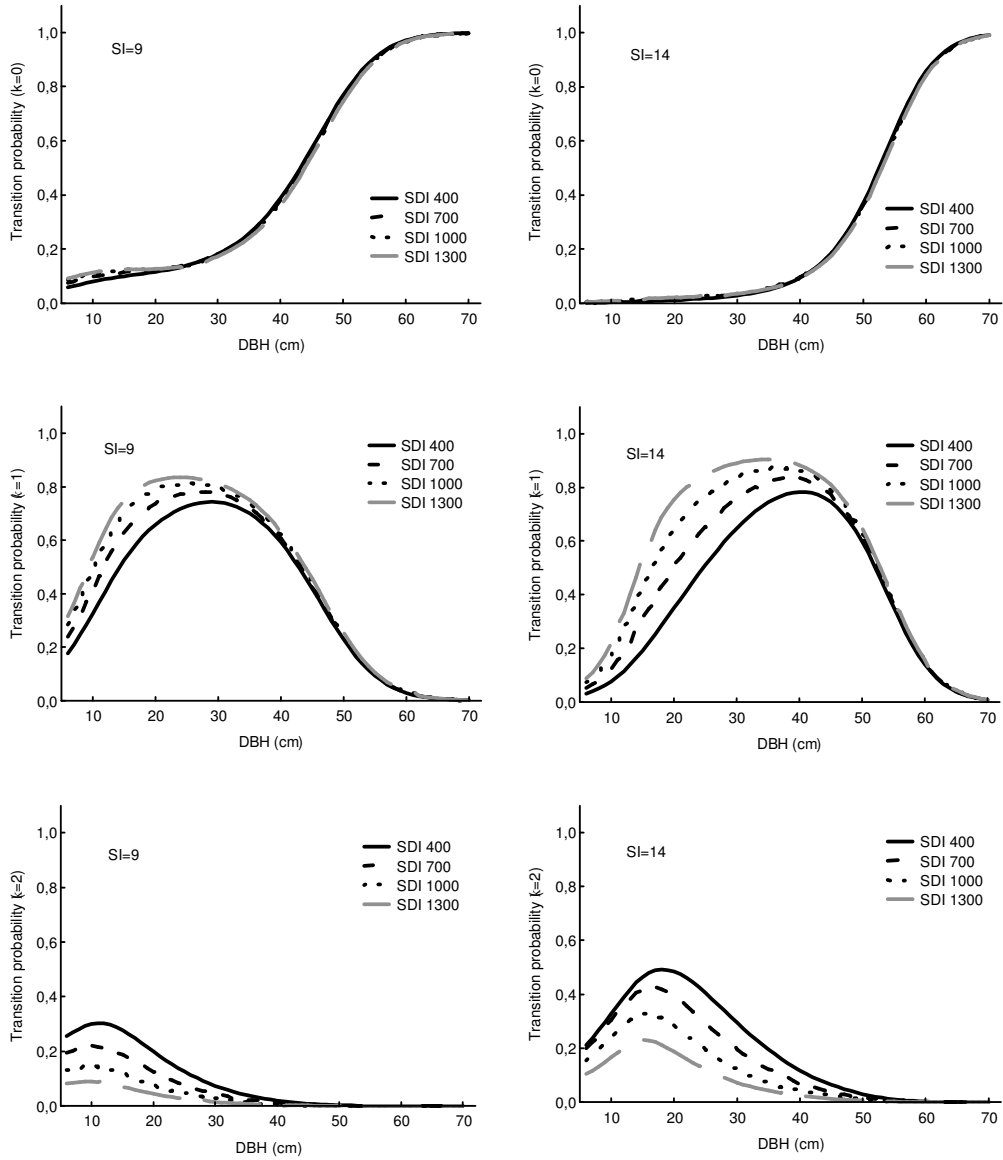
Figure 2. Continued.

The impact of site index was lower in Maritime pine stands than in Scots pine stands although the probability patterns were highly different. Our simulations supported by similar findings in other studies (Bravo and Montero, 2001; Bravo-Oviedo, Río et al., 2004) indicate that forest growth increases at the better sites. Transition probabilities in the stand simulations were higher in the better sites even though different stand densities occurred. For Scots pine in dense stands upgrowth probability increased for $k=1$ and decreased in the rest ($k=2$ and 3). For Maritime pine stands, curve shape of the transition probabilities was slightly similar among densities for each k-upgrowth simulated. No upgrowth ($k=0$) showed large differences in upgrowth prediction for both species.

A similar constant-parameter transition probability across diameter classes was observed by Hao (2005). As we expected, site quality is positively related to diameter growth, indicating that trees are attaining larger diameter growth in the better sites. Evidently, climate and soil differences may also cause variations in growth, some of which are not reflected by the site index variable used here. In general, low growth rates were observed in larger diameter trees, probably related to the tree maturity. In addition,

Scots pine trees showed a lower upgrowth rate because they are more sensitive to inter-tree competition. Low density of stands shows a higher upgrowth rate probability, so trees in these stands are more likely to grow by two or more classes.

A growth model for Scots pine and Maritime pine in Spain was fitted. Although further research is needed to assess more accurately growth dynamics in the studied stands (e.g., by introducing environmental change conditions), our results can help operational forestry to evaluate alternative management regimes. The determination of forest growth is of tremendous importance to the forest resource manager. It can be deduced from the results obtained that Model 1 should be used when DBH and D_{DBH} have been measured and calculated, respectively. When these variables previously described are not available, forest managers could run Model 2 to predict diameter growth in pine stands. Both models are acceptable because of their simplicity of interpretation and use. As in other empirical models, matrix model performance is strongly dependent on its structure, which should reflect tree growth theory adequately. However, it could benefit from the inclusion of variables based on biological findings. The model developed in the present study can be considered a useful tool for simulating tree growth. The main limitation of our models derives from the fact that we have only used individual diameters and some variables related to stand structure to predict the upgrowth transition rates by diameter classes. By including environmental variables that can represent climate change effects (temperature, rainfall, etc.), we could obtain a more robust model. Likewise, equations that represent mortality, harvest and ingrowth should be developed to improve the model. This approach has been used before to develop successful hybrid models and could improve matrix models in the near future.

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