Universidad de Valladolid

ECO - UVa

Topic 3

Production and costs

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Outline

1. Production:
   - The firm and the production function.
   - Total product, average product and marginal product.

2. The costs:
   - Total cost, average costs and marginal cost.
   - Costs in the short run.

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Production

- So far we have seen that producers “produce”, but at no point have we talked about **how much they produce**.

- We have also said that producers are willing to sell their goods and services **at a certain price**, but we have not said **what** or **where the P** at which they are willing to sell **comes from.**

The production function

- A **firm** is an organization that produces goods and services for sale:
  - It transforms **inputs into outputs**.
  - The quantity of output a firm produces depends on the quantity of inputs, and this is the **production function**.
- A **production function** is the relationship between the **quantity of inputs** a firm uses and the **quantity of output** it produces.

The production function

- Let us consider a model to represent the production function. Let's think of a farm:
  - **Only one output**. All workers are of the same quality. **Two inputs**: land and labor.
  - **Land**: Land cannot be bought or sold, so land is a **fixed input**: it is an input whose quantity is fixed for a period of time and cannot be varied.
  - **Labor**: people are hired according to needs. It is a **variable input**: it is an input whose quantity the firm can vary at any time.

- Whether an **input** is **variable** or **fixed** depends on the time horizon:
  - **Short run**: it is the time period in which at least one input is fixed.
  - **Long run**: it is the time period in which all inputs can be varied.

The production function

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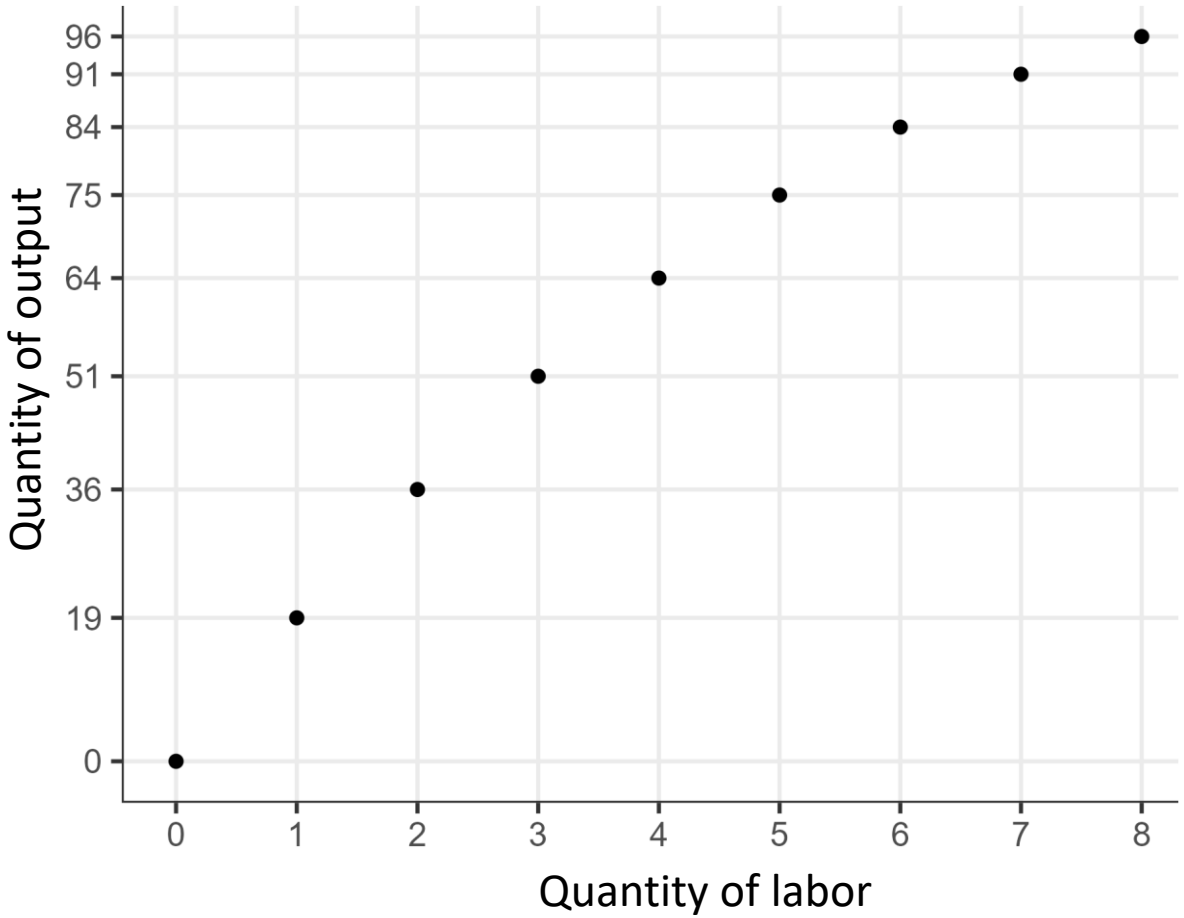
The production function

- On the farm, production depends on the **number of workers**:
  - If there is only one worker, all the land is cultivated, <u>but not very intensively</u>.
  - As <u>additional workers are employed</u>, the land is divided into equal shares and each worker cultivates a <u>smaller area, but more intensively</u>.

The production function

Labor (workers)	Quantity of output
0	0
1	19
2	36
3	51
4	64
5	75
6	84
7	91
8	96

The production function



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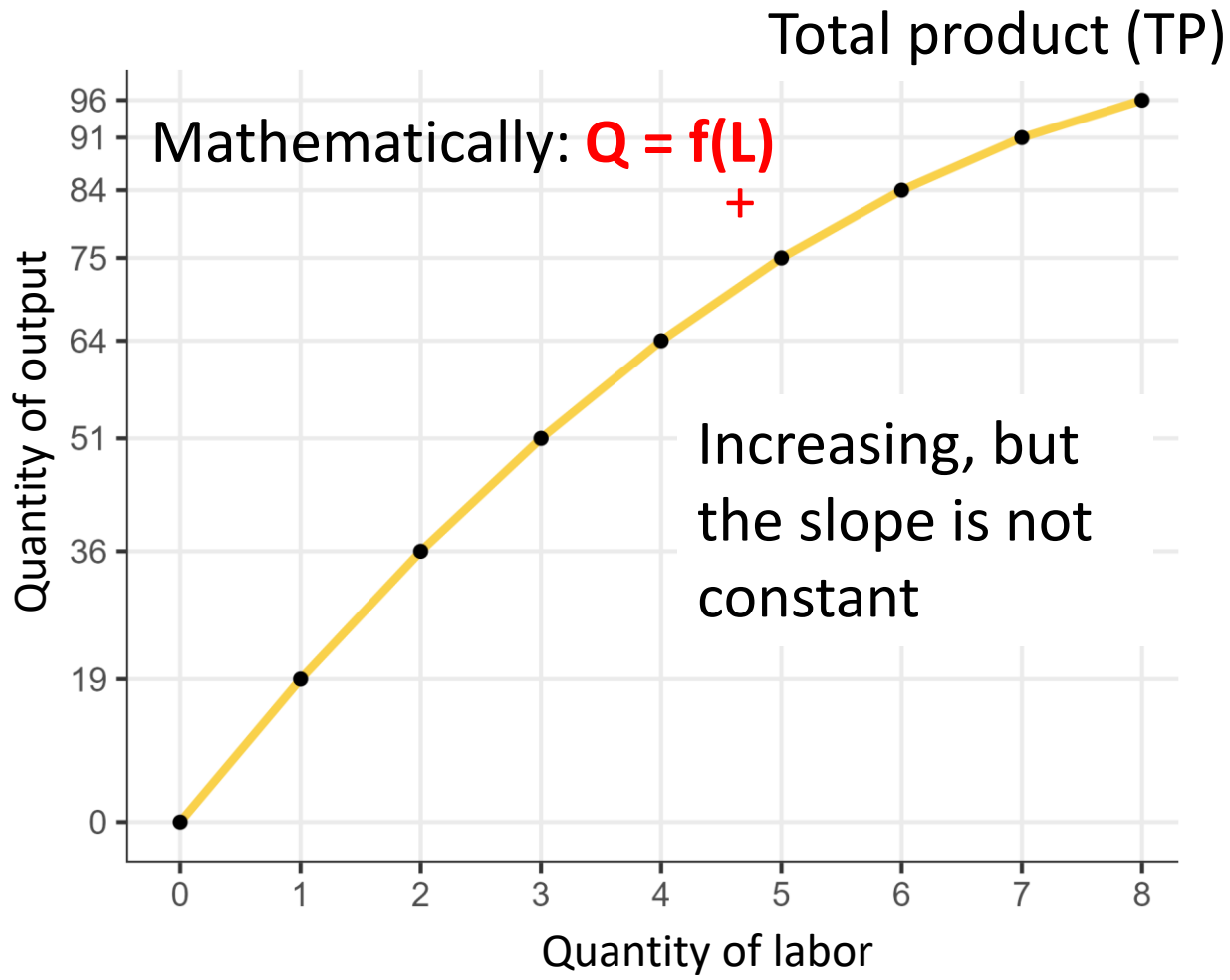
The production function



Labor (workers)	Quantity of output
0	0
1	19
2	36
3	51
4	64
5	75
6	84
7	91
8	96

The production function

- Why does the slope change? Because of **marginal product of labor** (**MPL**).
  - **Marginal product (MP)** of an input is the additional quantity of output that is produced by using one more unit of that input:

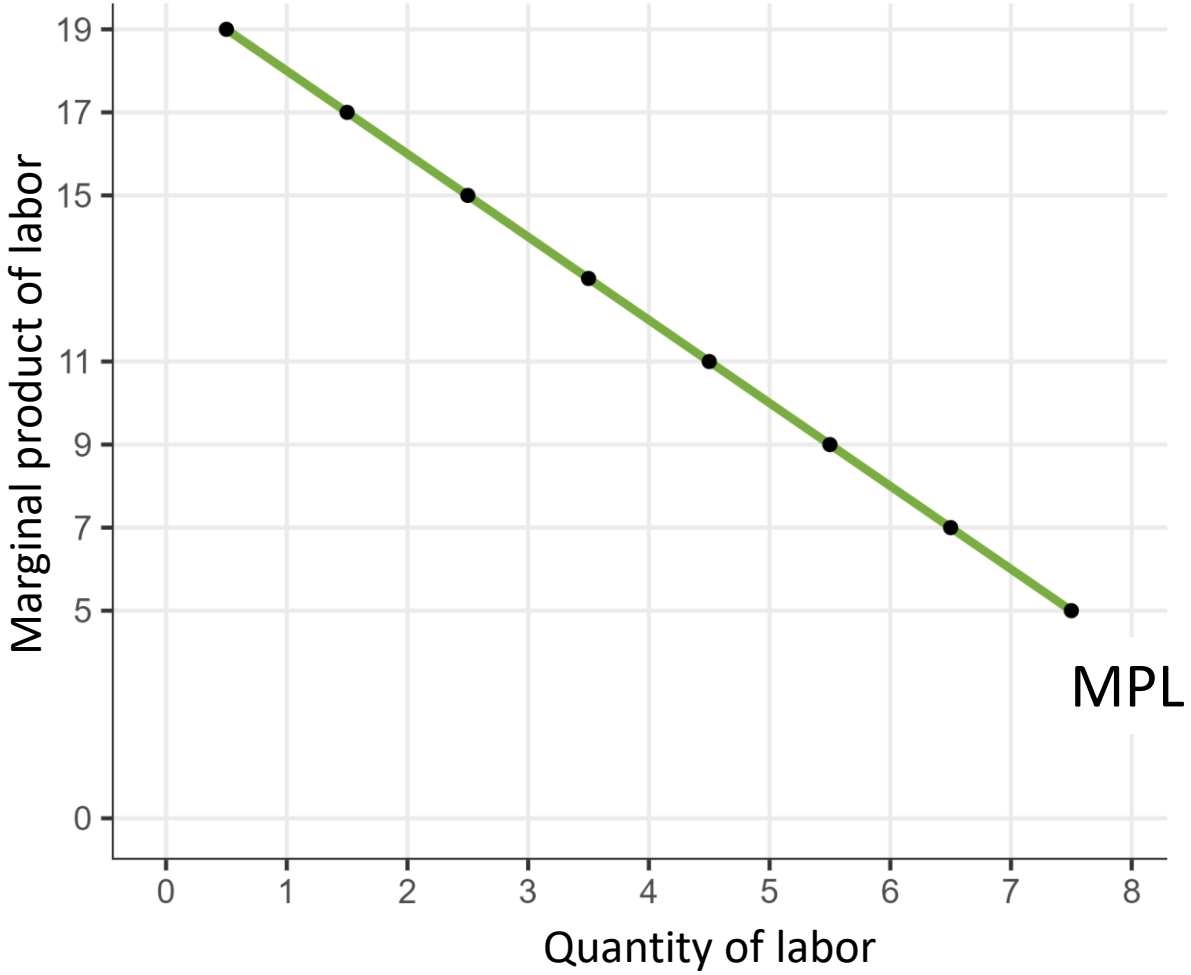
$$\mathrm{MPL} = \frac{\Delta \mathrm{Q}}{\Delta \mathrm{L}} = \frac{\partial Q(L)}{\partial L}$$

  - **Marginal**: one more.
  - The **slope** of the total product curve is equal to the **MP**.

Marginal product

Labor (workers)	Quantity of output	MPL
0	0	
		19
1	19	
		17
2	36	
		15
3	51	
		13
4	64	
		11
5	75	
		9
6	84	
		7
7	91	
		5
8	96	

Marginal product



Marginal product

- **MPL is decreasing**, i.e., each additional worker individually contributes less than the previous worker.
  - **MPL decreases as the number of workers increases**: there are diminishing returns to an input.
  - There are **diminishing returns to an input** when an increase in the quantity of that input, *ceteris paribus* (i.e., holding the levels of all other inputs fixed), leads to a decline in the MP of that input.

Average product

- **Average product** (**AP**) is the total output per unit of input used in its production.
- The average product of labor (**APL**) relates the total output and the number of workers:

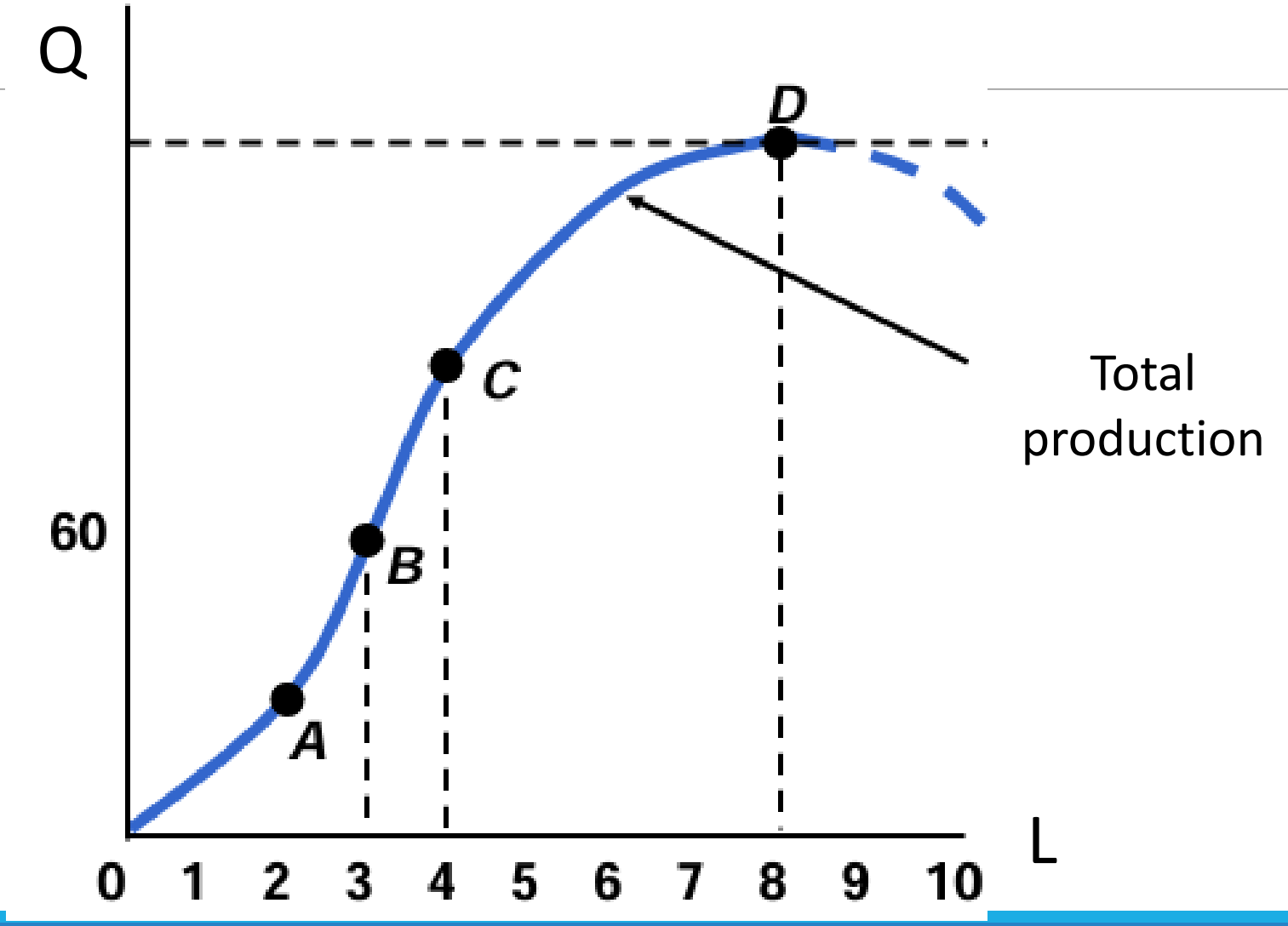
$$\mathrm{APL} = \frac{Q}{L}$$

- This is the average output produced by each worker.

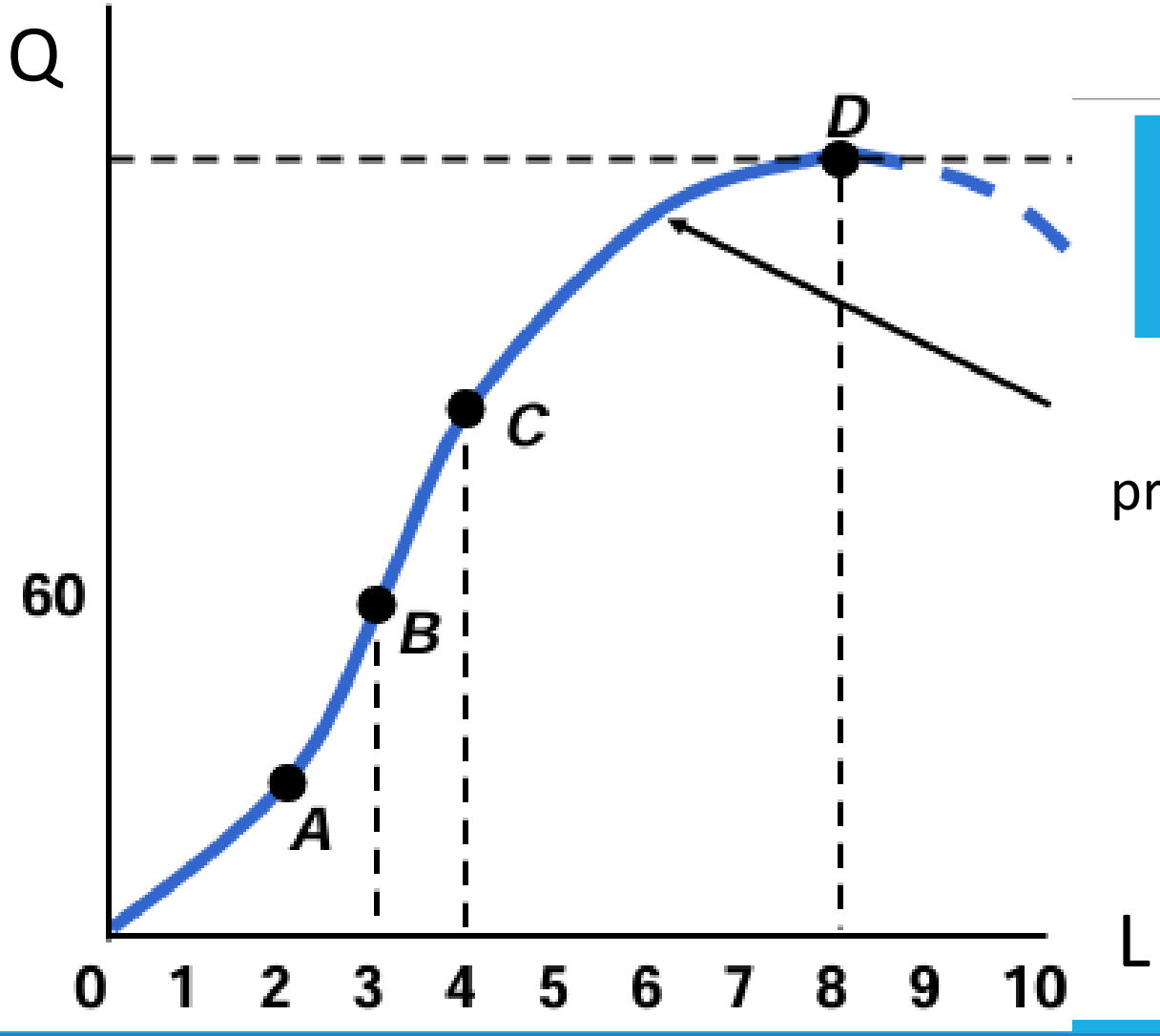
Average product

Labor (workers)	Quantity of output	APL
0	0	----
1	19	19
2	36	18
3	51	17
4	64	16
5	75	15
6	84	14
7	91	13
8	96	12

Short-run production



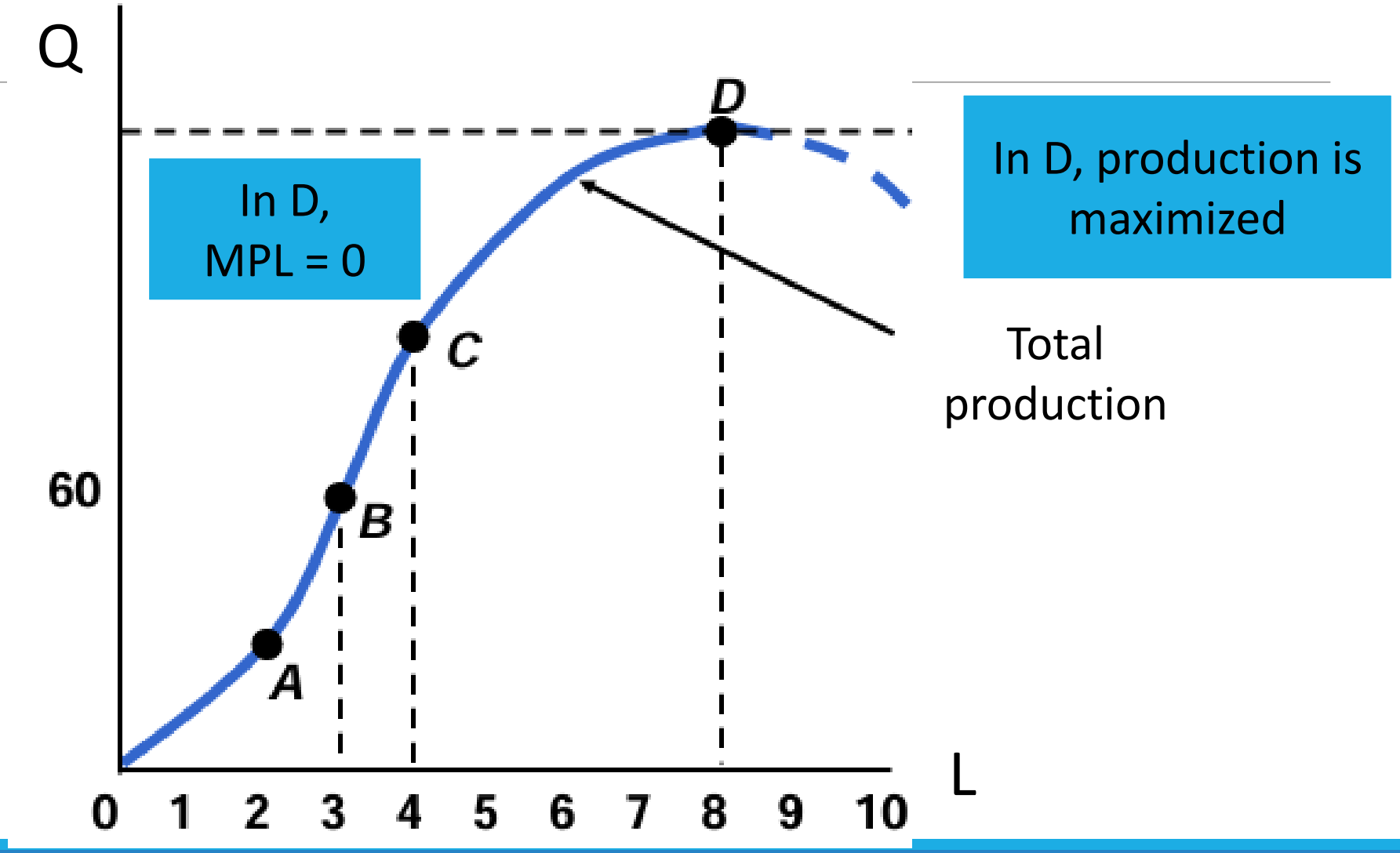
Short-run production



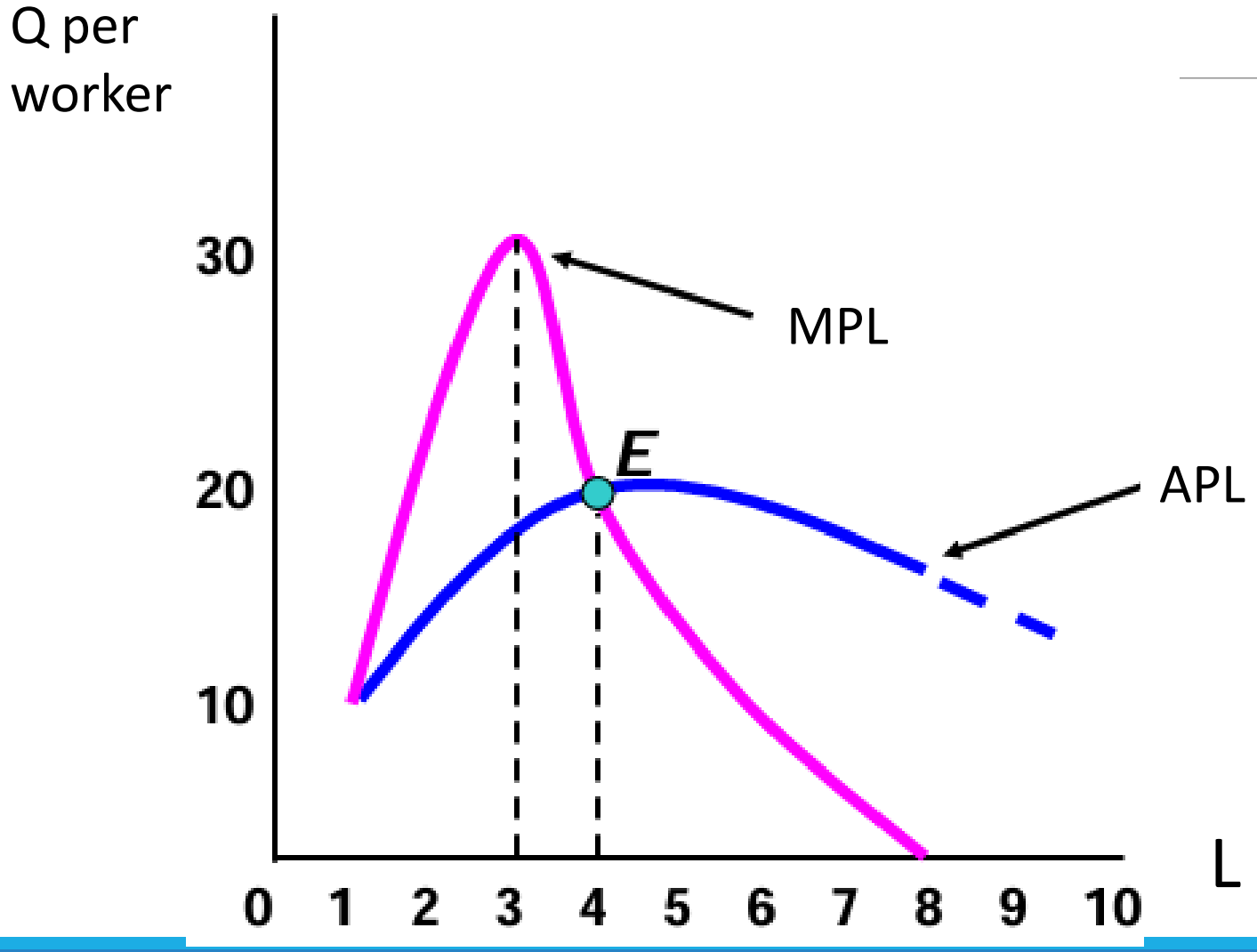
In D, production is maximized

Total production

Short-run production



Short-run production



- If MP > AP, AP increases.
- If MP < AP, AP decreases.
- In E, MP = AP and it is the maximum of the AP curve.
- When L = 8:
  - MPL = 0
  - Q is maximum

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1. Production:
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The total cost curve

- The **production function** is the basis for deriving the **cost curve**.
- In order to understand how to **maximize the profit**, it is necessary to relate **production and costs**.
- Continuing with the example, let's say the land costs $400, either because you rent it (**explicit cost**) or because you own it and do not earn from renting it (**implicit cost**).
  - **Explicit cost**: cost that involves an outlay of money.
  - **Implicit cost**: no outlay is required, but it is money foregone. It is an opportunity cost.

The total cost curve

- The rent, $400, would be a **fixed cost** because you pay it whether you produce or not.
  - **Fixed cost** (**FC**): it is a cost that does not depend on the quantity of output produced. It is *the cost of the fixed input*.

- Workers should be paid. Let's say $200 each. Since the cost of labor varies with the number of workers, it is a **variable cost**.
  - **Variable cost** (**VC**): it is a cost that depends on the quantity of output produced. It is *the cost of the variable input*.

- The **sum** of **VC** and **FC** results in the **Total Cost**.
  - **Total cost** (**TC**): The total cost of producing a given quantity of output is the sum of the fixed cost and the variable cost of producing that quantity of output. **TC = FC + VC**.

The total cost curve

Point on graph	Labor (workers)	Quantity of output
A	0	0
B	1	19
C	2	36
D	3	51
E	4	64
F	5	75
G	6	84
H	7	91
I	8	96

The total cost curve

Point on graph	Labor (workers)	Quantity of output	FC
A	0	0	400
B	1	19	400
C	2	36	400
D	3	51	400
E	4	64	400
F	5	75	400
G	6	84	400
H	7	91	400
I	8	96	400

FC = 400

The total cost curve

Point on graph	Labor (workers)	Quantity of output	FC	VC
A	0	0	400	0
B	1	19	400	200
C	2	36	400	400
D	3	51	400	600
E	4	64	400	800
F	5	75	400	1000
G	6	84	400	1200
H	7	91	400	1400
I	8	96	400	1600

salary = 200

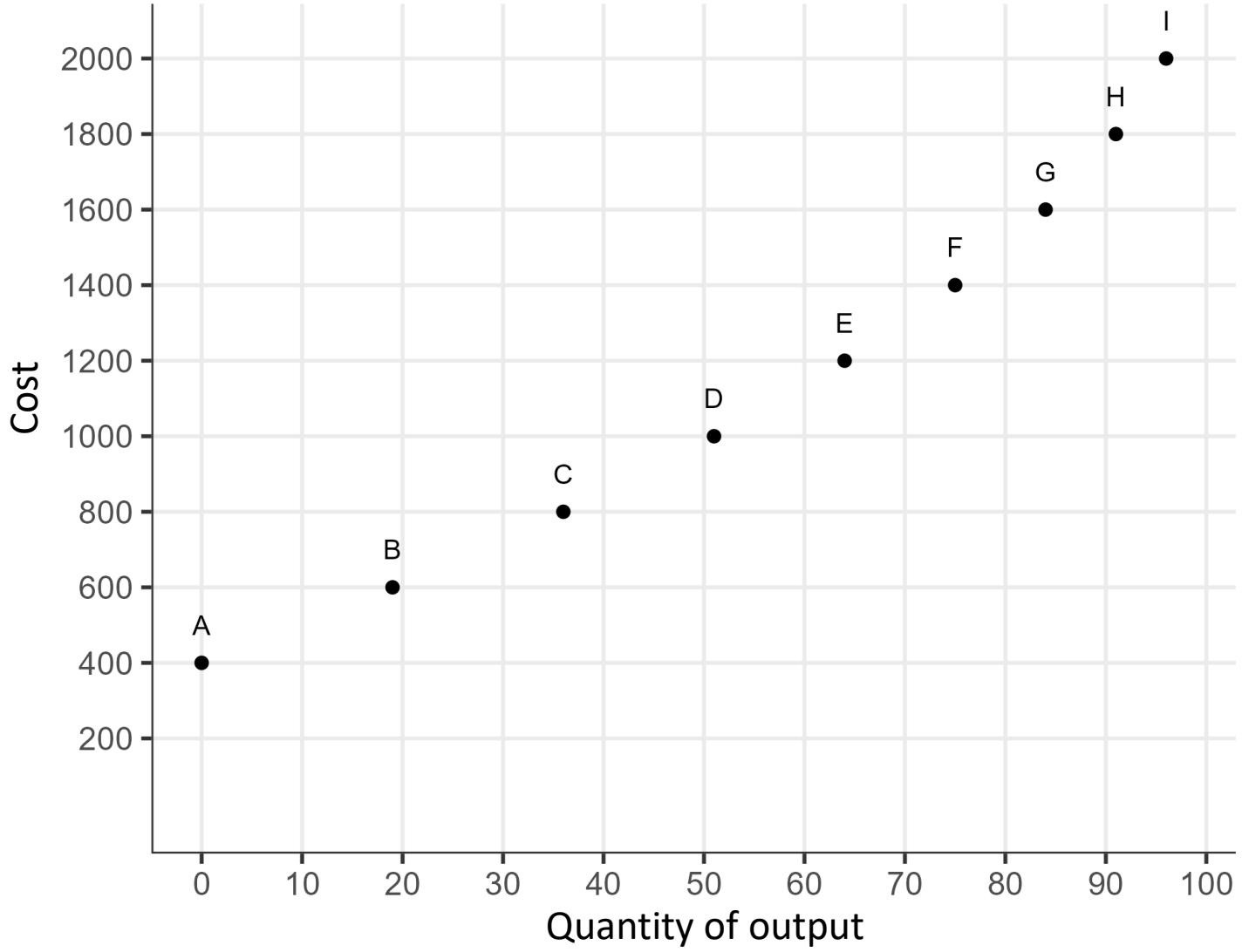
VC = workers * salary

The total cost curve

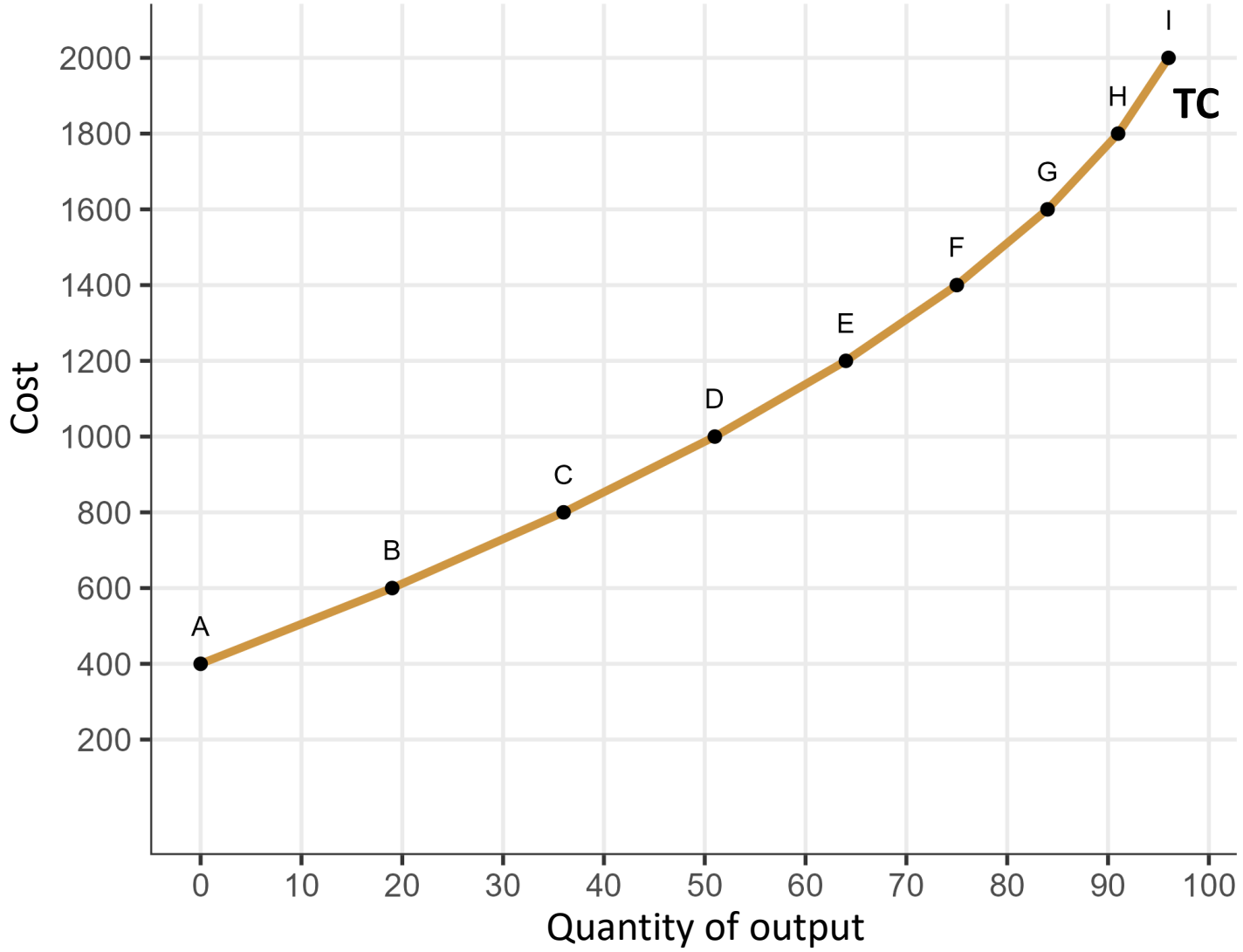
Point on graph	Labor (workers)	Quantity of output	FC	VC	TC
A	0	0	400	0	400
B	1	19	400	200	600
C	2	36	400	400	800
D	3	51	400	600	1000
E	4	64	400	800	1200
F	5	75	400	1000	1400
G	6	84	400	1200	1600
H	7	91	400	1400	1800
I	8	96	400	1600	2000

TC = FC + VC

The total cost curve



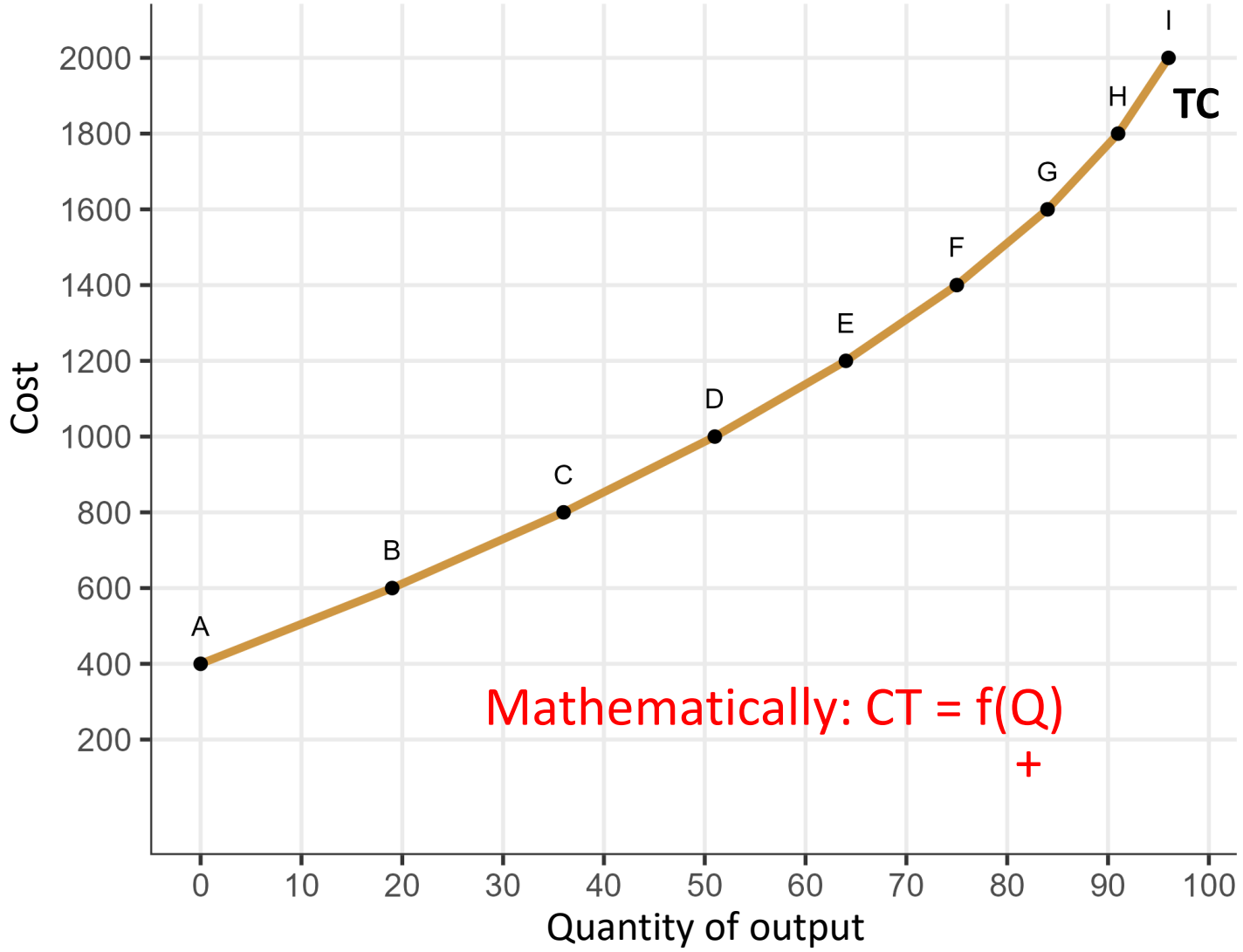
The cost curve



Total cost curve: it shows how total cost depends on the quantity of output.

TC is **increasing**, and **its slope is also rising**.

The cost curve



Total cost curve: it shows how total cost depends on the quantity of output.

TC is **increasing**, and **its slope is also rising**.

Marginal cost

- **Marginal cost** (**MC**): is the change in total cost generated by producing one more unit of output.
  - We have said that <u>the slope of the total cost curve is increasing</u>, that means that <u>the variation in the cost of producing one more unit is increasing</u>. Therefore, the **MC is increasing**.
  - $$\mathbf{MC} = \frac{\Delta \mathbf{TC}}{\Delta \mathbf{Q}} = \frac{\partial \mathbf{TC(Q)}}{\partial \mathbf{Q}}$$

Marginal cost

$$CMg = \frac{\Delta TC}{\Delta Q}$$

Point on graph	Labor (workers)	Quantity of output	FC	VC	TC
A	0	0	400	0	400
B	1	19	400	200	600
C	2	36	400	400	800
D	3	51	400	600	1000
E	4	64	400	800	1200
F	5	75	400	1000	1400
G	6	84	400	1200	1600
H	7	91	400	1400	1800
I	8	96	400	1600	2000

10,5

11,8

13,3

15,4

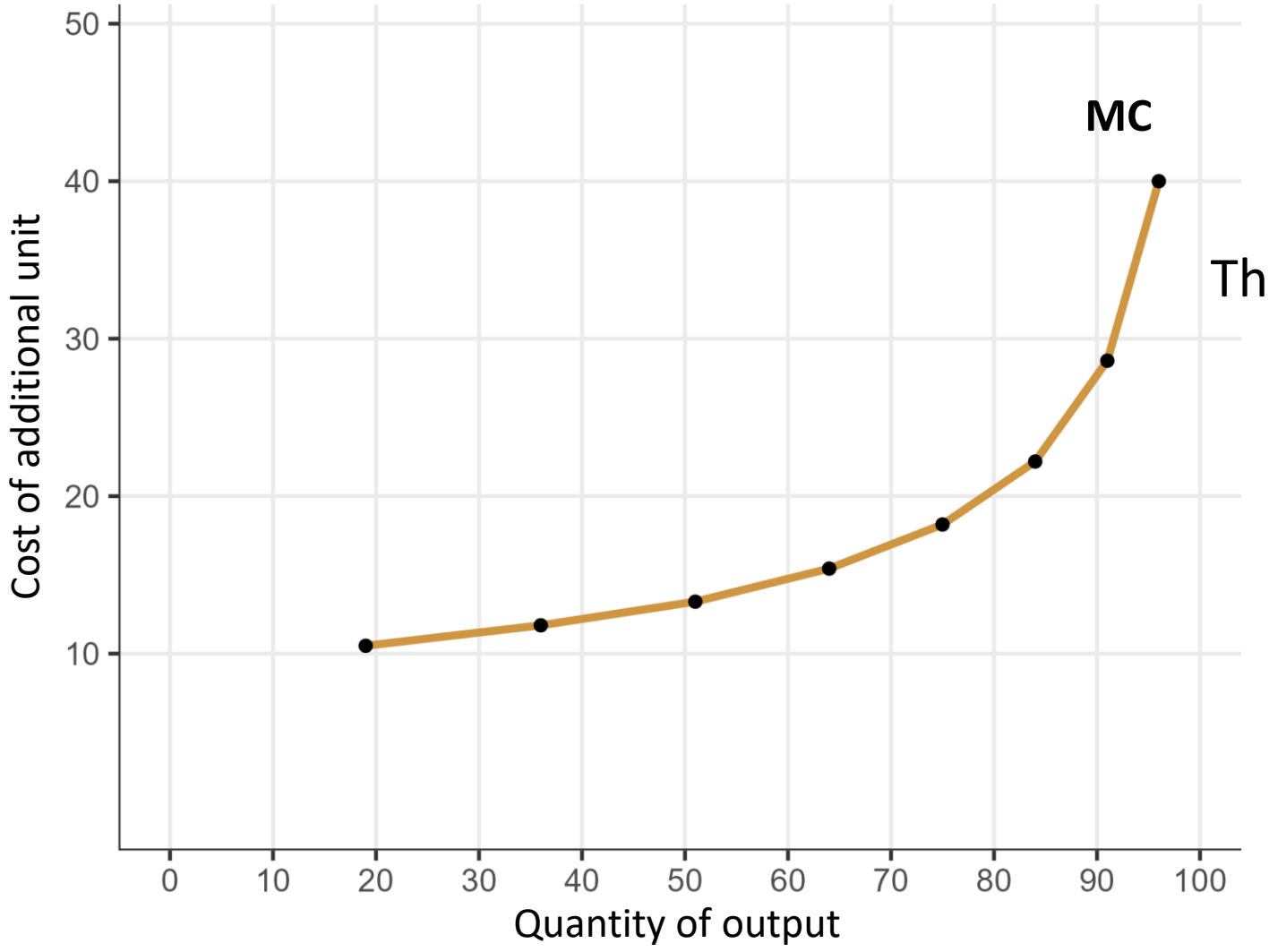
18,2

22,2

28,6

40,0

Marginal cost



The MC is increasing

Marginal cost

- Why is MC increasing? Because there are **diminishing returns to variable inputs**.
  - It is necessary to use an increasing amount of inputs to produce an additional unit. This causes the MC to increase with each additional unit produced.
- We will see later that MC is crucial in deciding how much to produce to maximize profits.

Average total cost

- **Average fixed cost** (**AFC**): it is the fixed cost per unit of output. **ACF =** $\frac{\mathbf{FC}}{\mathbf{Q}}$.
- **Average variable cost** (**AVC**): is the variable cost per unit of output. **AVC =** $\frac{\mathbf{VC}}{\mathbf{Q}}$.
- **Average total cost** (**ATC**): it is the total cost divided by quantity of output produced. **ATC =** $\frac{\mathbf{TC}}{\mathbf{Q}}$ = **AFC + AVC**.
- It is important as average costs inform how much it costs **on average** to produce one unit of product. The MC is another unit. They may sound similar, but they are not.

Average total cost

L	Q	FC	VC	TC
0	0	108	0	108
1	1	108	12	120
4	2	108	48	156
9	3	108	108	216
16	4	108	192	300
25	5	108	300	408
36	6	108	432	540
49	7	108	588	696
64	8	108	768	876
81	9	108	972	1080
100	10	108	1200	1308

Salary = 12

Average total cost

Q	FC	VC	TC	ATC
1	108	12	120	120.0
2	108	48	156	78.0
3	108	108	216	72.0
4	108	192	300	75.0
5	108	300	408	81.6
6	108	432	540	90.0
7	108	588	696	99.4
8	108	768	876	109.5
9	108	972	1080	120.0
10	108	1200	1308	130.8

$$ATC = \frac{TC}{Q}$$

Average total cost

Q	FC	VC	TC	ATC	AFC
1	108	12	120	120.0	108.0
2	108	48	156	78.0	54.0
3	108	108	216	72.0	36.0
4	108	192	300	75.0	27.0
5	108	300	408	81.6	21.6
6	108	432	540	90.0	18.0
7	108	588	696	99.4	15.4
8	108	768	876	109.5	13.5
9	108	972	1080	120.0	12.0
10	108	1200	1308	130.8	10.8

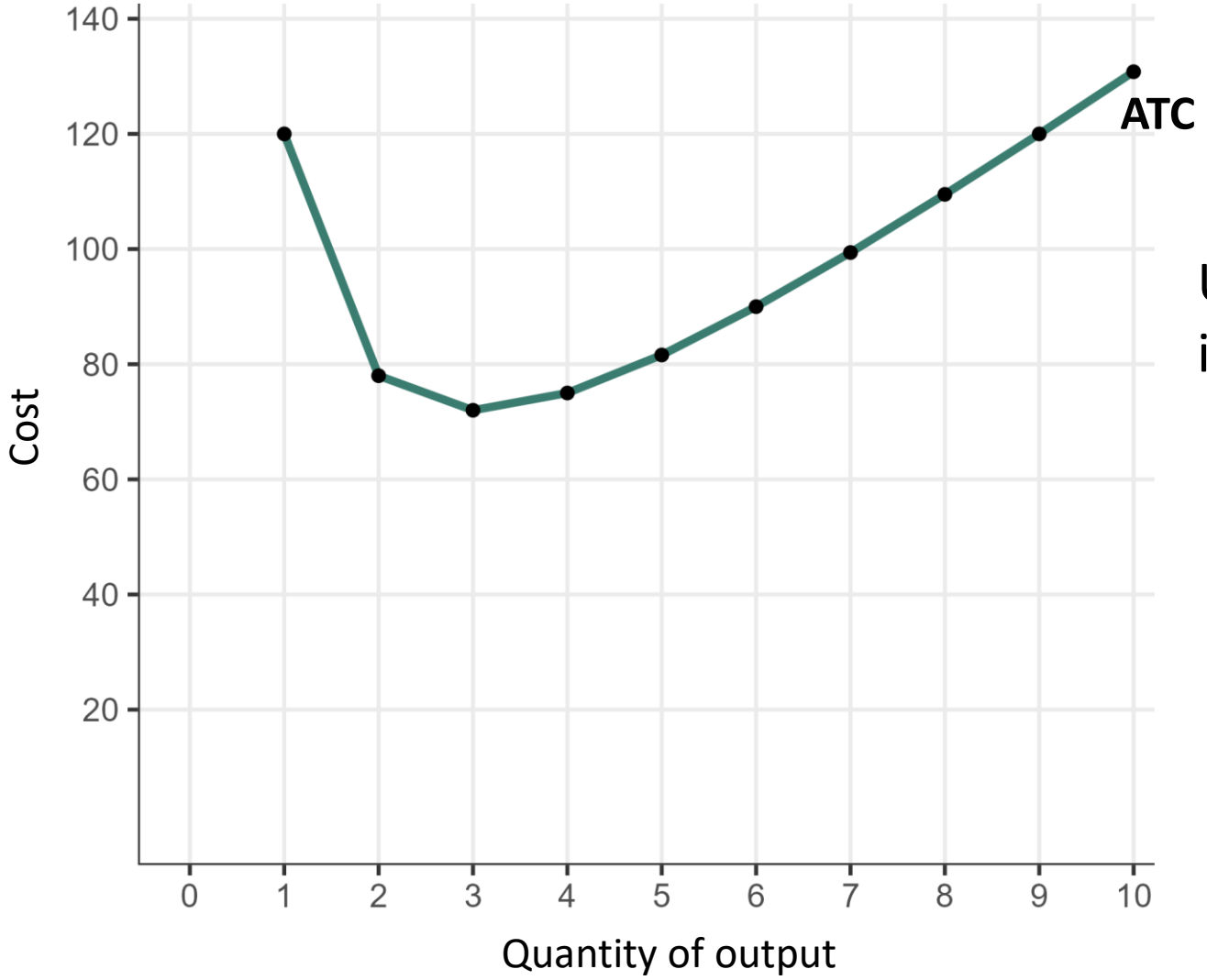
$$AFC = \frac{FC}{Q}$$

Average total cost

Q	FC	VC	TC	ATC	AFC	AVC
1	108	12	120	120.0	108.0	12
2	108	48	156	78.0	54.0	24
3	108	108	216	72.0	36.0	36
4	108	192	300	75.0	27.0	48
5	108	300	408	81.6	21.6	60
6	108	432	540	90.0	18.0	72
7	108	588	696	99.4	15.4	84
8	108	768	876	109.5	13.5	96
9	108	972	1080	120.0	12.0	108
10	108	1200	1308	130.8	10.8	120

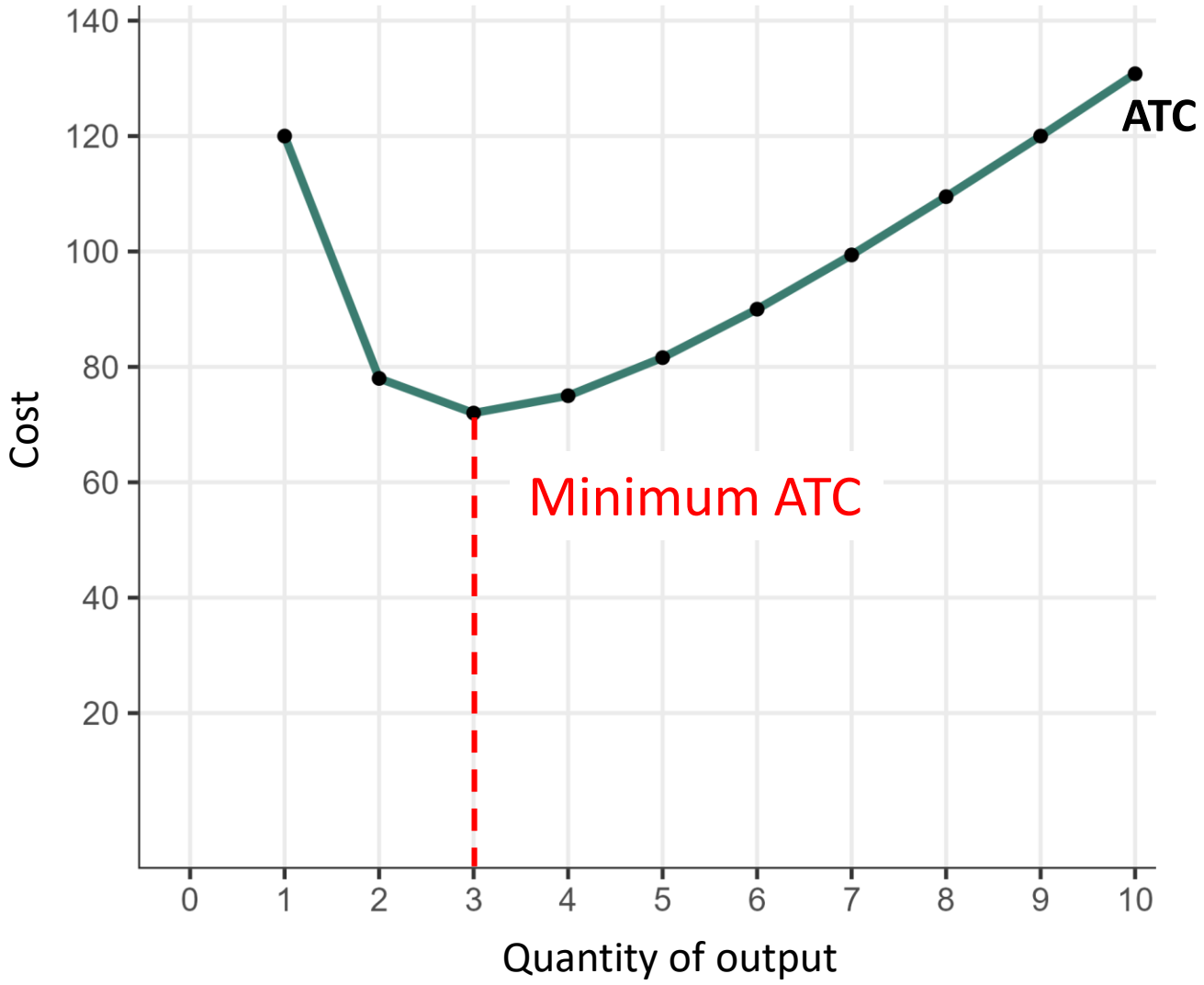
$$AVC = \frac{VC}{Q}$$

Average total cost

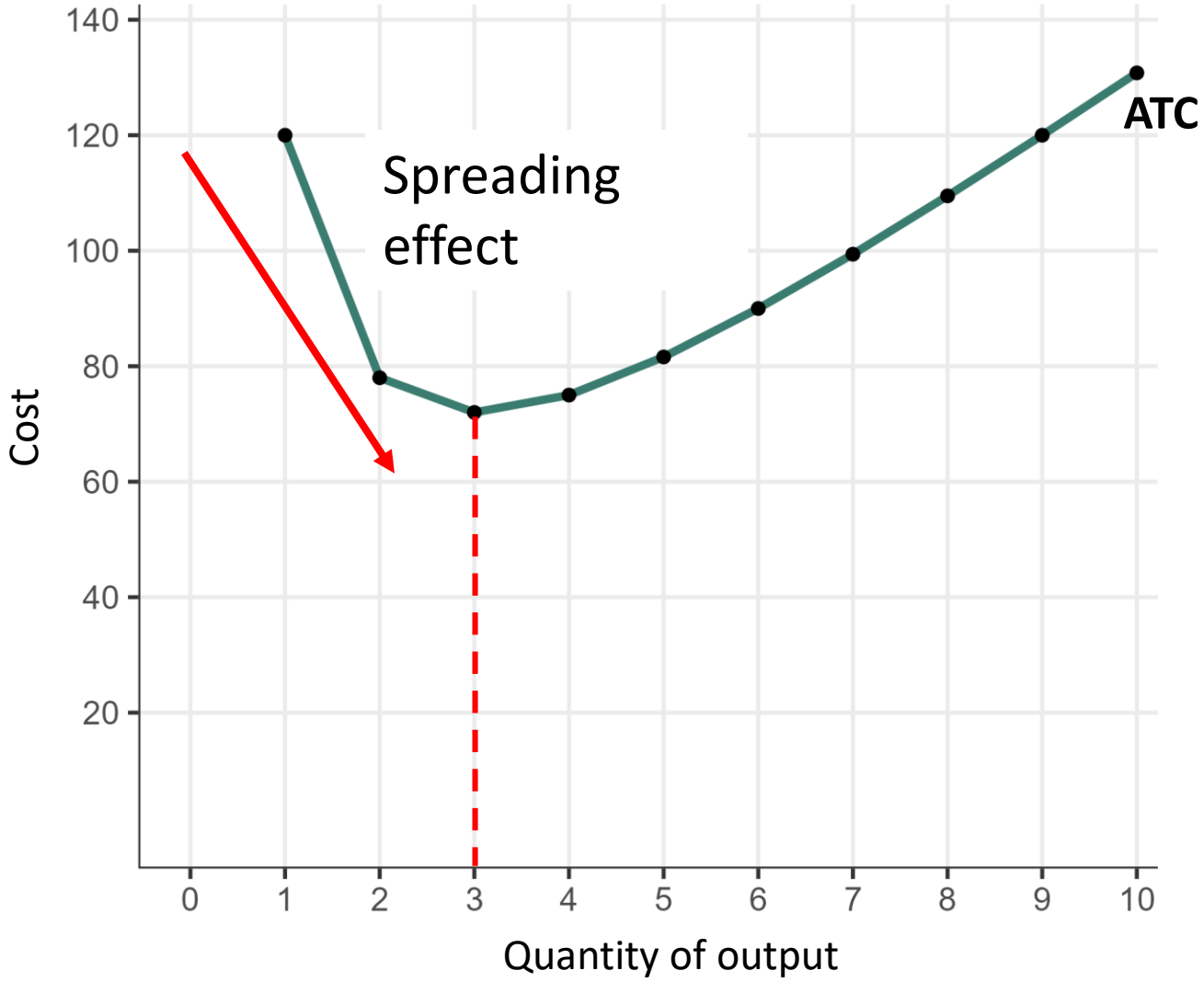


U-shape (or J-shape):
it reaches a minimum level

Average total cost



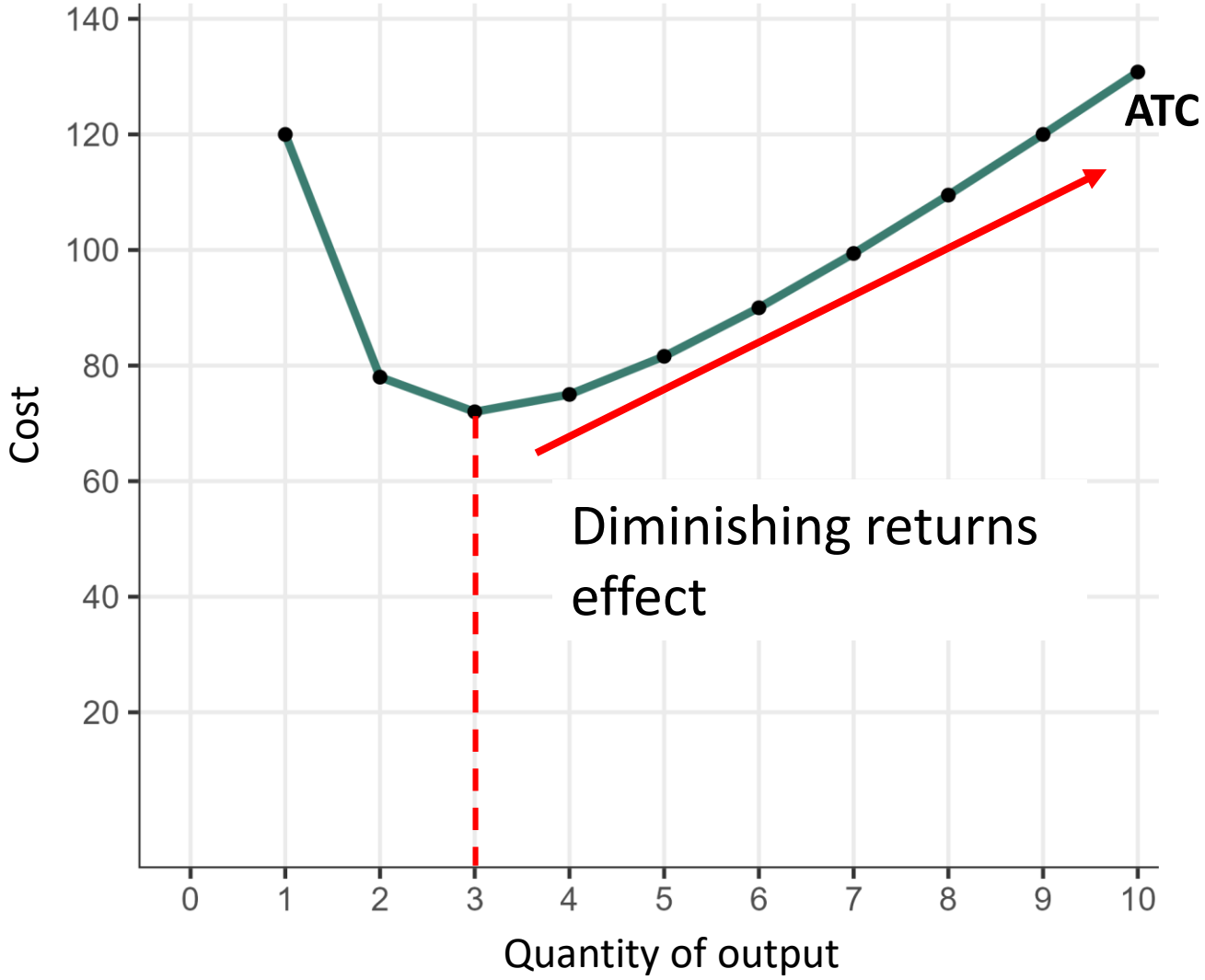
Average total cost



ATC decreases when output levels are low

<u>The spreading effect</u>: ATC decreases to the left of the minimum ATC level because the decrease in AFC dominates.

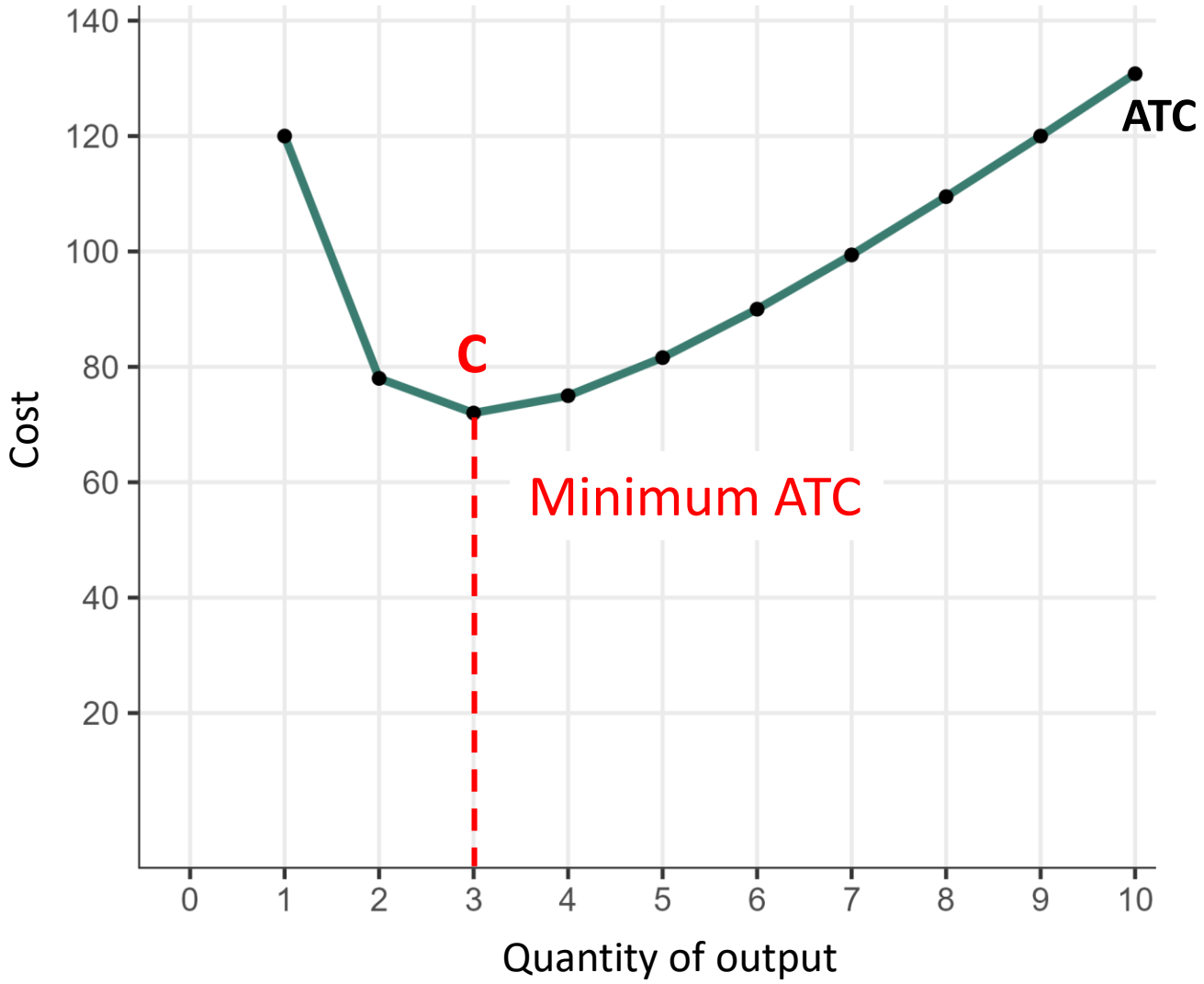
Average total cost



ATC increases when output levels are large

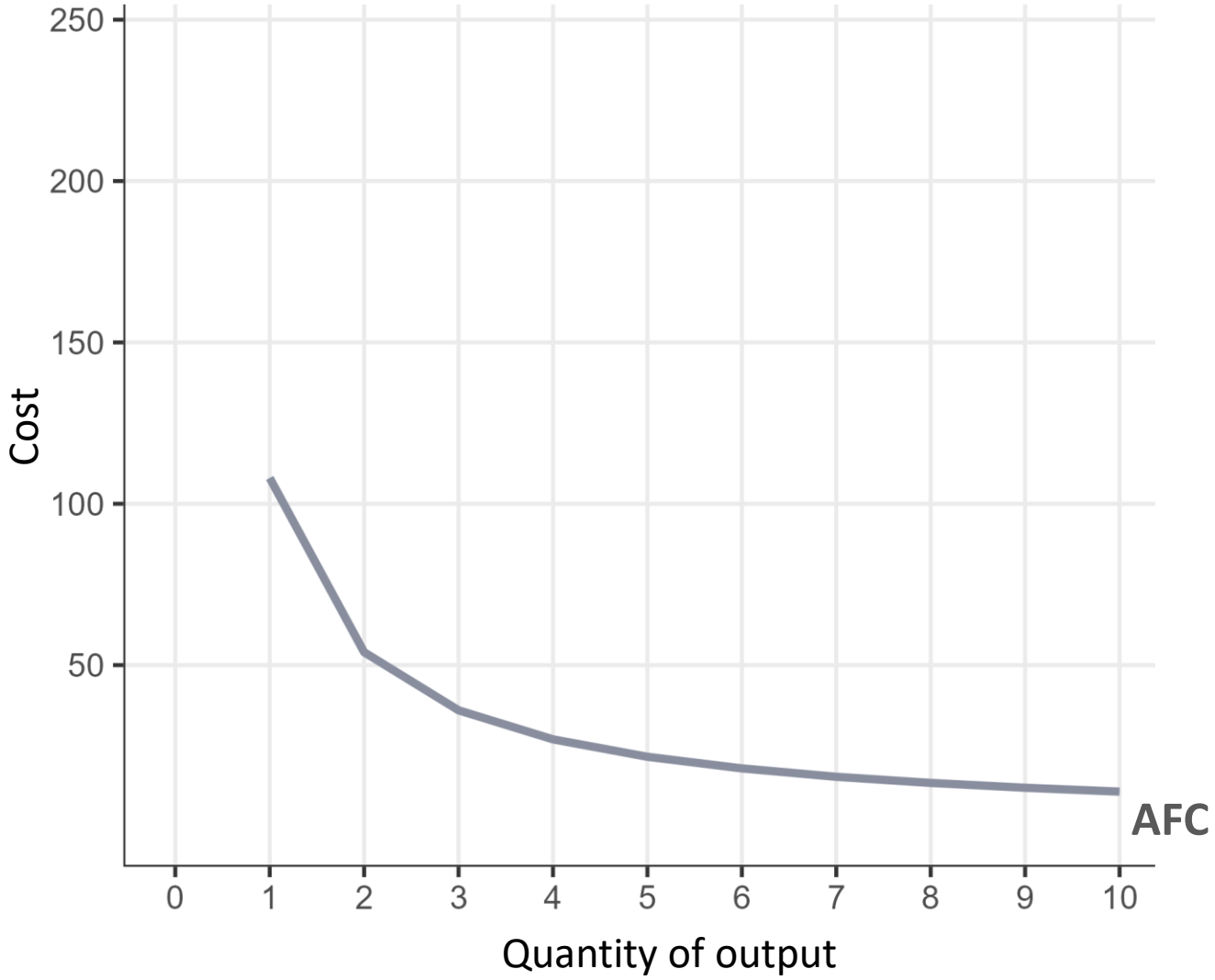
<u>The diminishing returns effect</u>: ATC increases to the right of the minimum ATC level because the increase in AVC dominates. More variable input is needed to produce more because there is a fixed input.

Average total cost

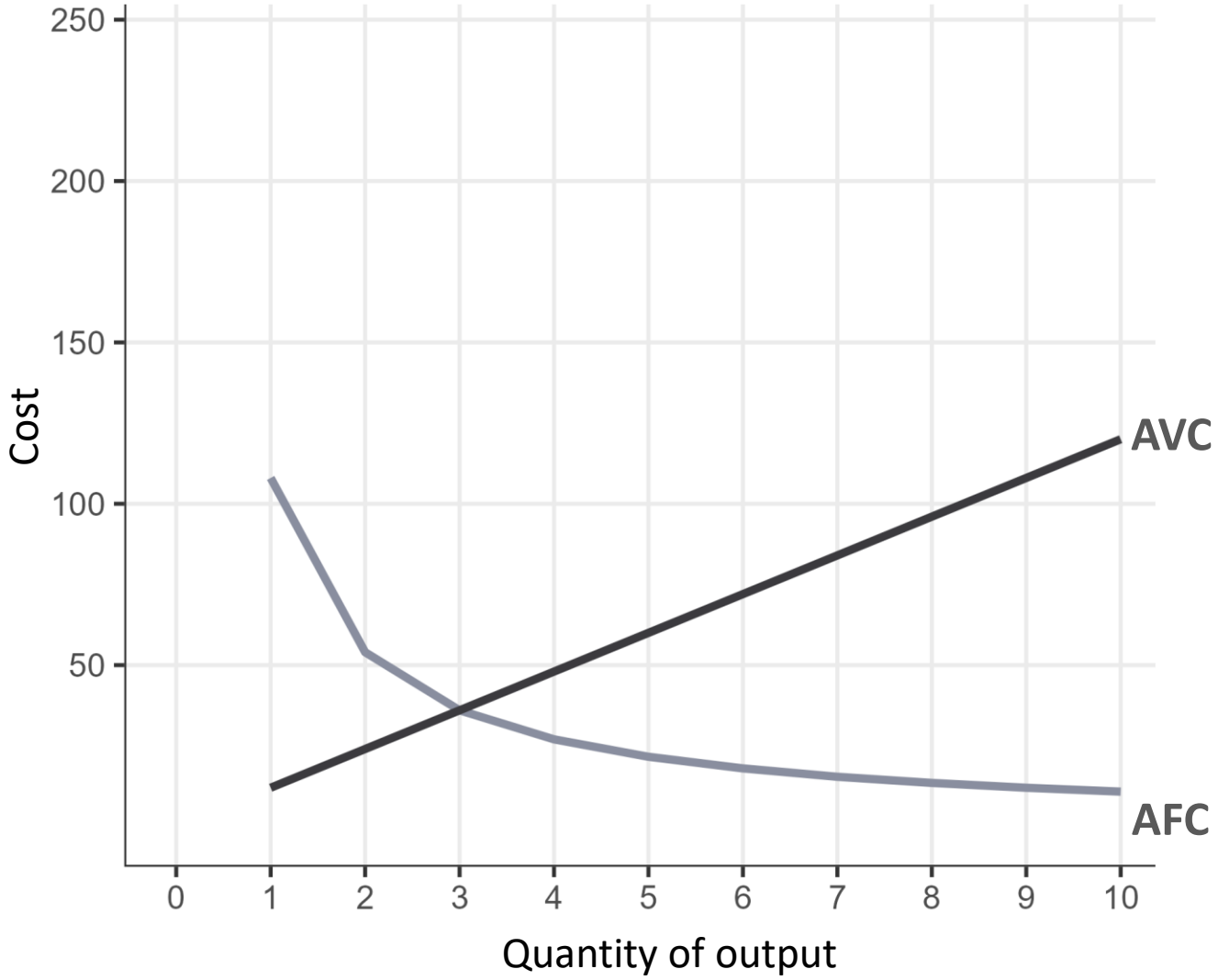


In C, the two effects exactly balance each other.
In C, ATC is at its minimum.

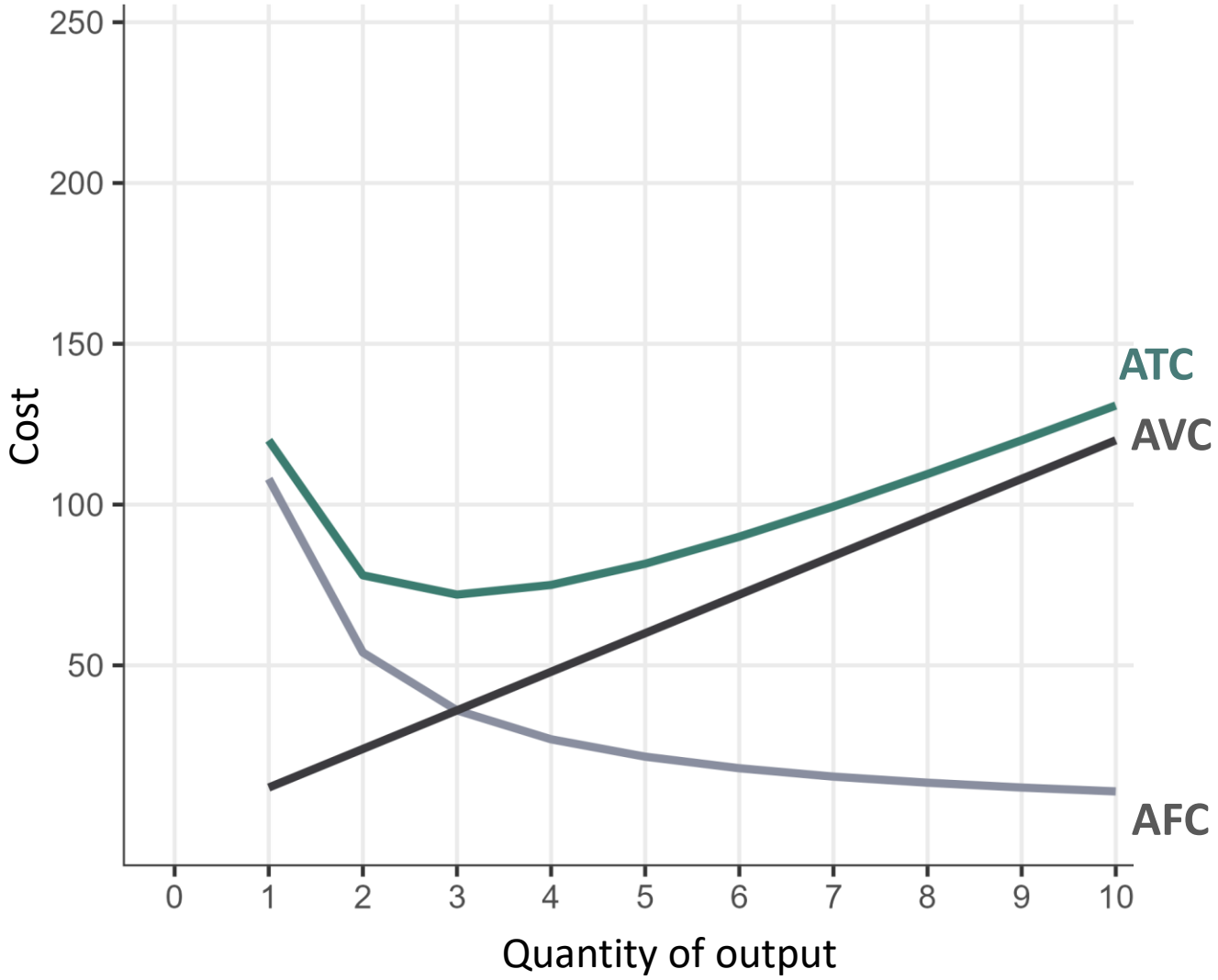
Average total cost



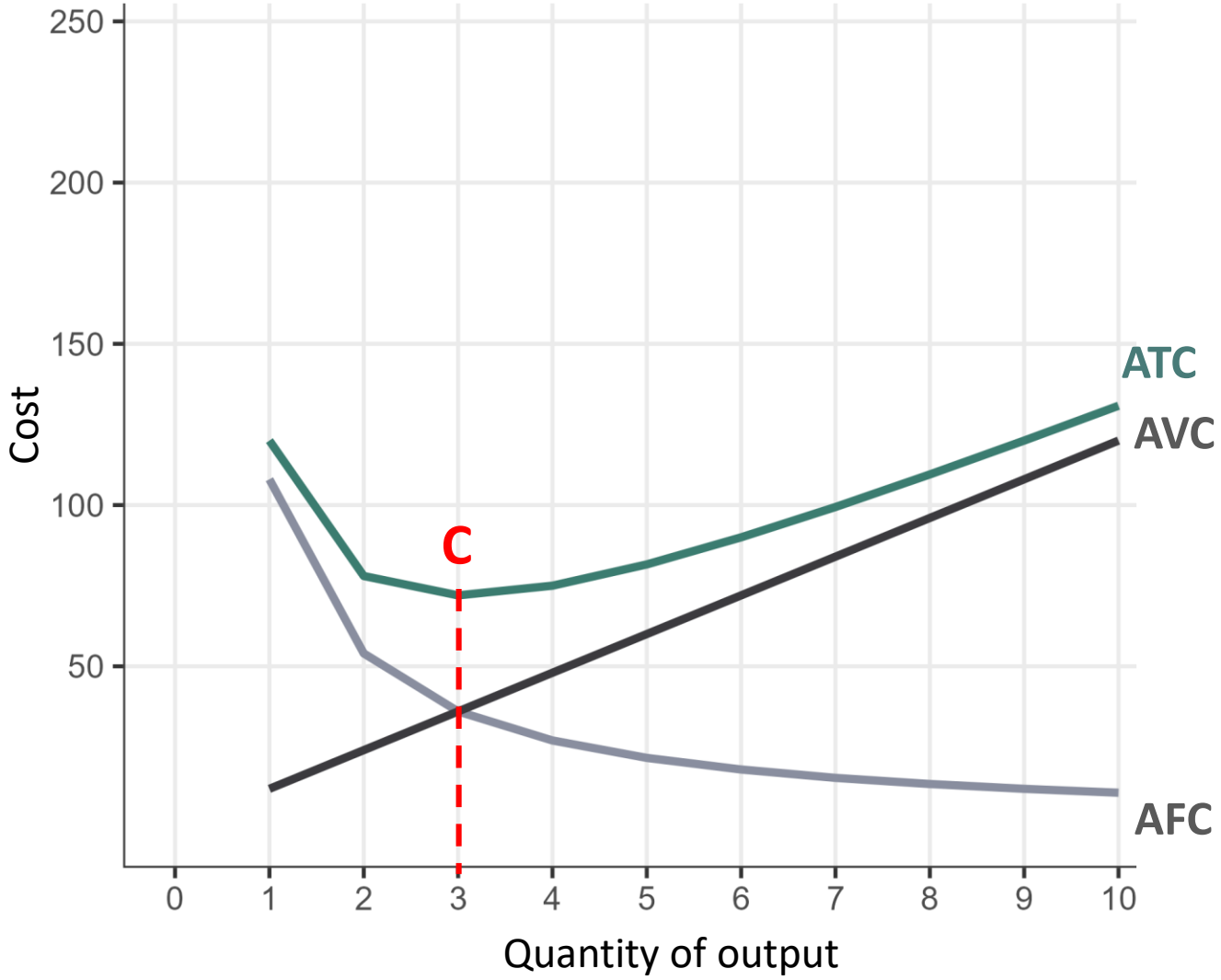
Average total cost



Average total cost



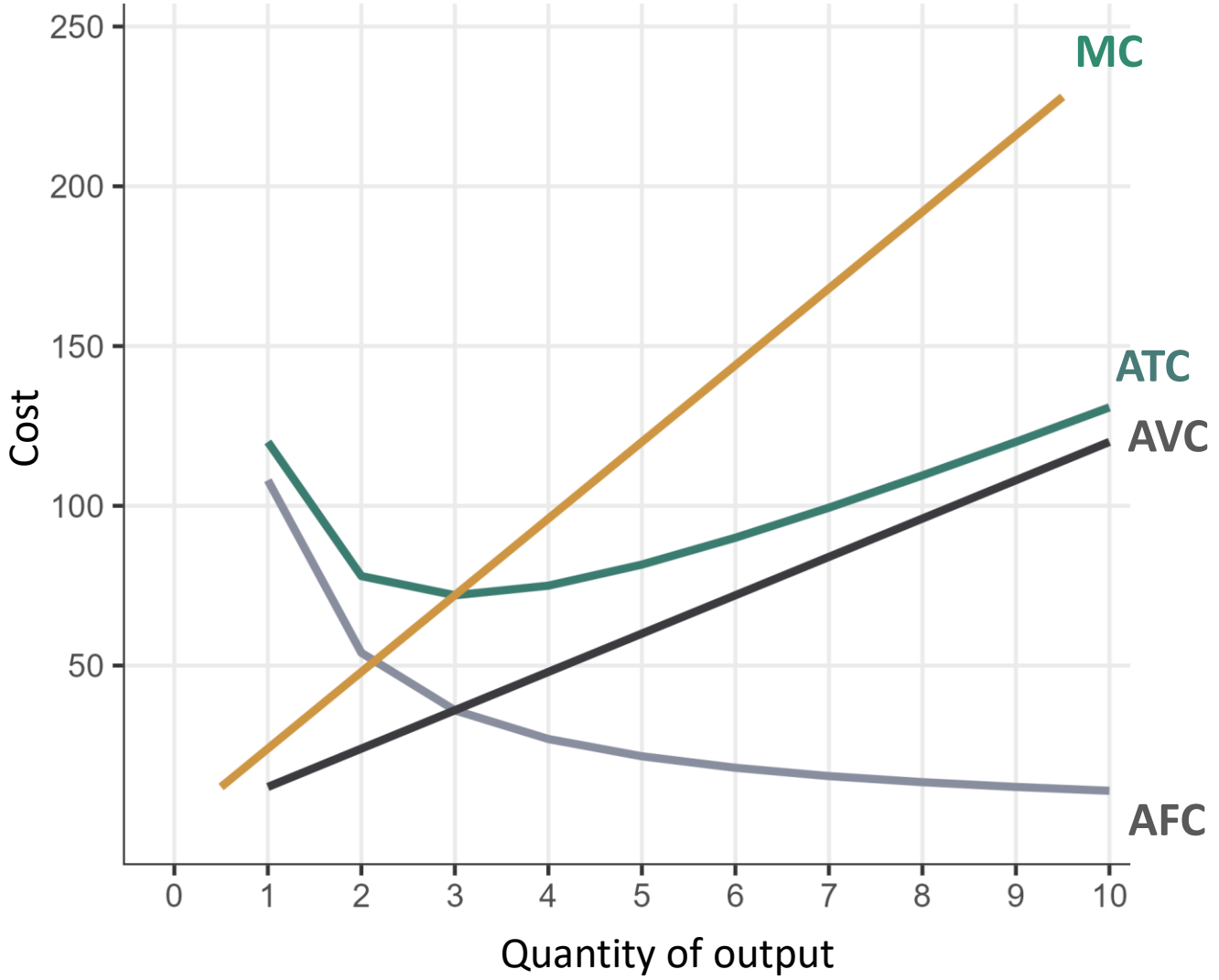
Average total cost



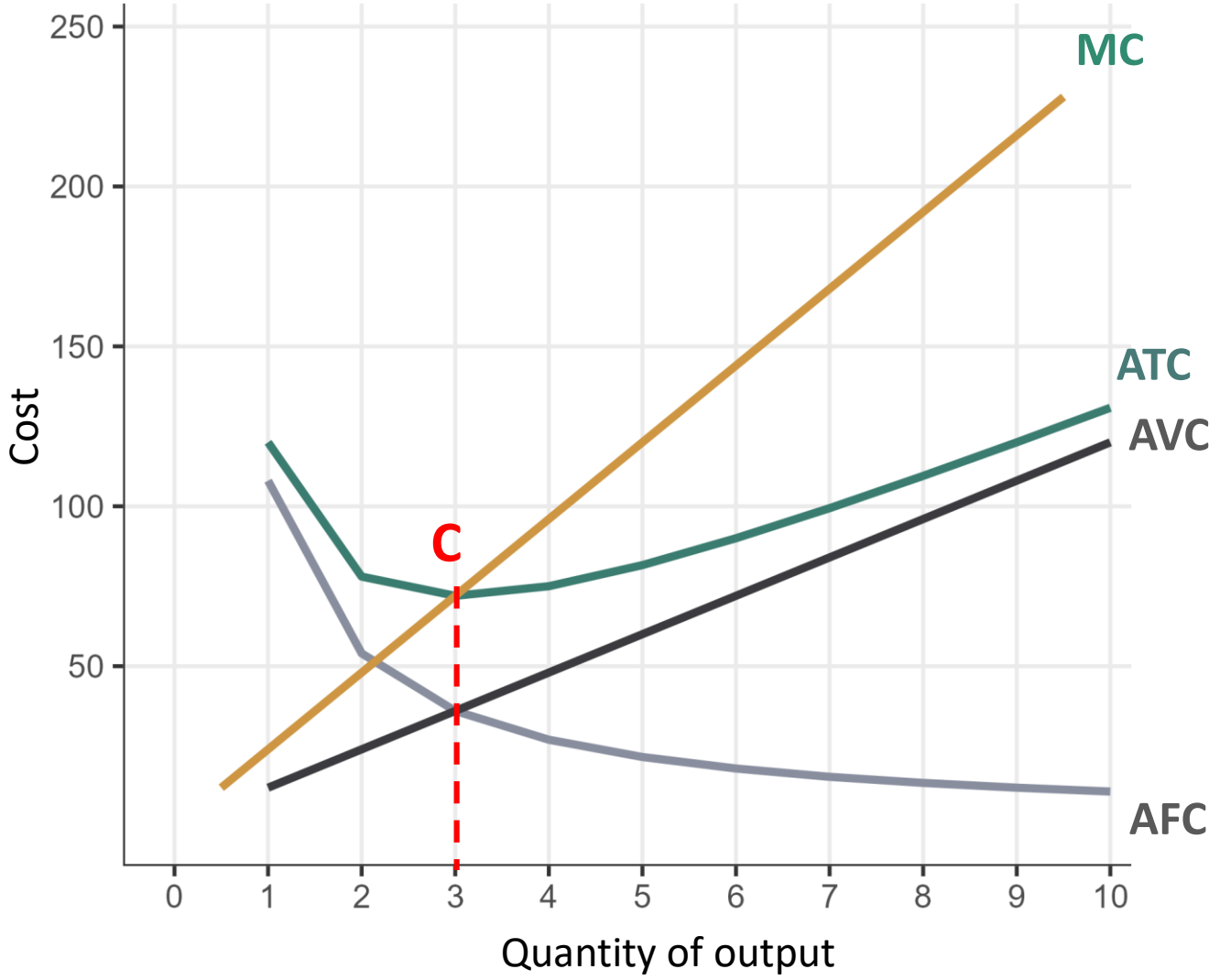
Marginal cost and average total cost

L	Q	FC	VC	TC	MC
0	0	108	0	108	
					12
1	1	108	12	120	
					36
4	2	108	48	156	
					60
9	3	108	108	216	
					84
16	4	108	192	300	
					108
25	5	108	300	408	
					132
36	6	108	432	540	
					156
49	7	108	588	696	
					180
64	8	108	768	876	
					204
81	9	108	972	1080	
					228
100	10	108	1200	1308	

Marginal cost and average total cost



Marginal cost and average total cost



The **MC** curve will cross the **ATC** curve at its minimum level: minimum-cost output.

Marginal cost and average total cost

To remember:

- The **MC** has a **positive slope** due to diminishing returns that make it increasingly costly to produce one more unit.
- The **AVC** has a **positive slope** due to the diminishing returns effect but its slope is lower than that of the MC because when calculating the AVC the cost of producing an additional unit is spread over all the units produced, not just the last one.
- The **AFC** has a **negative slope** due to the spreading effect.
- The **MC** curve **cross** the **ATC** curve at its **minimum level** (minimum-cost output):
  - If **MC** goes **above ATC**, ATC is increasing.
  - If **MC** goes **below ATC**, ATC is decreasing.

Short-run and long-run costs

- So far we have focused on the **short run**, so we have treated FCs as if they were outside the firm's control. In the **long run**, all inputs (and costs!) are variable.

- The firm **chooses** the **long-run FC** based on the quantity of output it expects to produce.

- Varying the fixed input affects costs in two ways:
  - When acquiring or buying machinery, there is an **increase in short-run FC**, so **costs increase**.
  - **Workers are more productive** with more machinery, so fewer workers will be needed, and **VC tends to decrease**, and so TC.

Returns to scale

- What determines the shape of the long-run average total cost curve?
  - The **scale** – the size of a firm's operations – is often an important determinant of its long-run costs of production.

1. **Increasing returns to scale**: when long-run average total cost decline as output increases.
2. **Decreasing returns to scale**: when long-run average total cost increases as output increases.
3. **Constant returns to scale**: when long-run average total cost is constant as output increases.

Short-run and long-run costs

Time horizon	Measurement	Definition	Mathematical term
Short run	Fixed cost	Cost that does not depend on the quantity of output produced	**FC**
	Average fixed cost	Fixed cost per unit of output	**AFC = FC/Q**
Short run and long run	Variable cost	Cost that depends on the quantity of output produced	**VC**
	Average variable cost	Variable cost per unit of output	**AVC = VC/Q**
	Total cost	Sum of fixed cost (short run) and variable cost	**TC = FC (short run) + VC**
	Average total cost (Average cost)	Total cost per unit of output	**ATC = AFC + AVC = TC/Q**
	Marginal cost	The change in total cost generated by producing one more unit of output	**MC = ΔTC/ΔQ**
Long run	Long-run average total cost	Average total cost when the fixed cost has been chosen to minimize average total cost for each level of output.	**LRATC**

Mandatory readings

- Krugman, P. and Wells, R. (2023). *Essentials of Economics*. MacMillan Learning. 6th edition.
  - Chapter 6: Behind the supply curve: Inputs and costs.

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End of Topic 3

Production and costs

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