



Topic 8

Introduction to the IS-LM model

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Outline

1. The Goods Market and the IS curve.
 - Shifts of the IS curve.
2. The Money Market and the LM curve.
 - Shifts of the LM curve.
3. IS-LM model.
 - Effects of fiscal policy and monetary policy in the IS-LM model.



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Goods market and the IS curve

- The **IS curve** shows combinations of **interest rates** and **output levels** such that **aggregate spending equals income**.
- The IS curve is derived in two steps:
 - Relationship between I and interest rate (i).
 - Include the demand for I in the DA.



Goods market and the IS curve

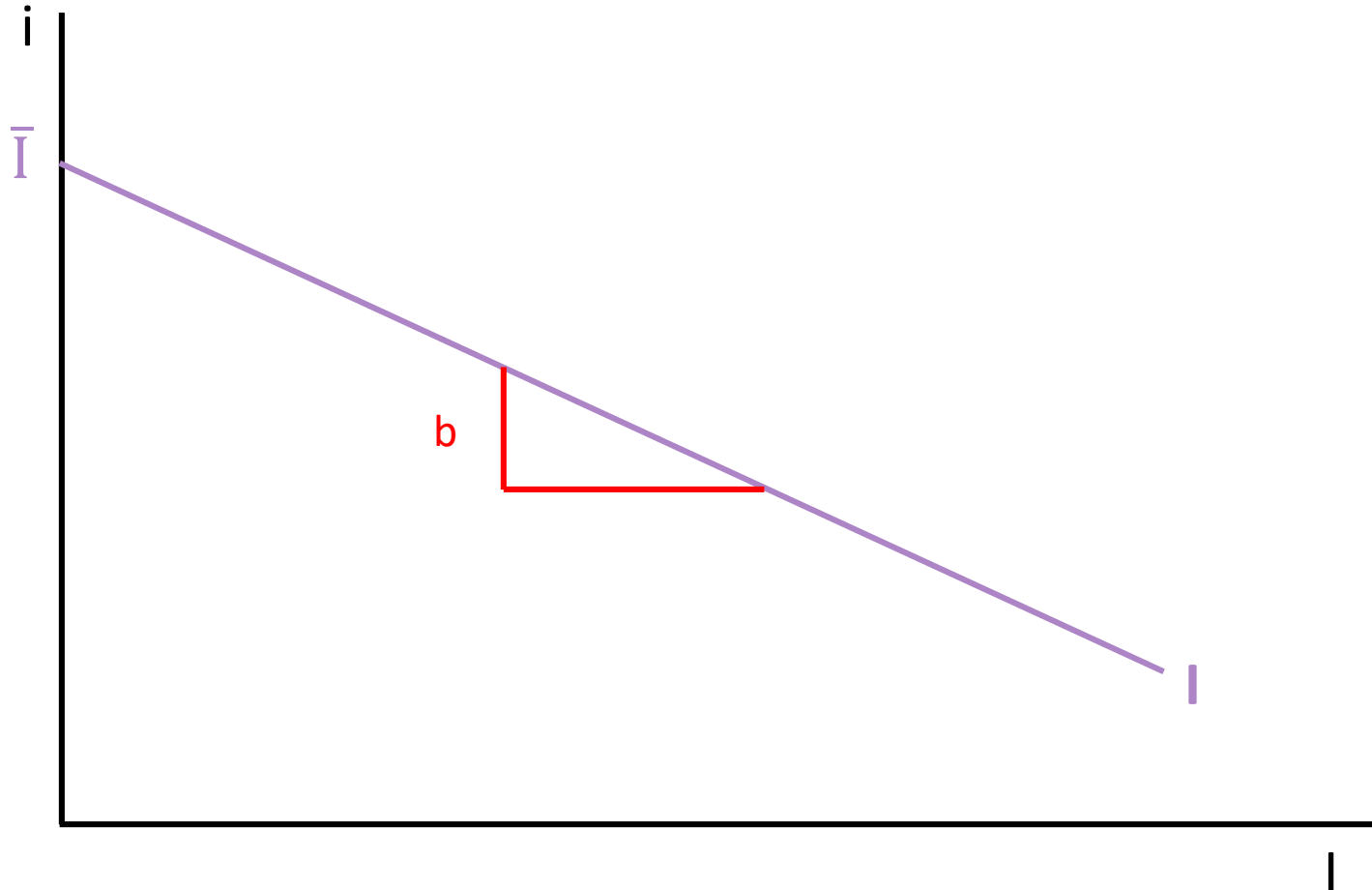
- Up to this point, investment has been exogenous. Now it becomes endogenous.
- Investment depends inversely on the interest rate. The investment function is then:

$$I = \bar{I} - bi$$

Where:

- b measures the sensitivity of investment with respect to the rate of interest (i). If b is large, a small increase in i leads to a large drop in I .
- $b > 0$. Note that b is the **slope** of the equation.

Investment and interest rate





Interest rate and aggregate demand: the IS curve

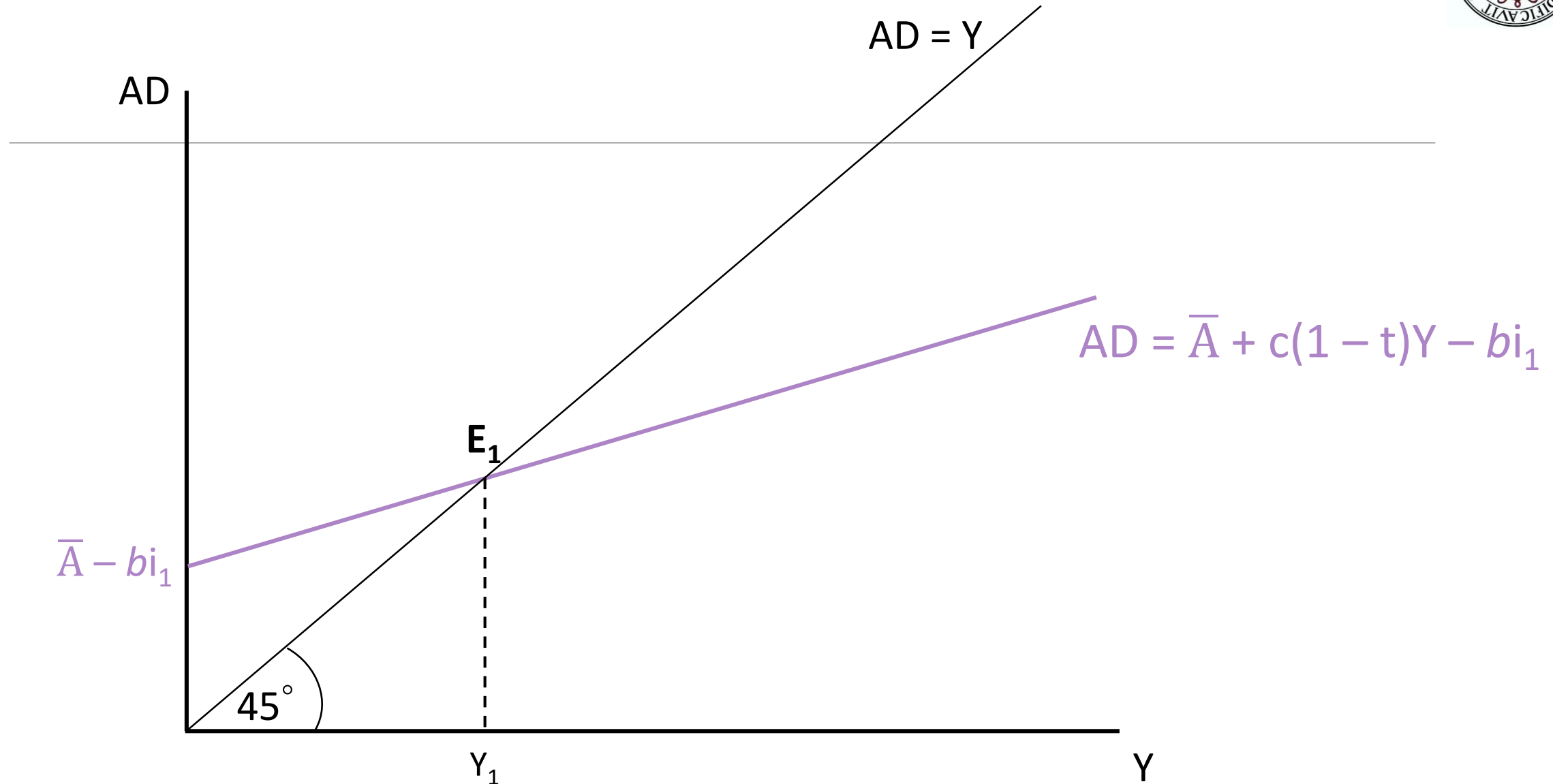
- Recall: $AD = C + I + G + NX$
- Replacing the equations of C and I :

$$AD = [\bar{C} + c\bar{TR} + c(1 - t)Y] + (\bar{I} - bi) + \bar{G} + \bar{NX}$$

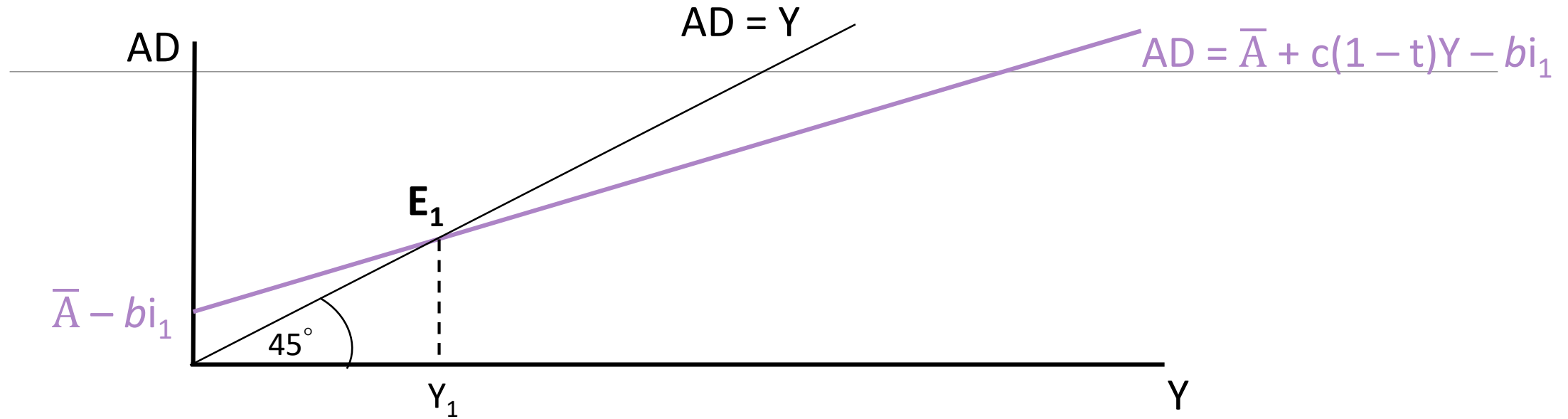
$$AD = \bar{A} + c(1 - t)Y - bi$$

$$\text{Where, } \bar{A} = [\bar{C} + c\bar{TR} + \bar{I} + \bar{G} + \bar{NX}]$$

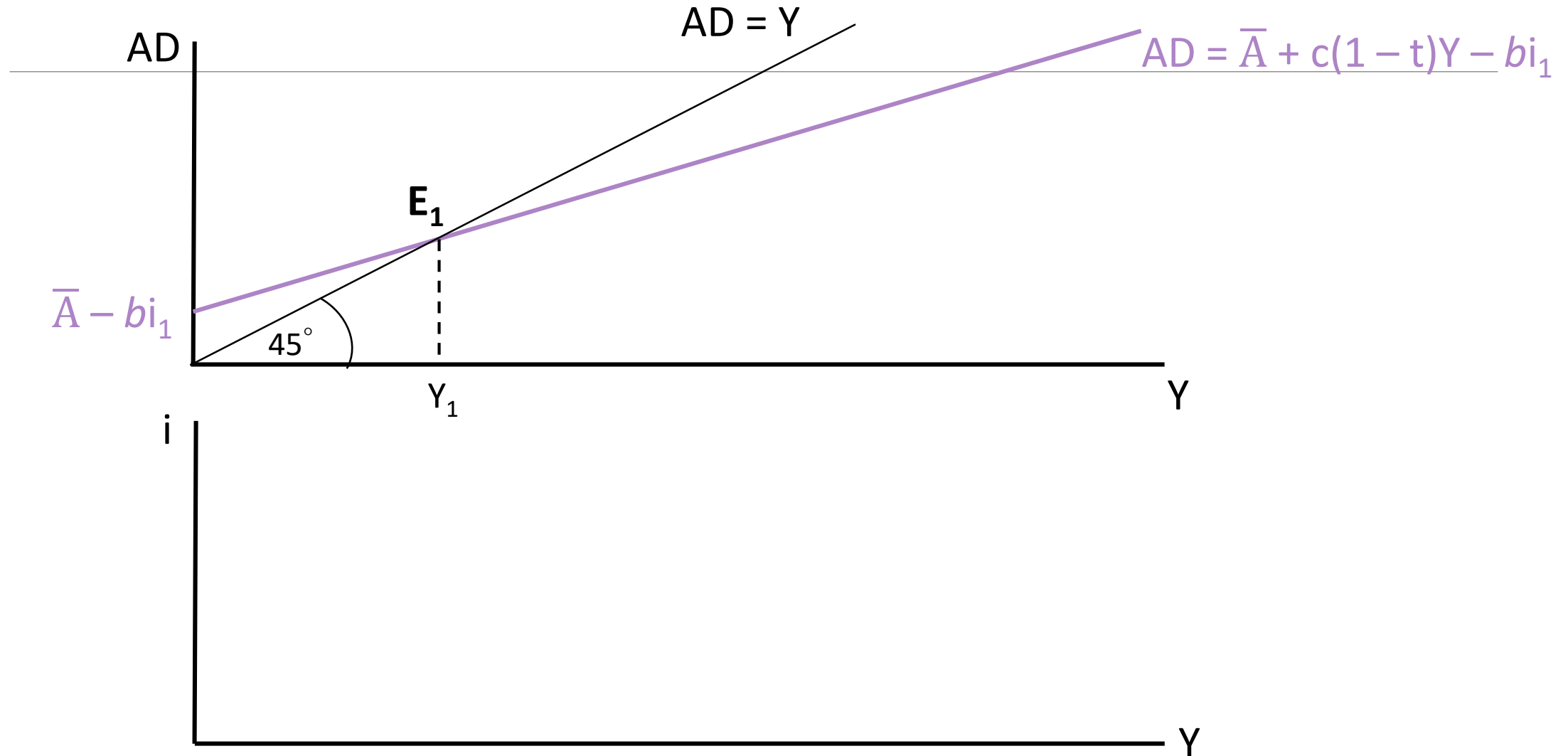
Interest rate and aggregate demand: the IS curve



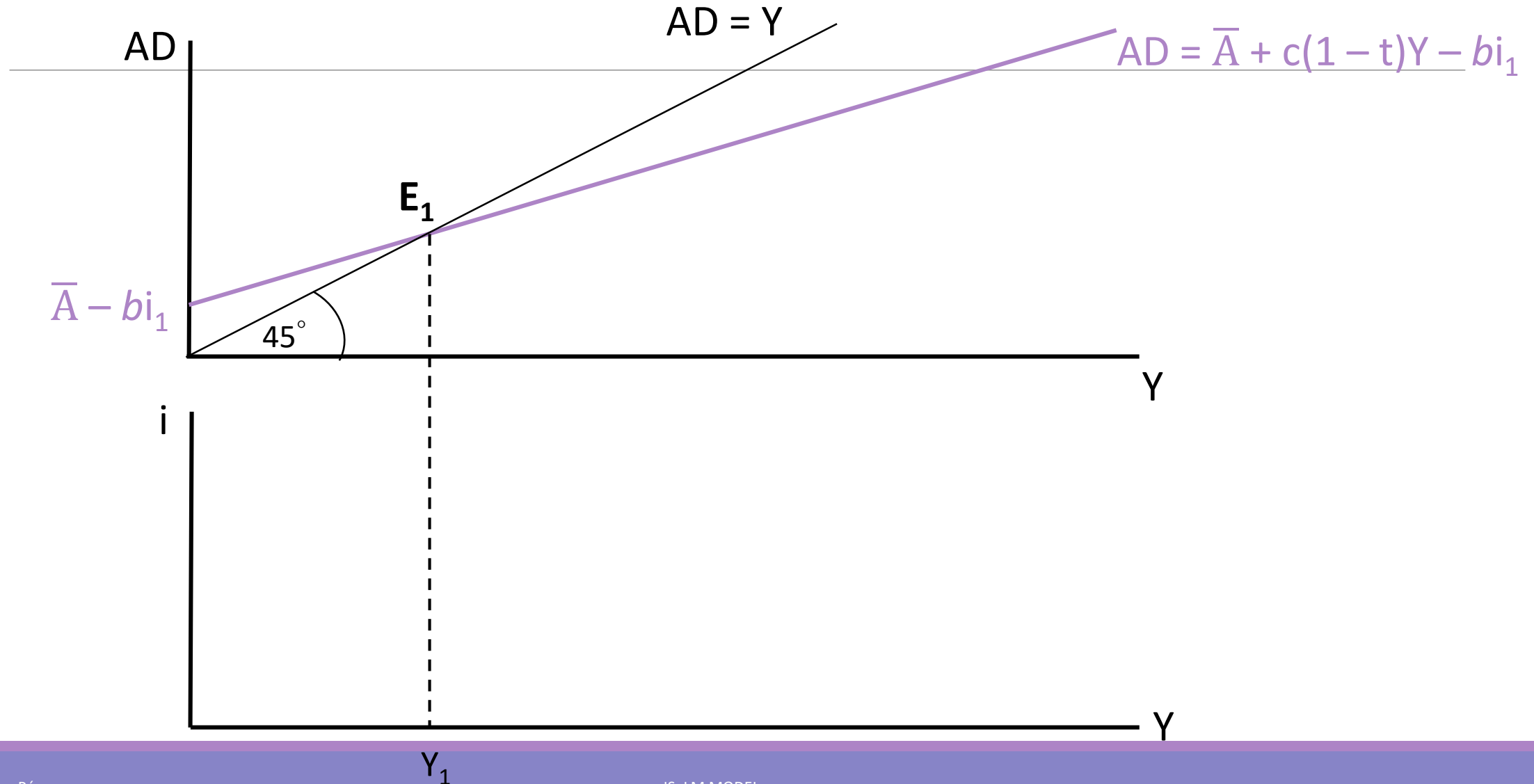
Interest rate and aggregate demand: the IS curve



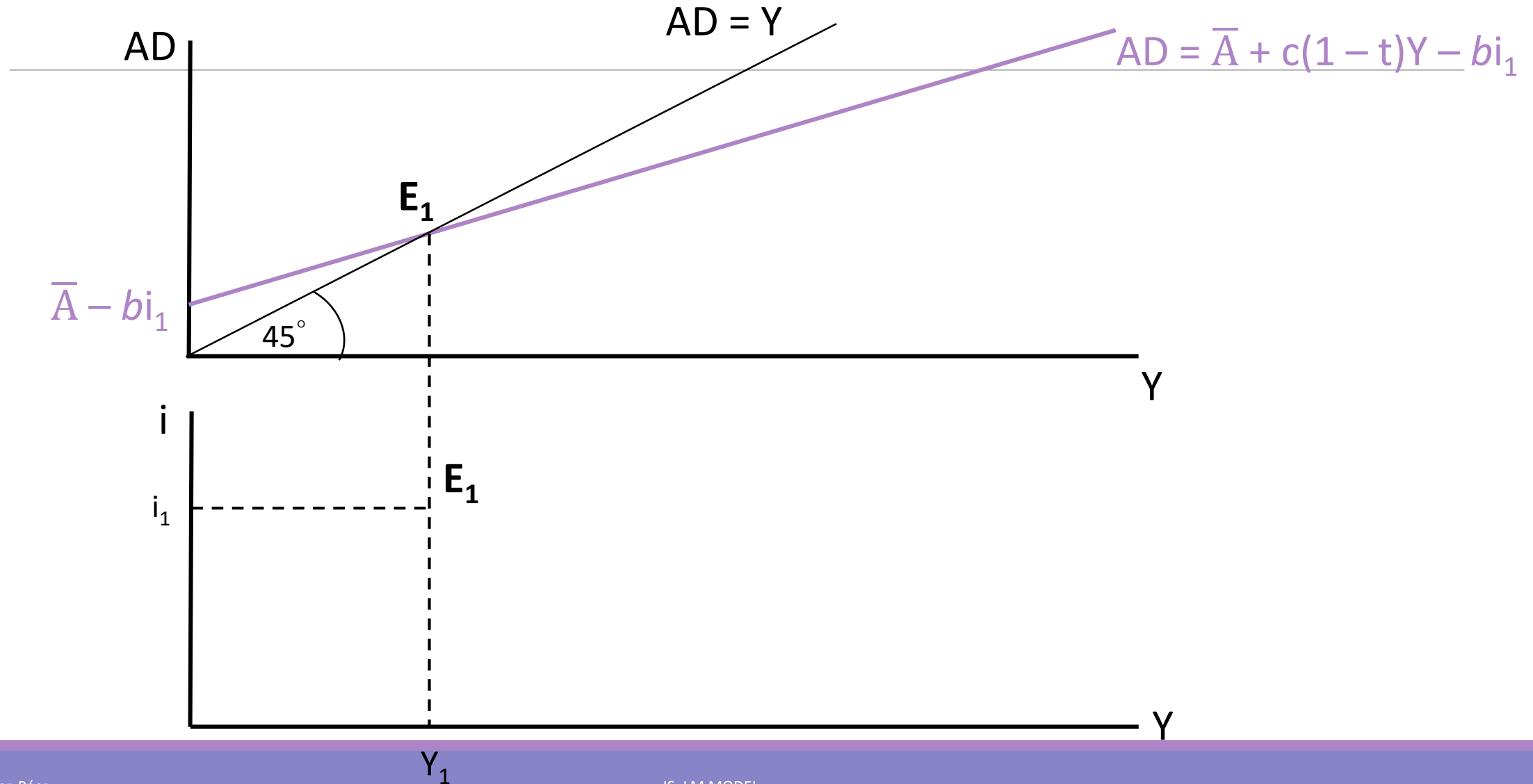
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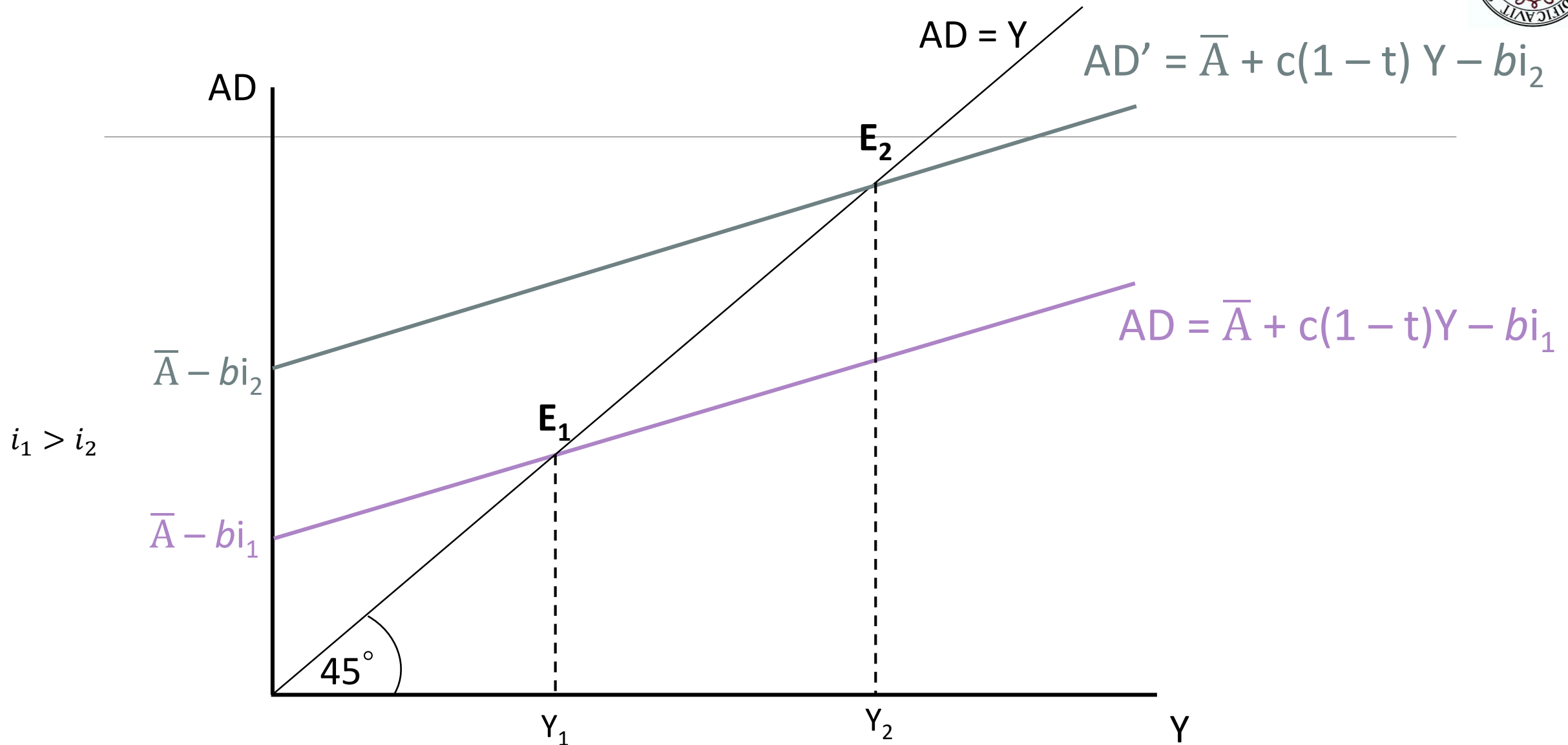
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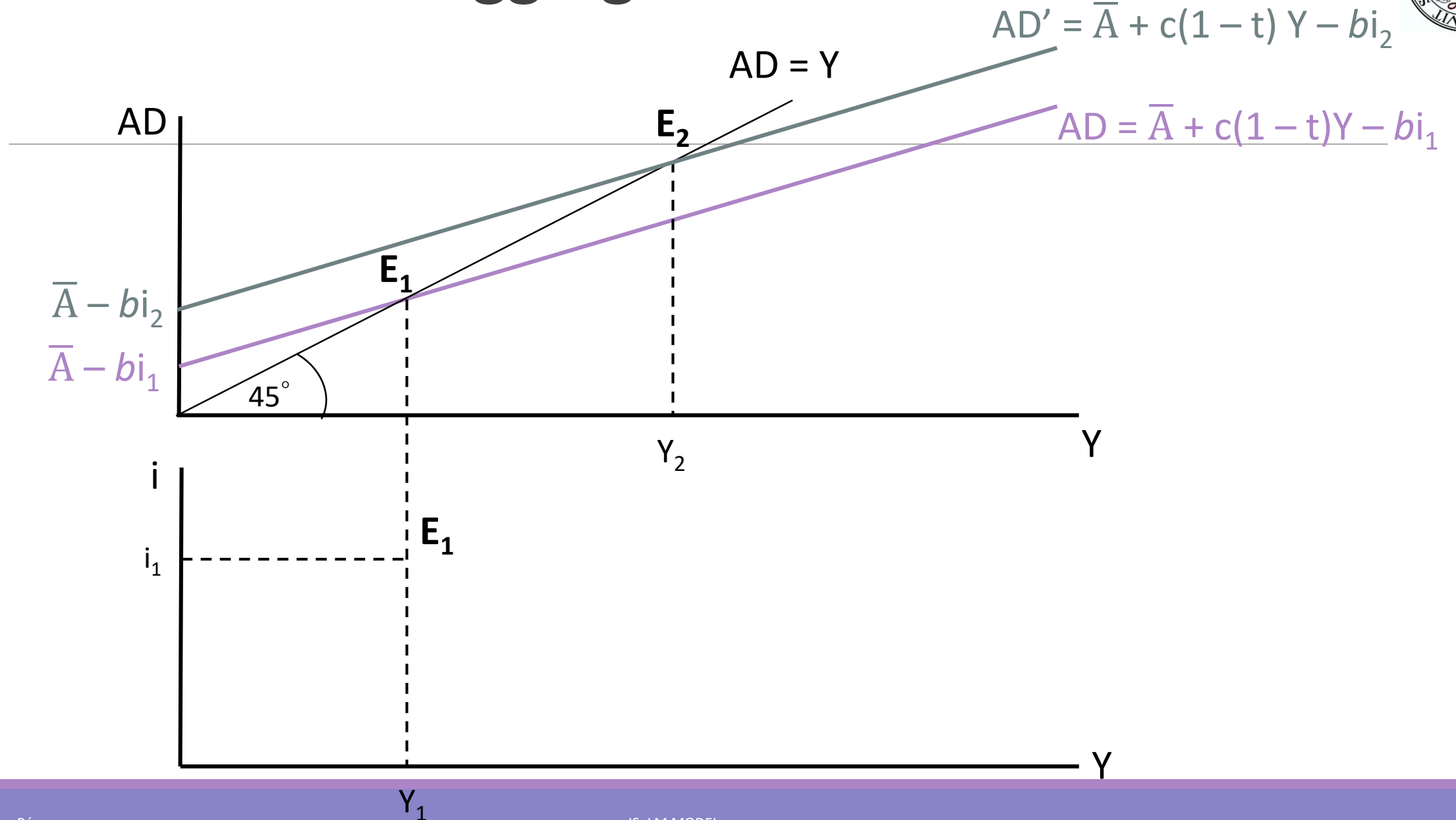
Interest rate and aggregate demand: the IS curve



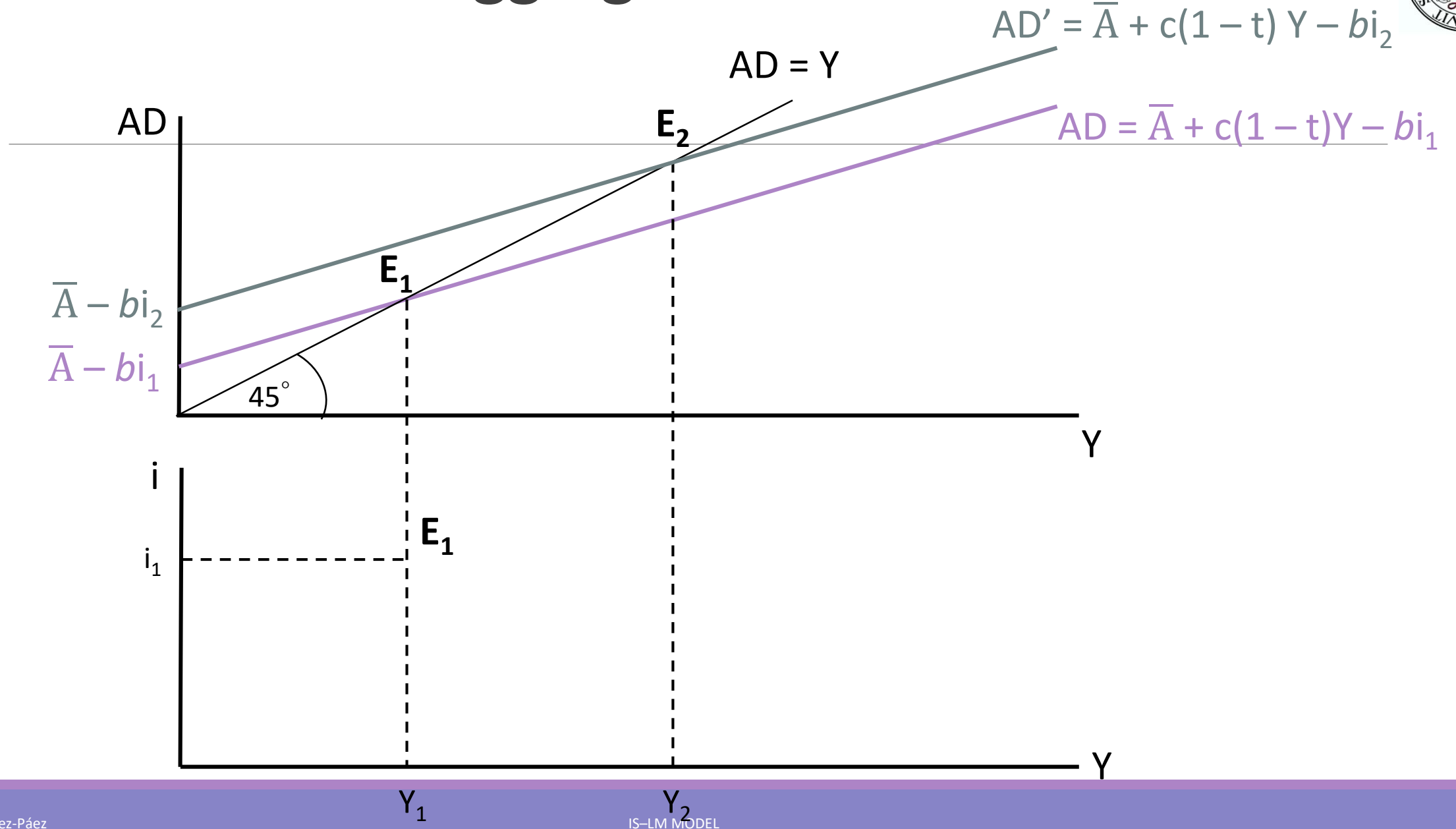
Interest rate and aggregate demand: the IS curve



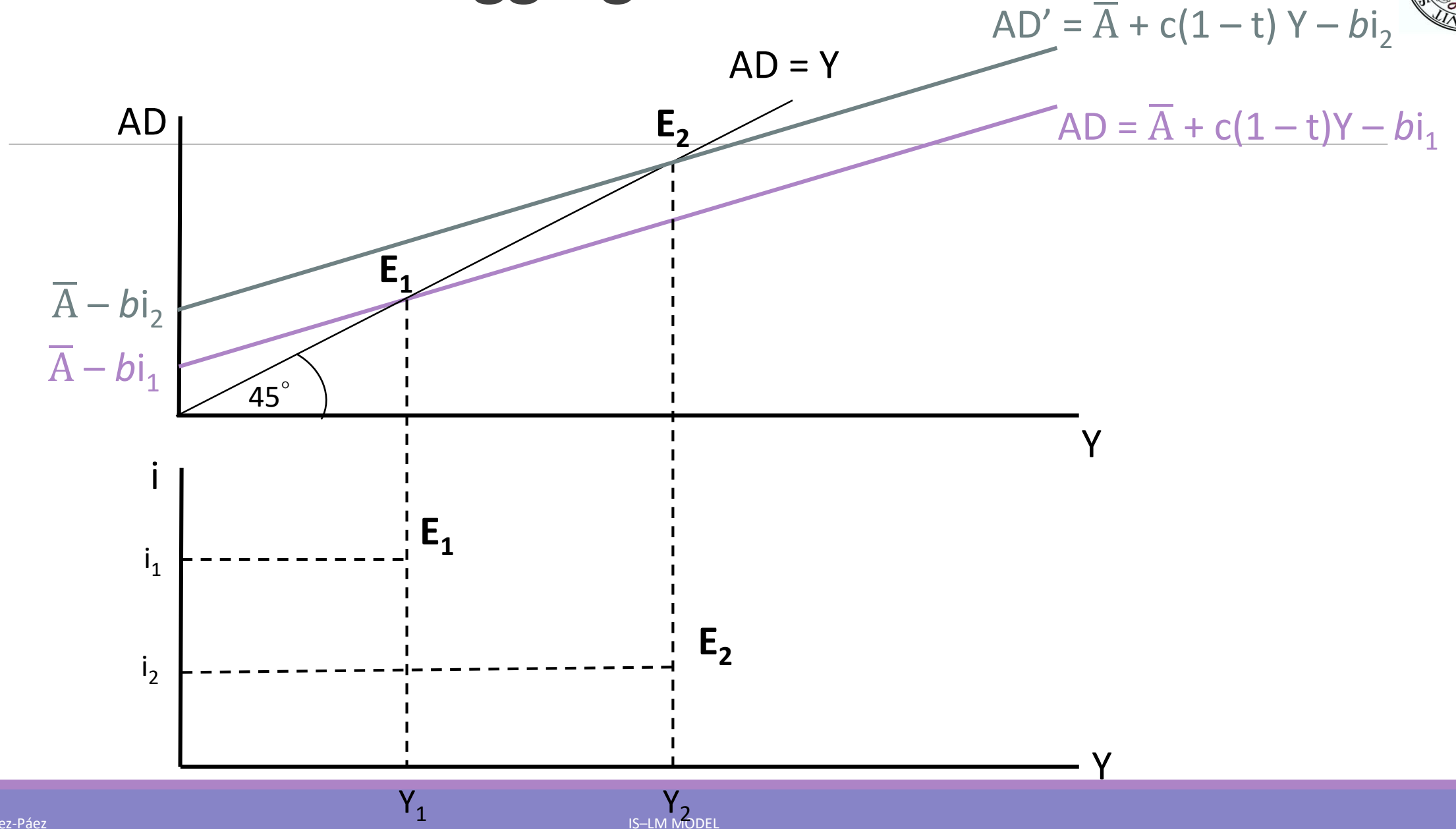
Interest rate and aggregate demand: the IS curve



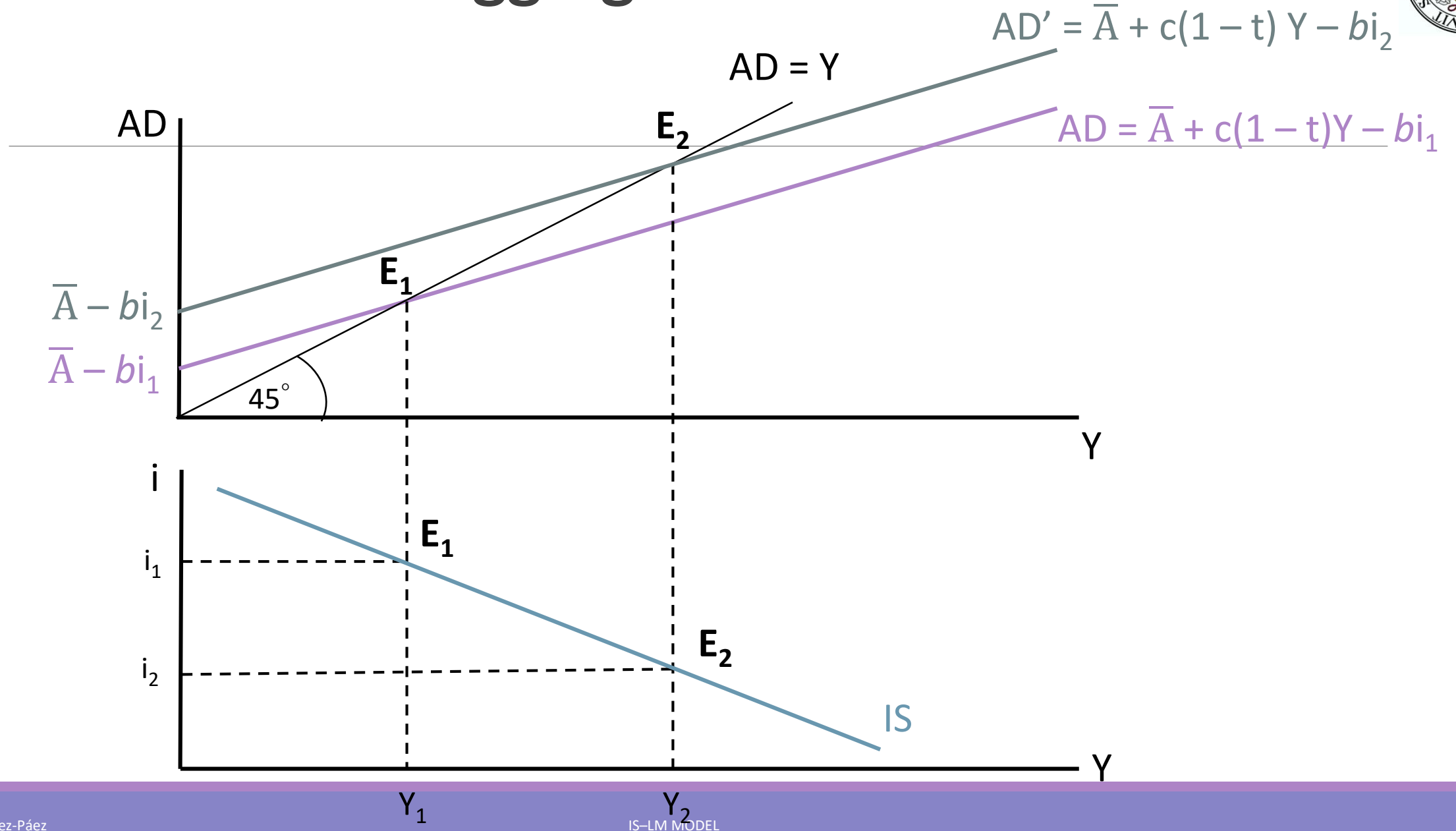
Interest rate and aggregate demand: the IS curve



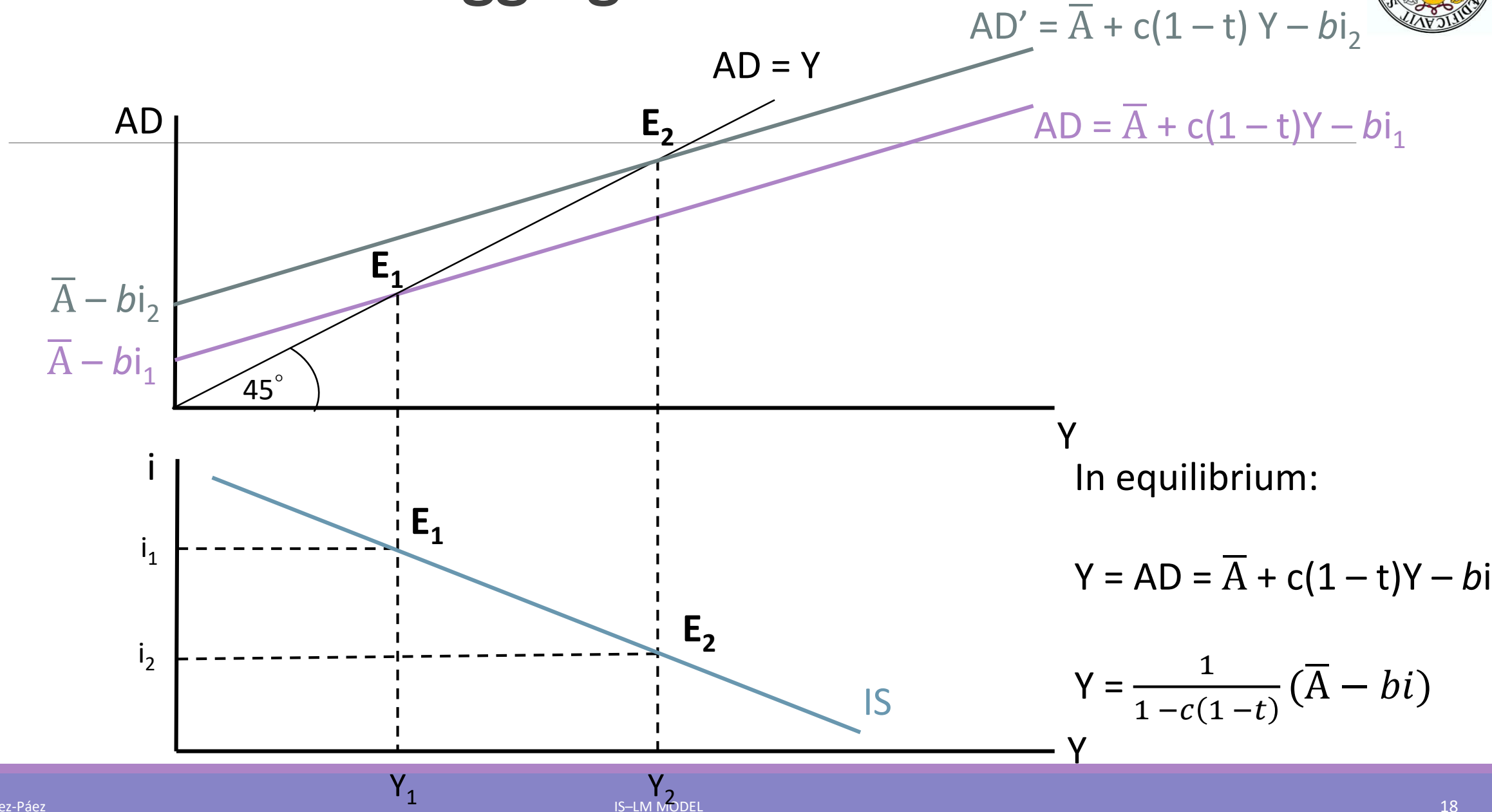
Interest rate and aggregate demand: the IS curve



Interest rate and aggregate demand: the IS curve



Interest rate and aggregate demand: the IS curve





The IS curve

Recall: $AD = C + I + G + NX$

$$C = \bar{C} + c\overline{TR} + c(1 - t)Y$$

$$I = \bar{I} - bi$$

$$G = \bar{G}$$

$$NX = \overline{NX}$$

Therefore,

$$AD = [\bar{C} + c\overline{TR} + c(1 - t)Y] + (\bar{I} - bi) + \bar{G} + \overline{NX}$$

The IS curve

If $\bar{A} = [\bar{C} + c\bar{TR} + \bar{I} + \bar{G} + \bar{NX}]$, then:

$$AD = \bar{A} + c(1 - t)Y - bi$$

In equilibrium it must be satisfied that

$$Y = AD = \bar{A} + c(1 - t)Y - bi$$

Therefore:

$$Y = \frac{1}{1 - c(1 - t)} (\bar{A} - bi)$$

The IS curve

- The Keynesian multiplier is:

$$\alpha = \frac{1}{1 - c(1 - t)}$$

- Therefore,

$$Y = \alpha(\bar{A} - bi)$$

- By subtracting i :

$$i = \frac{1}{b}\bar{A} - \frac{1}{\alpha b}Y$$

The IS curve

- The Keynesian multiplier is:

$$\alpha = \frac{1}{1 - c(1 - t)}$$

- Therefore,

$$Y = \alpha(\bar{A} - bi)$$

- By subtracting i :

$$i = \frac{1}{b}\bar{A} - \frac{1}{\alpha b}Y = \textbf{IS}$$

The IS curve: the slope

- The Keynesian multiplier is:

$$\alpha = \frac{1}{1 - c(1 - t)}$$

- Therefore,

$$Y = \alpha(\bar{A} - bi)$$

- By subtracting i :

Intercept of the IS

Slope of the IS

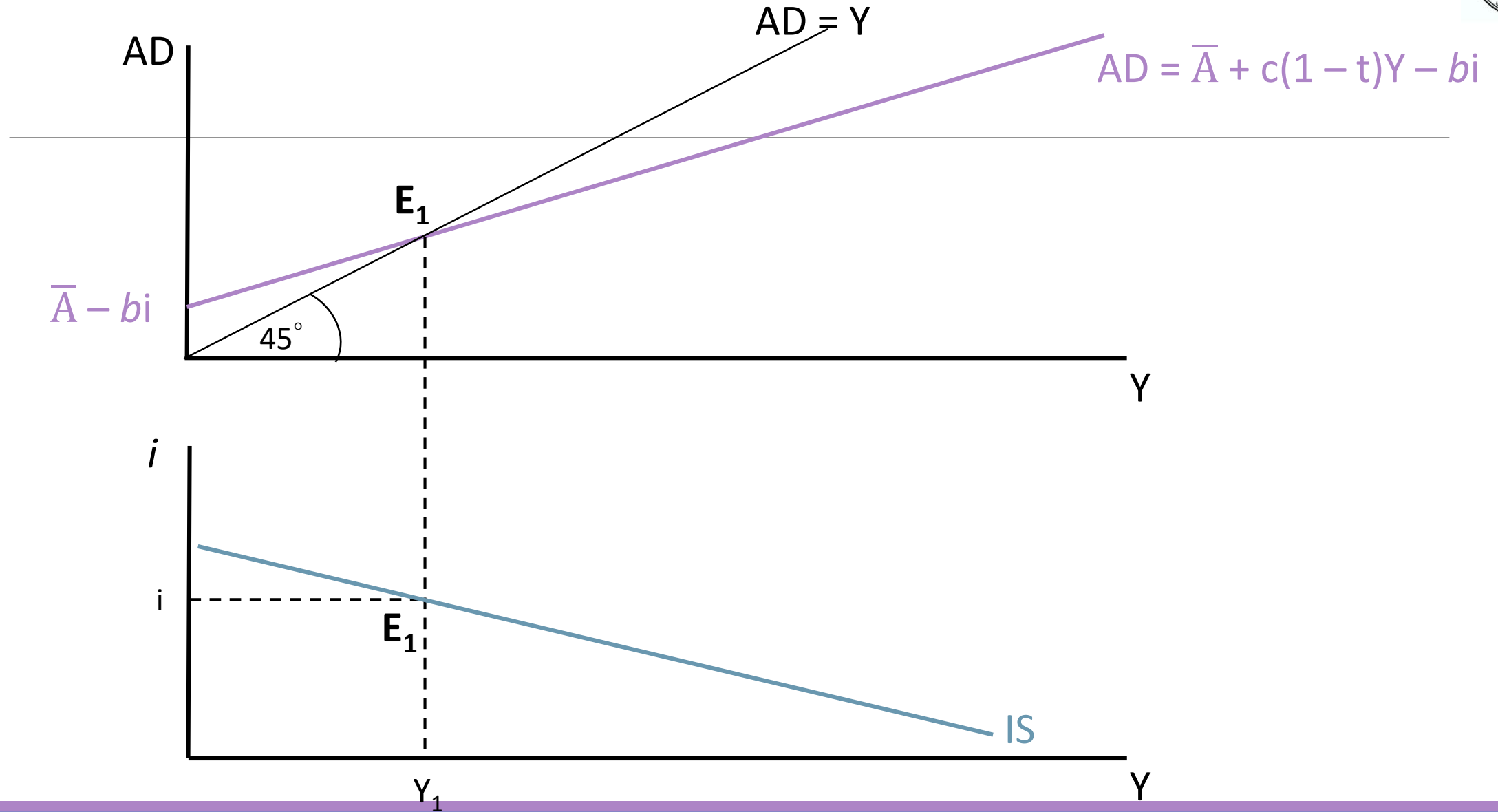
$$i = \boxed{\frac{1}{b}\bar{A}} - \boxed{\frac{1}{\alpha b}}Y = \text{IS}$$



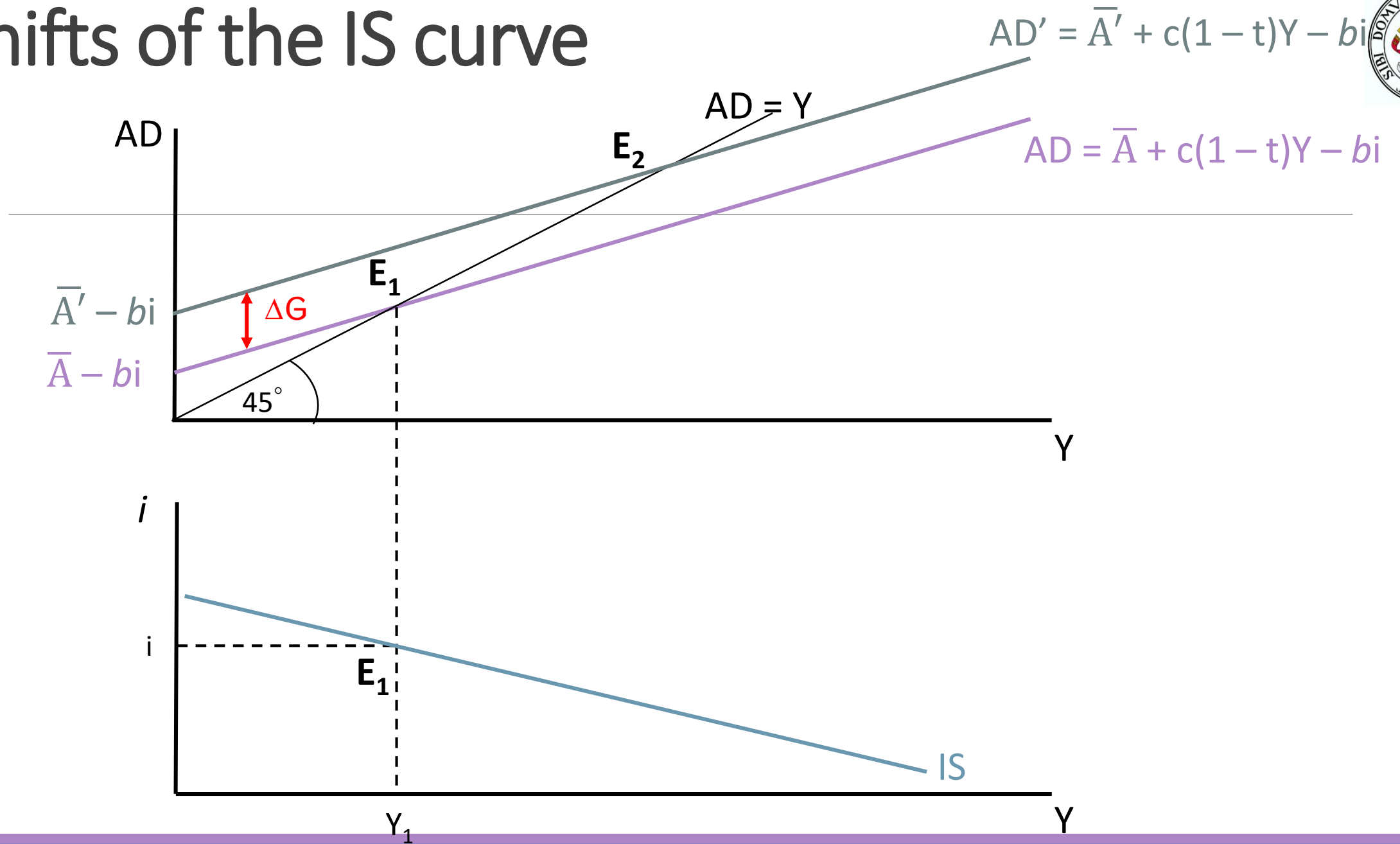
Shifts of the IS curve

- What happens if you vary any of the determinants of \bar{A} ?

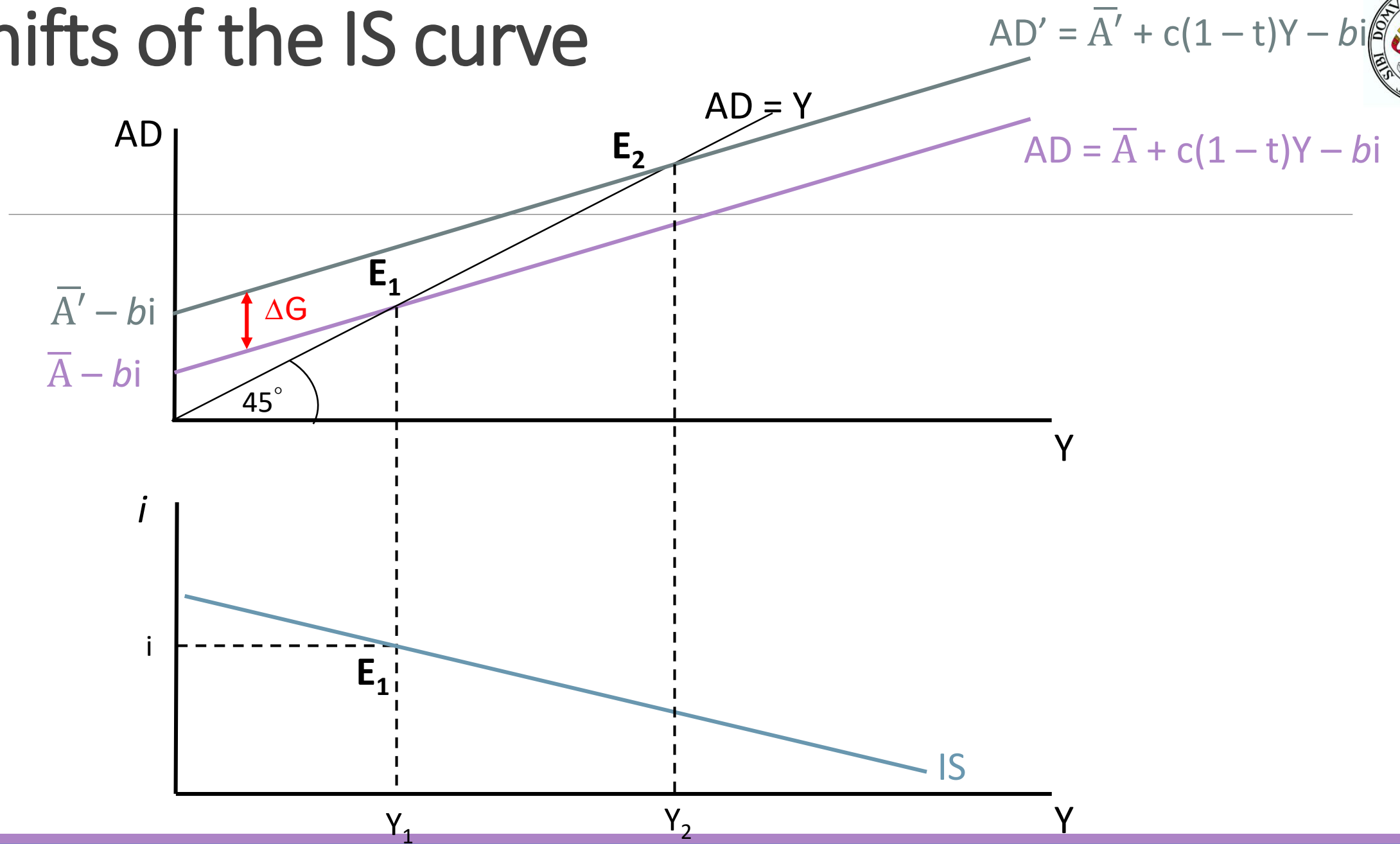
Shifts of the IS curve



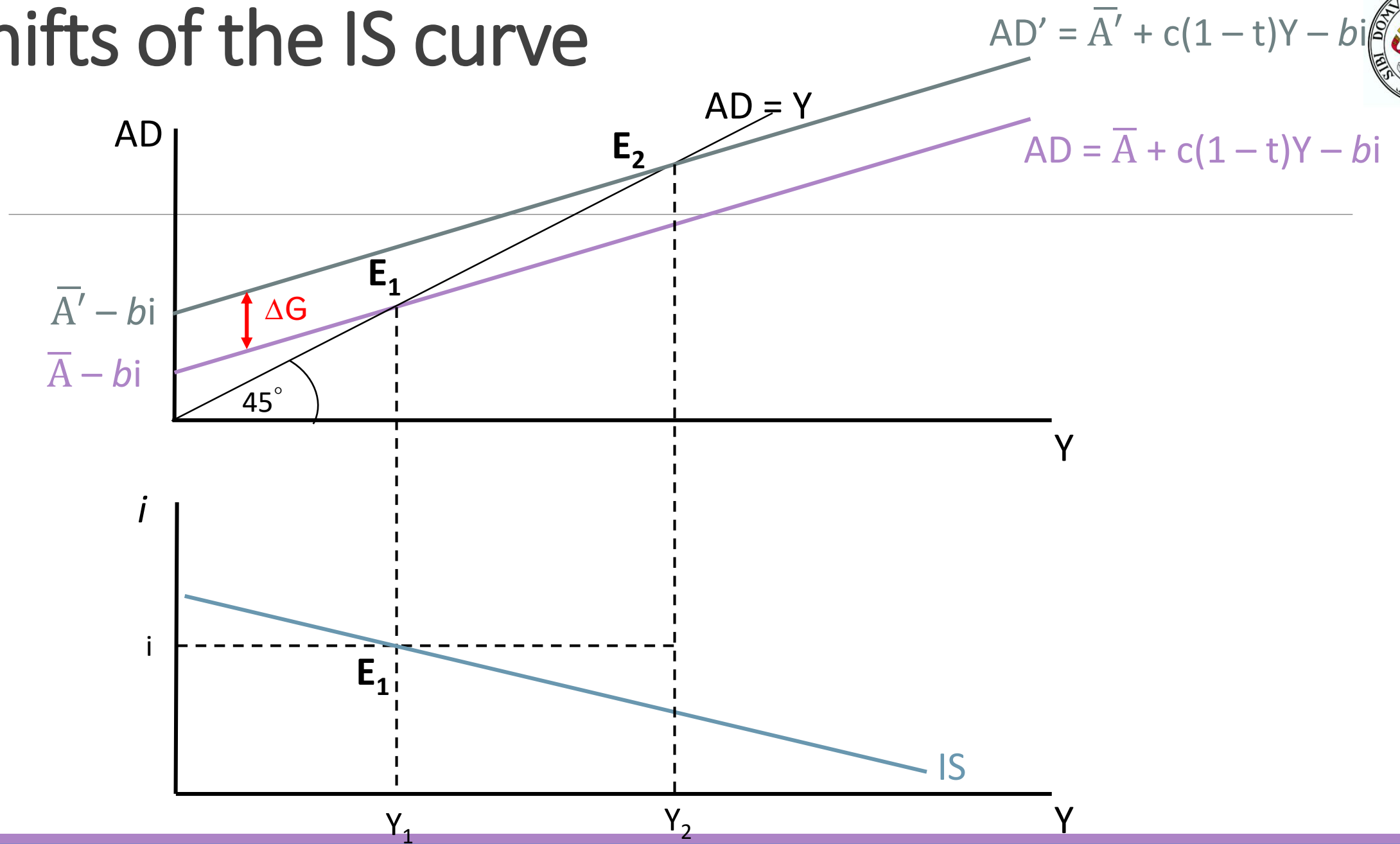
Shifts of the IS curve



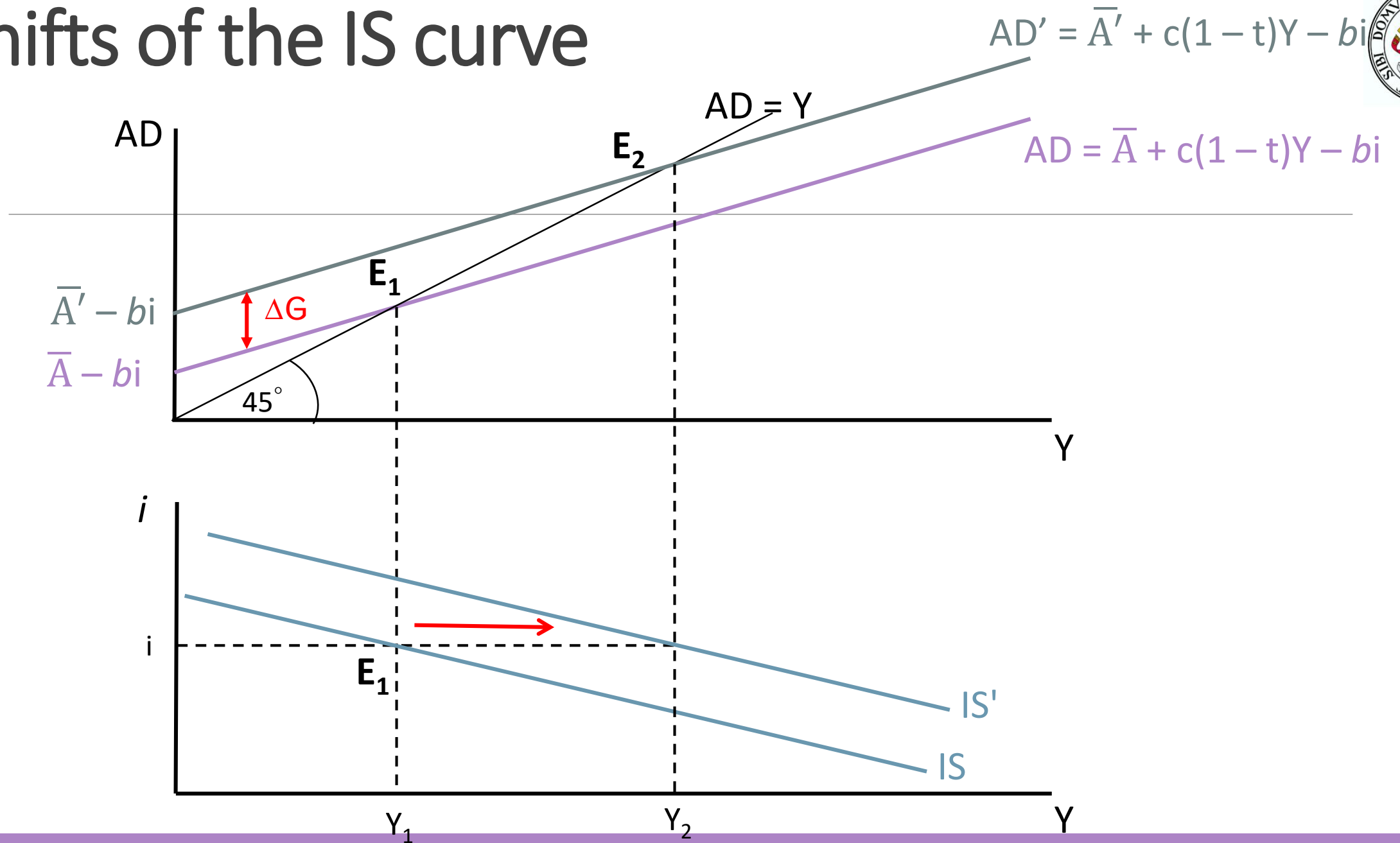
Shifts of the IS curve



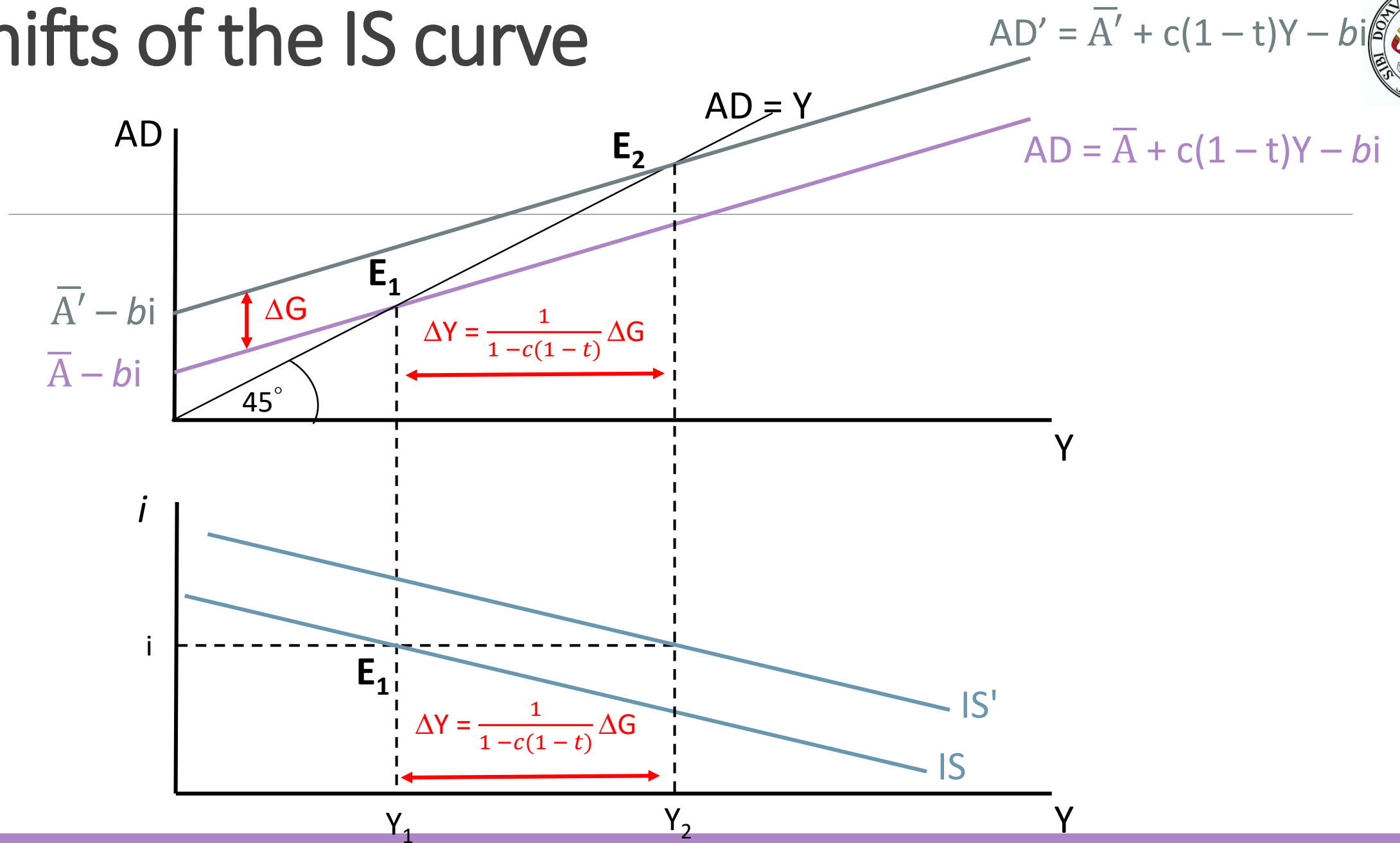
Shifts of the IS curve



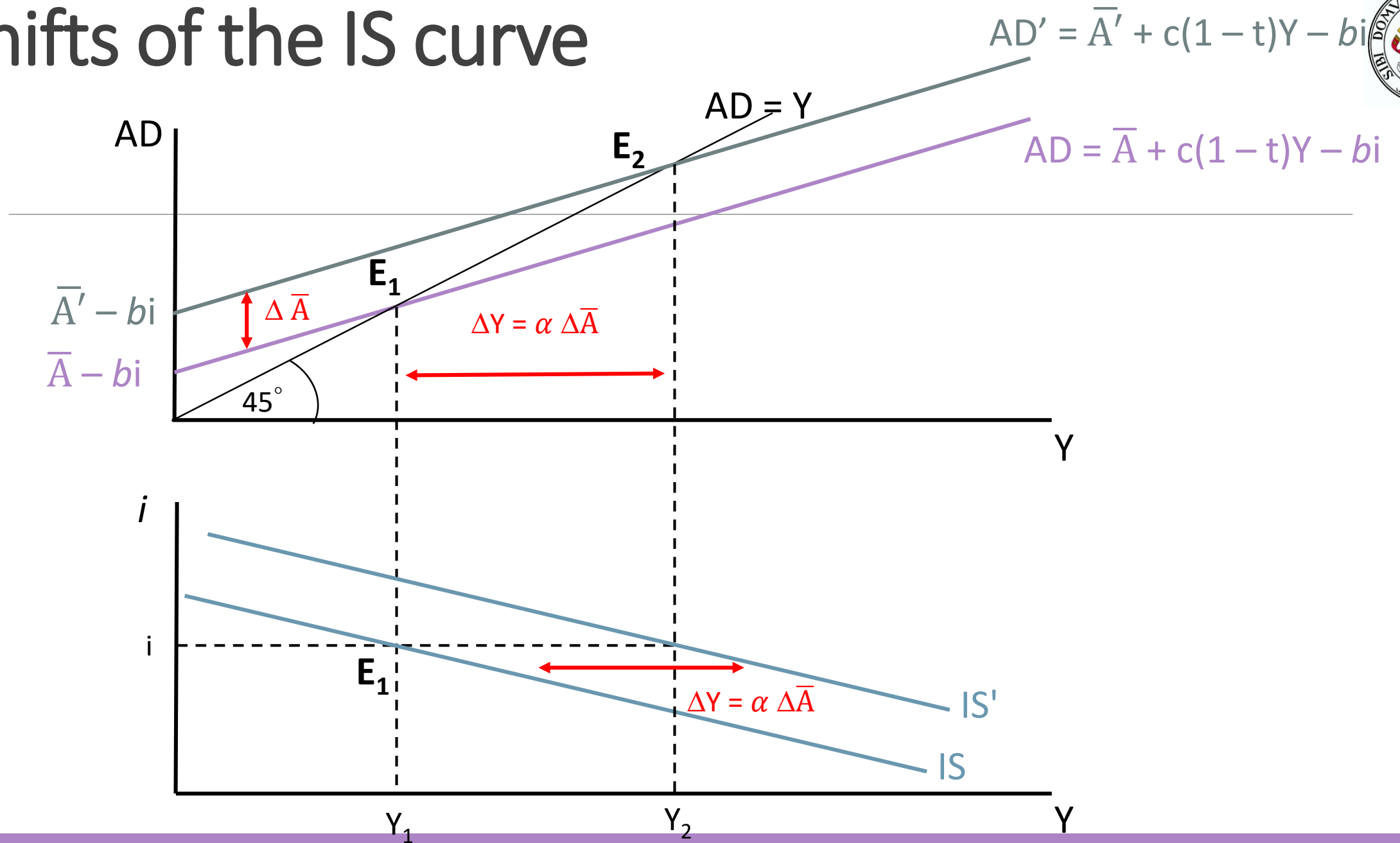
Shifts of the IS curve



Shifts of the IS curve



Shifts of the IS curve





In summary...

- The IS curve results from combinations of i and Y such as the goods market is in equilibrium.
- IS has a negative slope: an increase in i reduces I causing a reduction in AD, which causes Y to fall.
- It has a steeper slope the lower the multiplier and the less sensitive I (measured through b) is to changes in i .
- The IS curve is shifted by changes in \bar{A} . An increase in \bar{A} causes a shift to the right.



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Money market and the LM curve

- The **LM curve** shows the combinations of **interest rates** and **output levels** such that **the demand for real balances equals the supply**.
- The LM curve is derived in two steps:
 - Relationship between demand for money (***DM***), interest rate (***i***) and income (***Y***).
 - Equalize DM and fixed money supply (***MS***) for different combinations of ***Y*** and ***i***.



Money market and the LM curve

- Recall that **DM** is:

$$DM = kY - hi$$

Where,

$k > 0$: sensitivity of the demand for money with respect to income.

$h > 0$: sensitivity of the demand for money with respect to the interest rate.



Money market and the LM curve

- Recall that **MS** is constant and is expressed in real terms:

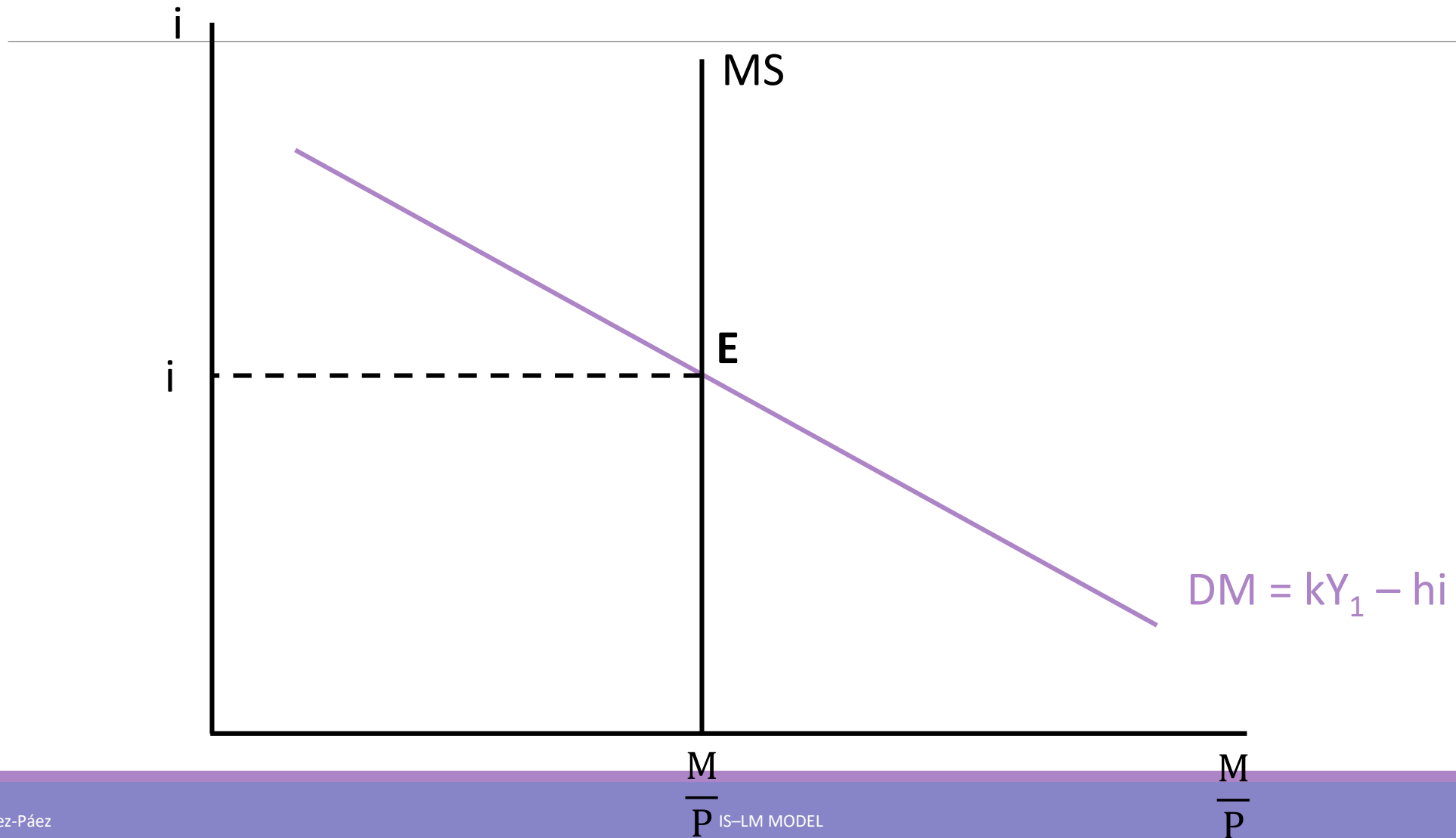
$$MS = \frac{M}{P}$$

Where,

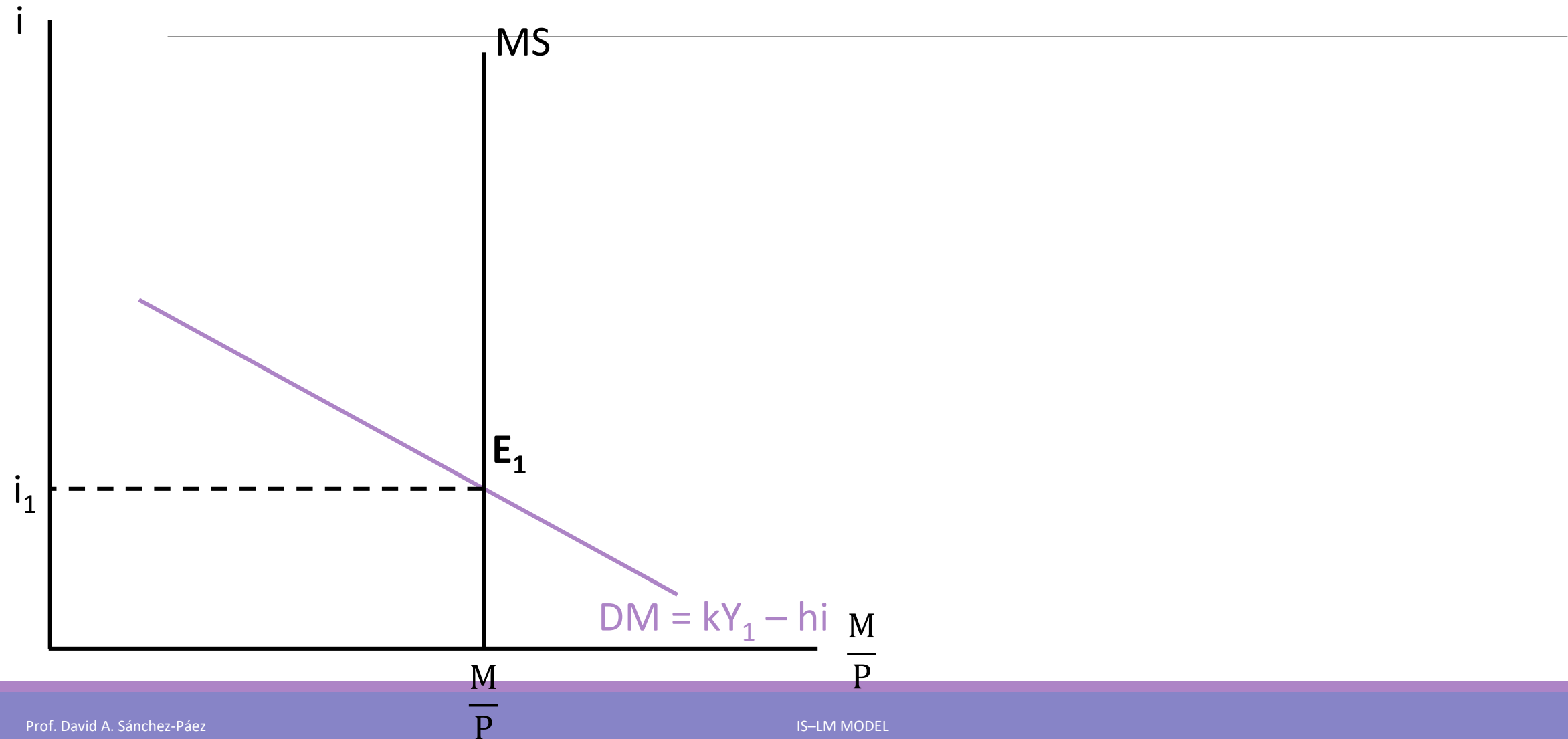
M : nominal quantity of money in the economy.

P : aggregate level of prices in the economy.

Money market equilibrium and LM curve

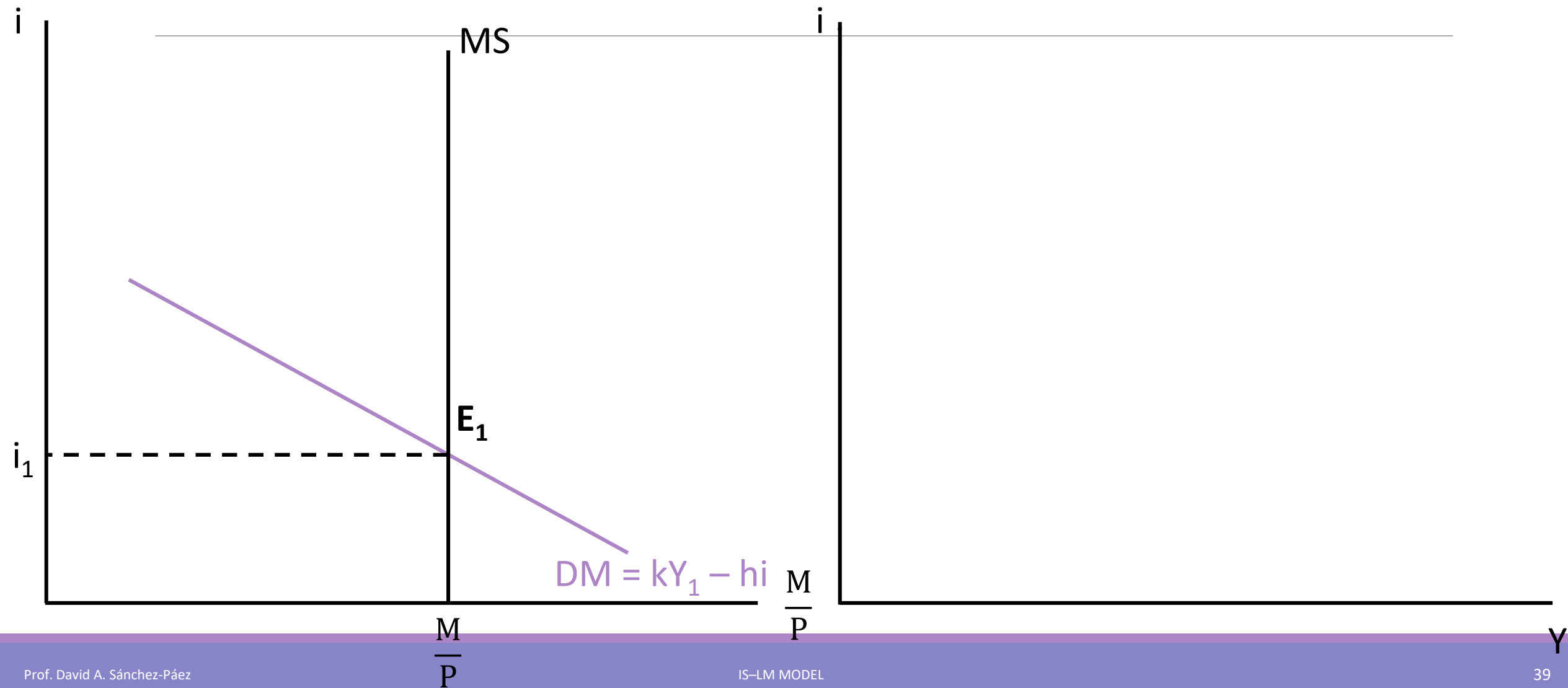


Money market equilibrium and LM curve

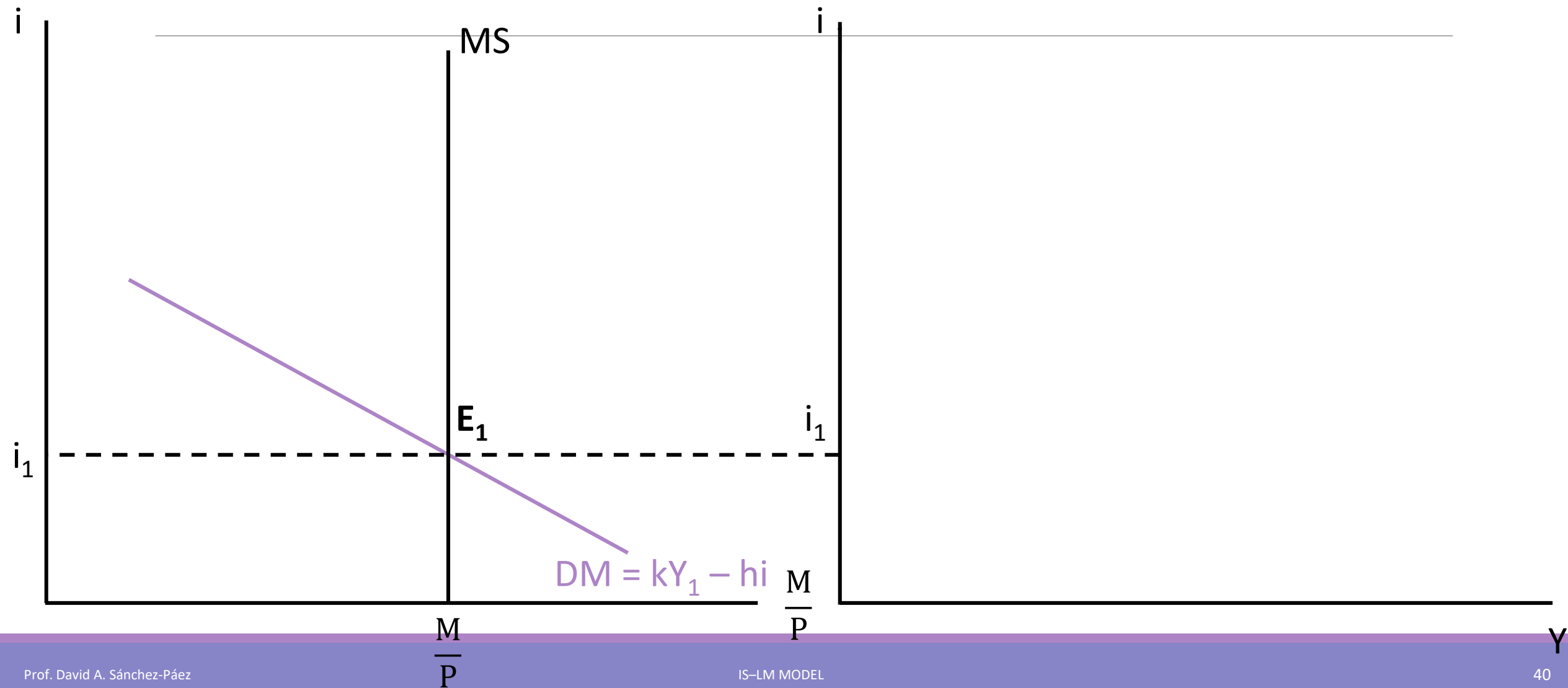




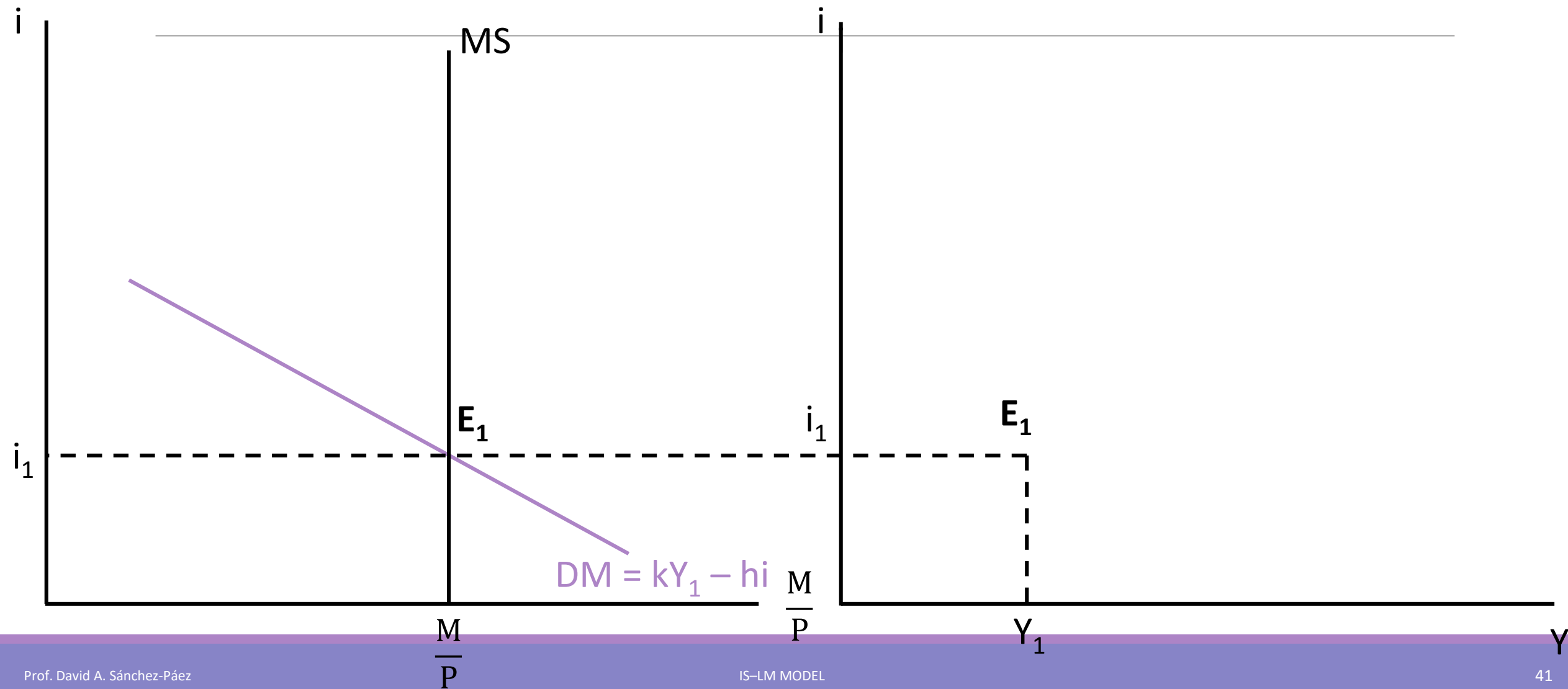
Money market equilibrium and LM curve



Money market equilibrium and LM curve

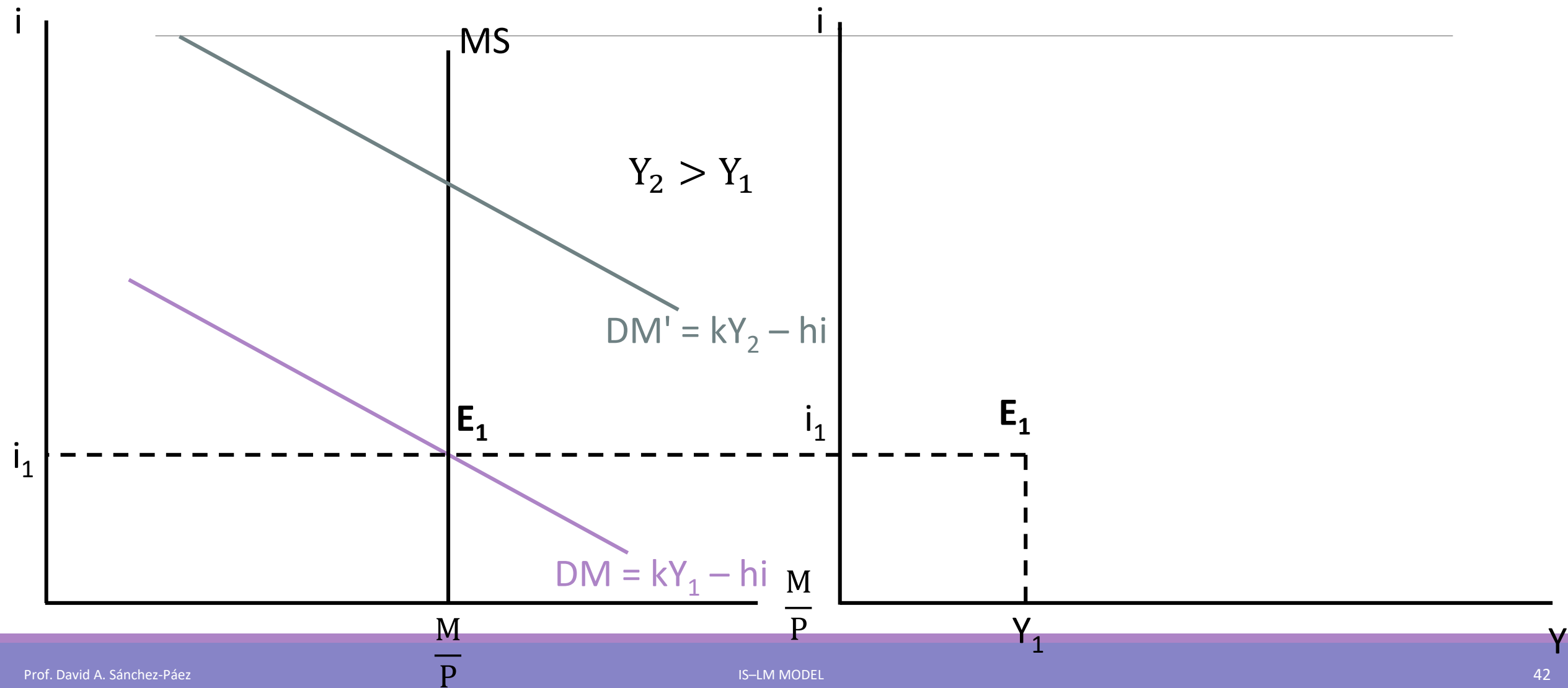


Money market equilibrium and LM curve

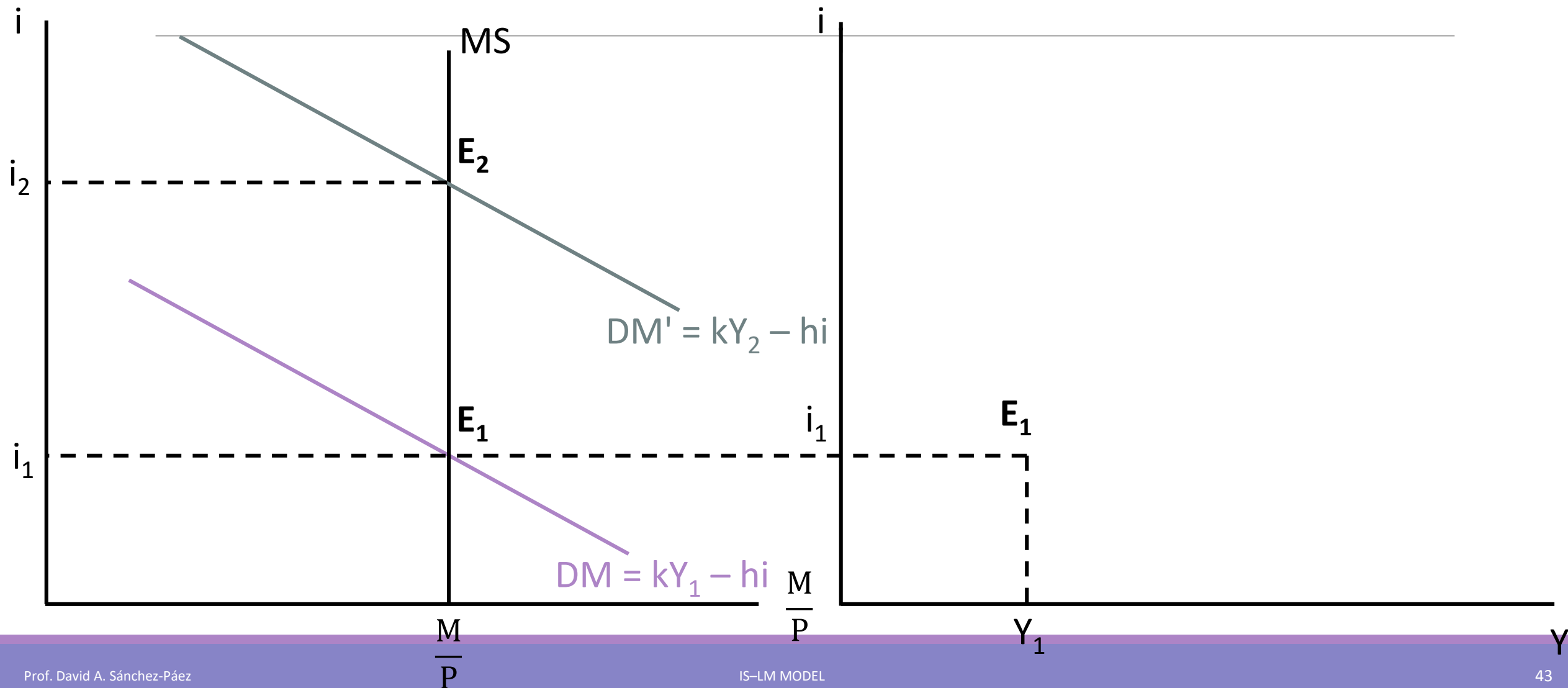




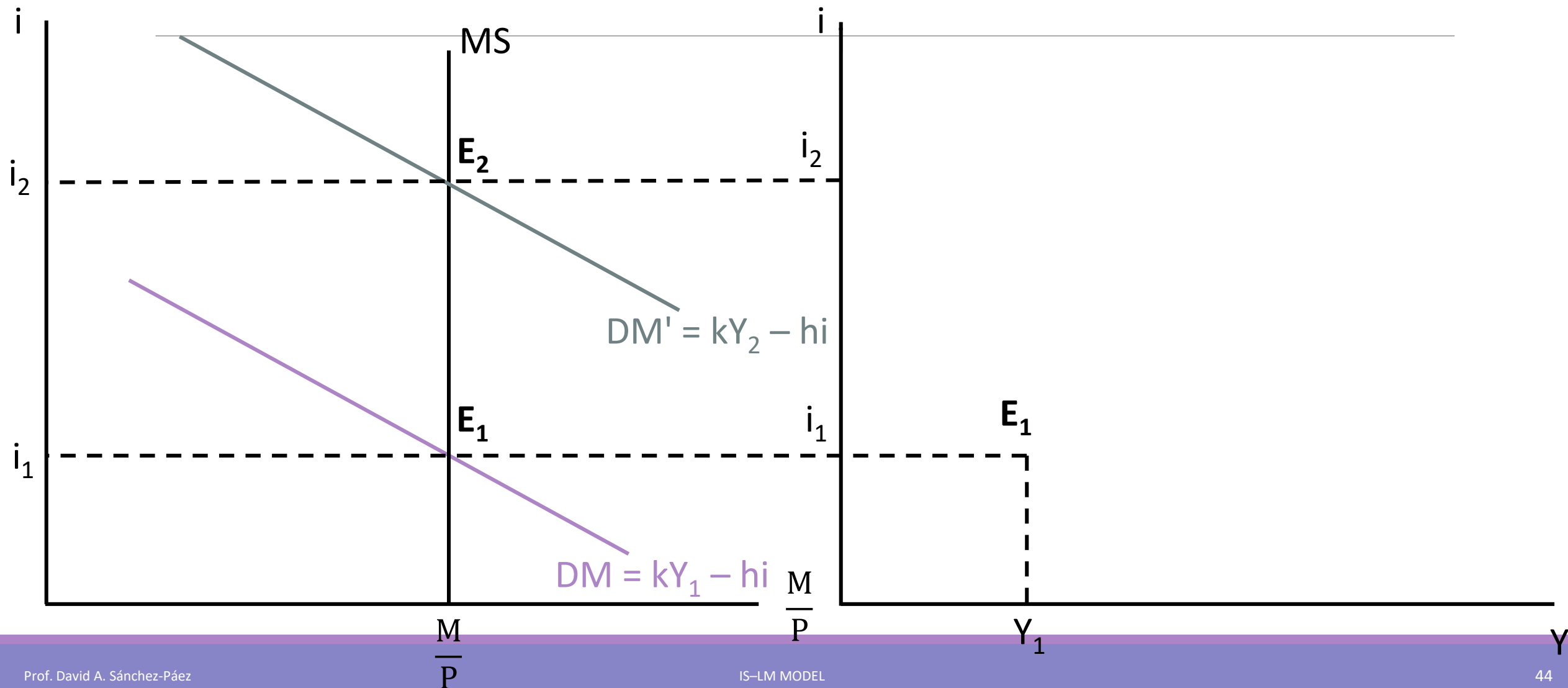
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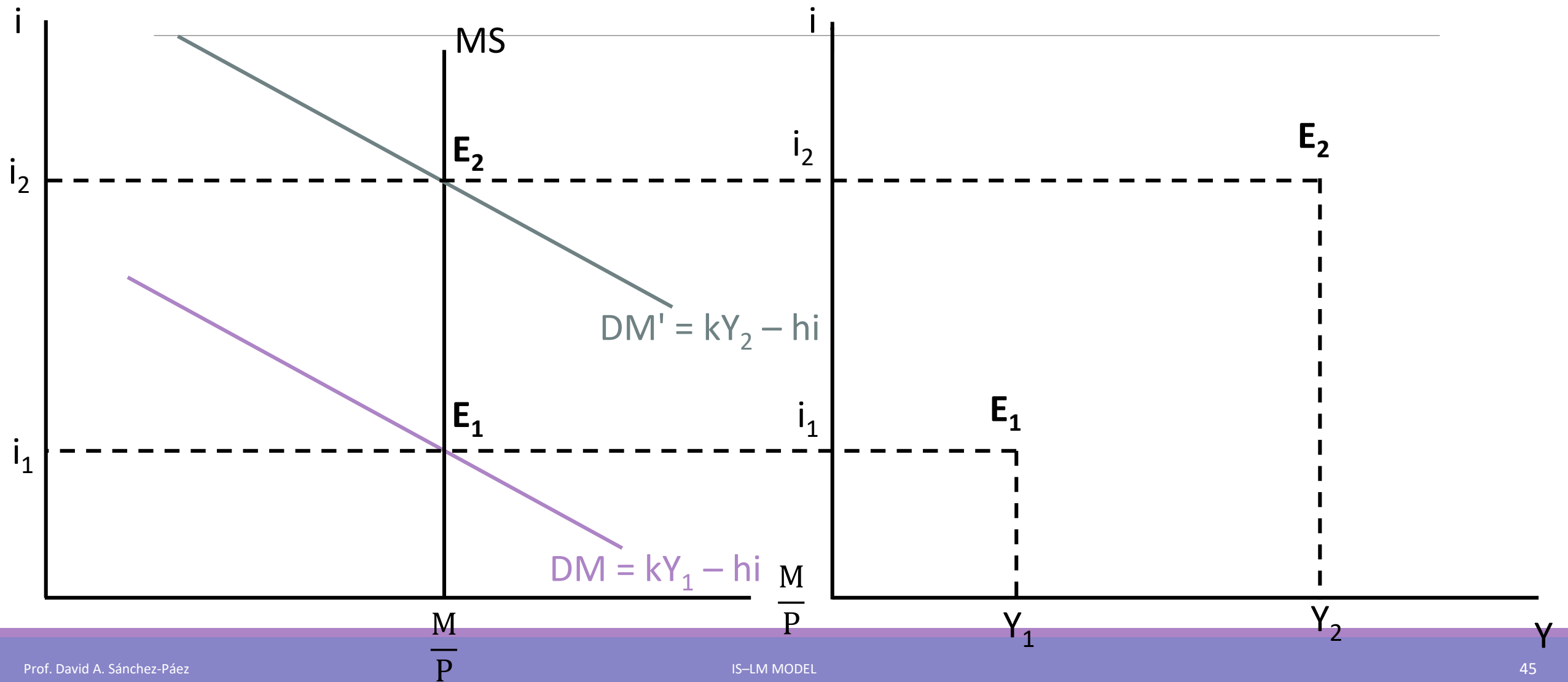
Money market equilibrium and LM curve



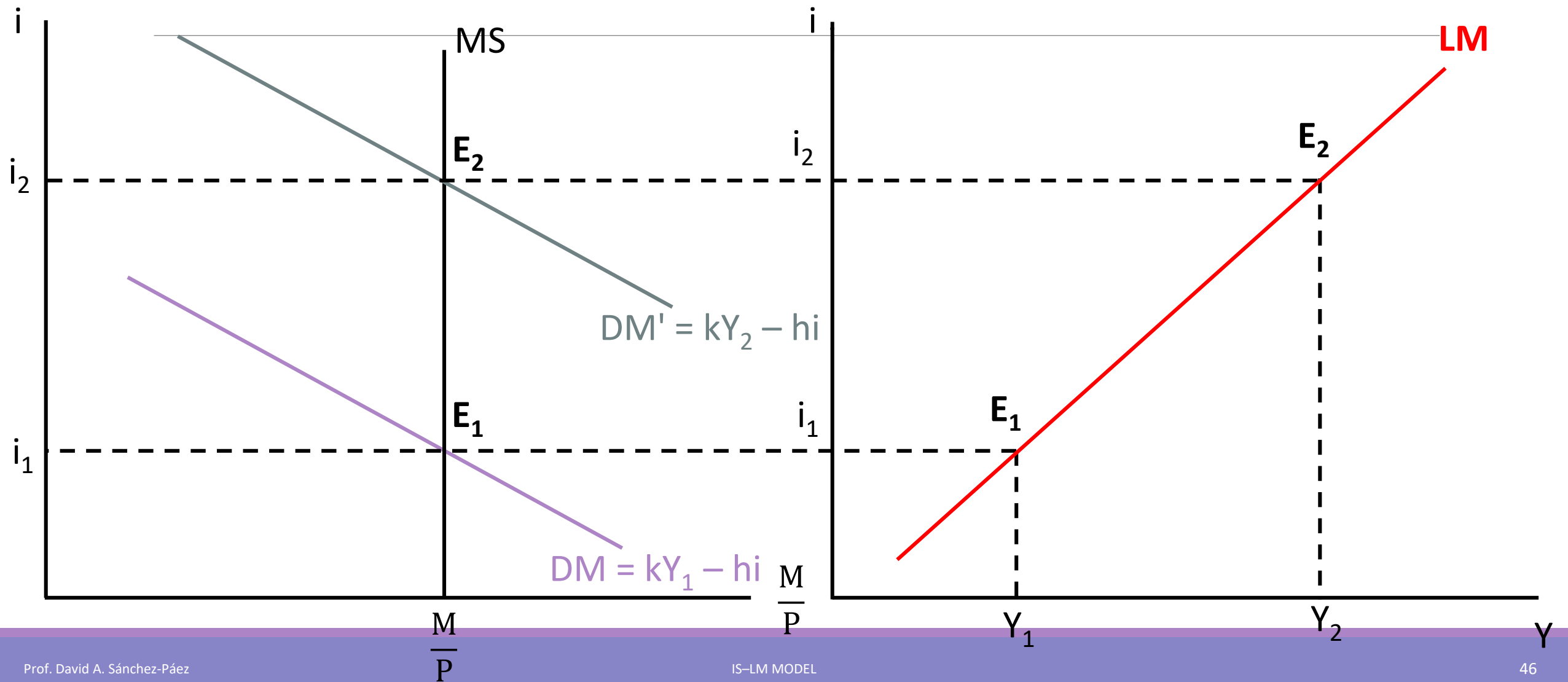
Money market equilibrium and LM curve



Money market equilibrium and LM curve



Money market equilibrium and LM curve



The LM curve

- In equilibrium:

$$DM = MS$$

- Therefore,

$$kY - hi = \frac{M}{P}$$

- By clearing i :

$$i = -\frac{1}{h} \frac{M}{P} + \frac{k}{h} Y$$

The LM curve

- In equilibrium:

$$DM = MS$$

- Therefore,

$$kY - hi = \frac{M}{P}$$

- By clearing i :

$$i = -\frac{1}{h} \frac{M}{P} + \frac{k}{h} Y = \mathbf{LM}$$

The LM curve: the slope

- In equilibrium:

$$DM = OM$$

- Therefore,

$$kY - hi = \frac{M}{P}$$

- By clearing i :

Intercept of the LM

Slope of the LM

$$i = -\frac{1}{h} \frac{M}{P} + \frac{k}{h} Y = \mathbf{LM}$$

The LM curve

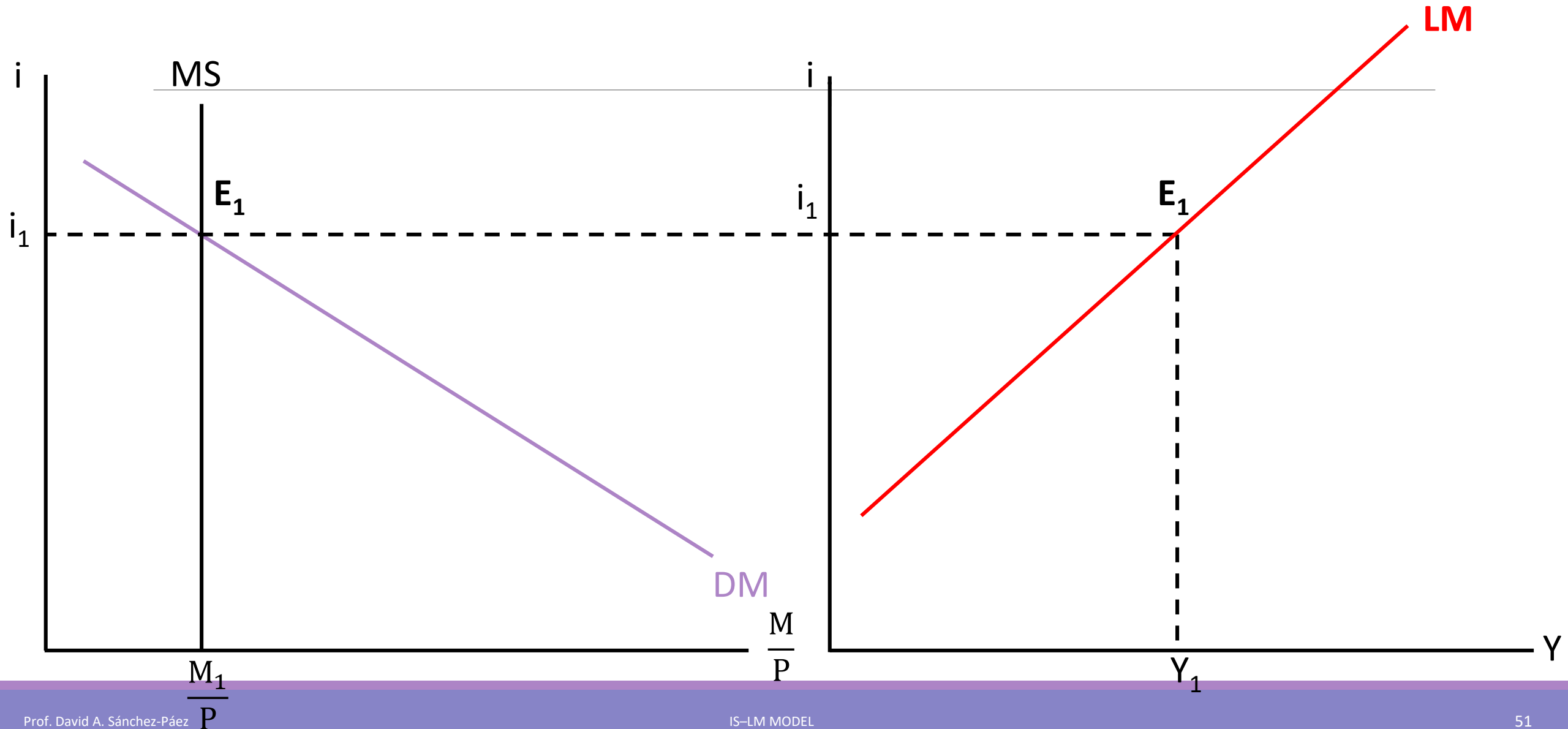
- The LM can also be expressed as:

$$LM = i = \frac{1}{h} \left(kY - \frac{M}{P} \right)$$

- The **greater the** sensitivity of the demand for money with respect to income (k), the **steeper the LM**.
- The **lower the** sensitivity of the demand for money with respect to the interest rate (h), the **steeper the LM**.
- The interest rate is affected more by changes in Y than in $\frac{M}{P}$.

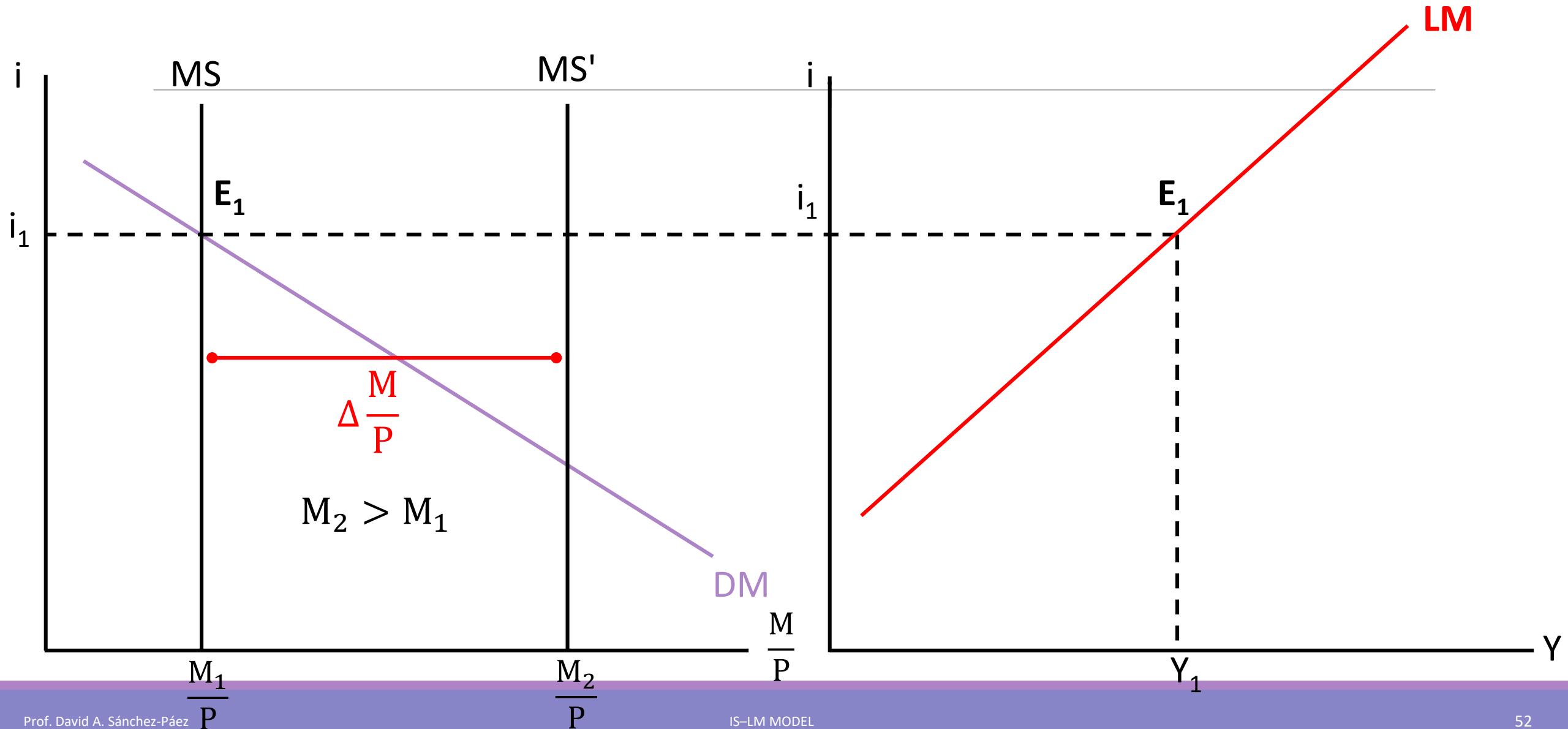


Shifts of the LM curve: increase in M



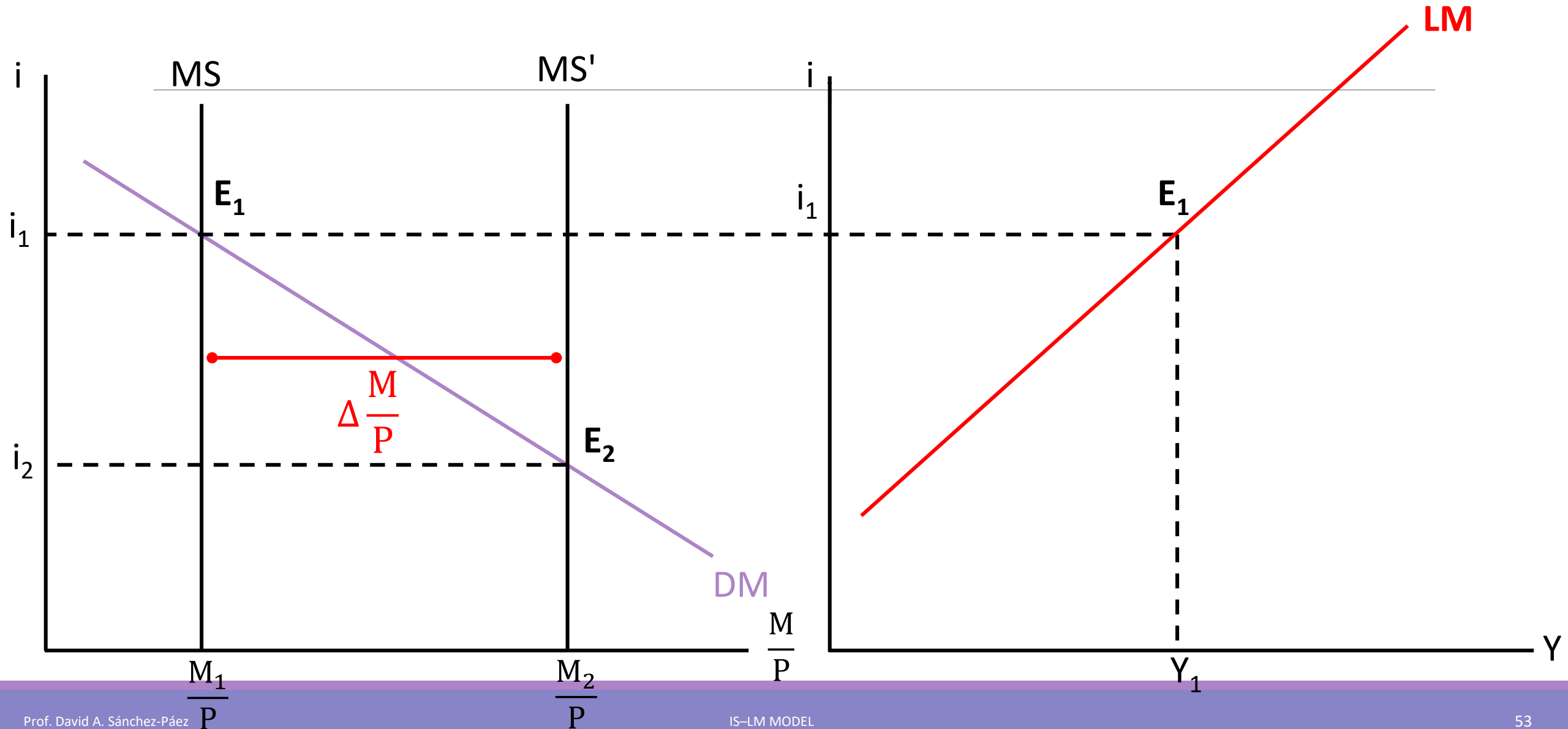


Shifts of the LM curve: increase in M



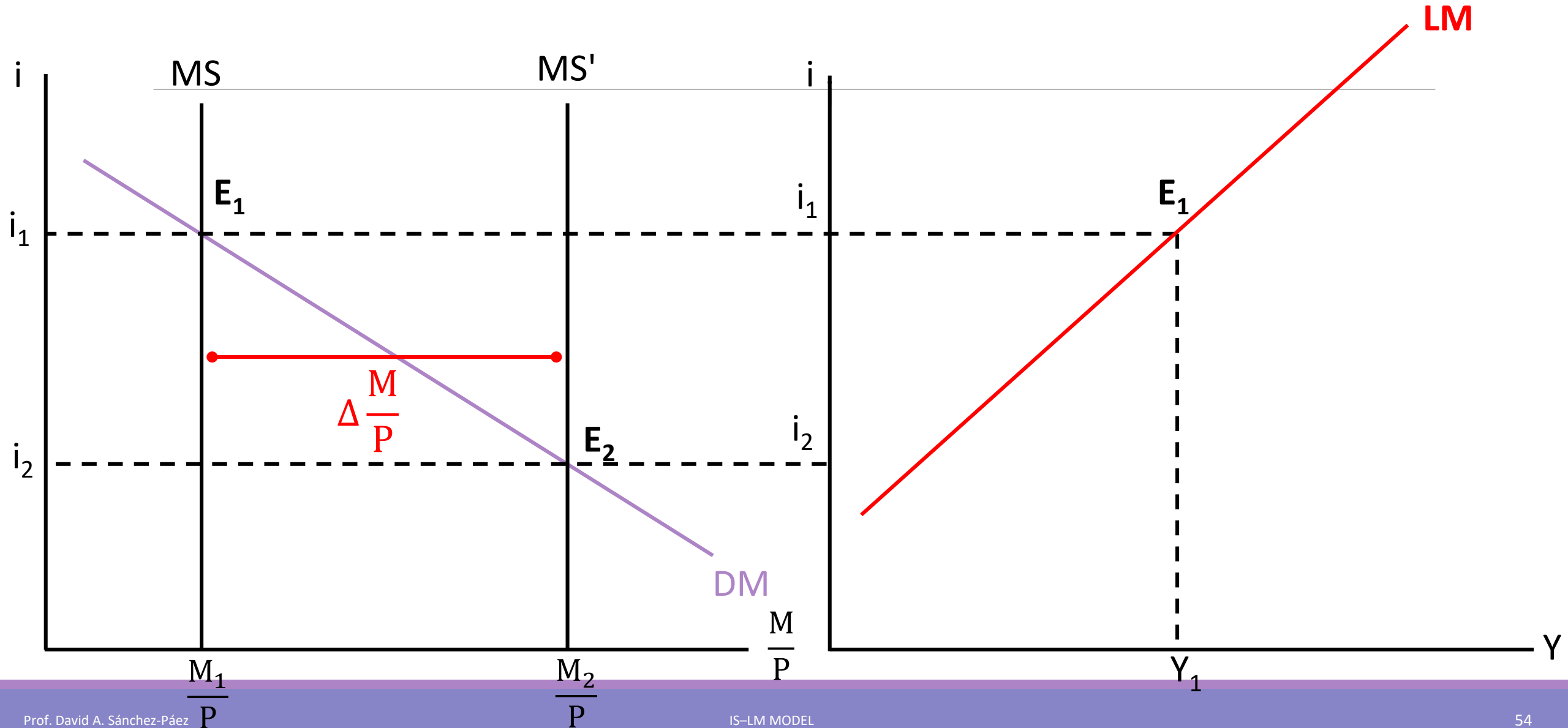


Shifts of the LM curve: increase in M



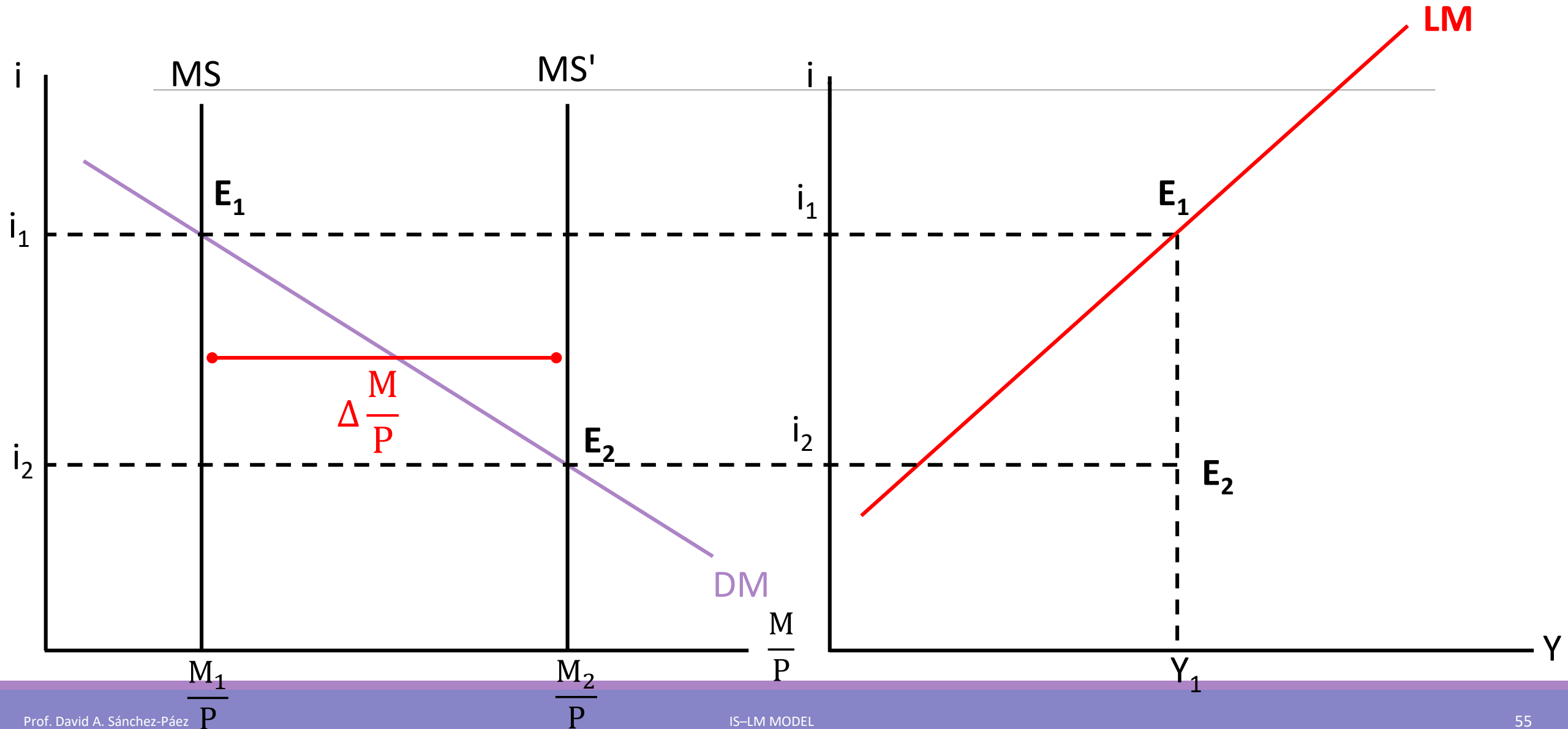


Shifts of the LM curve: increase in M



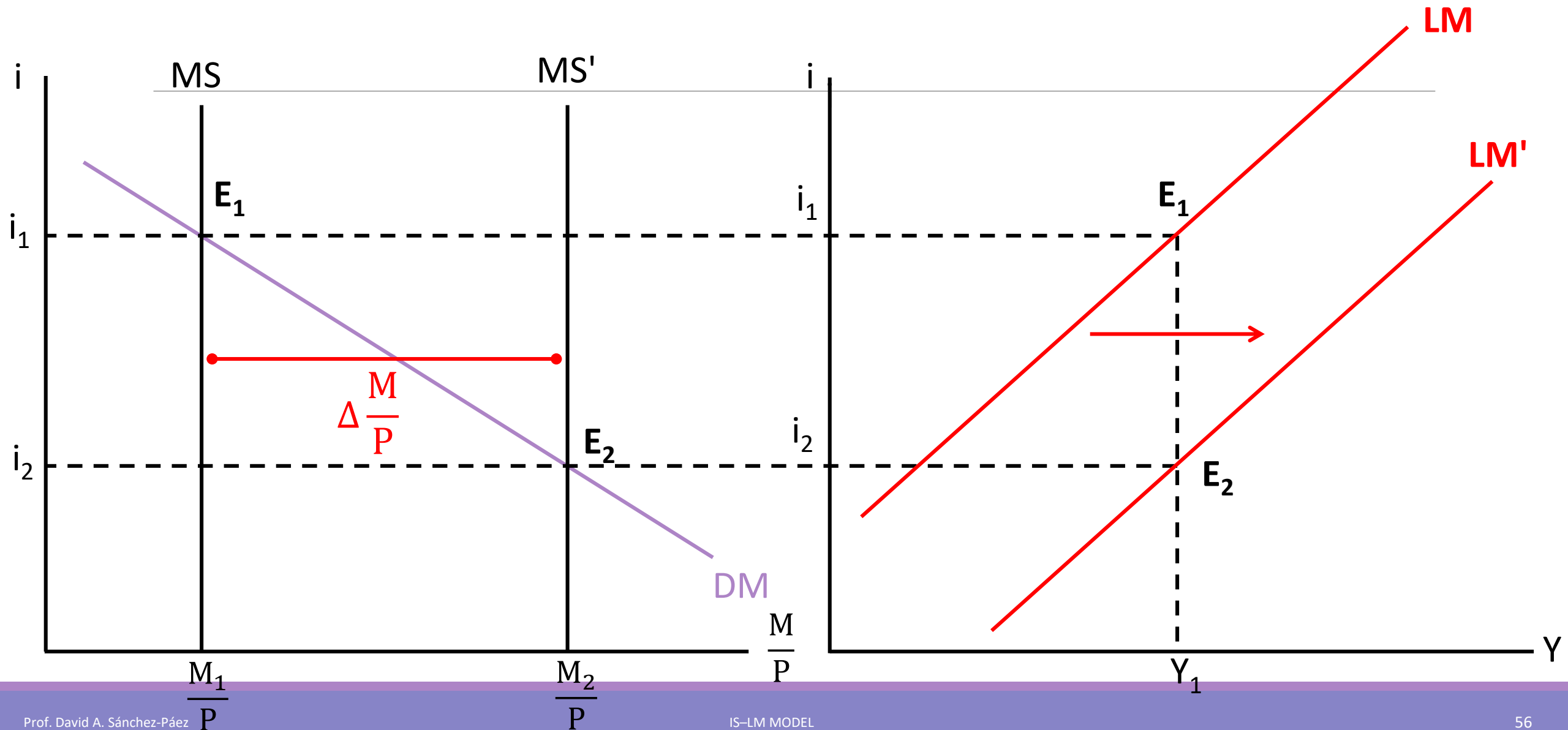


Shifts of the LM curve: increase in M



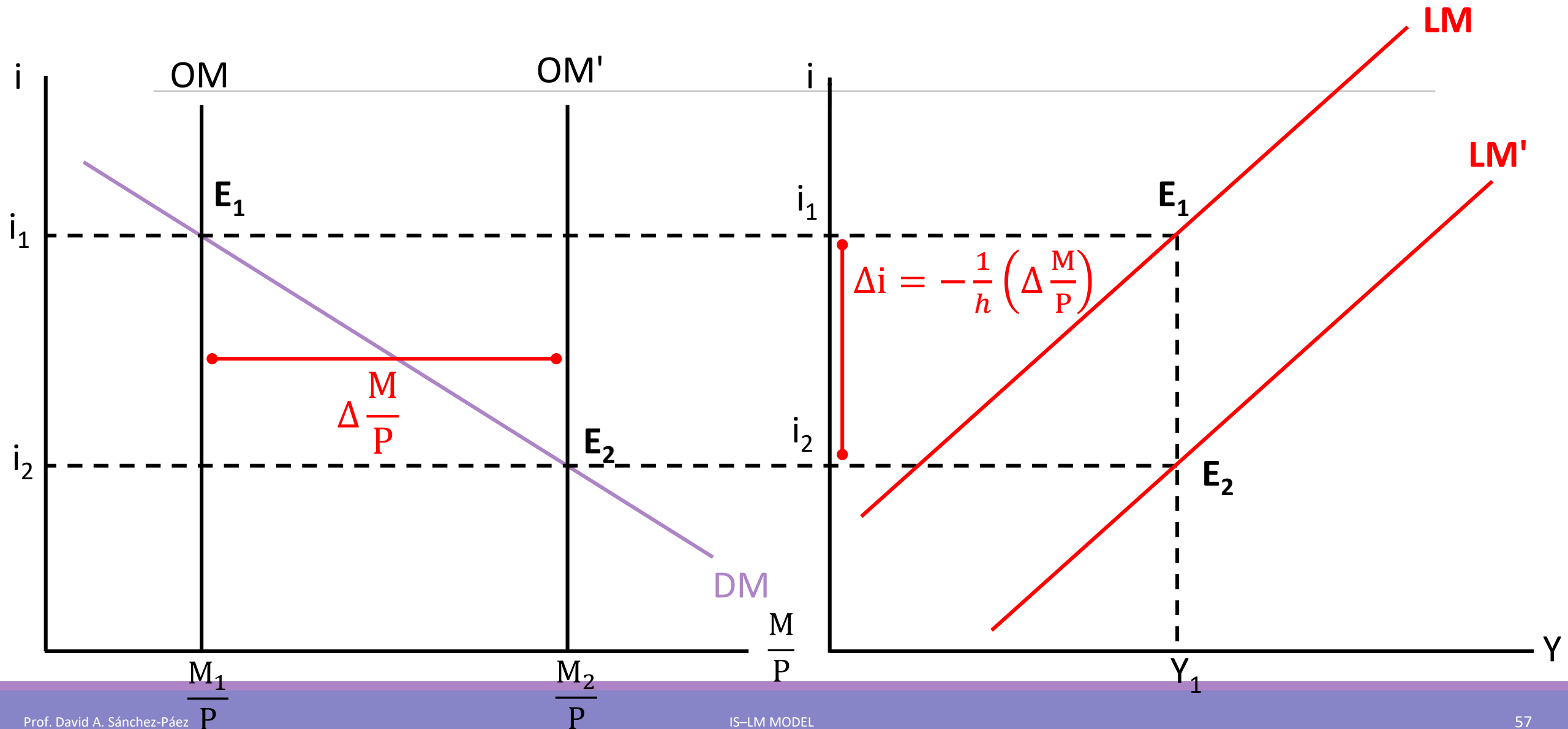


Shifts of the LM curve: increase in M





LM shifts: increase in M





In summary...

- The LM curve results from the combinations of i and Y such as the money market is in equilibrium.
- LM has a positive slope: since MS is fixed, an increase in Y increases DM causing i to increase. This reduces DM and the money market equilibrium is maintained.
- The slope is steeper when DM responds strongly to Y and weakly to i .
- The LM curve shifts due to changes in MS . An increase in M causes a shift to the right.



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Equilibrium in the goods and money markets

- The **IS** and **LM** curves summarize the conditions that have to be satisfied in order for the **goods and money markets to be in equilibrium**.
- **IS curve**: goods market equilibrium. **Aggregate demand** equals **aggregate supply** for given levels of interest rate and income.
- **LM curve**: money market equilibrium. **Money demand** equals **money supply** for given levels of interest rate and income.

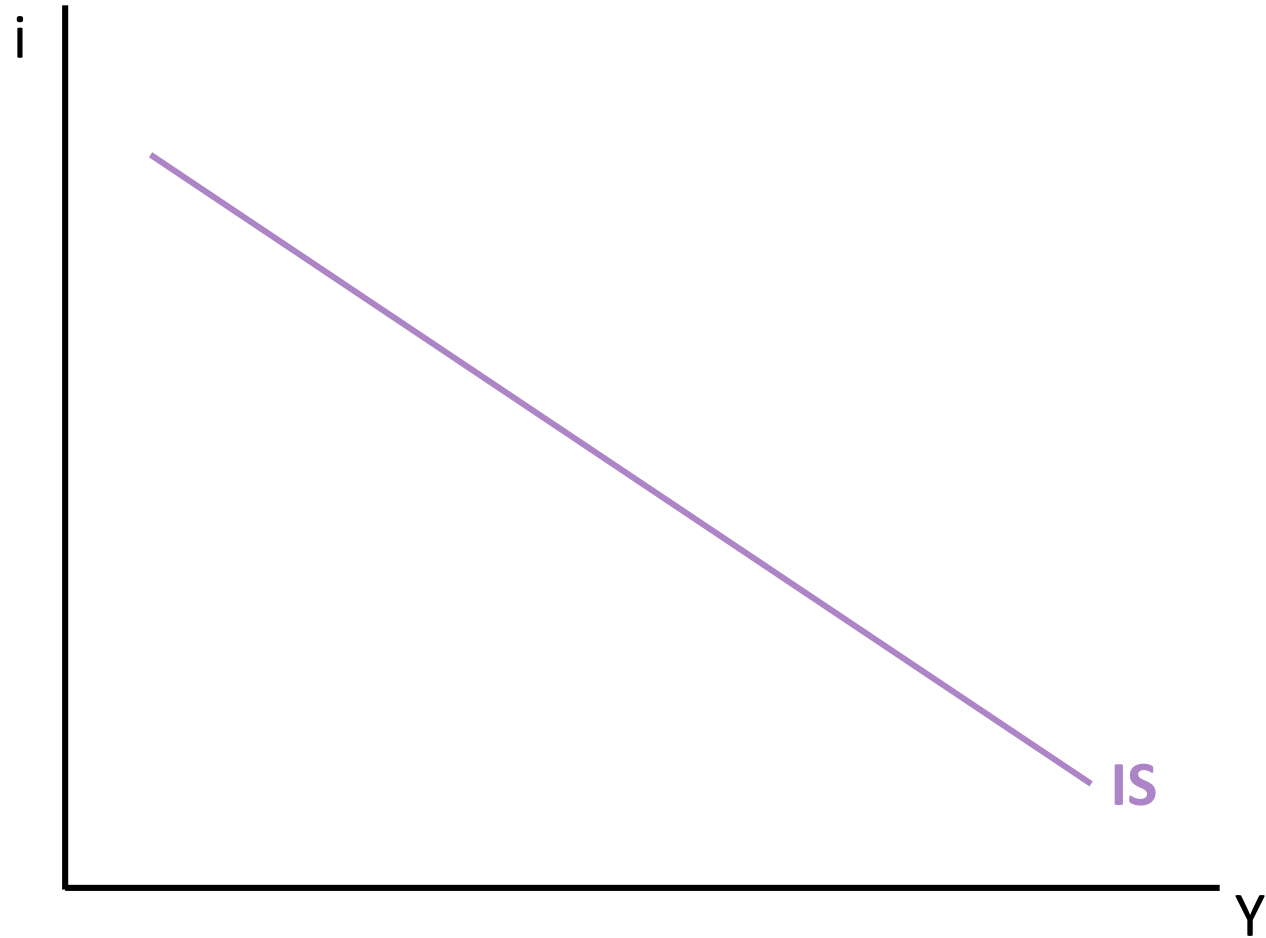


Equilibrium in the goods and money markets.

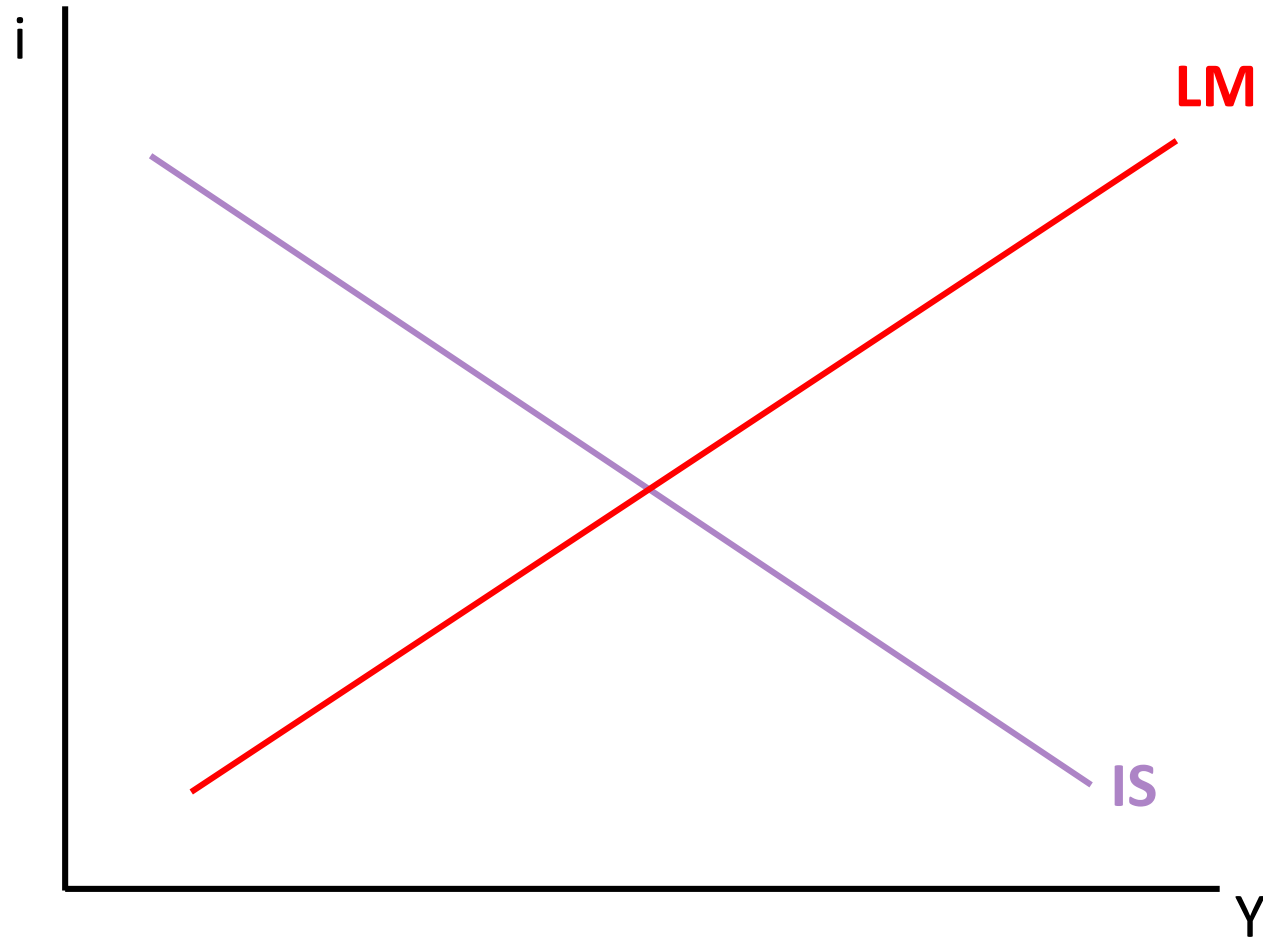
- Now, the task is to determine how these markets reach **simultaneous equilibrium**.
- The interest rate and income levels have to be such that **both** the goods market **and** the money market are in equilibrium.
- To find the equilibrium we join the IS and LM curves on the same graph.



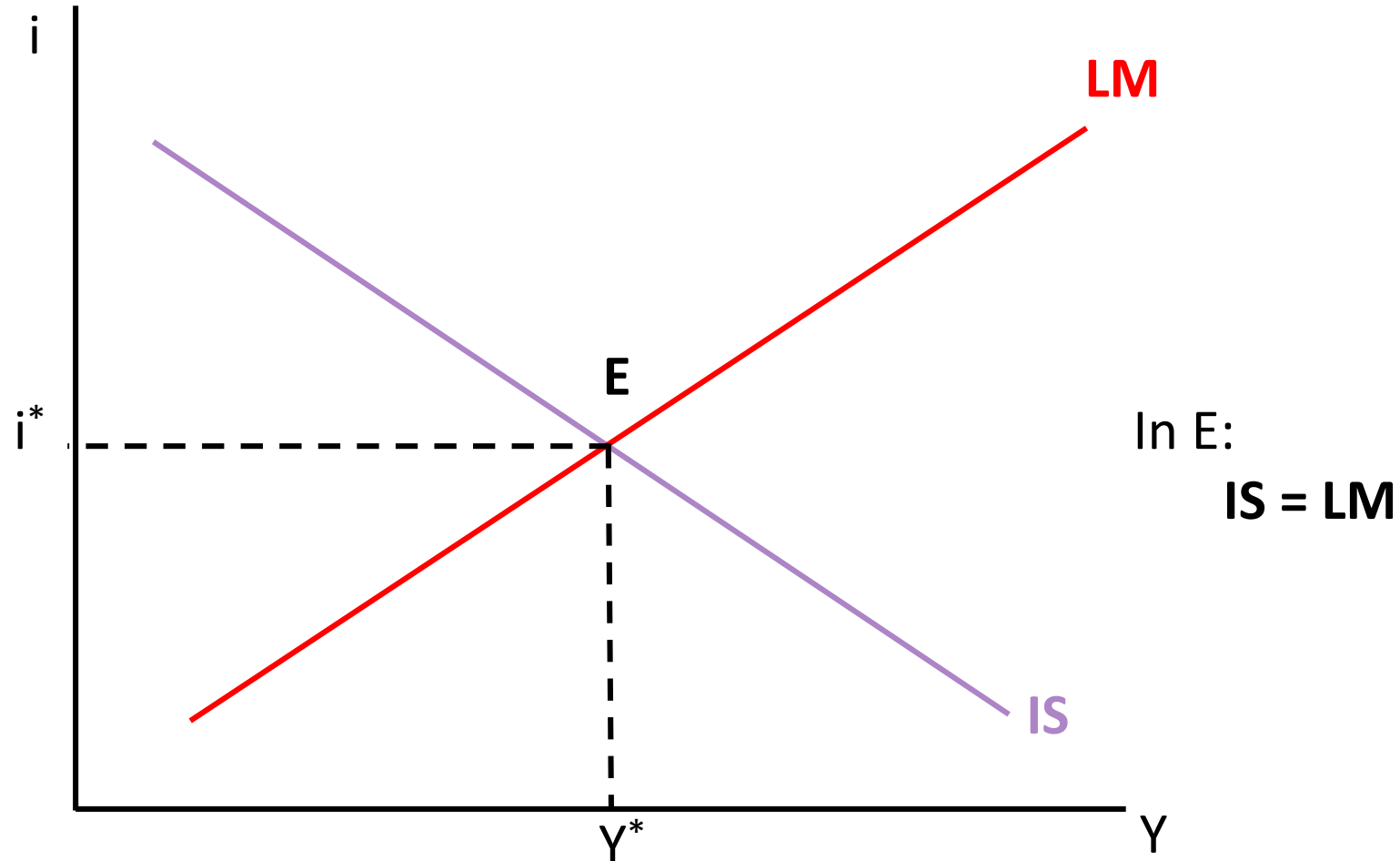
IS-LM Model



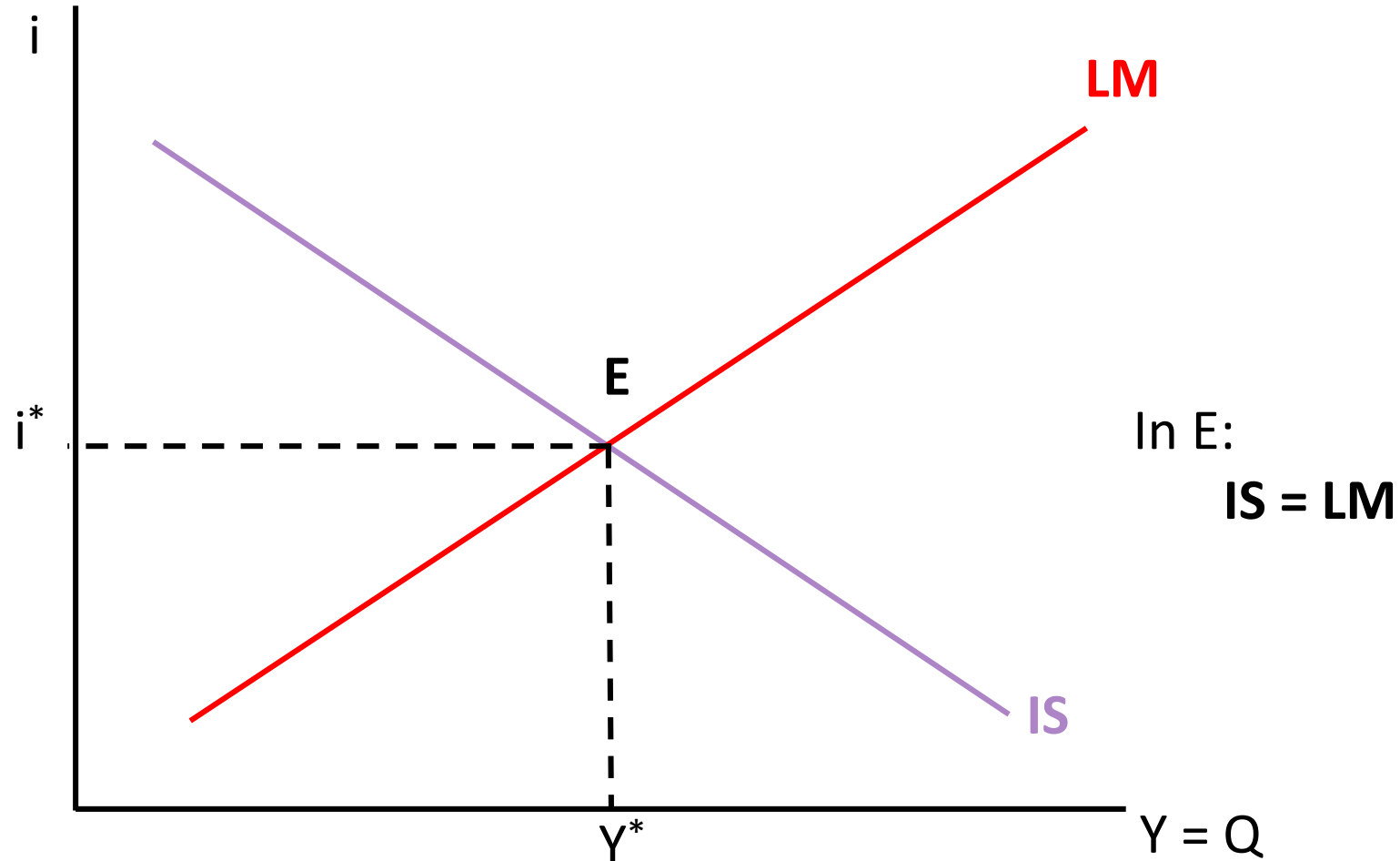
IS-LM Model



Goods and money market in equilibrium



Goods and money market in equilibrium



Equilibrium in the IS-LM model

- Mathematically:

$$\text{IS: } Y = \frac{1}{1 - c(1 - t)} (\bar{A} - bi)$$

$$\text{LM: } i = \frac{1}{h} \left(kY - \frac{M}{P} \right)$$

- The multiplier α can be replaced:

$$\alpha = \frac{1}{1 - c(1 - t)}$$



Equilibrium in the IS-LM model

- Therefore:

$$\text{IS: } Y = \alpha(\bar{A} - bi)$$

$$\text{LM: } i = \frac{1}{h} \left(kY - \frac{M}{P} \right)$$



Equilibrium in the IS-LM model

- In equilibrium:

$$IS = LM$$

- Therefore:

$$\text{IS: } Y = \alpha(\bar{A} - bi)$$

$$\text{LM: } i = \frac{1}{h} \left(kY - \frac{M}{P} \right)$$

Y and i are the same in both equations.

Equilibrium in the IS-LM model

- After solving the equations, we obtain:

$$Y = \gamma \bar{A} + \gamma \frac{b}{h} \frac{M}{P}$$

$$i = \frac{k}{h} \gamma \bar{A} - \gamma \left(\frac{1}{\alpha h} \right) \frac{M}{P}$$

Where,

$$\gamma = \frac{\alpha h}{h + \alpha b k} = \frac{\alpha}{1 + \frac{\alpha b k}{h}}$$

Equilibrium in the IS-LM model

$$Y = \gamma \bar{A} + \gamma \frac{b}{h} \frac{M}{P}$$

$$i = \frac{k}{h} \gamma \bar{A} - \gamma \left(\frac{1}{\alpha h} \right) \frac{M}{P}$$

Fiscal policy multiplier (public expenditure):

$$\frac{\Delta Y}{\Delta G} = \gamma$$



Equilibrium in the IS-LM model

$$Y = \gamma \bar{A} + \gamma \frac{b}{h} \frac{M}{P}$$

$$i = \frac{k}{h} \gamma \bar{A} - \gamma \left(\frac{1}{\alpha h} \right) \frac{M}{P}$$

Fiscal policy multiplier (public expenditure):

$$\frac{\Delta Y}{\Delta G} = \gamma$$

$$\gamma = \frac{\alpha}{1 + \frac{\alpha b k}{h}}$$



Equilibrium in the IS-LM model

$$Y = \gamma \bar{A} + \gamma \frac{b}{h} \frac{M}{P}$$

$$i = \frac{k}{h} \gamma \bar{A} - \gamma \left(\frac{1}{\alpha h} \right) \frac{M}{P}$$

Fiscal policy multiplier (transfers):

$$\frac{\Delta Y}{\Delta TR} = \gamma c$$



Equilibrium in the IS-LM model

$$Y = \gamma \bar{A} + \gamma \frac{b}{h} \frac{M}{P}$$

$$i = \frac{k}{h} \gamma \bar{A} - \gamma \left(\frac{1}{\alpha h} \right) \frac{M}{P}$$

Monetary policy multiplier (quantity of money):

$$\frac{\Delta Y}{\Delta \left(\frac{M}{P} \right)} = \gamma \frac{b}{h}$$



Equilibrium in the IS-LM model

$$Y = \gamma \bar{A} + \gamma \frac{b}{h} \frac{M}{P}$$

$$i = \frac{k}{h} \gamma \bar{A} - \gamma \left(\frac{1}{\alpha h} \right) \frac{M}{P}$$

Monetary policy multiplier (quantity of money):

$$\frac{\Delta Y}{\Delta \left(\frac{M}{P} \right)} = \gamma \frac{b}{h}$$

$$\gamma = \frac{\alpha}{1 + \frac{\alpha b k}{h}}$$



IS-LM model

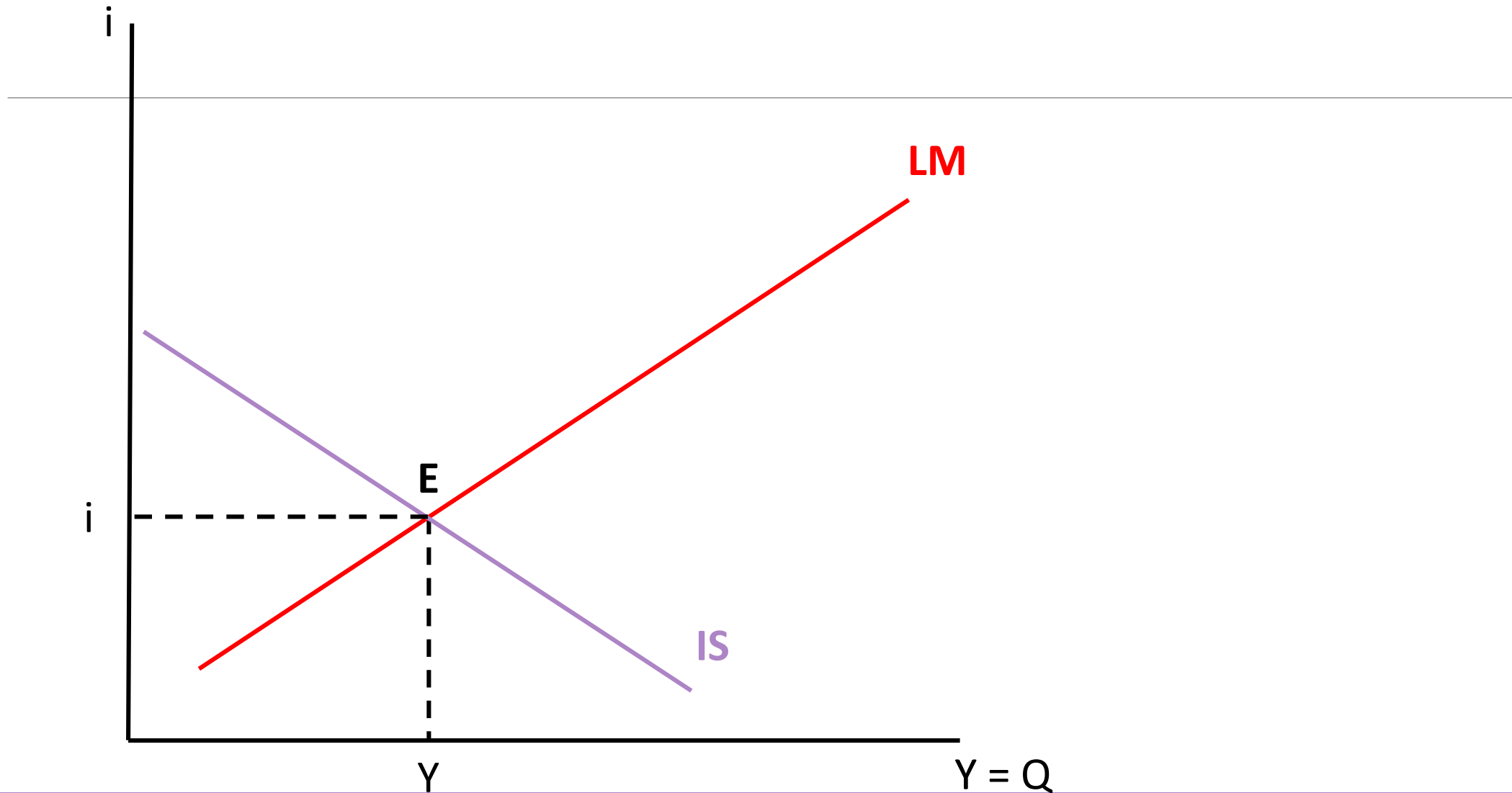
- As we can see, the **IS-LM model** is a useful model for evaluating changes in economic policy.
 - **Fiscal policy:** changes in public spending, transfers or taxes.
 - **Monetary policy:** changes in the quantity of money.



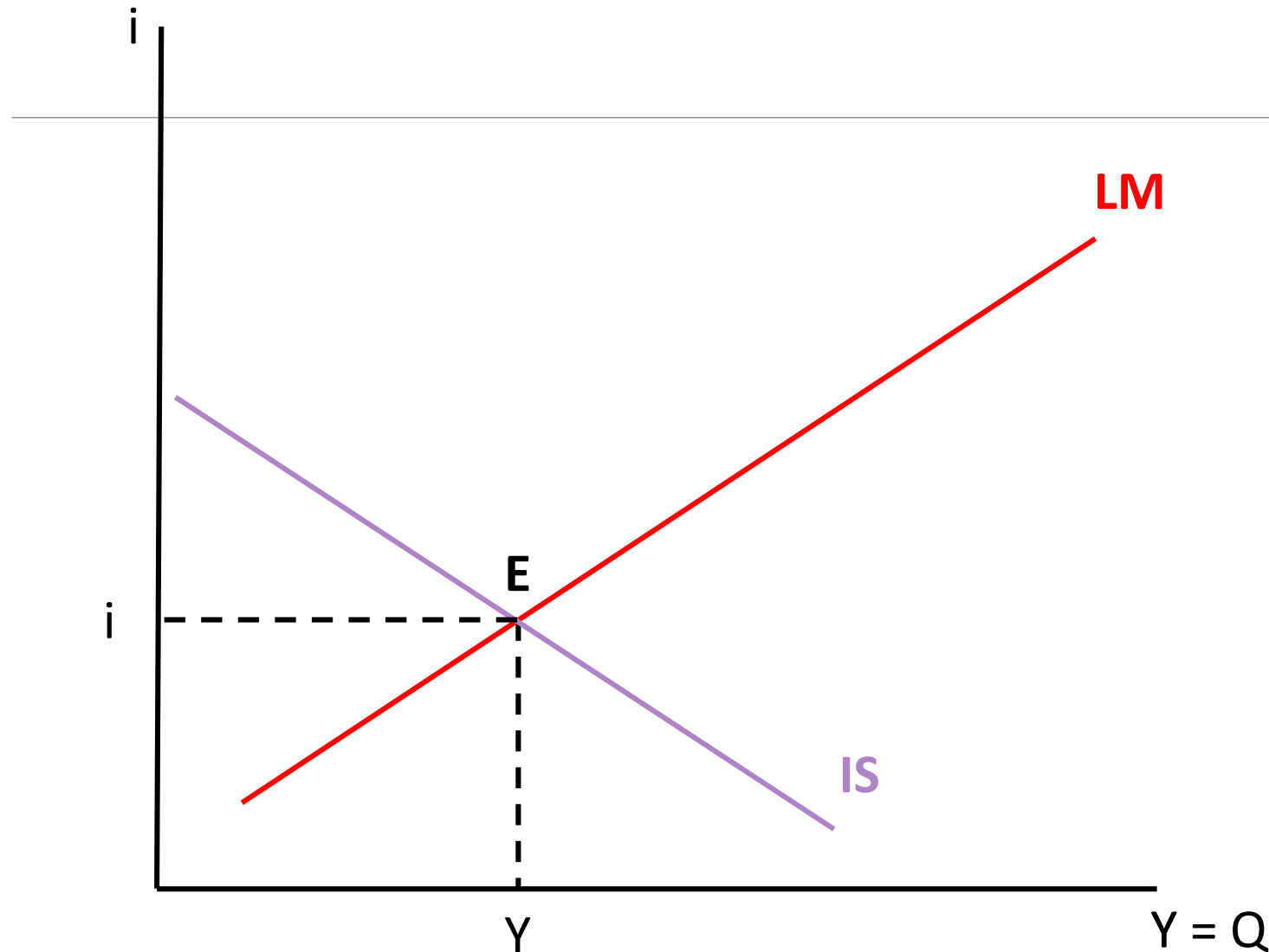
IS-LM model: fiscal policy

- **Expansionary fiscal policy:** increase in public spending or transfers, or reduction of taxes.
- **Contractionary fiscal policy:** decrease in public spending or transfers, or increase in taxes.

IS-LM model: expansionary fiscal policy



IS-LM model: expansionary fiscal policy



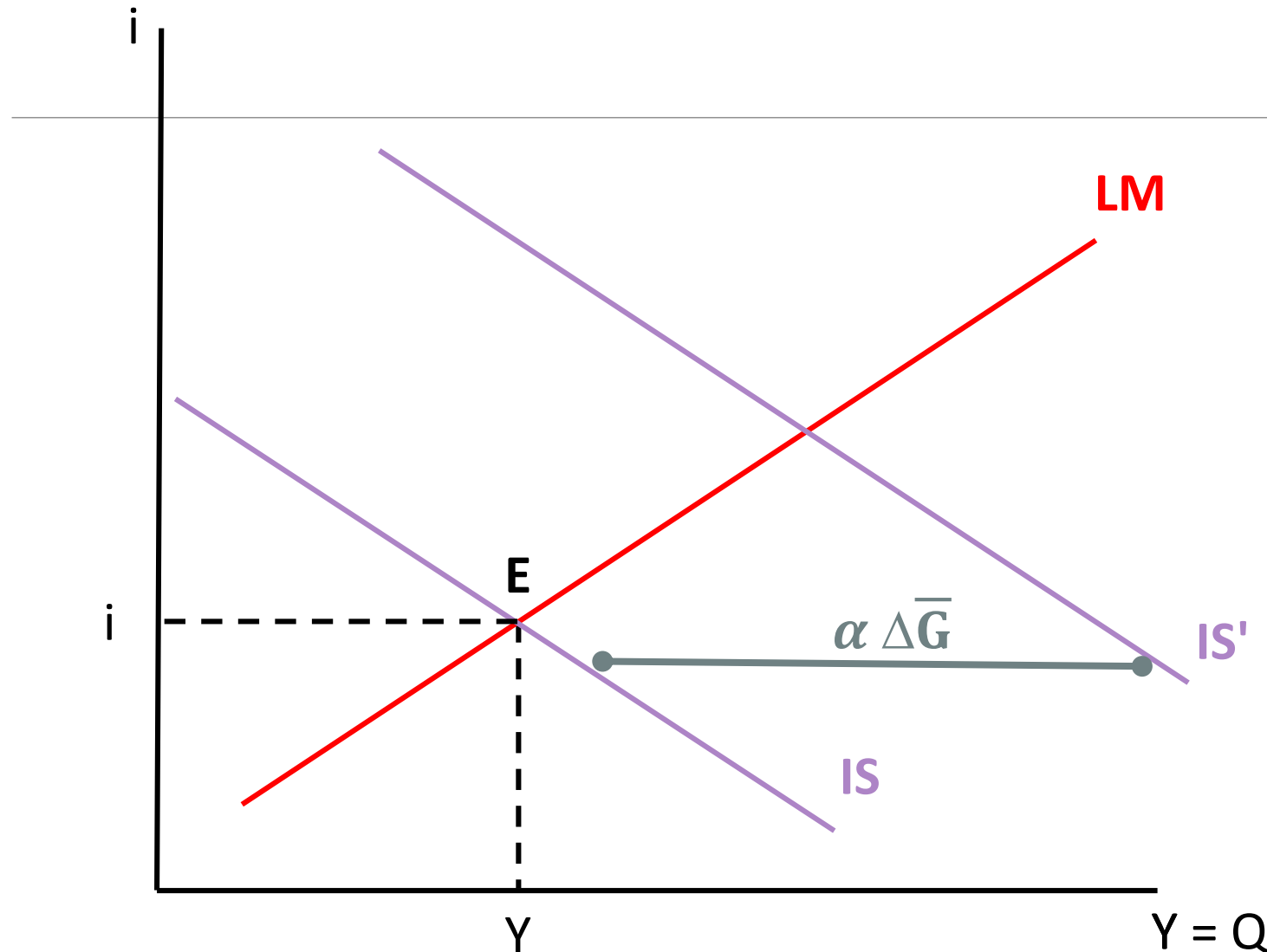
$$Y = \frac{1}{1 - c(1 - t)} (\bar{A} - bi)$$

$$\bar{A} = \bar{C} + c\bar{TR} + \bar{I} + \bar{G}$$

$$\Delta Y = \frac{1}{1 - c(1 - t)} \Delta \bar{G}$$

$$\Delta Y = \alpha \Delta \bar{G}$$

IS-LM model: expansionary fiscal policy



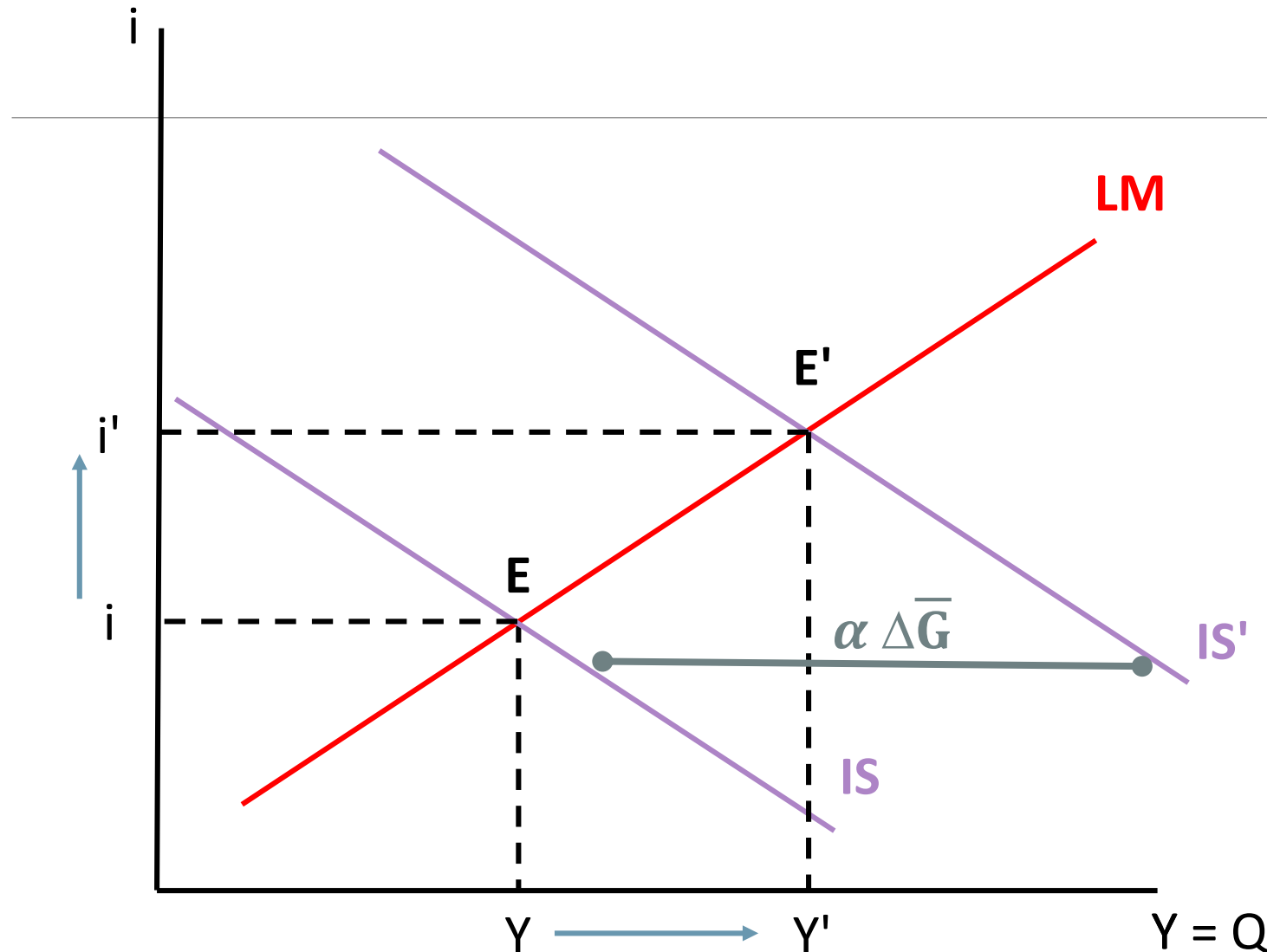
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$$\Delta Y = \alpha \Delta \bar{G}$$

IS-LM model: expansionary fiscal policy



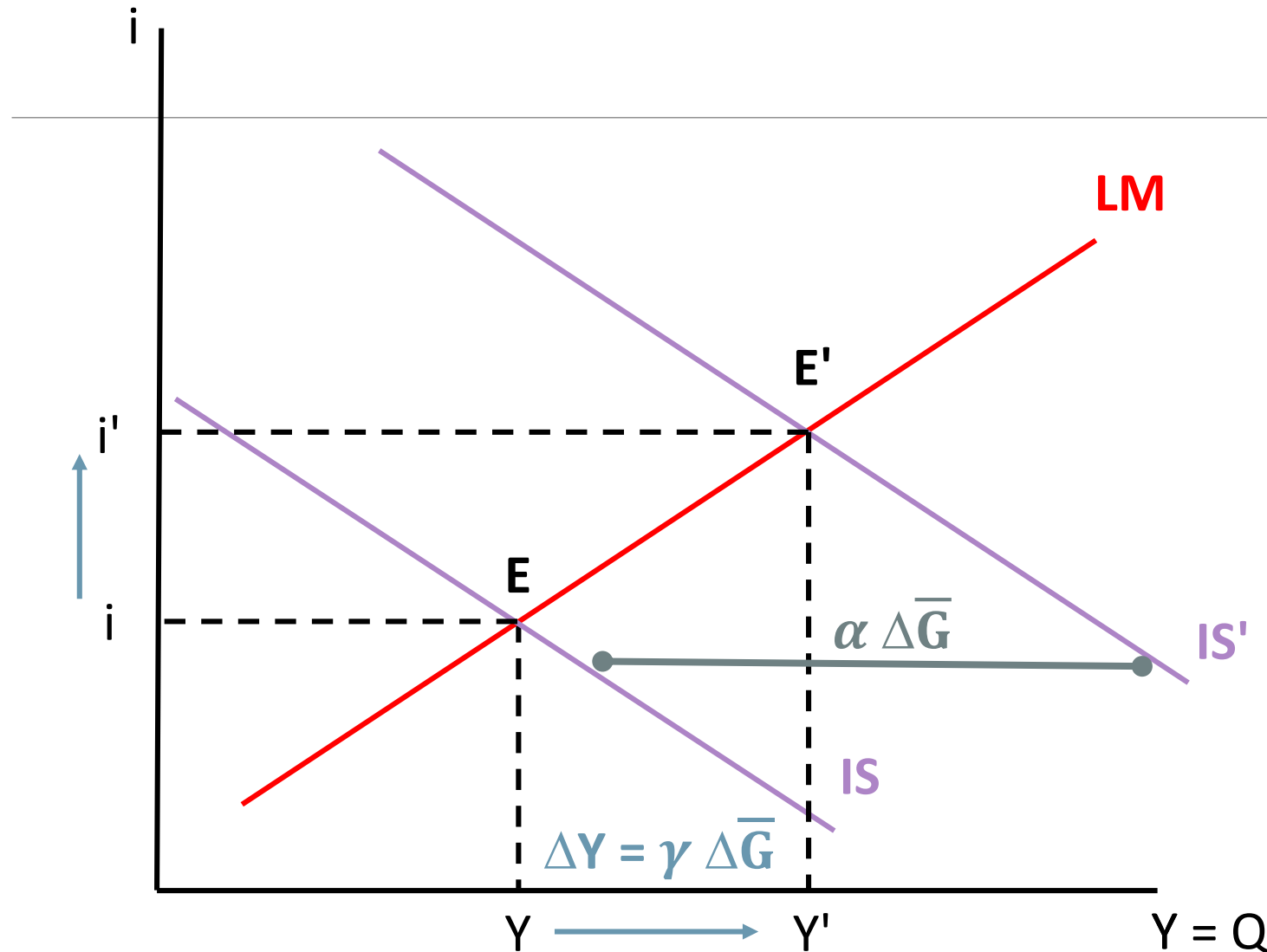
$$Y = \frac{1}{1 - c(1 - t)} (\bar{A} - bi)$$

$$\bar{A} = \bar{C} + c\bar{T}R + \bar{I} + \bar{G}$$

$$\Delta Y = \frac{1}{1 - c(1 - t)} \Delta \bar{G}$$

$$\Delta Y = \alpha \Delta \bar{G}$$

IS-LM model: expansionary fiscal policy



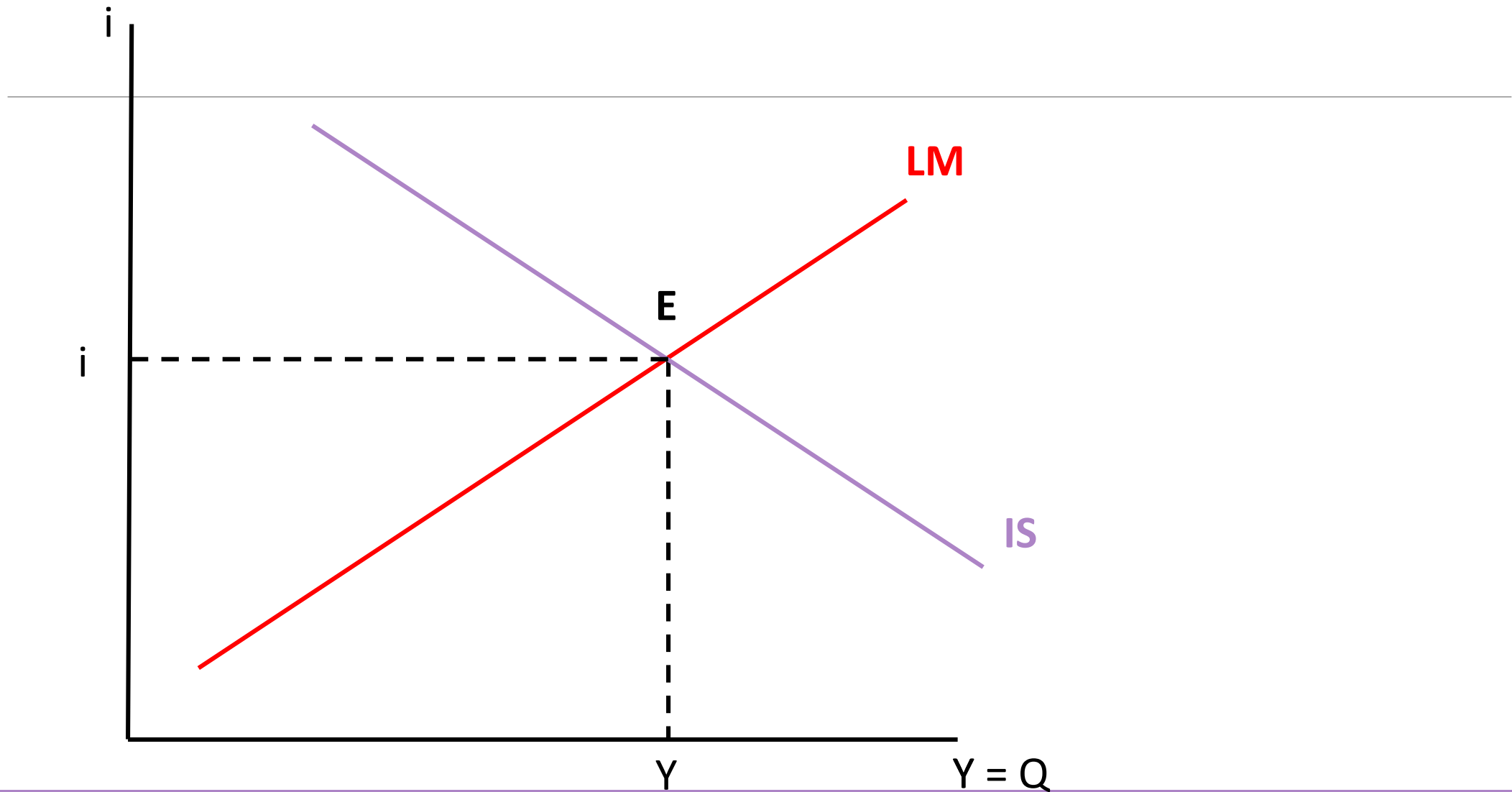
$$Y = \frac{1}{1 - c(1 - t)} (\bar{A} - bi)$$

$$\bar{A} = \bar{C} + c\bar{T}R + \bar{I} + \bar{G}$$

$$\Delta Y = \frac{1}{1 - c(1 - t)} \Delta \bar{G}$$

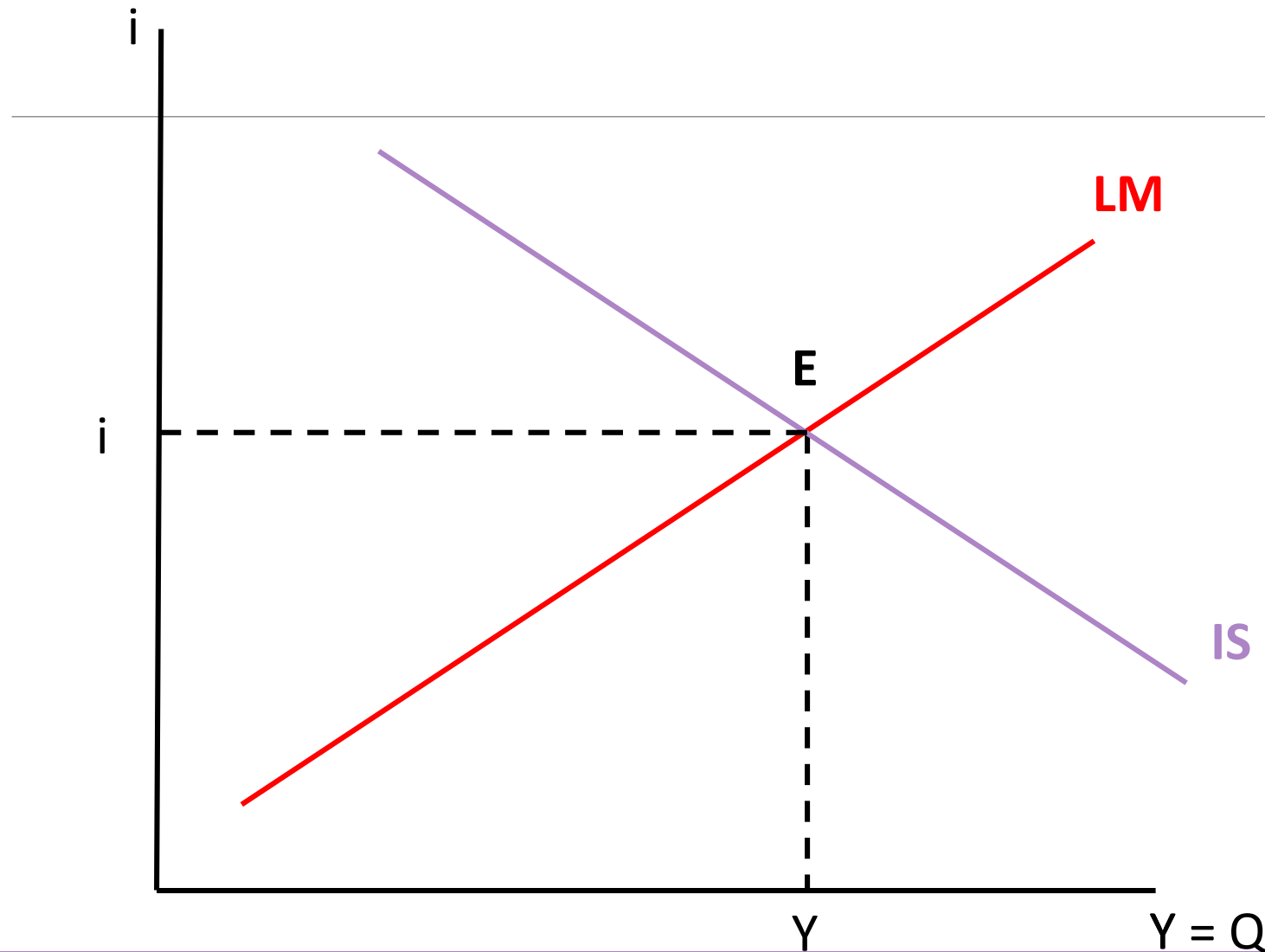
$$\Delta Y = \alpha \Delta \bar{G}$$

IS-LM model: contractionary fiscal policy





IS-LM model: contractionary fiscal policy



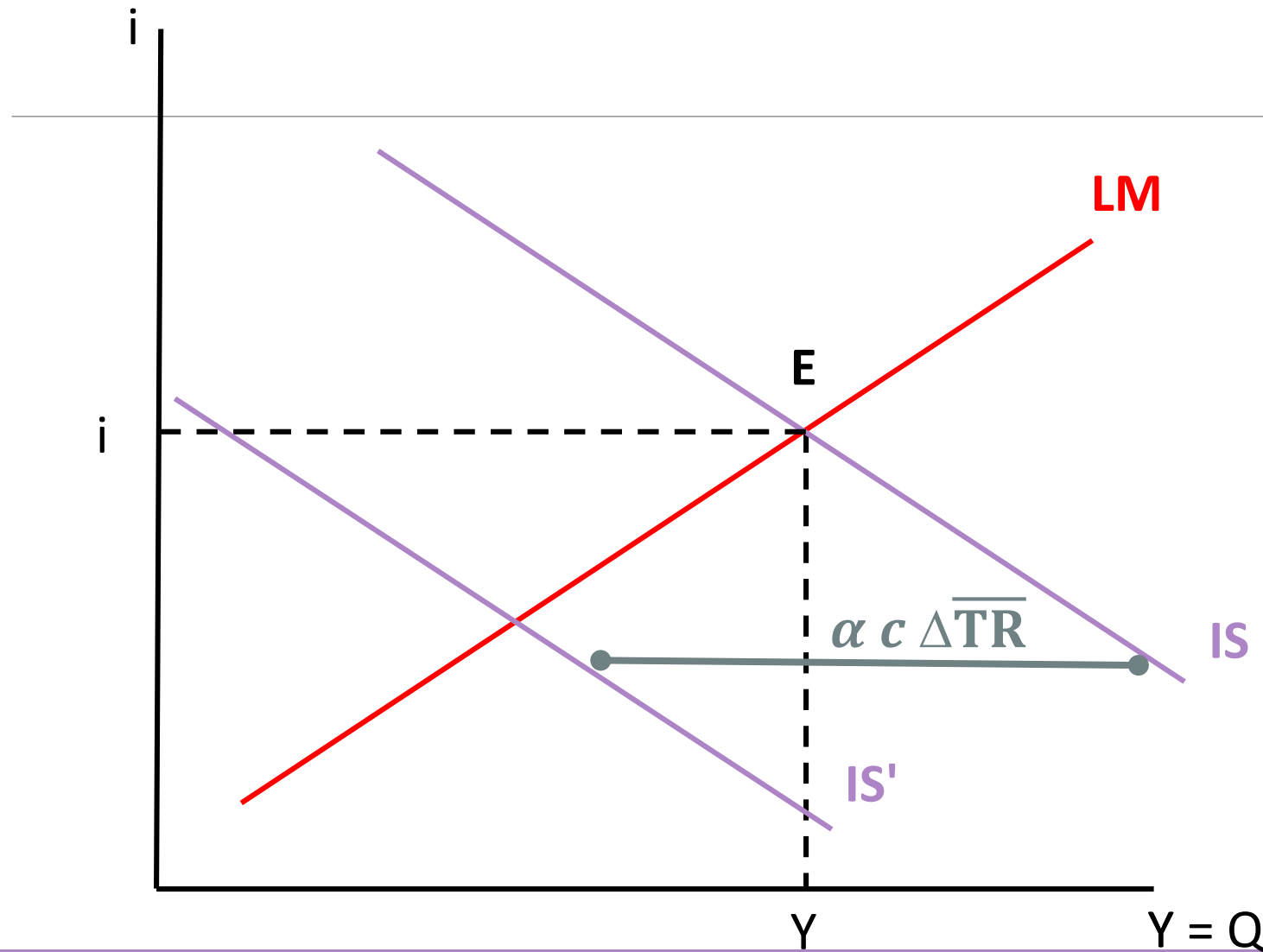
$$Y = \frac{1}{1 - c(1 - t)} (\bar{A} - bi)$$

$$\bar{A} = \bar{C} + c\bar{TR} + \bar{I} + \bar{G}$$

$$\Delta Y = \frac{1}{1 - c(1 - t)} c \Delta \bar{TR}$$

$$\Delta Y = \alpha c \Delta \bar{TR}$$

IS-LM model: contractionary fiscal policy.



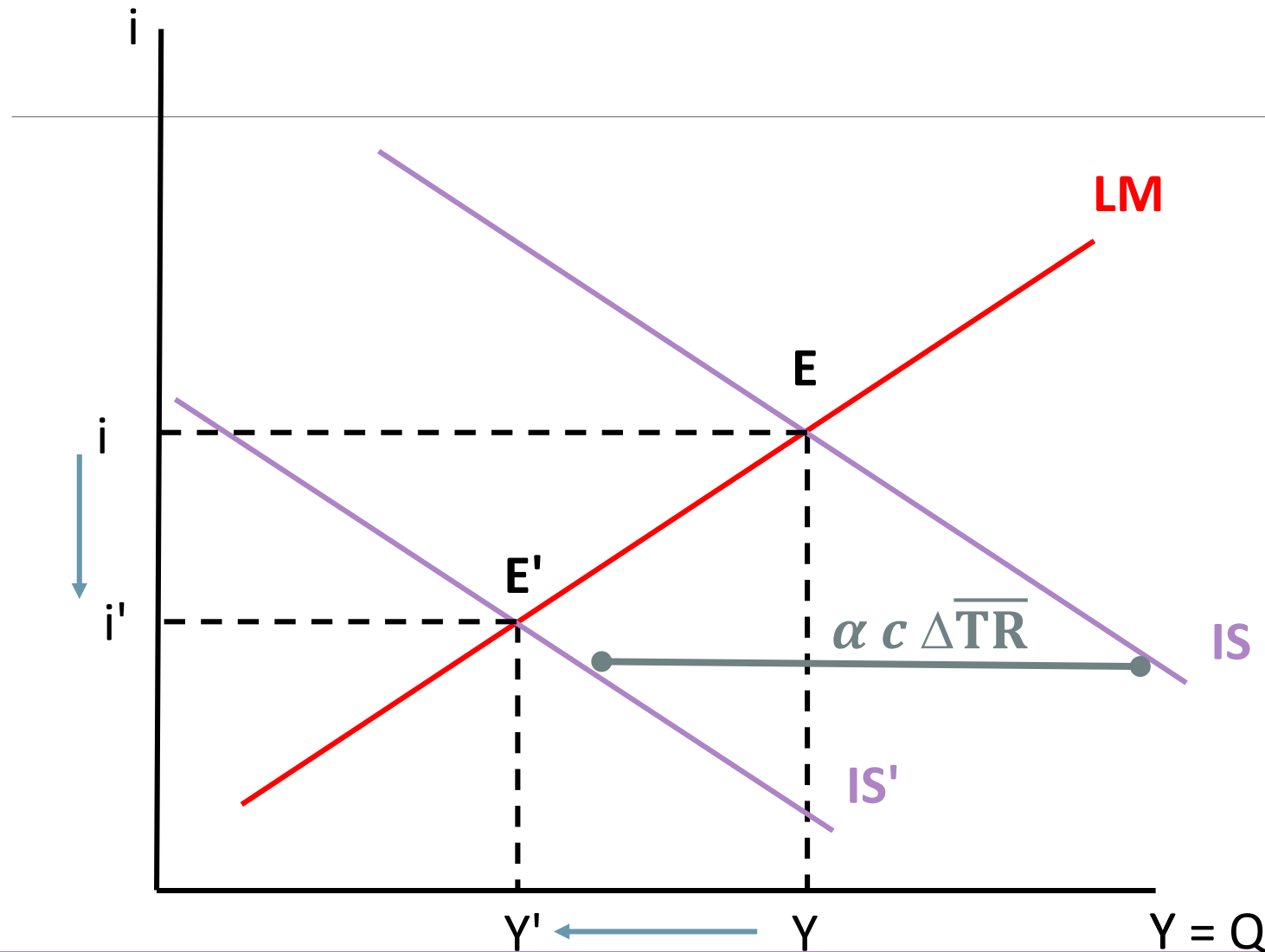
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IS-LM model: contractionary fiscal policy



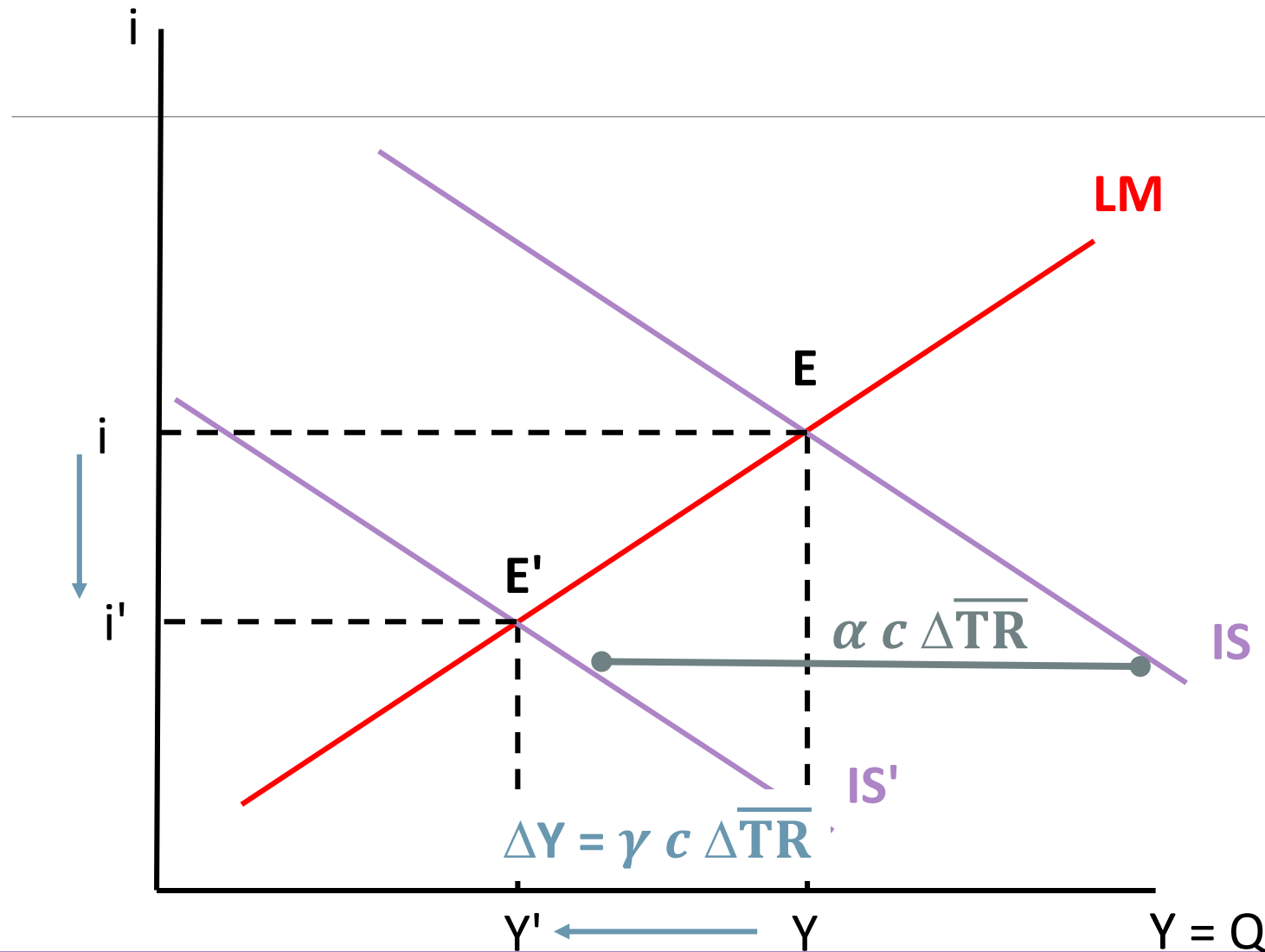
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IS-LM model: contractionary fiscal policy



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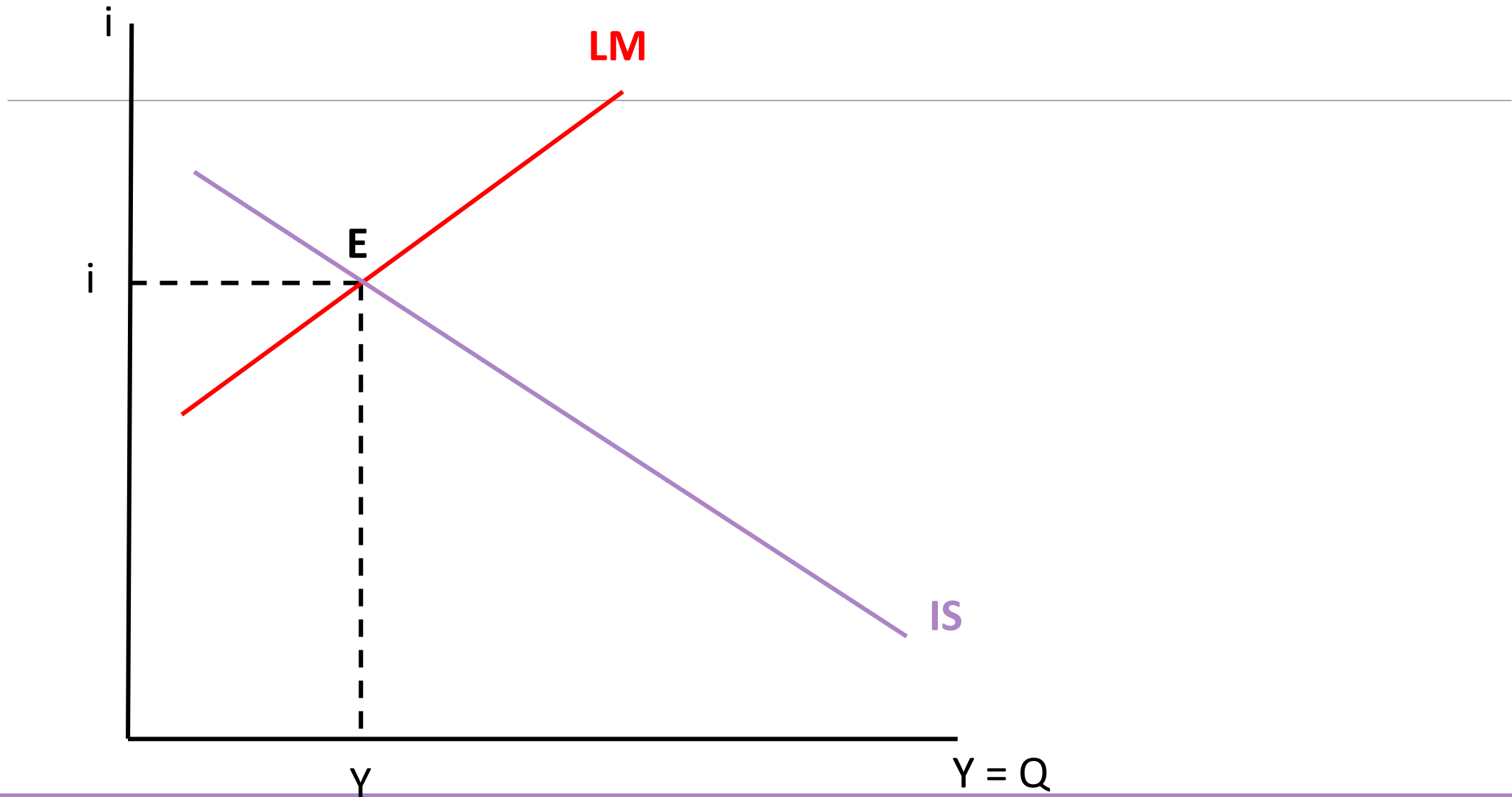


IS-LM model: monetary policy

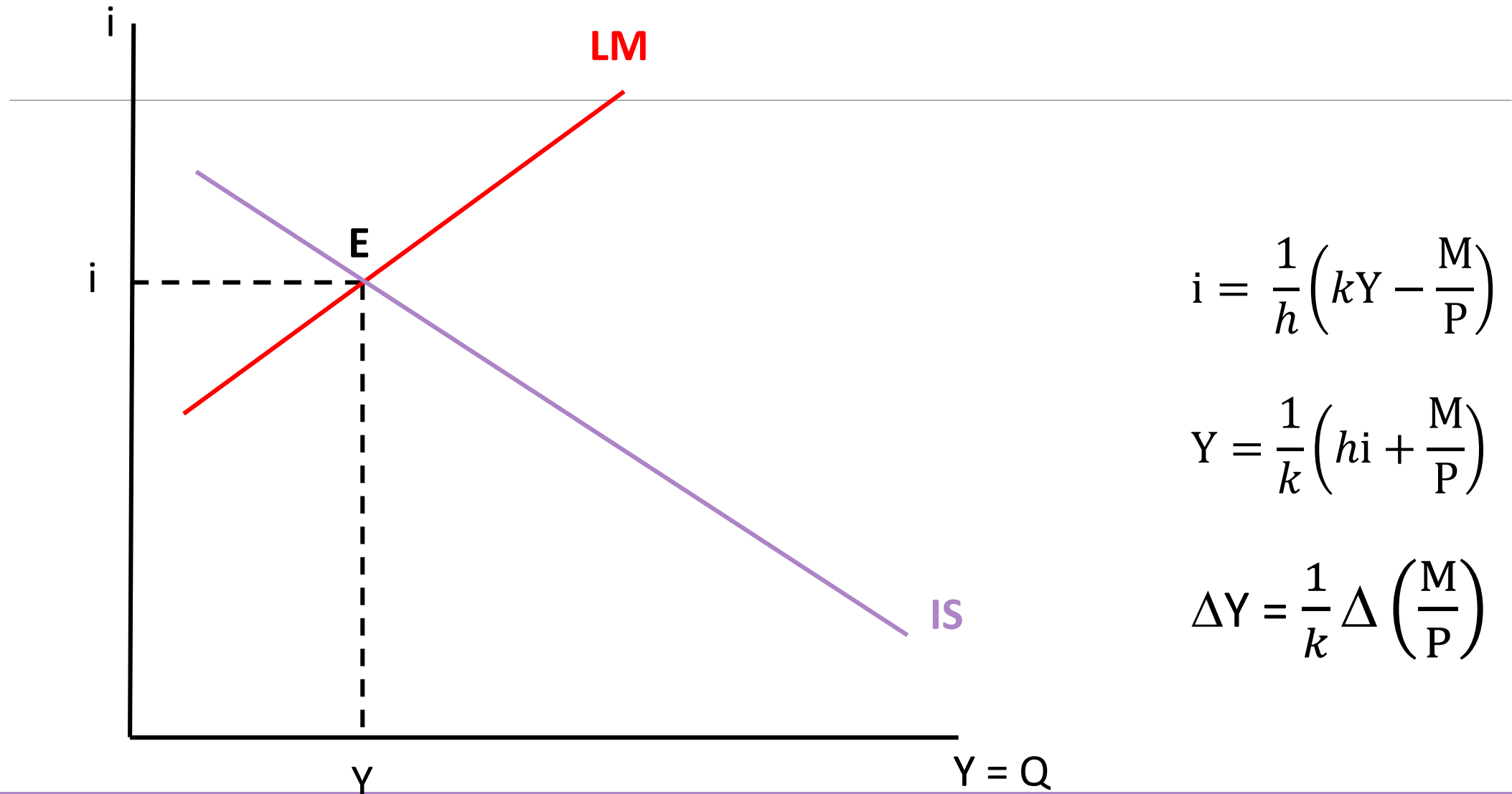
- **Expansionary monetary policy:** increase in the quantity of money.
- **Contractionary monetary policy:** decrease in the quantity of money.



IS-LM model: expansionary monetary policy



IS-LM model: expansionary monetary policy



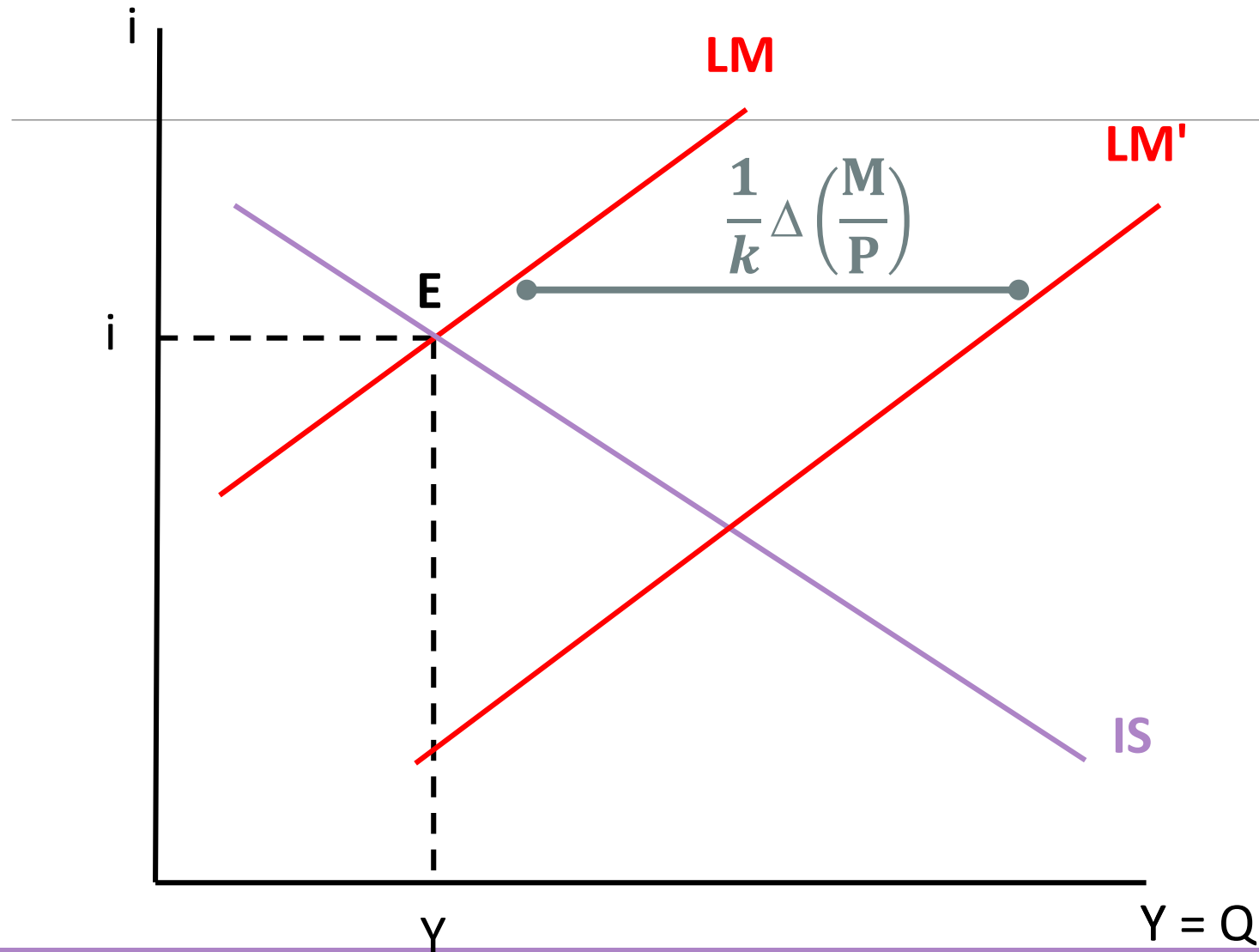
$$i = \frac{1}{h} \left(kY - \frac{M}{P} \right)$$

$$Y = \frac{1}{k} \left(hi + \frac{M}{P} \right)$$

$$\Delta Y = \frac{1}{k} \Delta \left(\frac{M}{P} \right)$$



IS-LM model: expansionary monetary policy

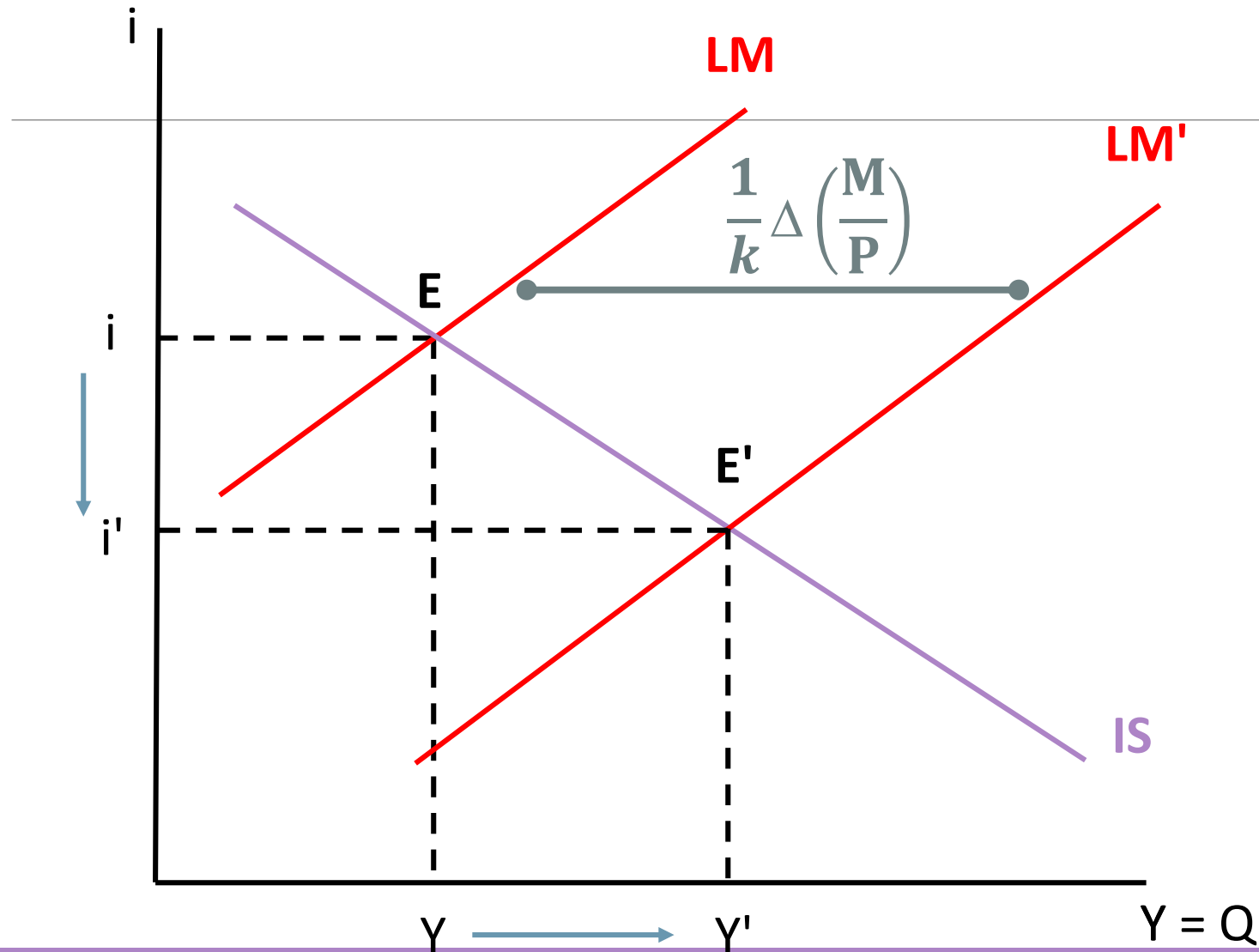


$$i = \frac{1}{h} \left(kY - \frac{M}{P} \right)$$

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IS-LM model: expansionary monetary policy



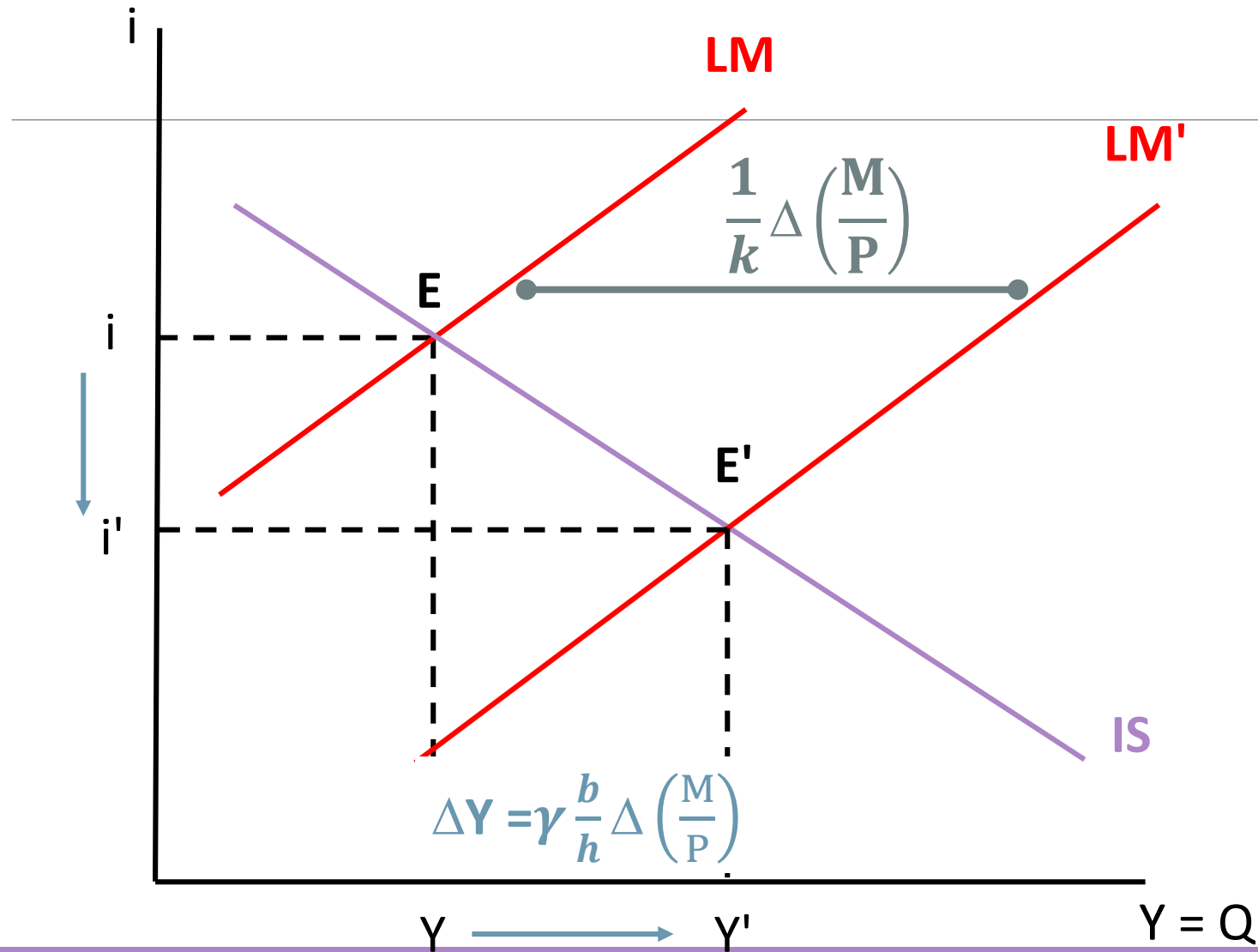
$$i = \frac{1}{h} \left(kY - \frac{M}{P} \right)$$

$$Y = \frac{1}{k} \left(hi + \frac{M}{P} \right)$$

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IS-LM model: expansionary monetary policy

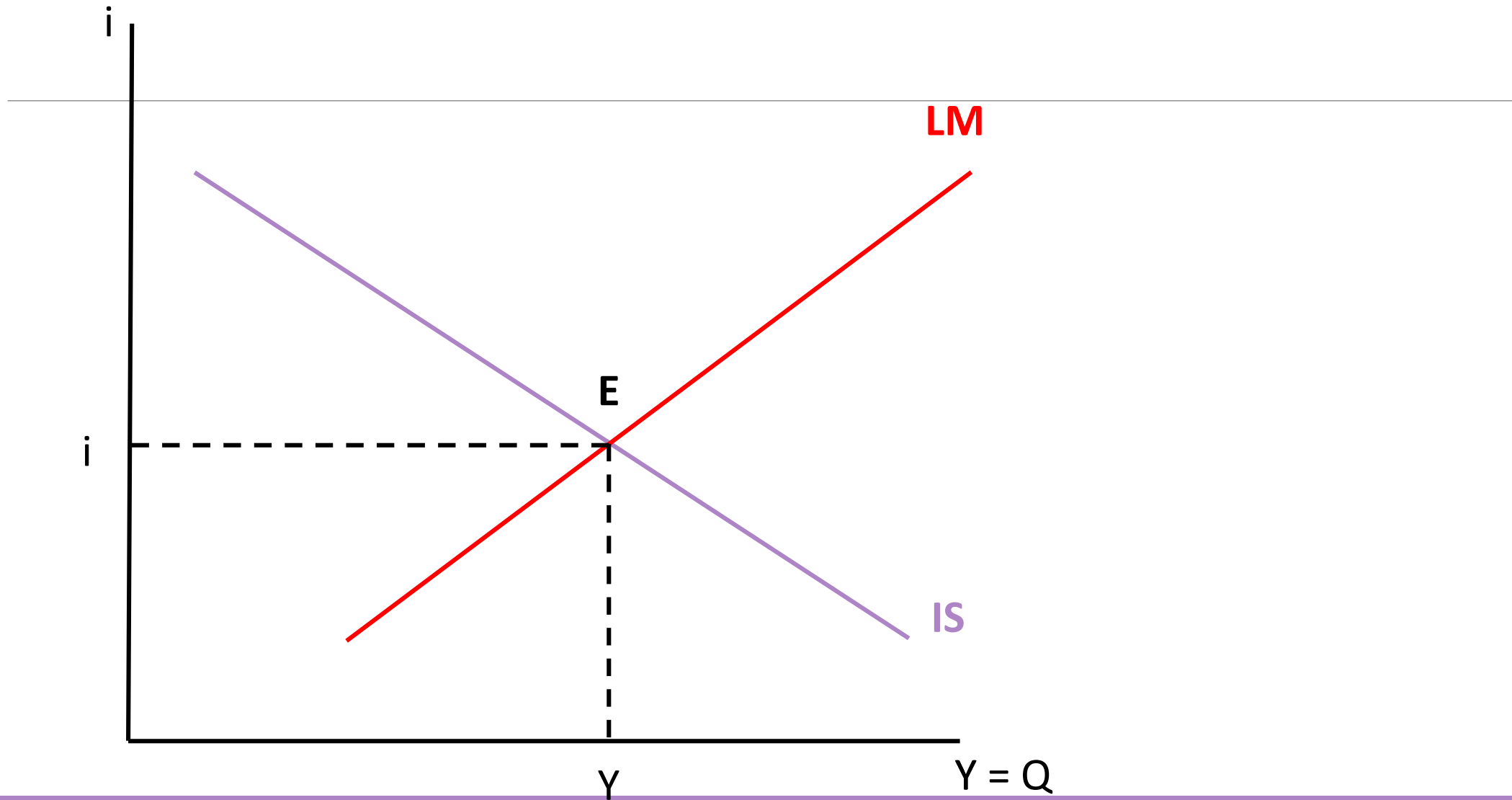


$$i = \frac{1}{h} \left(kY - \frac{M}{P} \right)$$

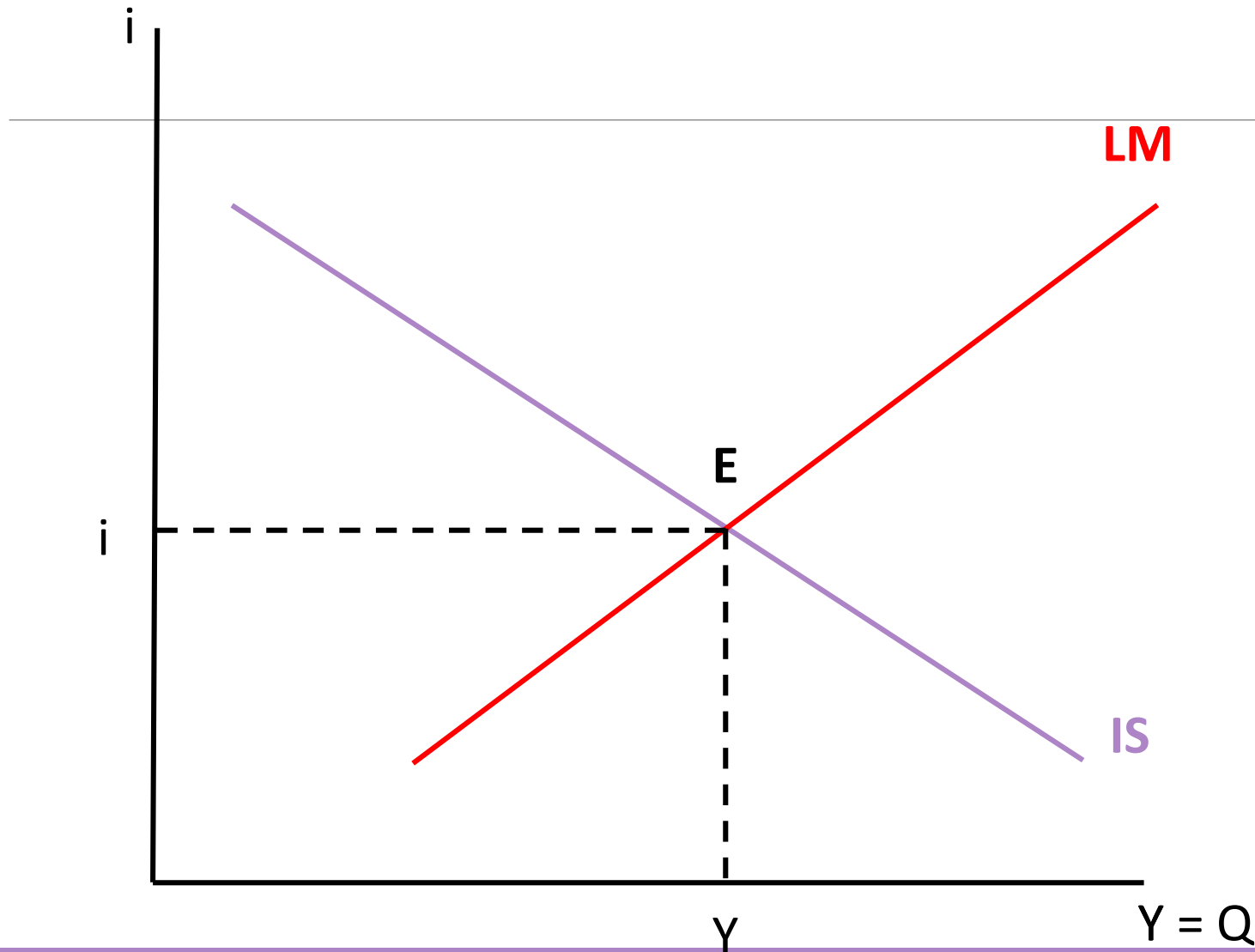
$$Y = \frac{1}{k} \left(hi + \frac{M}{P} \right)$$

$$\Delta Y = \frac{1}{k} \Delta \left(\frac{M}{P} \right)$$

IS-LM model: contractionary monetary policy



IS-LM model: contractionary monetary policy

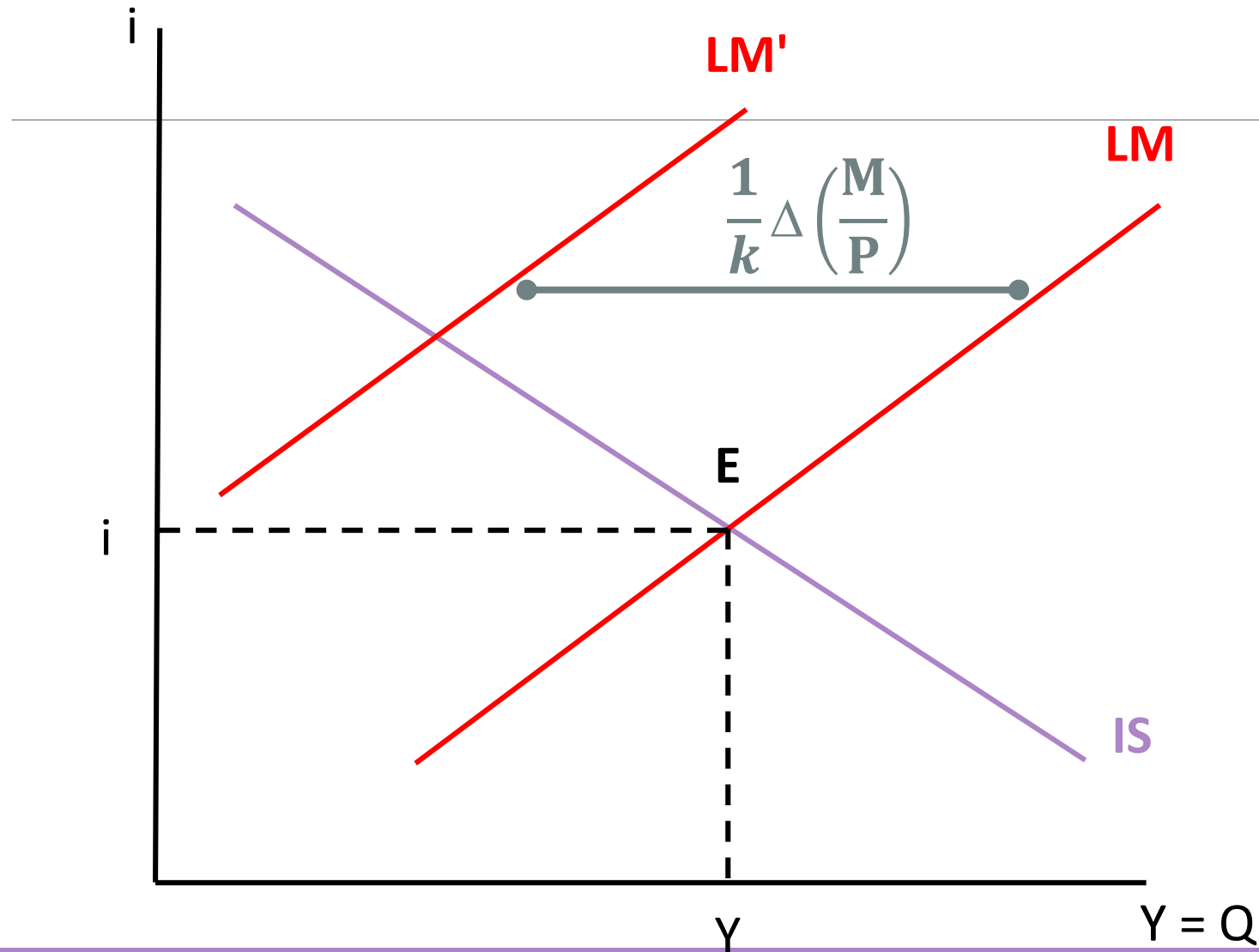


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IS-LM model: contractionary monetary policy

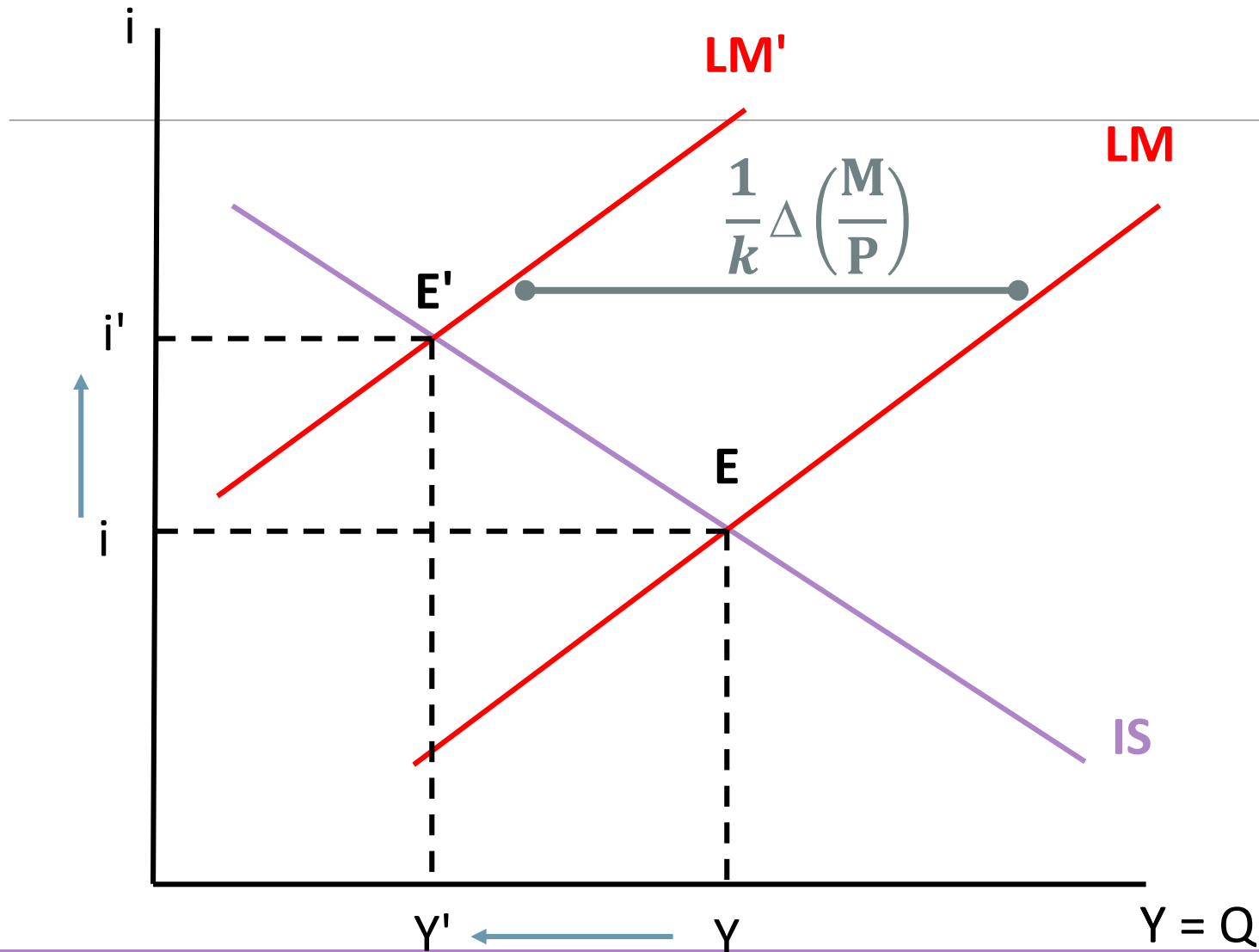


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IS-LM model: contractionary monetary policy

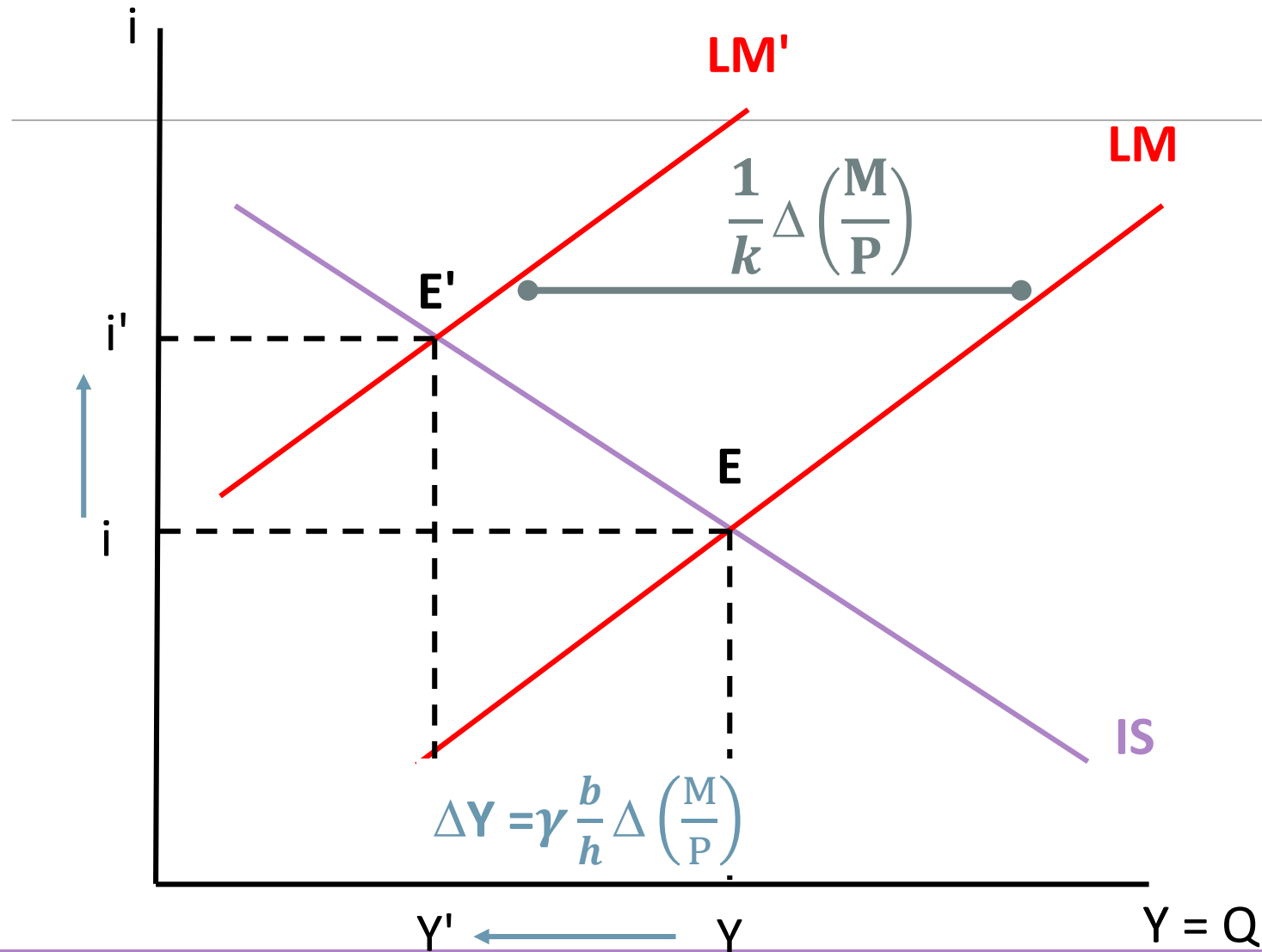


$$i = \frac{1}{h} \left(kY - \frac{M}{P} \right)$$

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IS-LM model: contractionary monetary policy



$$i = \frac{1}{h} \left(kY - \frac{M}{P} \right)$$

$$Y = \frac{1}{k} \left(hi + \frac{M}{P} \right)$$

$$\Delta Y = \frac{1}{k} \Delta \left(\frac{M}{P} \right)$$



IS-LM model: fixed interest rate

- In the past (and as we have seen so far), Central Banks focused on the money supply as a monetary policy variable.
- Nowadays, **they focus on the interest rate**.
- An **interest rate** is chosen as a **target** and they change the quantity of money to achieve it.



IS-LM model: fixed interest rate

- The LM would be horizontal since the interest rate is fixed.
- Therefore,

$$\text{IS: } Y = \alpha(\bar{A} - bi)$$

$$\text{LM: } i = \bar{i}$$

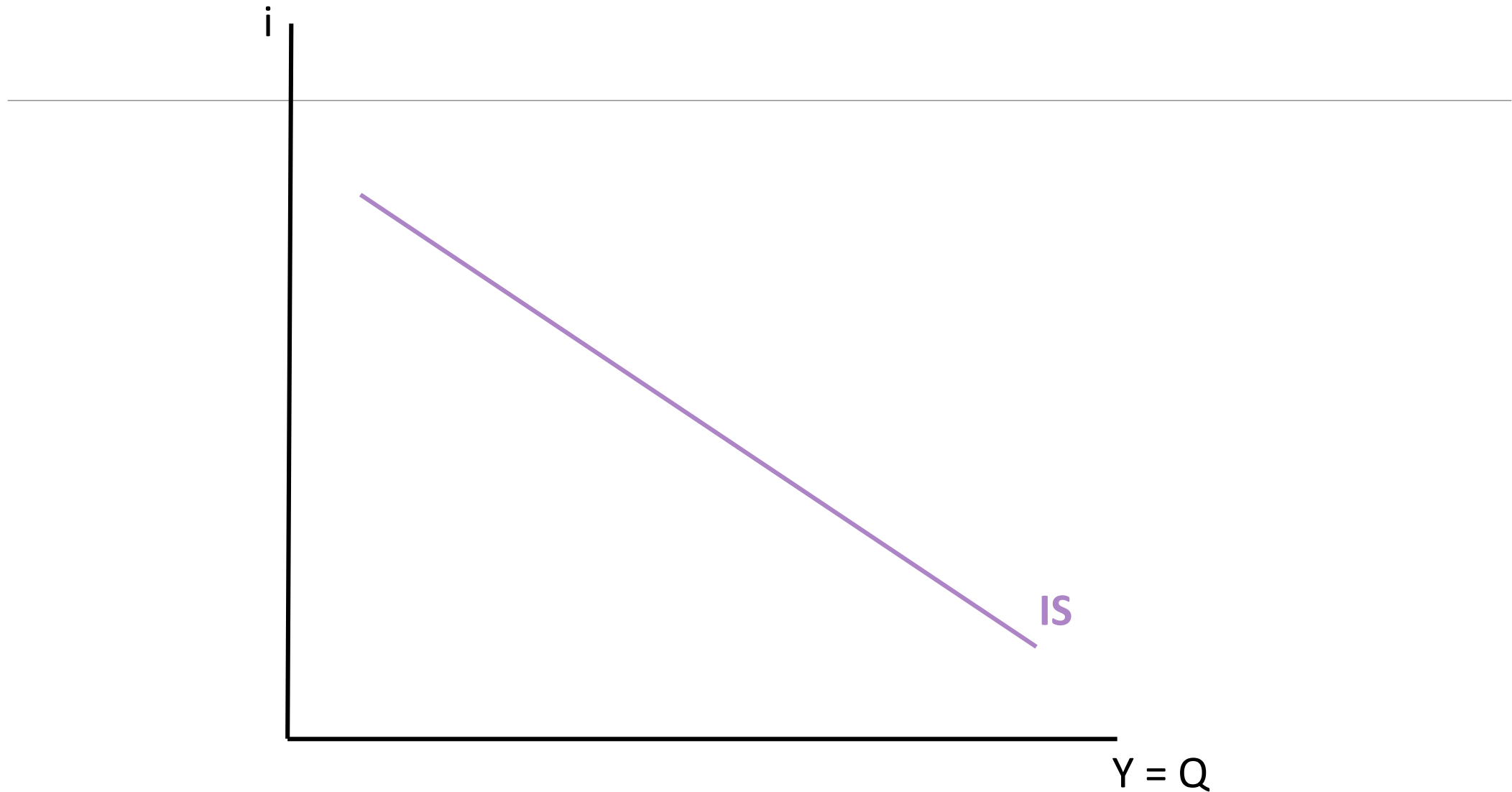
IS-LM model: fixed interest rate

- The equilibrium of the IS-LM model will again be given by:

$$IS = LM$$

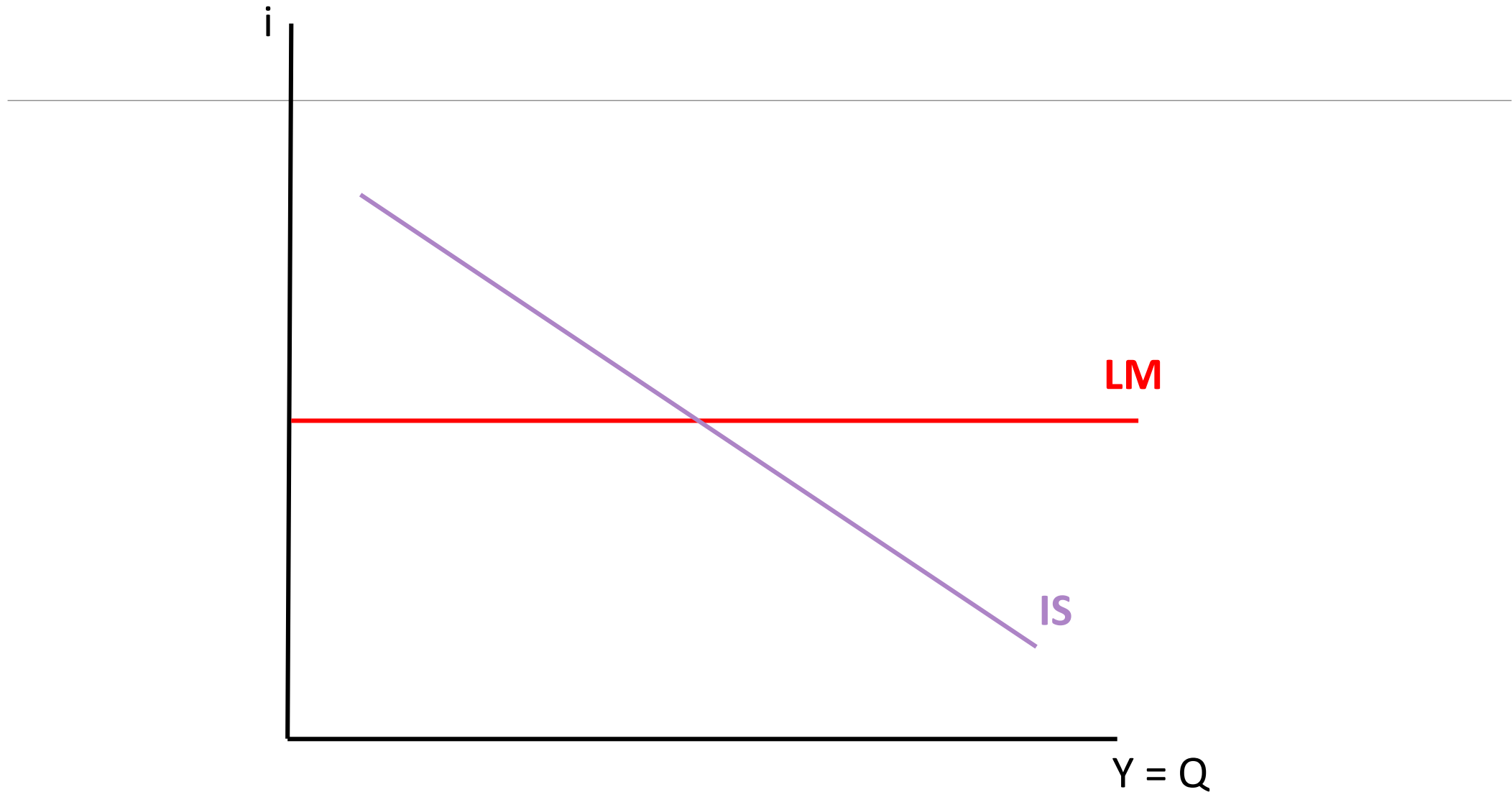


IS-LM model: fixed interest rate



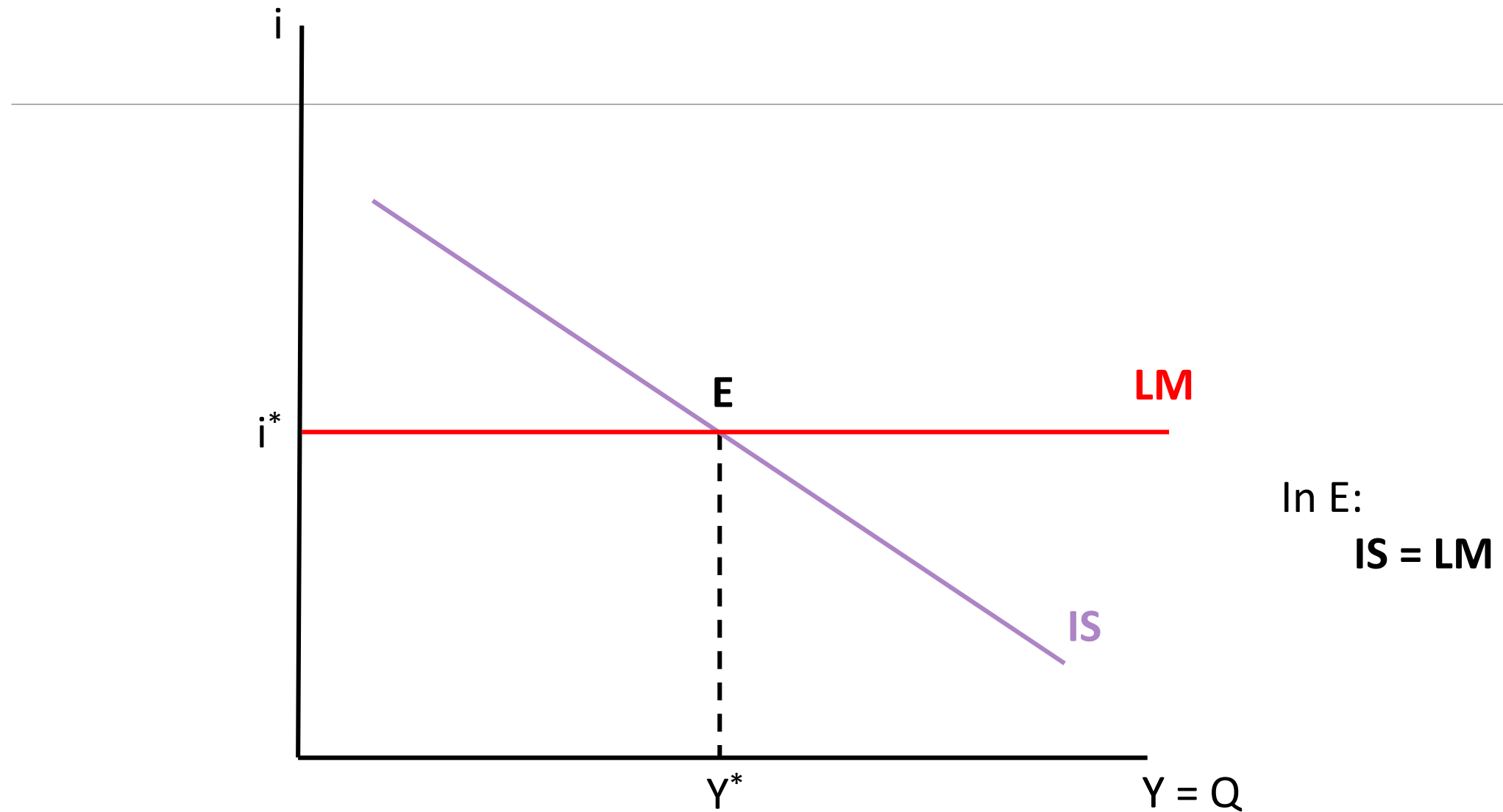


IS-LM model: fixed interest rate





IS-LM model: fixed interest rate

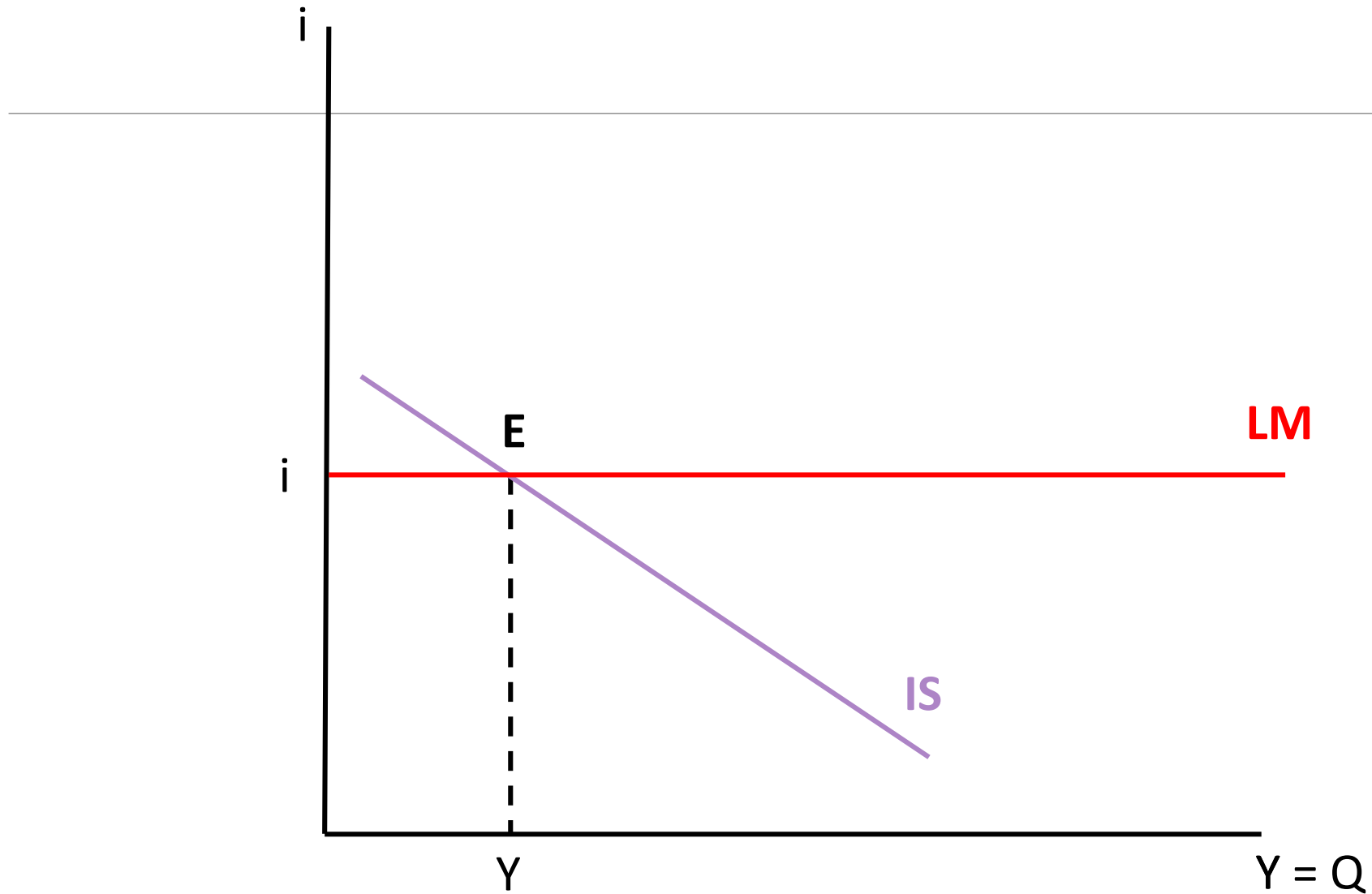




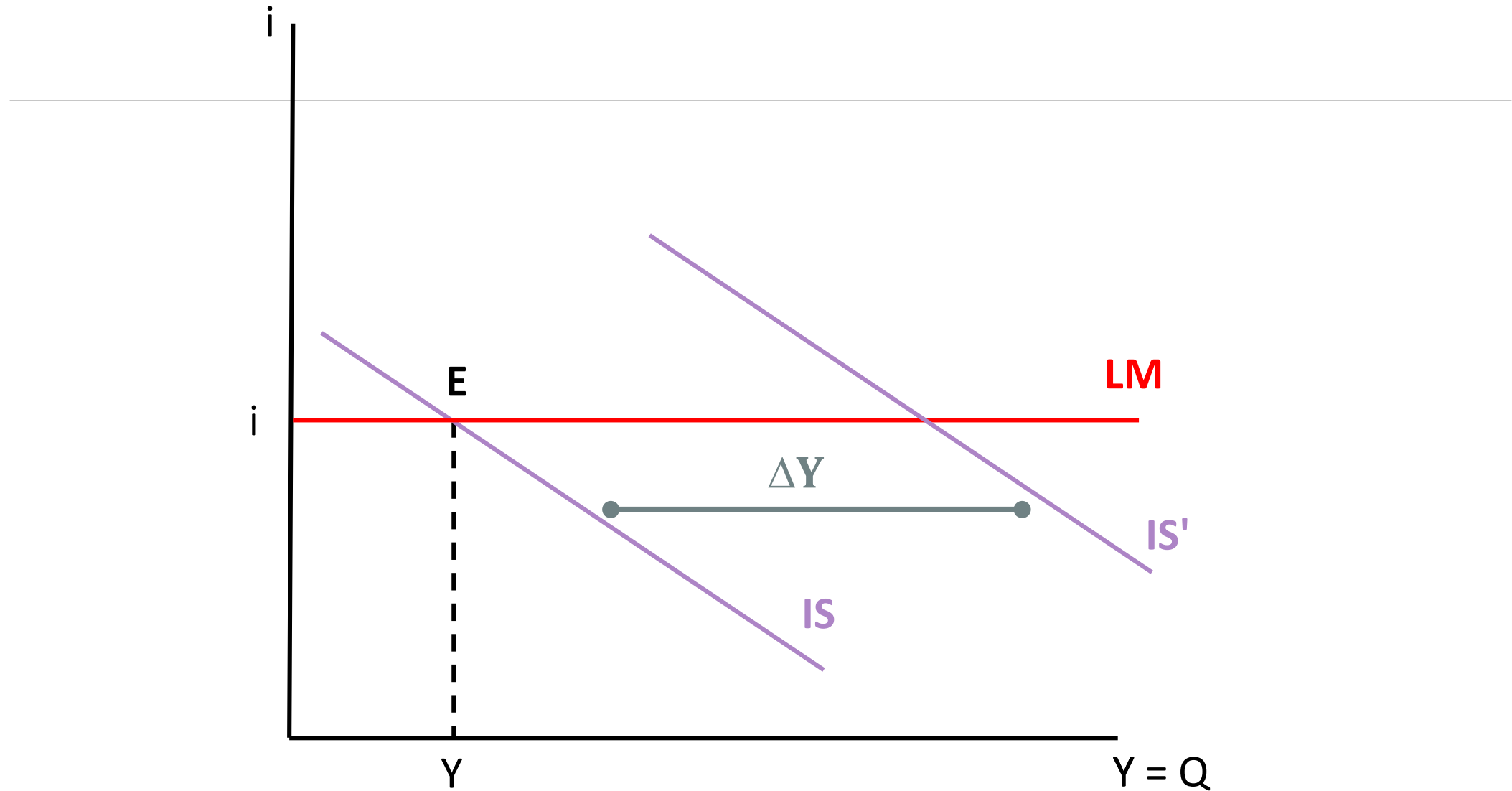
Fiscal policy with fixed interest rate

- **Expansionary fiscal policy:** increase in public spending or transfers, or reduction of taxes.
- **Contractionary fiscal policy:** decrease in public spending or transfers, or increase in taxes.

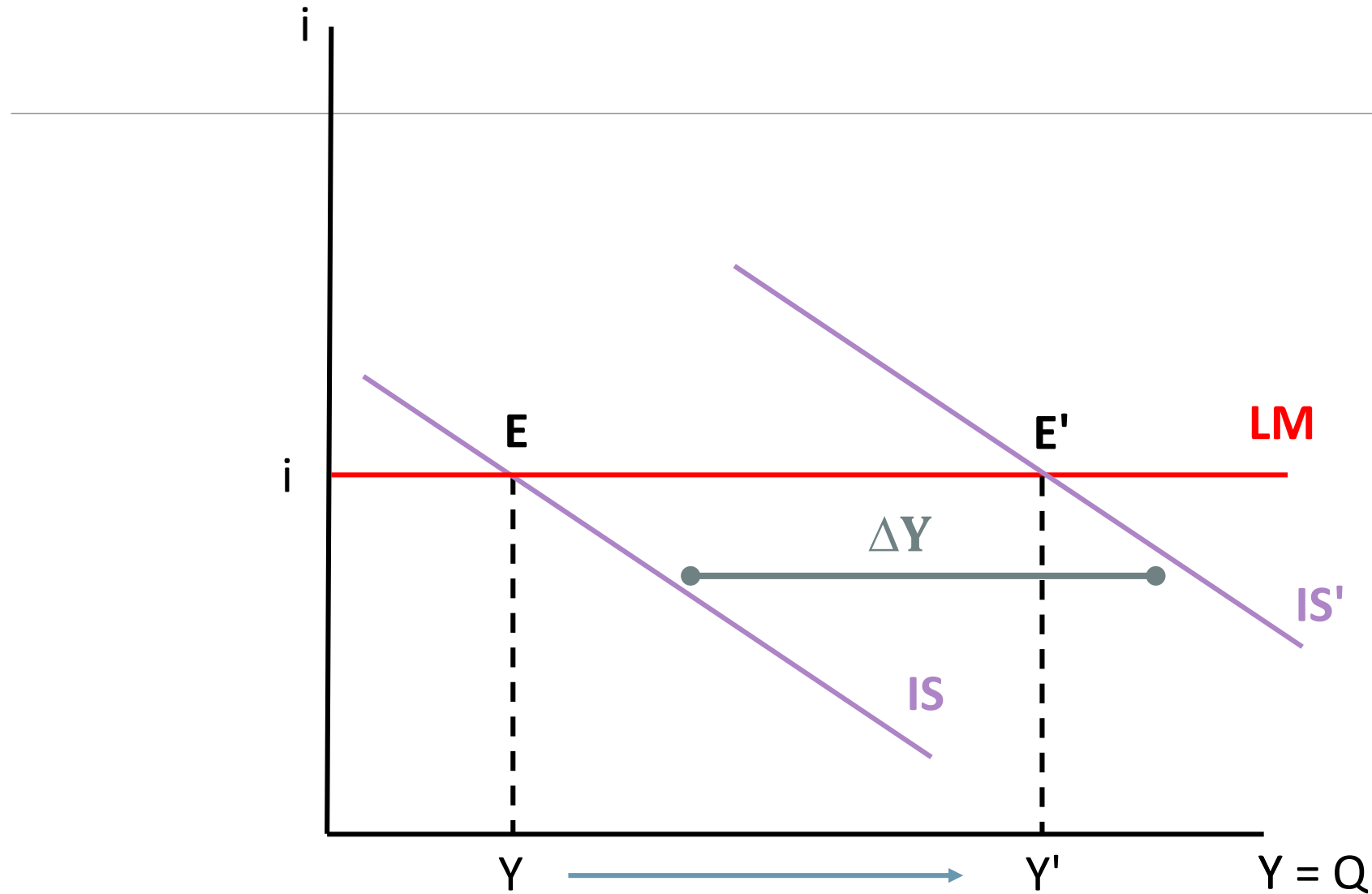
Expansionary fiscal policy



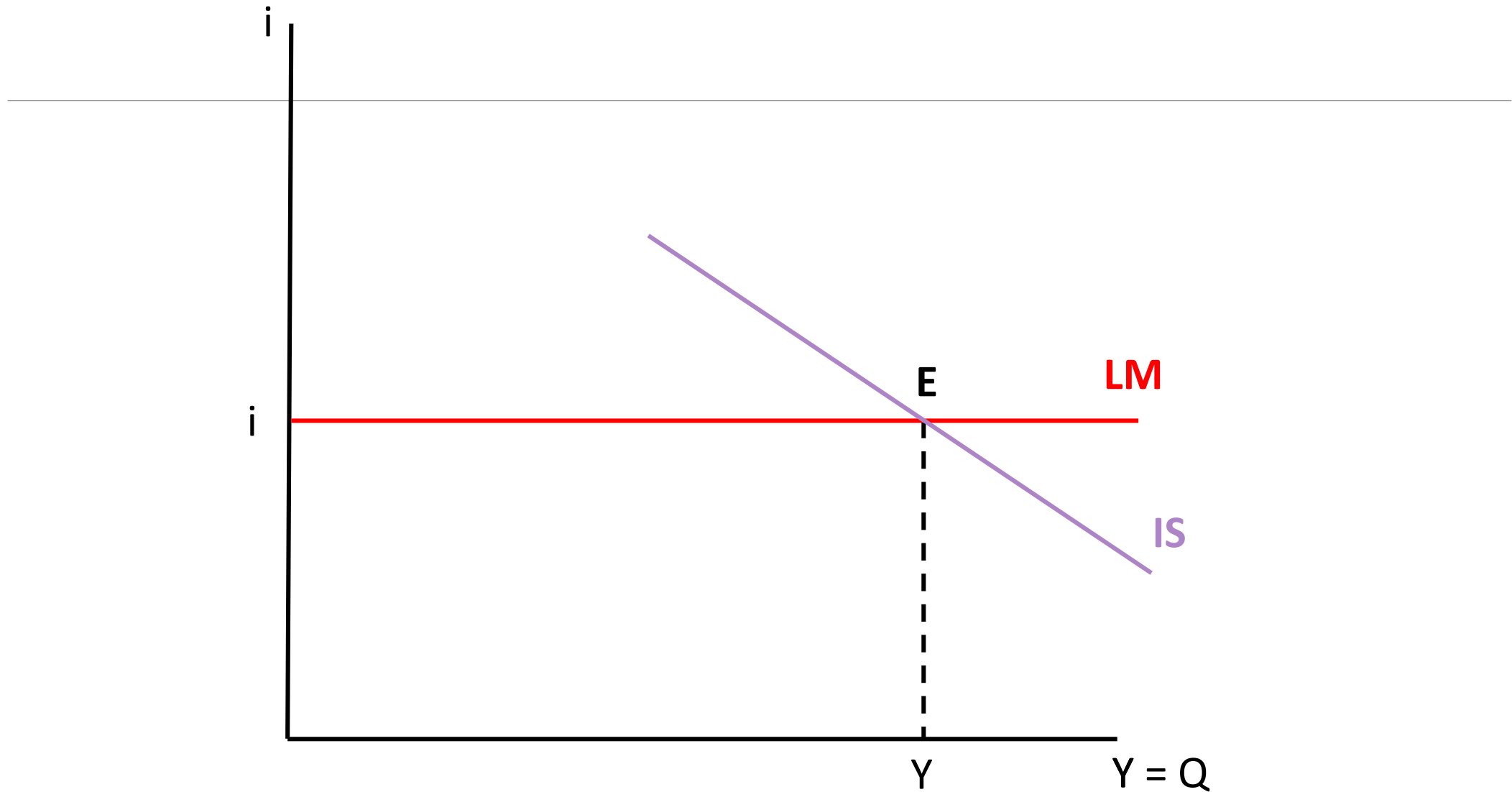
Expansionary fiscal policy



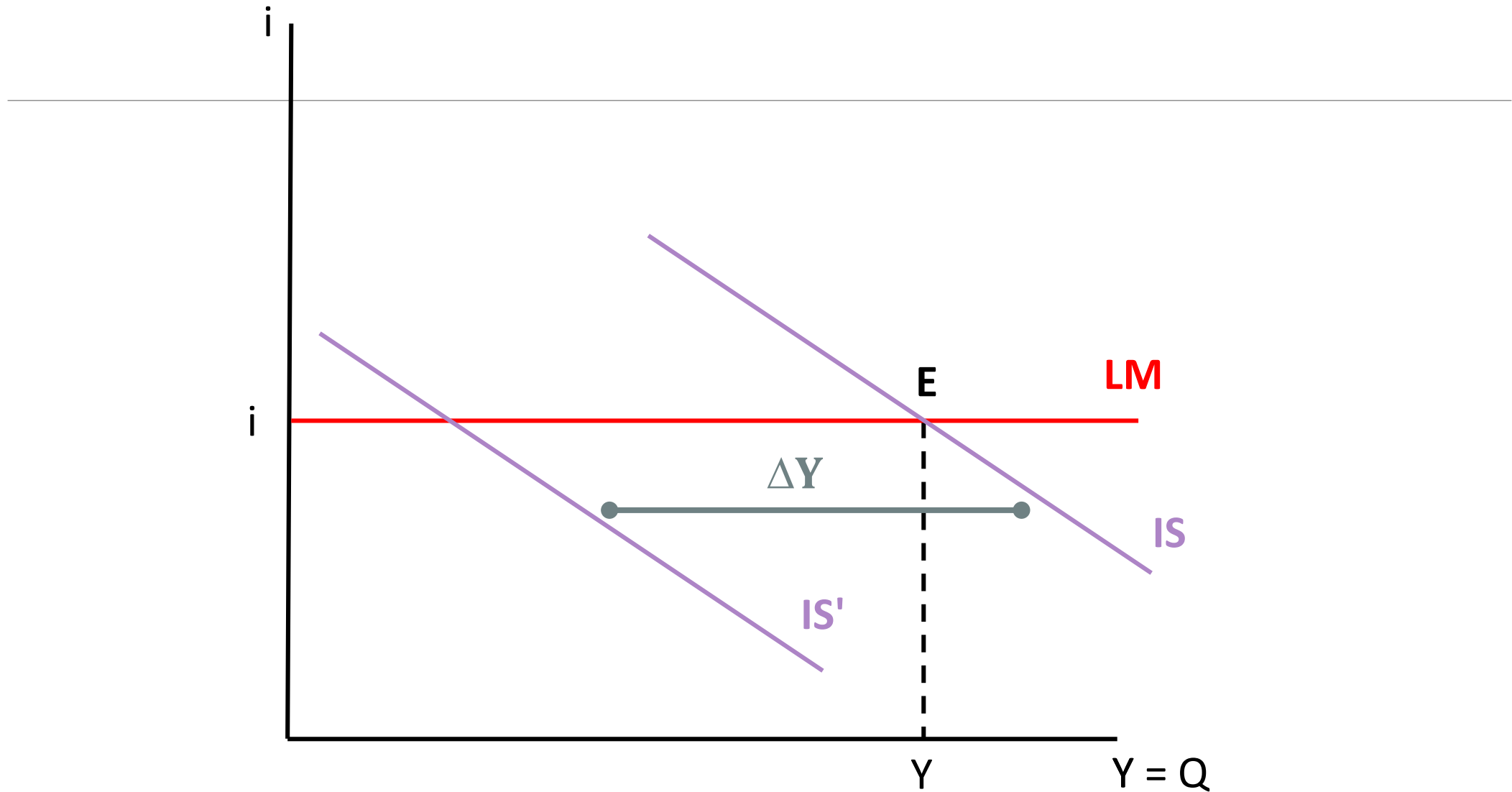
Expansionary fiscal policy



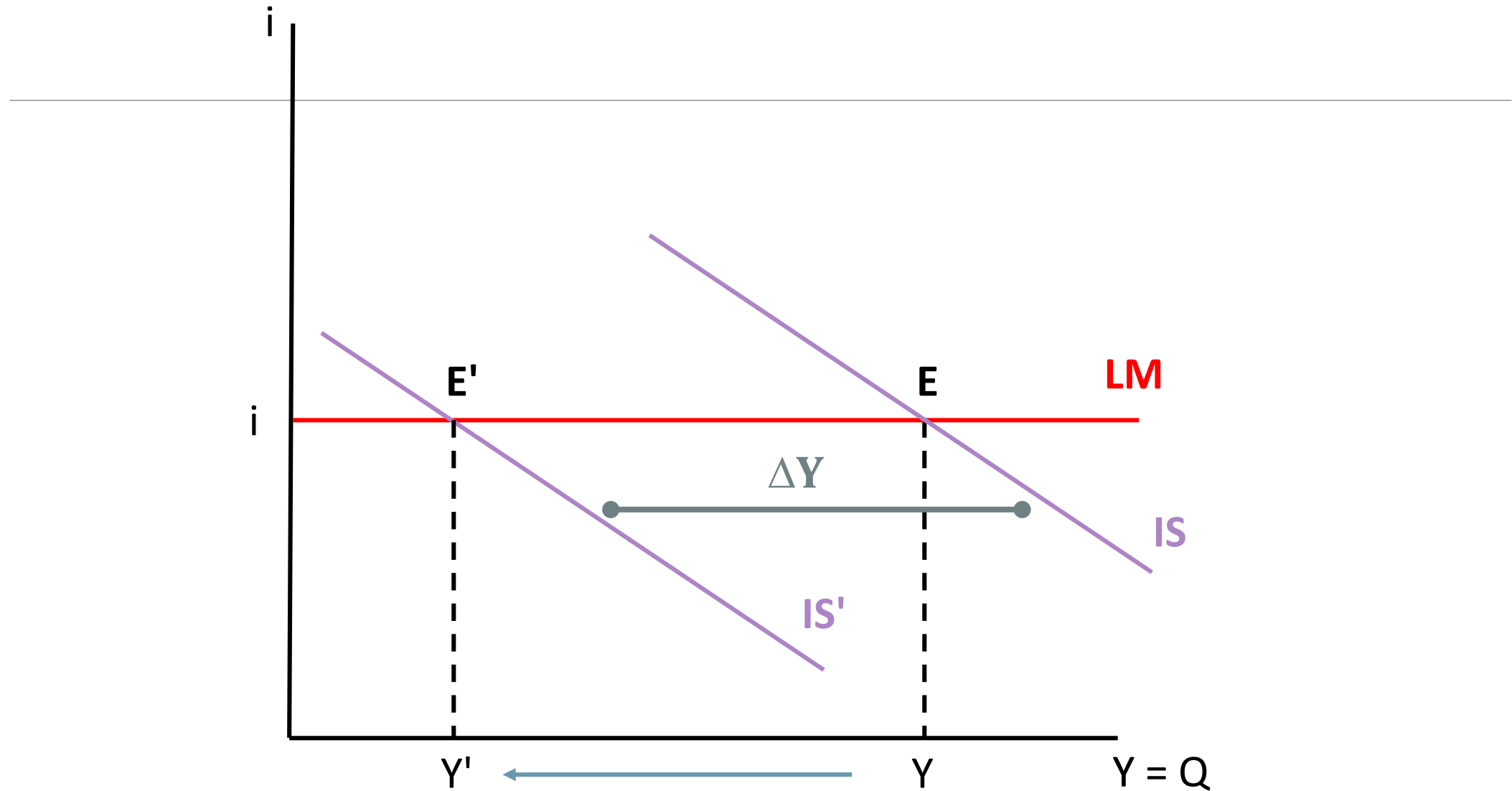
Contractionary fiscal policy



Contractionary fiscal policy



Contractionary fiscal policy

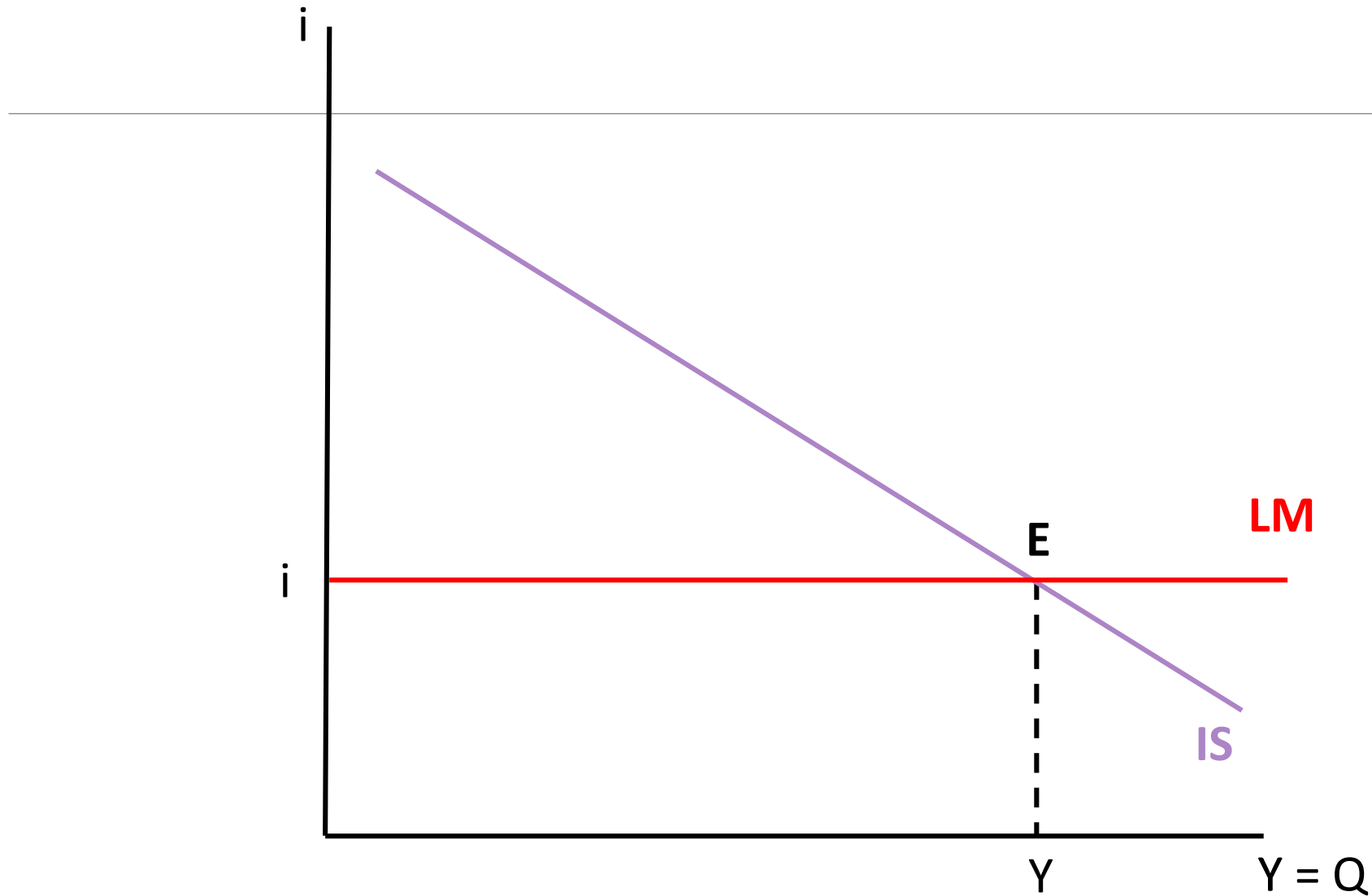




Monetary policy with fixed interest rate

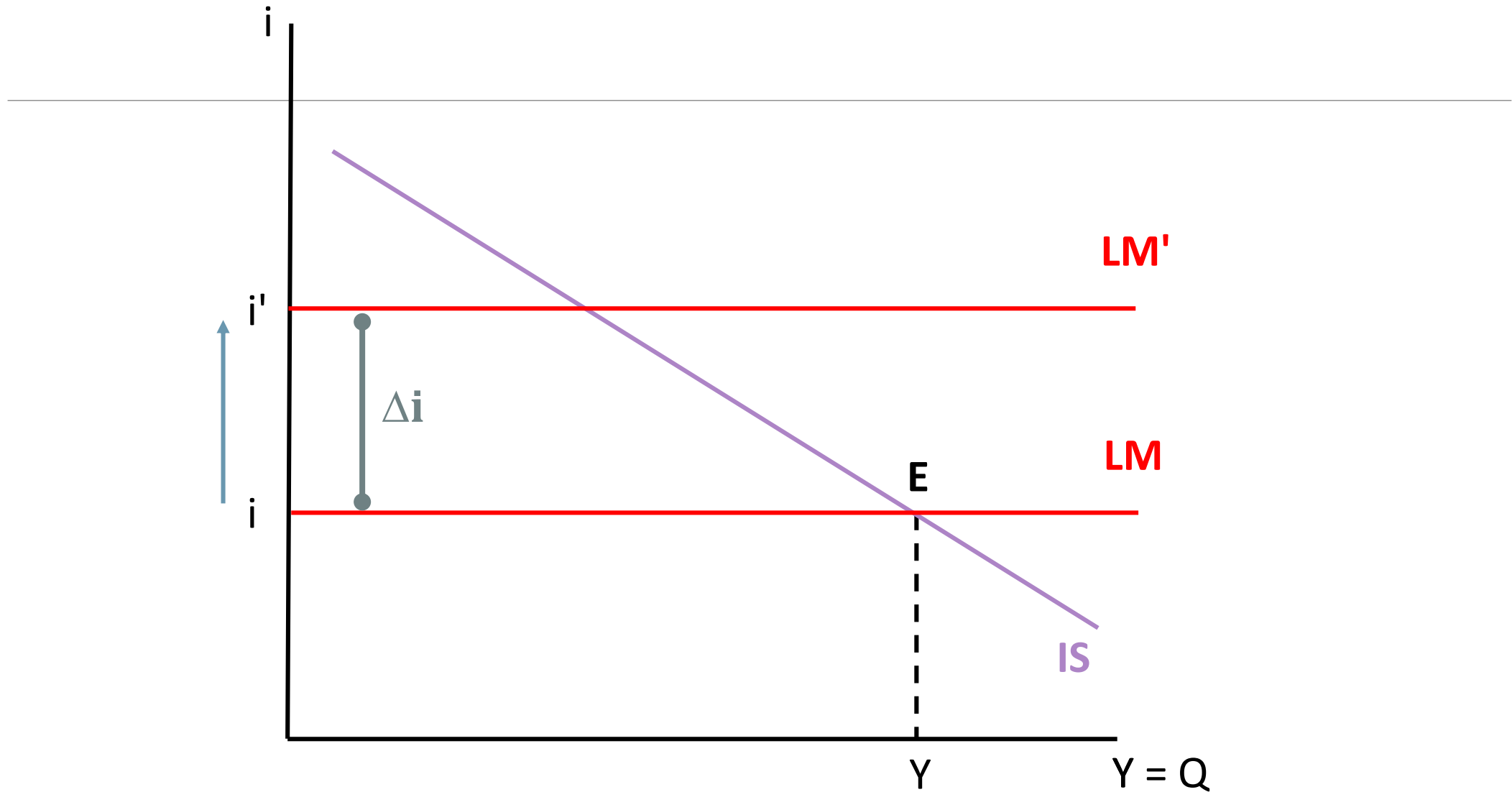
- **Expansionary monetary policy:** decrease in the interest rate.
- **Contractionary monetary policy:** increase in the interest rate.

Contractionary monetary policy

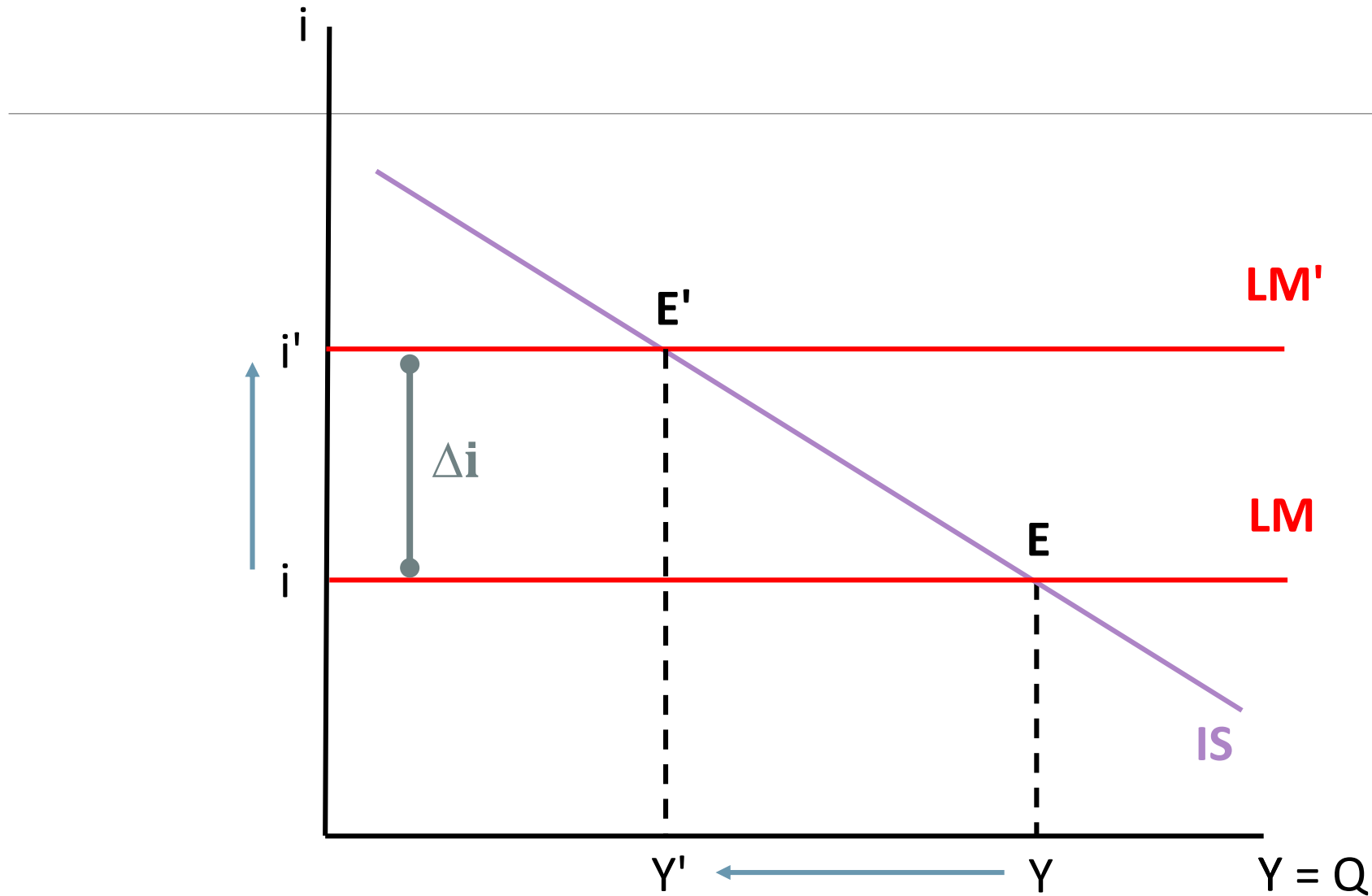




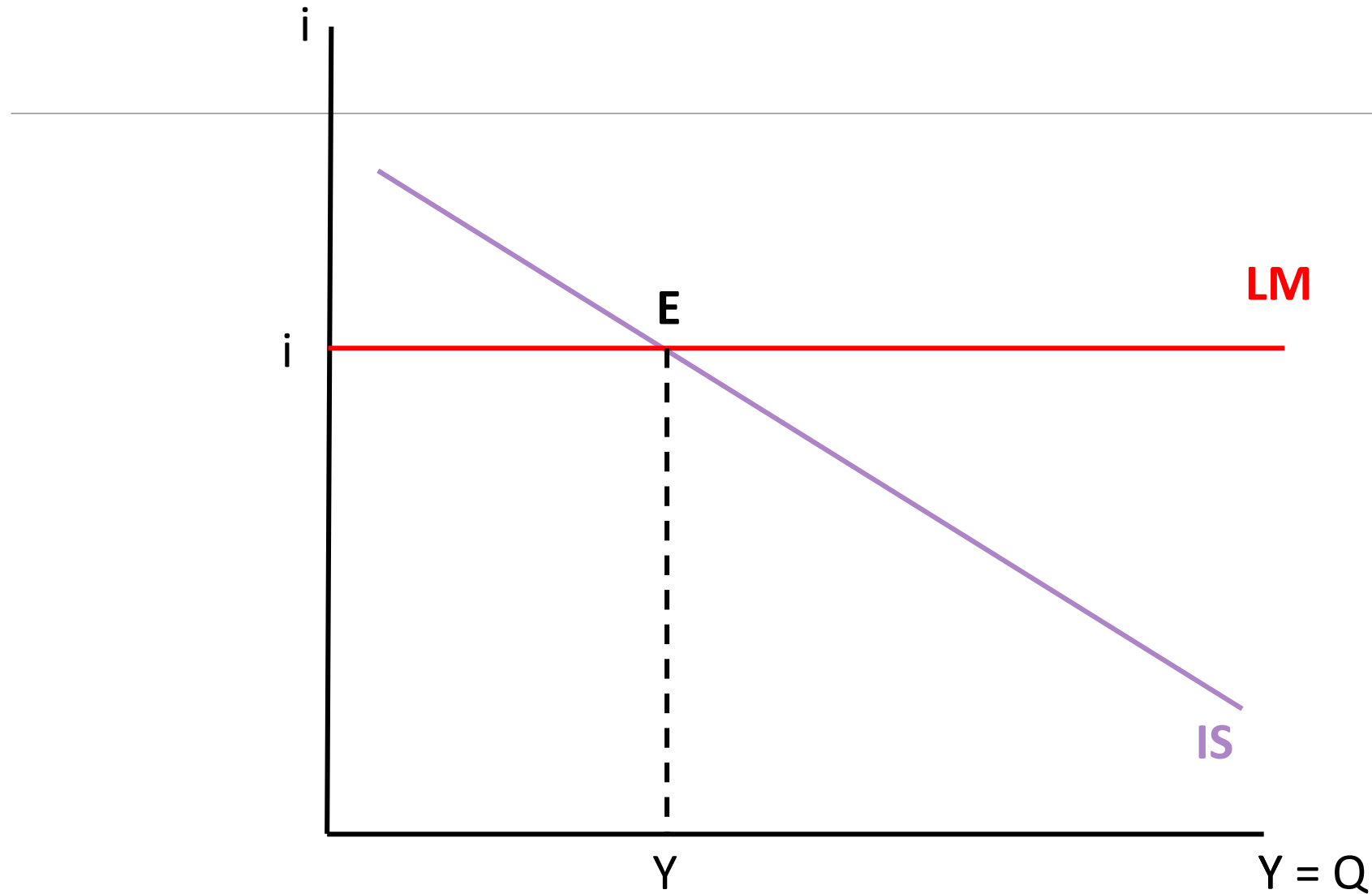
Contractionary monetary policy



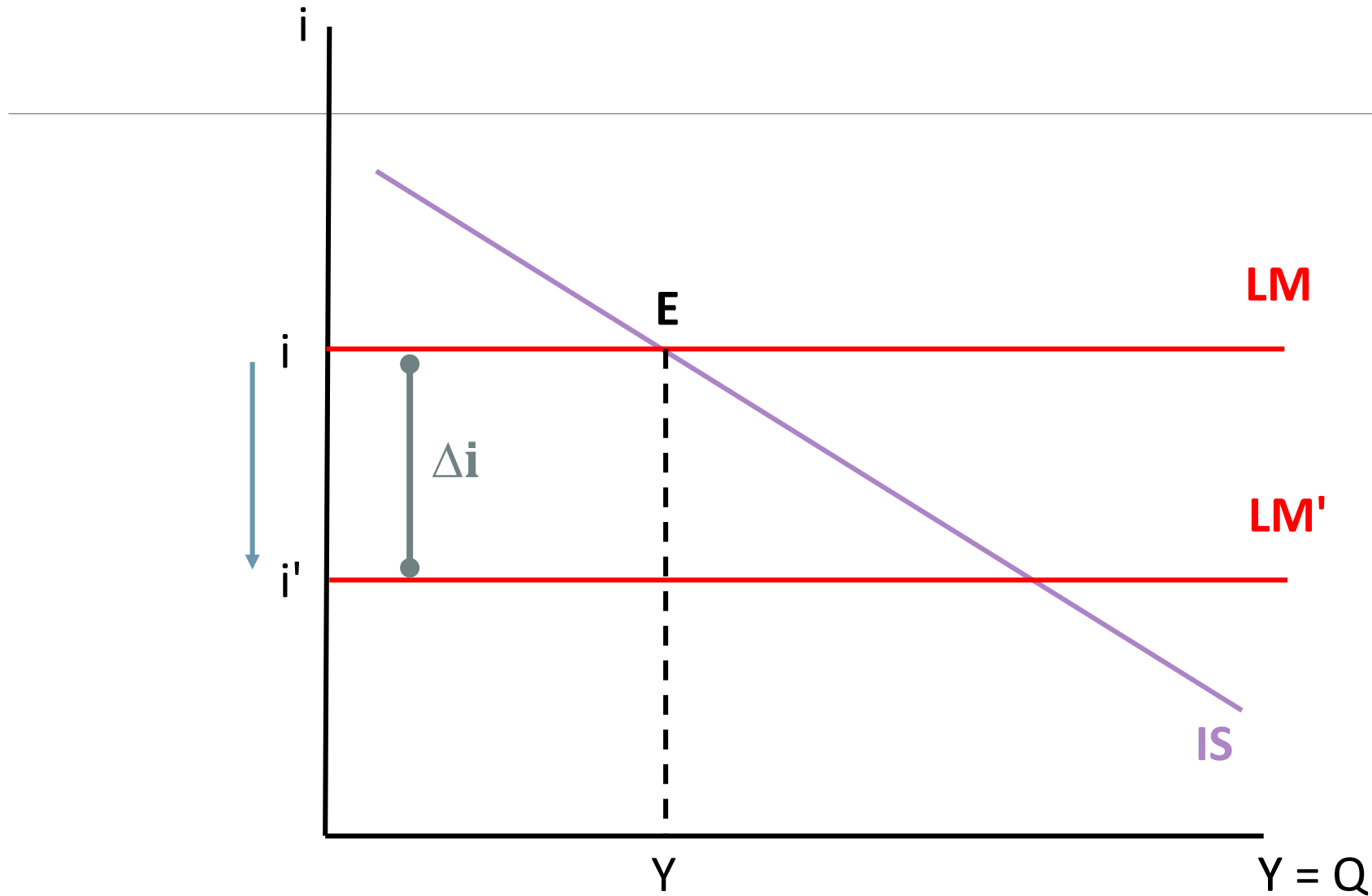
Contractionary monetary policy



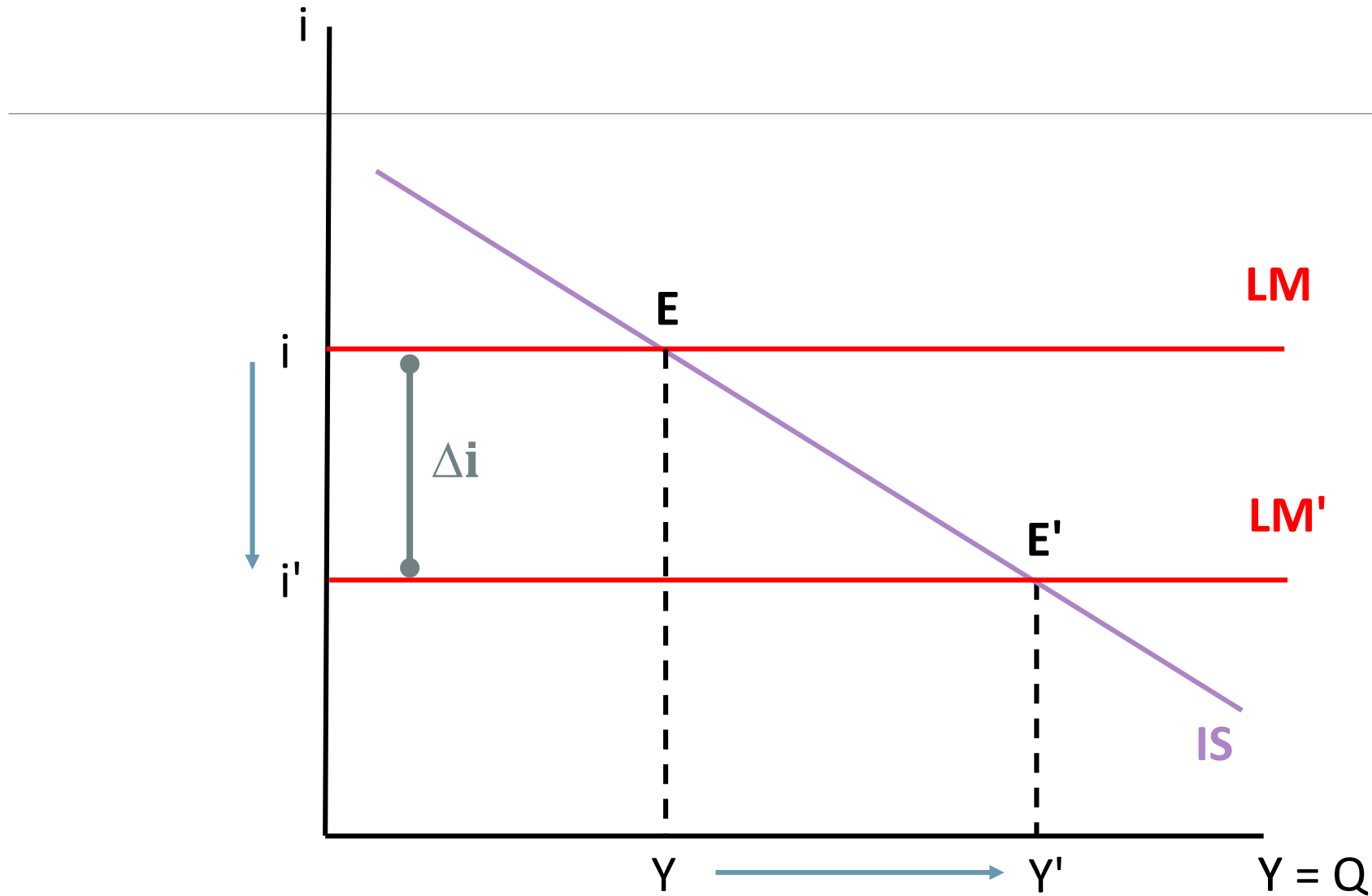
Expansionary monetary policy



Expansionary monetary policy



Expansionary monetary policy





In summary...

1. The IS-LM model is the basic model of aggregate demand that incorporates the goods market and the money market. It highlights the channels through which fiscal policy and monetary policy affect the economy.
2. Increases in i reduce AD by reducing I . At higher levels of i , levels of Y , at which the goods market is in equilibrium is lower: the IS curve slopes downward.
3. The demand for money is a demand for *real* balances. Demand for real balances increases with Y and decreases with i . With fixed (exogenous) MS, the LM curve is upward-sloping.



To recapitulate...

4. The level of i and $Q (=Y)$ are jointly determined by simultaneous equilibrium of the goods and money markets: $IS = LM$.
5. Monetary policy affects the economy first affecting i and then by affecting AD . An increase in MS reduces i , increases I (hence AD), and increases equilibrium output ($Q=Y$).
6. The IS and LM curves together determine the aggregate demand schedule.
7. Changes in monetary and fiscal policy affect the economy through the monetary and fiscal policy multipliers.



Mandatory readings

- Dornbusch, R., Fischer, S. and Startz, R. (2018). *Macroeconomics*. McGraw-Hill Education. 13th edition.
 - Chapter 10: Income and spending.
 - Chapter 11: Money, interest and income.
 - Chapter 12: Monetary and fiscal policy.



End of Topic 8

Introduction to the IS-LM model

Prof. David A. Sánchez-Páez



Mathematical Appendix to Topic 8

Introduction to the IS-LM model

Prof. David A. Sánchez-Páez



Equilibrium in the IS-LM model

- In equilibrium:

$$IS = LM$$

- Therefore:

$$\text{IS: } Y = \alpha(\bar{A} - b\dot{i})$$

$$\text{LM: } \dot{i} = \frac{1}{h} \left(kY - \frac{M}{P} \right)$$

Y and \dot{i} are the same in both equations.

Equilibrium in the IS-LM model

Replacing the **LM** in the **IS**:

$$Y = \alpha \left[\bar{A} - b \left(\frac{1}{h} \left(kY - \frac{M}{P} \right) \right) \right]$$

$$Y = \alpha \left[\bar{A} - \frac{b}{h} \left(kY - \frac{M}{P} \right) \right]$$

$$Y = \alpha \bar{A} - \alpha \frac{b}{h} kY + \alpha \frac{b}{h} \frac{M}{P}$$



Equilibrium in the IS-LM model

$$Y = \alpha \bar{A} - \alpha \frac{b}{h} k Y + \alpha \frac{b}{h} \frac{M}{P}$$

$$Y + \alpha \frac{b}{h} k Y = \alpha \bar{A} + \alpha \frac{b}{h} \frac{M}{P}$$

$$\left(1 + \frac{\alpha b k}{h}\right) Y = \alpha \bar{A} + \alpha \frac{b}{h} \frac{M}{P}$$

$$\left(\frac{h + \alpha b k}{h}\right) Y = \alpha \bar{A} + \alpha \frac{b}{h} \frac{M}{P}$$



Equilibrium in the IS-LM model

$$\left(\frac{h + \alpha bk}{h}\right) Y = \alpha \bar{A} + \alpha \frac{b}{h} \frac{M}{P}$$

$$Y = \alpha \left(\frac{h}{h + \alpha bk}\right) \bar{A} + \alpha \frac{b}{h} \left(\frac{h}{h + \alpha bk}\right) \frac{M}{P}$$

$$Y = \frac{\alpha h}{h + \alpha bk} \bar{A} + \frac{b}{h} \frac{\alpha h}{h + \alpha bk} \frac{M}{P}$$



Equilibrium in the IS-LM model

$$Y = \frac{\alpha h}{h + \alpha b k} \bar{A} + \frac{b}{h} \frac{\alpha h}{h + \alpha b k} \frac{M}{P}$$

$$Y = \gamma \bar{A} + \gamma \frac{b}{h} \frac{M}{P}$$

Where,

$$\gamma = \frac{\alpha h}{h + \alpha b k}$$



Equilibrium in the IS-LM model

$$Y = \frac{\alpha h}{h + \alpha b k} \bar{A} + \frac{b}{h} \frac{\alpha h}{h + \alpha b k} \frac{M}{P}$$

$$Y = \gamma \bar{A} + \gamma \frac{b}{h} \frac{M}{P}$$

Where,

$$\gamma = \frac{\alpha h}{h + \alpha b k}$$

Equilibrium in the IS-LM model

$$Y = \frac{\alpha h}{h + \alpha b k} \bar{A} + \frac{b}{h} \frac{\alpha h}{h + \alpha b k} \frac{M}{P}$$

$$Y = \gamma \bar{A} + \gamma \frac{b}{h} \frac{M}{P}$$

Where,

$$\gamma = \frac{\alpha h}{h + \alpha b k}$$

Recall that:

$$\bar{A} = [\bar{C} + c\bar{T}\bar{R} + \bar{I} + \bar{G} + \bar{N}\bar{X}]$$



Equilibrium in the IS-LM model

Finally, the equilibrium interest rate is obtained from the equilibrium Y equation.

$$Y = \gamma \bar{A} + \gamma \frac{b}{h} \frac{M}{P}$$

$$\text{LM: } i = \frac{1}{h} \left(kY - \frac{M}{P} \right)$$



Equilibrium in the IS-LM model

Y is replaced in the **LM**:

$$i = \frac{1}{h} \left[k \left(\gamma \bar{A} + \gamma \frac{b}{h} \frac{M}{P} \right) - \frac{M}{P} \right]$$

$$i = \frac{1}{h} \left[k\gamma \bar{A} + k\gamma \frac{b}{h} \frac{M}{P} - \frac{M}{P} \right]$$

$$i = \frac{1}{h} \left[k\gamma \bar{A} + k \left(\frac{\alpha h}{h + \alpha b k} \right) \frac{b}{h} \frac{M}{P} - \frac{M}{P} \right]$$

Equilibrium in the IS-LM model

$$i = \frac{1}{h} \left[k\gamma\bar{A} + k \left(\frac{\alpha h}{h + \alpha b k} \right) \frac{b}{h} \frac{M}{P} - \frac{M}{P} \right]$$

$$i = \frac{1}{h} \left[k\gamma\bar{A} + k \left(\frac{\alpha}{h + \alpha b k} \right) b \frac{M}{P} - \frac{M}{P} \right]$$

$$i = \frac{1}{h} \left[k\gamma\bar{A} + \left(\frac{\alpha b k}{h + \alpha b k} \right) \frac{M}{P} - \frac{M}{P} \right]$$

$$i = \frac{1}{h} \left[k\gamma\bar{A} + \left(\frac{\alpha b k}{h + \alpha b k} - 1 \right) \frac{M}{P} \right]$$

Equilibrium in the IS-LM model

$$i = \frac{1}{h} \left[k\gamma\bar{A} + \left(\frac{\alpha bk}{h + \alpha bk} - 1 \right) \frac{M}{P} \right]$$

$$i = \frac{1}{h} \left[k\gamma\bar{A} + \left(\frac{\alpha bk - h - \alpha bk}{h + \alpha bk} \right) \frac{M}{P} \right]$$

$$i = \frac{1}{h} \left[k\gamma\bar{A} + \left(\frac{-h}{h + \alpha bk} \right) \frac{M}{P} \right]$$

$$i = \frac{1}{h} k\gamma\bar{A} + \frac{1}{h} \left(\frac{-h}{h + \alpha bk} \right) \frac{M}{P}$$



Equilibrium in the IS-LM model

$$i = \frac{1}{h} k \gamma \bar{A} + \frac{1}{h} \left(\frac{-h}{h + \alpha b k} \right) \frac{M}{P}$$

$$i = \frac{k}{h} \gamma \bar{A} - \frac{1}{h + \alpha b k} \frac{M}{P}$$

$$i = \frac{k}{h} \gamma \bar{A} - \frac{1}{h + \alpha b k} \left(\frac{\alpha h}{\alpha h} \right) \frac{M}{P}$$

$$i = \frac{k}{h} \gamma \bar{A} - \frac{\alpha h}{h + \alpha b k} \left(\frac{1}{\alpha h} \right) \frac{M}{P}$$



Equilibrium in the IS-LM model

$$i = \frac{k}{h} \gamma \bar{A} - \frac{\alpha h}{h + \alpha b k} \left(\frac{1}{\alpha h} \right) \frac{M}{P}$$

$$\mathbf{i} = \frac{\mathbf{k}}{\mathbf{h}} \gamma \bar{\mathbf{A}} - \gamma \left(\frac{\mathbf{1}}{\alpha \mathbf{h}} \right) \frac{\mathbf{M}}{\mathbf{P}}$$



Equilibrium in the IS-LM model

$$i = \frac{k}{h} \gamma \bar{A} - \frac{\alpha h}{h + \alpha b k} \left(\frac{1}{\alpha h} \right) \frac{M}{P}$$

$$\mathbf{i} = \frac{k}{h} \gamma \mathbf{\bar{A}} - \gamma \left(\frac{1}{\alpha h} \right) \mathbf{\frac{M}{P}}$$

Equilibrium in the IS-LM model

- Summarizing,

$$Y = \gamma \bar{A} + \gamma \frac{b}{h} \frac{M}{P}$$

$$i = \frac{k}{h} \gamma \bar{A} - \gamma \left(\frac{1}{\alpha h} \right) \frac{M}{P}$$