Environmental Regulation and Inspection Delegation with Stock Pollution *

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Abstract

In this paper, we model a differential game played *à la Stackelberg* between a regulator and a polluting firm in a stock pollution context. The regulator can be a single body deciding on the emission standard and the probability of inspection overtime as functions of the pollution stock. Alternatively, the regulator can delegate the inspection activities to a local agency that maximizes revenues coming from fines net of inspection costs. Although the objective of the agency departs from social welfare, decentralization can be welfare improving, depending on the type of strategic interaction between the local agency and the polluting firm, as well as on the firm anticipating the effects of current pollution decisions on future regulatory policy. Up to our knowledge, this is the first paper dealing with hierarchical regulation in a stock pollution context.

Key words: Emission standards; Monitoring; Non-compliance; Stock Pollution; Hierarchical Governments; Stackelberg differential game.

JEL Codes: C61, C73, K32, K42, L51, Q28.

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Declarations

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1 Introduction

In many environmental contexts—such as those involving wastewater discharges—emissions from polluting facilities accumulate over time, with environmental damages depending on the resulting stock of pollution. To correct the associated negative externalities, the design of optimal environmental policies in such dynamic settings requires to explicitly consider the intertemporal nature of pollution. When emissions cannot be measured without incurring monitoring costs, the environmental policy must be complemented by enforcement tools—such as inspection probabilities, monetary sanctions for non-compliance, or both. Although emission standards and fines are typically set by a central regulator, enforcement through inspections may be delegated to local entities, often attributed to informational advantages at the local level. To the best of our knowledge, the hierarchical nature of environmental regulation in dynamic contexts involving pollution accumulation has not yet been formally examined in the literature. This paper aims to fill that gap.

Examples such as the contamination of rivers and lakes caused by industrial saline wastewater discharges from sectors like agro-food, petroleum, textiles, leather, mining, and power industries illustrate the regulatory setting described above (see, for example, Lefebvre and Moletta, 2006). Specific case studies include the regulation of saline pollution in the Hunter River, the Great Lakes, the Chesapeake Bay, and the Ganges River, among others. Wastewater discharges are known to adversely affect aquatic ecosystems, water potability, and agricultural productivity, and most legal frameworks around the world include provisions to mitigate the environmental impacts of saline pollution (prominent examples include the Protection of the Environment Operations Act 1997 in New South Wales, Australia, the US Clean Water Act, 1972, or the EU Water Framework Directive 2000/60/EC). Typically, these regulations are implemented through pollution permits that define maximum allowable salt concentrations at each point source, conditional on the background salinity level in the receiving water body. Regulatory agencies typically operate these schemes using a broad range of enforcement mechanisms, including monetary penalties, formal warnings, official cautions, license conditions, notices, directions, and prosecutions. In some cases, such as the Hunter River Salinity Trading Scheme in New South Wales, the regulatory system operates under a one-layer model, where a single authority—the NSW Environmental Protection Authority (EPA)—is responsible for both setting discharge limits and enforcing compliance through monitoring and inspections. This centralized structure contrasts with two-layer regulatory systems such as those under the US Clean Water Act or the EU Water Framework Directive, where enforcement responsibilities are delegated to state, regional or local agencies. This delegation allows for more flexible and locally adapted implementation, and may also help reduce monitoring costs by taking advantage of local knowledge and administrative capacity.

In spite of the enforcement efforts, full compliance is not always guaranteed. For example, in the Hunter River, several companies have been alleged by the NSW EPA to have breached environmental legislation (EPA NSW, n.d.). In the United States, energy company Duke Energy pleaded guilty to criminal violations of the Clean Water Act after a 2014 coal ash spill released 39,000 tons of waste into the Dan River, resulting in over $100 million in fines and restitution (U.S. Department of Justice, 2015). More recently, Phillips 66 was indicted for discharging approximately 790,000 gallons of industrial wastewater with excessive oil and grease into the Los Angeles County sewer system, violating federal water pollution regulations (U.S. Attorney's Office, 2024). In the European context, several water companies in the United Kingdom have come under investigation by the Office for Environmental Protection for recurrent illegal discharges into rivers, prompting broader scrutiny of compliance with water quality standards (The Guardian, 2023).

The objective of this paper is to develop a theoretical model that integrates the features of the regulatory context described above—namely, pollution stock dynamics, emission standards, costly monitoring, the possibility of non-compliance and potential decentralization of inspection activities. Within this framework, we aim to characterize the socially optimal combination of emission standards and inspection probabilities as functions of the pollution stock. Our focus is on identifying the conditions under which delegating monitoring responsibilities to a local agency may be socially preferable to a centralized enforcement regime. While cost advantages commonly associated with local agencies can partly justify decentralization, our analysis highlights two additional critical factors: the strategic interaction between the local agency and the regulated firm, and the firm's forward-looking behavior in anticipation of future environmental regulation. These elements are intrinsic to the dynamic nature of the problem and cannot be adequately captured within the static frameworks that are prevalent in the existing literature up to date.

For this purpose, we model a differential game played à la Stackelberg between a representative firm (which acts as the follower) and the regulatory body (the leader). The game is played over an infinite time horizon, allowing for dynamic strategic interactions between the firm and the regulator over time. The firm decides on instantaneous emissions, which accumulate over time and generate a negative externality on society. To correct this problem and induce firms to internalize this externality, the regulatory body, in turn, chooses the emission standard and the inspection probability, with non-compliance being sanctioned if detected. The regulatory body can be either a unique central regulator (centralized or one-layer case) or a central regulator plus a local inspection agency (decentralized or two-layer case). In the one-layer specification, both the standard and the inspection probability are jointly determined by a social welfare maximizer regulator. In the two-layer specification, the standard is determined by the regulator, and the probability of inspection is chosen by the local agency.

We first consider a baseline scenario where the central regulator is a social welfare maximizer, but the inspection agency (which finances the monitoring activity with a fix proportion of all the collected fines) maximizes the revenues coming from fines net of the inspection costs.[1] The regulator acts as the leader with a follower agency, and we analyze two alternative scenarios regarding the relation between the agency and the firm. In the first one, denoted two-layer Stackelberg game, the agency plays the role of the leader in a nested hierarchical game with the firm. This corresponds to a situation where the agency can commit to the announced inspection probability to induce the desired reaction from the firm. By contrast, the alternative scenario is one where such commitment is not possible, denoted two-layer Nash game, where the local inspection agency and the firm play simultaneously. Both modelling alternatives have been explored widely in the environmental enforcement

[1] The objective of the local agency may vary depending on the case. While those in developed countries often aim to improve economic and environmental outcomes, agencies in developing contexts may prioritize fine collection—as observed in water pollution control efforts in Southern China (BSR, n.d.) or by India's National Green Tribunal in the Ganges basin (NGT, n.d.). Our baseline model adopts a revenue-maximizing agency objective, similar to that used in the tax evasion literature, representing a conservative benchmark for assessing decentralization. We later extend the model to include environmental concerns in the agency's objective function.

literature in the static context, as we discuss in the literature review section. In the different regulatory experiences around the world, the most common relation between local agencies and polluting firms seems to be the Nash mode, in the sense that firms are not announced any inspection frequencies. This, however, does not preclude firms from forming beliefs according to their size or past history, among others.

For a given pollution stock, instantaneous emissions are lowest in the one-layer case and highest in the two-layer Stackelberg. The pollution standard and inspection probability act as substitute instruments: the one-layer case features the most lenient standard and highest inspection probability, while the two-layer Stackelberg shows the opposite. This configuration leads to imperfect compliance in both two-layer settings, with the greatest non-compliance in the Stackelberg case. There, the agency behaves dynamically, reducing monitoring to widen current non-compliance and hence revenues, and also to accelerate pollution accumulation and increase future non-compliance—and hence revenues—unlike in the Nash game, where the agency acts myopically and these effects do not arise. Because of these strategic effects, the Nash interaction between the local agency and the firm consistently yields higher social welfare than the Stackelberg case.

Regarding the social preference for decentralization relative to the one-layer case—assuming equal monitoring costs for the local agency and the central regulator—we find that both two-layer configurations can outperform centralization. This is a surprising result, given the sharp contrast in objectives: the central regulator seeks to maximize social welfare, while the local agency is driven solely by net fine revenues. Moreover, decentralization may still be socially preferred even when the local agency faces higher monitoring costs than the central regulator—particularly when inspection costs or environmental damages are high, fines are relatively low, and the agency retains a smaller share of the collected fines.

Next, we extend our analysis to the case in which the local agency cares not only about its net revenues but also about environmental damages. Under Nash behavior, this modification does not alter the preference for decentralization. However, under Stackelberg, the effect is non monotone. Initially, giving some weight to environmental concerns increases the likelihood that decentralization outperforms centralization. However, as the agency places greater emphasis on environmental damages, this preference diminishes.

Summarizing, we find that enforcement decisions can sometimes be optimally delegated (more likely when the local agency and the firm play *à la Nash* rather than *à la Stackelberg*), even if the objectives of the local agency diverge from those of the central regulator. Key to our results is our dynamic regulatory framework, particularly the fact that the firm takes into account how its decision on emissions affects the stock of pollution, and hence future regulation. This facilitates a proactive and engaging environment, which helps to reduce monitoring costs, as compared to a case where the firm is not farsighted.[2] Simpler scenarios where the firm acts myopically (or even a fully static case where pollution does not accumulate overtime) result in delegation always being worse-off. In the fully dynamic context, decentralization is more likely to be the socially preferred option under large monitoring costs, large environmental damages, small fine per unit of non-compliance, or small percentage of the collected fines passed to the local agency as revenue.

The remainder of the paper is organized as follows. In Section 2, we position our paper with respect to the existing literature. In Section 3, we present a full description of the dynamic model, including the two regulatory versions, one-layer versus two-layer. In Section 4 we derive the results and find the conditions under which the two-layer case dominates the one-layer specification in social welfare terms. In Section 5, we present simpler versions of the model, to analyze the impact in the results of the different players being farsighted or myopic. In Section 6, we extend the model to consider that the local agency departs from strict net revenue maximization. We conclude in Section 7. The Appendix contains additional technical material.

2 Literature Review

Our work is related to the literature on fiscal federalism in environmental policy. This literature examines how environmental responsibilities are distributed across different levels of

[2]A prominent example of such farsighted behavior is the case of Bengalla Mining Company, part of the broader coal mining sector in the Hunter Valley, which has historically been under scrutiny for its environmental practices. In recent years, Bengalla has adopted measures to reduce water contamination from its operations, aligning with regulatory standards and actively participating in the Hunter River Salinity Trading Scheme. This proactive stance has not only contributed to mitigating the company's environmental footprint but also positioned the firm favorably in the public eye, potentially shaping future regulatory outcomes (New Hope Group, 2023).

government and discusses the welfare implications of decentralization (Oates 1999, 2004). The debate often concerns whether pollution standards and regulatory programs are best set centrally or locally (Dalmazzone, 2006), especially in federal countries, where interjurisdictional competition can lead to variation in environmental standards and enforcement (Gupta, 2014). Theoretical and empirical studies show that decentralization can lead to more tailored and efficient policies by considering local conditions and preferences, but it also brings some challenges like free riding, border effects, and underinvestment in key evaluations such as cost-benefit analysis, among others (Destandau et al., 2014; Sigman, 2005). Huntington and Kennedy (2008, 2013) find that while decentralized governance can improve environmental outcomes by better reflecting local preferences, it may also result in inconsistent enforcement and varying regulatory quality. Similarly, Veld and Shogren (2012) highlight how decentralization interacts with strategic behavior among jurisdictions, which can both improve or worsen environmental outcomes depending on the context. Some studies emphasize the need for financial transfers and incentives aligned with local preferences to achieve social efficiency (Silva and Caplan, 1997; Nagase and Silva, 2000). Overall, these studies suggest that decentralization's success depends largely on institutional design, local capacity, proper incentives, and ways to manage cross-jurisdictional impacts. We contribute to this literature by showing that the specific type of strategic interaction between local agencies and firms, as well as firms' forward-looking behavior regarding the impact of their decisions on future regulation, play a critical role in shaping the outcomes of decentralization.

Methodologically, our work is close to the literature on the modelling of optimal environmental policies in dynamic settings. However, to the best of our knowledge, there is no study on the hierarchical nature of environmental regulations to tackle stock pollution problems. Alternative studied issues are the asymmetric information between regulators and polluting sources (Hoel and Karp, 2001); when firms undertake investment decisions that affect future abatement costs (Biglaiser et al., 1995; Karp and Zhang, 2016); in non-point pollution problems with heterogeneous sources (Xabadia et al., 2008; Goetz and Xabadia, 2015); or under imperfect compliance (Arguedas et al., 2020), which is the closest study to ours. Arguedas et al. (2020) model the dynamic interaction between a representative polluting firm and a regulator, where the former chooses emissions that accumulate overtime, and the latter sets an emission standard and a fine in case of non-compliance. Both instruments are set by a unique regulatory body and monitoring effort is assumed to be exogenous. In contrast, the current paper treats monitoring as an endogenous component of the regulatory framework, thereby raising the question of whether this activity should be delegated.

More specifically, the theoretical microeconomic literature that models the hierarchical setting of environmental regulations does it mainly in static contexts (see for example, Grieson and Singh, 1990, Jones and Scotchmer, 1990, Decker, 2007 or Arguedas and Rousseau, 2015). Interestingly, Grieson and Singh (1990) find, as we do, that the enforcement decision can sometimes be optimally delegated when the local agency and the firms play *à la Nash*, even if the objectives of the local agency diverge from those of the central regulator. This can be achieved by the regulator's appropriate choice of two instruments: the fee for being monitored and the sanction in case of non-compliance. There are, however, two key differences between our approach and theirs. First, in our case, the final outcome in the one-layer case is second best and delegation can achieve a superior outcome, while in Grieson and Singh (1990), both outcomes can be first best. And second, our regulatory framework is dynamic, while Grieson and Singh (1990)'s is static. This particularly means that the firm is farsighted and takes into account how its decision on emissions affects the stock of pollution, and hence future regulation. This enables delegation to be superior in our context with just one instrument chosen by the regulator (the emission standard), in contrast to the two instruments required in Grieson and Singh (1990)—the fee for being monitored and the fine).

Another related strand of the environmental enforcement literature in static settings deals with the specific strategic relation between the regulator and the polluting firms (Stackelberg versus Nash). The Stackelberg case corresponds to a situation where the agency can commit to the announced inspection probability to induce the desired reaction from the firm, while the Nash assumption reflects the scenario where such commitment is not possible. The case where the enforcement agency plays first taking the reaction of polluters into account is probably the most common in the literature, see, for example, Harford (1978), Beavis and Walker (1983), Garvie and Keeler (1994), Bose (1995), Stranlund and Dhanda (1999), Macho-Stadler and Perez-Castrillo (2006), or Arguedas and Rousseau (2015), among others.

However, there are notable alternative modellings where the agency and the firms play a simultaneous Nash game (Grieson and Singh, 1990, Franckx, 2002), or where the agency plays afterwards in response to firm's behavior (Friesen, 2006, Maxwell and Decker, 2006, or Chen et al., 2013).

Finally, the literature on environmental enforcement is not unanimous about the objective function for the local agency. For example, Grieson and Singh (1990) consider two alternatives, either social welfare as the central regulator's (that is, firms' costs, environmental damages and monitoring costs), or an alternative that places some weight on the net revenues of the local agency (fines collected minus monitoring costs). Keeler (1995) or Arguedas and Rousseau (2015) assume different weights placed by the local agency and the central regulator on firms' costs. Macho-Stadler and Perez-Castrillo (2006) assume that the agency only cares about the environment and aims to minimizes overall pollution. This reflects the heterogeneity in real world examples, where the objective of the local agency very much depends on the case. Our specific choice of the objective function for the local agency in our baseline model is closer to the one considered in the tax evasion literature (see, for example, Melumad and Mookherjee, 1989, or Sanchez and Sobel, 1993, among others). Even in this context where the objectives of the central regulator and the local agency are so different, we have found that decentralization can improve social welfare, and we later extend the analysis to cases where the local agency cares about the environment, such as in Keeler (1995) or Arguedas and Rousseau (2015).

3 The Model

A representative firm produces a consumption good with emissions as the only input. Let $Y(t)$, $E(t)$ respectively denote production and emissions of the facility at time t. For mathematical convenience, we assume that production is quadratic in emissions as follows:[3]

$$Y(E(t)) = \sigma E(t) - \frac{E^2(t)}{2}, \quad \sigma > 0, \tag{1}$$

that is, production is strictly concave in emissions, and it is maximum for $E(t) = \sigma$. This corresponds to the level of *business as usual* emissions, or optimal emissions without regulation.[4]

Emissions accumulate over time as a stock of pollutants, $S(t)$, according to the dynamic rule:

$$\dot{S}(t) = E(t) - \delta S(t), \quad S(0) = S_0, \tag{2}$$

where $\delta > 0$ is the degree of assimilative capacity of the environment, and $S_0 \geq 0$ is the stock of pollution at $t = 0$.

From the point of view of the firm, emissions are free in the absence of any regulation. However, the stock of pollution causes environmental damages, given by:

$$D(S(t)) = \gamma \frac{S^2(t)}{2}, \tag{3}$$

where $\gamma > 0$ measures the intensity of environmental damages. This damage function measures, in monetary units, the damage that a given stock of pollution represents for society. The quadratic formulation reflects the assumption of increasing marginal damages from pollution, which is common in the literature since the seminal papers on dynamic games and pollution stock by Ploeg and Zeeuw (1992) and Dockner and Long (1993).

We assume that an environmentally concerned regulator seeks to make the firm (partially) responsible for the environmental damages caused. To internalize the environmental damage, the regulator imposes an emission target or limit which can vary across time, $Q(t)$. To verify if the firm complies with the emission target at time t, the regulator monitors emissions

[3] This assumption on the production function is common in the differential game literature. An alternative formulation would be to consider emissions as a by-product of output, at the expense of losing the linear-quadratic structure. This would make the analysis more difficult, requiring the development of specific numerical methods. To recover the linear-quadratic structure we would need a change of variable, which would bring us back to the current formulation.

[4] As will be shown later, this level is never attained under regulation, and the equilibrium always lies in the increasing returns region.

by means of a time-dependent inspection probability $P(t)$. Inspection activities are costly. They require personnel, equipment and have associated administrative and legal costs. We assume that these costs increase marginally with the required monitoring frequency.[5] Hence monitoring costs can be represented by:

$$M(P(t)) = \alpha_L \frac{P^2(t)}{2}, \tag{4}$$

with $\alpha_L > 0$.

If the firm is found to be exceeding the standard at time t, that is, if $E(t) > Q(t)$, then the firm must pay an amount ϕ per unit of the violation:

$$F(E(t), Q(t)) = \max\{\phi[E(t) - Q(t)], 0\}, \tag{5}$$

where $\phi > 0$ describes the intensity of the fine.[6] A fine is not imposed if $E(t) \leq Q(t)$.

The two regulatory decision variables are the emission limit $Q(t)$ and the inspection probability $P(t)$, while the level of emissions $E(t)$ is the only decision variable of the firm. We distinguish between two cases. In the first one, the same regulatory body chooses the two regulatory instruments $Q(t)$, $P(t)$. We refer to this case as the *one-layer* specification. In the second case, namely the *two-layer* case, the regulator chooses the standard $Q(t)$ and a local agency chooses the inspection probability $P(t)$. We explain the main differences between the two cases next. Additional technical details for these two cases can be found, respectively, in Appendices A and B.

3.1 One-layer regulation

In this case, a single regulatory body chooses the two policy variables, $Q(t)$, $P(t)$, while the polluting firm chooses emissions $E(t)$. The interaction between the regulator and the firm is described as a Stackelberg differential game in which the regulator takes the role of the leader and the firm acts as the follower. As it is common in the literature, we consider stagewise

[5]Examples of quadratic costs in economic problems, analyzed as differential games, can be found in Dockner *et al.* (2000).

[6]This corresponds to a hybrid system of regulation, which combines a standard, Q, and a tax on emissions (above the standard).

feedback Stackelberg strategies, where the regulator has a stagewise first-mover advantage. Likewise, for differential games with an infinite time horizon, it is usually assumed that agents employ stationary strategies, i.e, their strategies and value functions do not explicitly depend on time, but exclusively on the pollution stock (see, for example, Dockner et al. 2000). We assume that the regulatory policy is set and announced by the leading-regulator as dependent on the actual stock of pollution, S. Correspondingly, the follower-firm chooses emissions, taking the regulatory policy and the pollution stock into consideration. This mode of play, which guarantees time-consistency, only requires instantaneous commitment from the leader.

We assume that the regulator behaves as a benevolent central planner. Thus, the revenues received from collected fines are assumed to be fully transferred to society in a lump-sum way. In this setting, the regulator is concerned about the firm's income from production (1), environmental damages (3) and inspection costs (4), while the firm cares about income from production (1) net of the expected penalty for non-compliance, that is, the probability of inspection times the fine in the event of non-compliance, given by (5).

To characterize the stagewise feedback Stackelberg solution, the firm chooses emissions to maximize the present value of expected profits[7] over an infinite time horizon, taking the time evolution of the pollution stock into account. Considering the consumption good as the *numéraire*, and taking expressions (1) and (5) into account, the dynamic maximization problem for the firm is the following:[8]

$$\begin{aligned} &\max_{E} \int_{0}^{\infty} \left[\sigma E - \frac{E^2}{2} - P\phi(E-Q) \right] e^{-\rho t} dt, \\ &\text{s.t.: } \dot{S} = E - \delta S, \quad S(0) = S_0 \end{aligned} \tag{6}$$

where $\rho > 0$ is the discount factor and $(Q,\, P)$ are chosen and announced by the regulator to

[7] A firm is monitored with probability P, and in case it is non-compliant, it is discovered and fined. By contrast, the firm is not audited and therefore not sanctioned with probability $1-P$. Thus, expected profits read:

$$P\left[\sigma E - \frac{E^2}{2} - \phi(E-Q)\right] + (1-P)\left[\sigma E - \frac{E^2}{2}\right] = \sigma E - \frac{E^2}{2} - P\phi(E-Q).$$

[8] The time argument is omitted here and henceforth when no confusion can arise. As a general principle, upper-case letters denote time-dependent (either state or control) variables, while lower-case and Greek letters denote time-independent parameters.

the firm. The firm's best-response function is given by $\widehat{E}^1(S;Q,P)$, a function of the state variable S and the regulatory variables (Q,P).[9]

Since the fine for non-compliance is linear in the standard, and this standard has no effect on the pollution stock accumulation, the firm chooses emissions as a function of P and S only (emissions do not depend on the standard Q):

$$\widehat{E}^1(S;P) = \sigma - \phi P + (V_F^1)'(S), \tag{7}$$

where $V_F^1(S)$ stands for the value function of the firm in the one layer specification, and hence, $(V_F^1)'(S)$ represents the marginal (negative) value of an additional unit of pollution stock for the firm.

The regulator decides upon the optimal regulatory instruments, taking into account the firm's best response, $\widehat{E}^1(S;P)$ given in (7). Assuming a feedback information structure, strategies are settled by the regulator as functions of the pollution stock. Since the regulator cares about firm's income from production (1), net of environmental damages (3) and inspection costs (4), the regulator then faces the following dynamic problem:

$$\begin{aligned}
&\max_{Q,P} \int_0^\infty \left[\sigma E - \frac{E^2}{2} - \gamma\frac{S^2}{2} - \alpha_L \frac{P^2}{2}\right] e^{-\rho t} dt,\\
&\text{s.t.: } \dot{S} = E - \delta S, \quad S(0) = S_0,\\
&E = \widehat{E}^1(S;P),
\end{aligned} \tag{8}$$

where $\widehat{E}^1(S;P)$ is given by (7).

Notice that the regulator can only determine the probability of inspection, P, because the standard does not have any influence on his objective function or the pollution dynamics. Once the optimal inspection probability, $P^1(S)$, is determined, the corresponding firm's optimal emissions, $E^1(S)$ can be obtained.

Given that the standard is undetermined, for simplicity we assume that the pollution standard always equals emissions, $Q^1(S) = E^1(S)$, so that full compliance is always achieved.

[9] Superscript 1 stands for one-layer scenario, and a hat is used to denote best-response functions.

Hence, the solution of problem (8) is given by the following two expressions:

$$Q^1(S) = E^1(S) = \sigma + (V_F^1)'(S) - \frac{\phi^2}{\alpha_L + \phi^2}[(V_F^1)'(S) - (V_R^1)'(S)], \tag{9}$$

$$P^1(S) = \frac{\phi}{\alpha_L + \phi^2}[(V_F^1)'(S) - (V_R^1)'(S)], \tag{10}$$

where $V_R^1(S)$ denotes the value function of the regulator and $(V_R^1)'(S)$ is the associated marginal value.

3.2 Two-layer regulation

In the two-layer specification, we assume that the regulator delegates the inspection activity to a local agency. We assume that the monitoring costs faced by the local agency are the following:

$$M_A(t) = \alpha_A \frac{P(t)^2}{2}, \tag{11}$$

with $\alpha_A > 0$. A similar interpretation to the one given for the monitoring costs at the central level applies here as well.

In order to externalize the inspection activity, the regulator pays a share β, with $0 < \beta < 1$, of the total collected fines from non-compliance to the agency. Hence, the regulator does no longer care about monitoring costs, but about the money transfer to the local agency. The regulator's problem consists of choosing the pollution standard Q as follows:

$$\begin{aligned} &\max_Q \int_0^\infty \left[\sigma E - \frac{E^2}{2} - \gamma\frac{S^2}{2} - \beta P\phi(E - Q)\right] e^{-\rho t} dt, \\ &\text{s.t.: } \dot{S} = E - \delta S, \quad S(0) = S_0, \end{aligned} \tag{12}$$

taking the best responses of the firm and the local agency into account. These optimal responses depend critically on the strategic interaction between these two agents, and two scenarios are possible. In the first scenario, the local agency plays *à la Stackelberg* acting as the leader against the follower-firm. In the second scenario, the local agency and the polluting firm play simultaneously *à la Nash*.

In the first scenario, namely *two-layer Stackelberg*, the agency takes the standard Q

selected by the regulator as given, and selects the probability of inspection, P to maximize the part of the collected fines passed by the regulator to the agency minus the monitoring costs, taking the best response of the firm into account. The firm in this case solves exactly the same problem as the one presented in (7), since both Q and P are given for the firm regardless of who decides what. Hence, the firm's best response is[10]

$$\widehat{E}^{2S}(S;P) = \sigma - \phi P + (V_F^{2S})'(S), \tag{13}$$

where $V_F^{2S}(S)$ stands for the value function of the firm in the two-layer Stackelberg scenario. Note that this is exactly the same functional form as the one expressed in (7). However, firm's valuation of the environment differs between the two scenarios.

The standard Q is given for the local agency, which takes the firm's best response into account when choosing the probability of inspection, P as follows:

$$\begin{aligned} &\max_P \int_0^\infty \left[\beta P \phi (E - Q) - \alpha_A \frac{P^2}{2}\right] e^{-\rho t} dt, \\ &\text{s.t.: } \dot{S} = E - \delta S, \quad S(0) = S_0, \end{aligned} \tag{14}$$

where $E = \widehat{E}^{2S}(S;P)$, given in (13) .[11] The agency chooses the probability of inspection, P, as a function of the pollution stock and the standard:

$$\widehat{P}^{2S}(S;Q) = \phi \frac{\beta(\sigma - Q) + \beta (V_F^{2S})'(S) - (V_A^{2S})'(S)}{\alpha_A + 2\beta\phi^2}, \tag{15}$$

where $V_A^{2S}(S)$ is the value function of the agency.

Finally, knowing the reaction functions of both the agency and the firm, $\widehat{E}^{2S}(S;P)$ and $\widehat{P}^{2S}(S;Q)$ respectively, the regulator determines the standard Q as a function of the pollution stock S, which now enters into the welfare function and the pollution dynamics through its effect on the probability of inspection and hence, on emissions. Mathematically, the regulator solves problem (12) taking (13) and (15) into consideration. Following this resolution process, the optimal policy, $Q^{2S}(S)$, is obtained, which induces the optimal inspection probability,

[10]Superscript $2S$ stands for the two-layer Stackelberg scenario.

[11]An extended model where the local agency considers environmental damages is discussed in Section 6.

$P^{2S}(S)$, and both induce the corresponding firm's optimal emissions, $E^{2S}(S)$.

The analytical solution of the two layer regulation when both the local agency and the firm play *à la Stackelberg* is the following:

$$
\begin{aligned}
E^{2S}(S) &= \sigma + (V_F^{2S})'(S) - \frac{\phi^2}{\phi^2 + 2(\alpha_A + \beta\phi^2)}[(V_F^{2S})'(S) - (V_R^{2S})'(S) - (V_A^{2S})'(S)],\\
P^{2S}(S) &= \frac{\phi}{\phi^2 + 2(\alpha_A + \beta\phi^2)}[(V_F^{2S})'(S) - (V_R^{2S})'(S) - (V_A^{2S})'(S)],\\
Q^{2S}(S) &= \sigma + \frac{(2\beta-1)\left(\alpha_A + \beta\phi^2\right)(V_F^{2S})'(S) + \left(\alpha_A + 2\beta\phi^2\right)(V_R^{2S})'(S) - \left(\alpha_A + \phi^2\right)(V_A^{2S})'(S)}{\beta\left(2\alpha_A + (2\beta+1)\phi^2\right)},
\end{aligned}
$$

with $V_R^{2S}(S)$ being the value function of the regulator.

In the alternative setting, namely *two-layer Nash*, the regulator plays *à la Stackelberg*, acting as the leader, against the agency and the firm, which compete *à la Nash*. The firm and the local agency choose emissions and the probability of inspection, respectively, solving problems (6) and (14) simultaneously, from which we obtain the best responses to a given emission standard, $\widehat{E}^{2N}(S;Q)$ and $\widehat{P}^{2N}(S;Q)$, respectively:[12]

$$
\widehat{E}^{2N}(S;Q) = \frac{\alpha_A(\sigma + (V_F^{2N})'(S)) + \beta\phi^2 Q}{\alpha_A + \beta\phi^2}, \tag{16}
$$

$$
\widehat{P}^{2N}(S;Q) = \phi\frac{\beta(\sigma - Q) + \beta(V_F^{2N})'(S)}{\alpha_A + \beta\phi^2}, \tag{17}
$$

where $V_F^{2N}(S)$ is the value function of the firm.

Given the two reaction functions of both the firm and the agency, the regulator determines the optimal standard Q as a function of the pollution stock, S, by solving problem (12). From this solution process, we obtain the optimal standard $Q^{2N}(S)$, which induces the firm's optimal emissions, $E^{2N}(S)$ and the agency's optimal inspection probability, $P^{2N}(S)$. The corresponding equilibrium expressions are:

$$
\begin{aligned}
E^{2N}(S) &= \sigma + (V_F^{2N})'(S) - \frac{\phi^2}{\phi^2 + 2\alpha_A}[(V_F^{2N})'(S) - (V_R^{2N})'(S)],\\
P^{2N}(S) &= \frac{\phi}{\phi^2 + 2\alpha_A}[(V_F^{2N})'(S) - (V_R^{2N})'(S)],\\
Q^{2N}(S) &= \sigma + \frac{(2\beta-1)\alpha_A(V_F^{2N})'(S) + \left(\alpha_A + \beta\phi^2\right)(V_R^{2N})'(S)}{\beta\left(\phi^2 + 2\alpha_A\right)},
\end{aligned}
$$

[12] Superscript $2N$ stands for the two-layer Nash scenario.

with $V_R^{2N}(S)$ being the value function of the regulator.

The main difference between these two alternative settings in the two layer scenario lies on the fact that in the Stackelberg game the local agency is aware of how the probability of inspection affects firm's emissions, $\widehat{E}^{2S}(S;P)$, while in the Nash game it is not. Thus, the agency acts as a dynamic player in the Stackelberg game, in contrast to its behavior in the Nash game, where it acts myopically. This has a twofold effect. On the one hand, in the Stackelberg case the agency knows that a higher inspection probability, by reducing emissions, slows down the accumulation of the pollution stock. Additionally, it can also assess that a higher inspection probability narrows the degree of non-compliance and, correspondingly, this represents an indirect negative effect on its revenues, which are a proportion of the collected fines.

This twofold effect appears when comparing the corresponding agency's best-response functions in (15) and (17). Assuming, as it will be shown later, that the agency's marginal valuation of pollution is positive, $(V_A^{2S})'(S) > 0$, it has a double incentive to reduce monitoring when playing *à la Stackelberg* versus playing *à la Nash*. First, less monitoring speeds up pollution accumulation, which is positively valued by a dynamic agency, and this is reflected by introducing the term $(V_A^{2S})'(S) > 0$ in (15). Second, less monitoring widens non-compliance, and the corresponding effect on revenues is anticipated by an agency playing *à la Stackelberg*, but not if it plays *à la Nash*. This is reflected by a higher denominator in (15) than in (17). Finally, the incentives to reduce monitoring activities by the agency in the Stackelberg game, induce higher emissions on the firm.

4 Results

In this section, we present the results of the different scenarios described previously, as well as a thoughtful comparison among them. Our research task is to analyze the conditions under which it is socially beneficial to delegate the monitoring activity to the local agency, even when its objective departs sharply from social welfare maximization. Interestingly, we find that this importantly depends on the type of strategic interaction between the local agency and the polluting firm.

Name	Meaning	Benchmark value	Feasible range
σ	economy size	1	$\sigma > 0$
α_A	agency monitoring cost parameter	1	$\alpha_A > 0$
ϕ	intensity of the fine	1	$\phi \geq 0.02$
β	share of collected fines transferred to the agency	0.6	$\beta \in [0.43, 1]$
γ	environmental damage parameter	0.005	$\gamma \in (0, 0.007]$
δ	pollution stock depreciation rate	0.05	$\delta \in [0.04, 1]$
ρ	time discount rate	0.05	$\rho \in [0.02, 1]$

Table 1: Description of model parameters

The complete characterization of the equilibrium strategies in the different scenarios requires the computation of the different agents' value functions. Given the linear-quadratic structure of the game, we focus on linear strategies and, in order to find the solution associated with a stable steady state which gives the highest value function to the regulator, among the different candidates for a solution, we need to rely on numerical simulations.

We have carried out a numerical analysis with the parameter values of our problem, given in Table 1. For the parameter values in this table the solution is feasible in all three scenarios. Feasibility is characterized by positive emissions, positive and less than one inspection probability, and positive emission limit lower than actual emissions. Moreover, both the firm and the agency obtain positive profits at any time. We define the feasible set as the set where all parameters take their benchmark values (given in the third column of Table 1), except one of them which moves in its feasible range (given in the last column of this table).

Parameter σ is a scale parameter which represents the economy size and it is normalized to one, with no qualitative effect on the results. Parameter α_A, which measures how costly monitoring activities are for the agency is also normalized to one. Parameters ϕ, the fine per unit, is normalized to one. and β are linked to the penalization mechanism. The share of the collected fines transferred to the agency, β, takes an intermediate value (0.6), between the minimum compatible with a implementable quota, $\beta = 0.43$ and the maximum possible share, 1. The last three rows of the table refer to the dynamics of the model. The parameter measuring the environmental damage for society, γ, takes an intermediate value between 0 and the maximum compatible with a implementable quota. $\gamma = 0.005$ helps us highlight the main result. The pollution depreciation rate and the discount rate are sufficiently large to ensure a credible quota ($\delta > 0.04$ and $\rho > 0.02$) and sufficiently small to have persistent problem with agents concerned about the future. Additionally, we carry out an exhaustive sensitivity analysis across the feasible ranges for all parameters in Table 1.

In Subsection 4.1, we present the case in which both the regulator and the local agency face exactly the same monitoring costs. In Subsection 4.2, we analyze the case of different inspection costs for the two regulatory bodies.

4.1 Equal monitoring costs

We start with the case where the two regulatory bodies, namely, the central regulator and the local agency, face exactly the same monitoring costs, that is, $\alpha_L = \alpha_A$.

The following results summarize the comparisons of the three alternative scenarios along the different dimensions for the set of parameter values presented in Table 1.

Result 1. *For any value of the pollution stock $S \geq 0$, the equilibrium standard and inspection probability under the alternative specifications with one layer (1), two-layer Stackelberg (2S) and two-layer Nash (2N) relate as follows:*

$$Q^1(S) > Q^{2N}(S) > Q^{2S}(S),$$
$$P^1(S) > P^{2N}(S) > P^{2S}(S),$$

resulting in the following ranking of emissions and degree of non-compliance:

$$E^1(S) < E^{2N}(S) < E^{2S}(S),$$
$$0 = E^1(S) - Q^1(S) < E^{2N}(S) - Q^{2N}(S) < E^{2S}(S) - Q^{2S}(S).$$

When the inspection activity is decentralized, the resulting inspection probability is lower than when the monitoring activity is centralized. The reduction of the inspection probability with decentralization is particularly salient in the two-layer Stackelberg case, where the agency takes the reaction of the firm into account when selecting the inspection probability. As explained in the previous section, the agency is farsighted and takes into account that

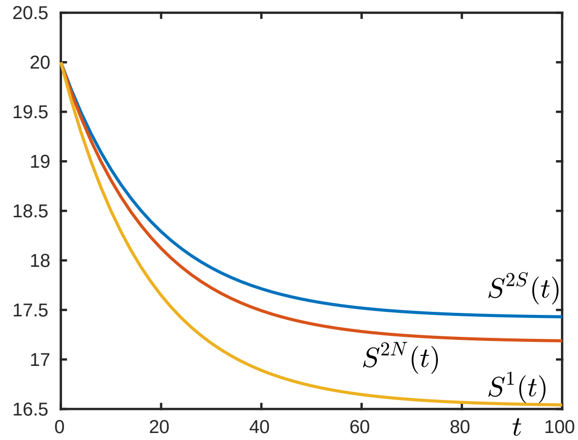

Figure 1: Optimal time paths for the pollution stock.

non-compliance (and hence revenues) is larger by reducing monitoring. Note also that the ranking of emissions and the degree of non-compliance is opposite to the ranking of inspection probabilities.

Figure 1 displays the optimal time paths for the pollution stock for the three alternative specifications taking into account the parameter values in the third column in Table 1 and an initial condition $S_0 = 20$. The figure illustrates that pollution decreases the most in the one layer case and the least in the two-layer Stackelberg case. This implies that the pollution at the steady state is highest in Stackelberg and lowest with a single regulator. Interestingly, the comparisons in Result 1 are valid at any time across the optimal temporal trajectories. Again, all this holds varying one parameter as described in the last column in Table 1 considering all other parameters fixed.

Now we turn to the comparison of social welfare levels across the three scenarios for an initial level of the pollution stock. Social welfare is defined as the discounted accumulated amounts of final output minus the damage from pollution and the cost of inspection. This corresponds to the value function of the regulator in the one-layer case, and the sum of the regulator and agency's value functions in the two-layer case. The result is the following:

Result 2. *For any initial value of the pollution stock $S_0 \geq 0$, the corresponding social welfare levels in the one-layer and two-layer scenarios relate as follows:*

1. $SW^{2S}(S_0) < SW^{2N}(S_0)$, for all parameters in the feasible set.

2. (a) $SW^{1}(S_0) < SW^{2S}(S_0)$, for all parameters in the feasible set, except:

(b) $SW^1(S_0) \in [SW^{2S}(S_0), SW^{2N}(S_0)]$, if $\alpha_A \leq 0.65$ or $\beta \geq 0.72$ or $\phi \geq 0.62$ or $\gamma \in [0.0023, 0.0037]$ or $\delta \in [0.063, 0.087]$ or $\rho \in [0.072, 0.11]$.

(c) $SW^{2N}(S_0) < SW^1(S_0)$, if $\gamma < 0.0023$ or $\delta > 0.087$ or $\rho > 0.11$,

where $SW^1(S_0) = V_R^1(S_0)$, $SW^{2S}(S_0) = V_R^{2S}(S_0) + V_A^{2S}(S_0)$, $SW^{2N}(S_0) = V_R^{2N}(S_0) + V_A^{2N}(S_0)$.

The first part of Result 2 states that from a social welfare point of view, playing *à la Nash* is preferred to playing *à la Stackelberg*. This is due to the fact that strategies under one-layer are closer to those under two-layer Nash than under two-layer Stackelberg, as shown in Result 1. Under Stackelberg the agency knows that the inspection probability negatively affects emissions, and hence, the level of non-compliance and the accumulation of the pollution stock. As it will be explained in Section 5, this gives the agency an extra-incentive to reduce monitoring effort which is not present under Nash.

The second part of Result 2 compares social welfare in the one-layer and the two-layer cases. Part 2.a states that decentralization can be better-off under Stackelberg and under Nash, even under the assumption of equal monitoring costs. This result is surprising, since the central regulator and the local agency face very different objectives. Under decentralization, the agency sets a lower inspection probability and hence the firm chooses higher emissions than their corresponding values in the one-layer specification. The positive effects associated with lower inspection probability (less monitoring cost) and larger emissions decrease at the margin, while the negative effect of higher pollution marginally increases. When the distance between the strategies under one- and two-layer specifications is not too large, the positive effects dominate and decentralization is social welfare improving. Part 2.b shows that decentralization is worse under Stackelberg, when the monitoring cost, the environmental damage, the depreciation rate or the discount rate are small or when the intensity of the fine or the share of the fine transferred to the agency are large. Finally, point 2.c highlights that decentralization under Nash can also be worse for society than the one-layer case, when the environmental problem is very mild, or alternatively, when the depreciation or the discount rates are sufficiently large.[13]

[13]Recall that we consider parameter values in Table 1, moving only one parameter at a time. The result that decentralization under Nash is not preferred to one-layer also occurs for more harmful environmental problems, when moving other parameters (i.e. increasing β or ϕ or reducing α_A).

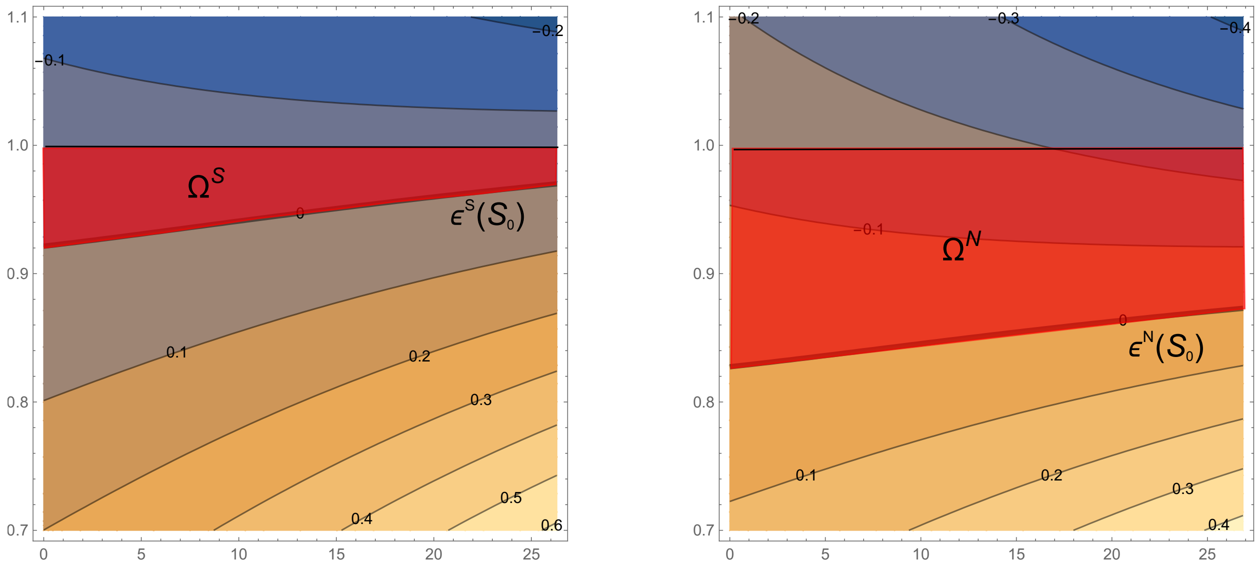

Figure 2: Social welfare: one-layer versus two-layer Stackelberg (left); Nash (right).

4.2 Different monitoring costs

In this section, we relax the assumption of identical monitoring costs for the agency and the regulator. We assume that the central regulator's monitoring costs are ε times those faced by the local agency, that is:

$$\alpha_L = \varepsilon \alpha_A. \tag{18}$$

The case $\varepsilon > 1$ corresponds to the situation when the regulator has a cost disadvantage in monitoring activities. The regulator and the agency are equally efficient in these activities for $\varepsilon = 1$, which is the case we have discussed in the previous section. Finally, $\varepsilon < 1$ corresponds to the case when the regulator is actually more efficient than the agency.

Figure 2 depicts the level curves of the gap between social welfare in the one-layer and the decentralized cases, as a function of the initial pollution stock, S_0 in the x-axis, and parameter ε that measures the monitoring cost difference between the two regulatory bodies in the y-axis. The curves are computed taking into account parameters in Table 1. The initial pollution stock runs from 0 to its maximum value compatible with feasible strategies. The left panel of Figure 2 depicts the Stackelberg case, while the right panel shows the Nash case. The one-layer case provides higher (lower) social welfare than the two-layer case for positive (negative) level curves. Note that in the case of identical monitoring costs for the regulator and the agency, $\varepsilon = 1$, Figure 2 is consistent with point 2.a in Result 2: For $\varepsilon = 1$, level curves are negative for equal monitoring costs both under Stackelberg and Nash.

Let $\varepsilon^i(S_0)$, $i \in \{S, N\}$ denote the zero level curve, that is, the locus where one- and two-layer provide identical social welfare. Decentralization is better (worse)-off above (below) this curve. Interestingly, in both figures, the zero level curves, $\varepsilon^S(S_0)$ and $\varepsilon^N(S_0)$, lie below 1. This means that decentralization is social welfare improving, even if the regulator has a cost advantage over the agency. Areas Ω^S and Ω^N depict the regions (under Stackelberg and Nash, respectively) where society is better-off under decentralization even when the regulator has a cost advantage. Note that the area is wider under Nash than under Stackelberg. That is, decentralization is welfare improving for a higher regulator's cost advantage under the Nash case than the Stackelberg case. Moreover, under both scenarios the lower the initial pollution stock (or the less worrying the environmental problem), the larger the cost advantage compatible with decentralization being better-off.

5 Why can decentralization be social welfare improving?

This section shows that the crucial reason why decentralization is welfare improving (even when the local agency faces an inspection cost disadvantage) is that the game is played in a dynamic setting. Our analysis revolves around three key issues presented, respectively, on Subsections 5.1 to 5.3. In Subsection 5.1, we analyze the impact of assuming a dynamic instead of a myopic firm on the results, both under Nash and Stackelberg. In Subsection 5.2, we study the impact of considering a dynamic versus a myopic agency, exclusively in the Stackelberg case (in the Nash scenario, the local agency is, by definition, myopic). In Subsection 5.3, assuming that both the firm and the agency are myopic, we analyze the impact of having a leadership advantage (i.e., the difference between Stackelberg and Nash when both players are myopic). In Subsection 5.4, we consider all the three elements together to visualize the reasons why the two-layer scenarios can be welfare improving. Appendix C presents a robustness analysis with respect to the main parameters of the model. Table 3 in this Appendix summarizes the effect of the main parameters on the three impacts explained in this section.

5.1 Dynamic versus myopic firm: firm's proactive engagement

In this subsection, we analyze the impact on the results when the firm shifts from dynamic to myopic behavior, while the agency's mode of play remains unaltered. We first consider the two-layer Stackelberg case, and we then analyze the two-layer Nash scenario.

When the agency and the firm play *à la Stackelberg*, the local agency cannot be myopic if the firm behaves dynamically. This is so because the agency incorporates the firm's state-dependent reaction function into its own optimization problem. In consequence, to characterize the role of the dynamic behavior of the firm, we compare the cases where both the firm and the agency are dynamic (case DD) with the case where only the firm switches to myopic behavior (case DM).[14] The main difference between the two cases is that a dynamic firm knows that the regulatory policy depends on the pollution stock and, hence, it anticipates that current emissions accumulate as stock pollutant influencing the regulatory policy in the future. Thus, firm's emissions differ when the firm acts in a farsighted or a myopic way, as follows:[15]

$$\widehat{E}^{2S}_{DD}(S;Q) = \frac{\phi^2\beta Q + (\alpha_A + \beta\phi^2)[\sigma + (V^{2S}_{FDD})'(S)] + \phi^2 (V^{2S}_{ADD})'(S)}{\alpha_A + 2\beta\phi^2}, \tag{19}$$

$$\widehat{E}^{2S}_{DM}(S;Q) = \frac{\phi^2\beta Q + (\alpha_A + \beta\phi^2)\sigma + \phi^2 (V^{2S}_{ADM})'(S)}{\alpha_A + 2\beta\phi^2}. \tag{20}$$

When the firm is farsighted and anticipates that current emissions will imply a tighter future policy, it has an incentive to behave proactively and self-regulate. The term that reflects this anticipation is $(V^{2S}_{FDD})'(S) < 0$, which suggests that the firm values pollution negatively. Thus, a self-regulation term appears in (19) but not in (20). The intensity of the

[14] In this section, D and M are used to describe dynamic and myopic behavior, respectively. The first entry always refers to the local agency, while the second entry refers to the firm.

[15] These expressions correspond to the best-response emissions anticipated by the regulator. Expressions (19) and (20) are obtained by substituting the agency's best-response inspection probability into the firm's best-response emissions.

firm's self-regulation mechanism reads:

$$FSR^S = \frac{\alpha_A + \beta\phi^2}{\alpha_A + 2\beta\phi^2}.$$

Hence, *ceteris paribus*, for the same standard and agency's valuation of pollution, we have $\widehat{E}^{2S}_{DD}(S;Q) < \widehat{E}^{2S}_{DM}(S;Q)$, which allows the agency to reduce monitoring activities when the firm is farsighted, i.e., $\widehat{P}^{2S}_{DD}(S;Q) < \widehat{P}^{2S}_{DM}(S;Q)$.

Conversely, when the agency and the firm play *à la Nash*, the agency never behaves dynamically. To analyze the role of the dynamic behavior of the firm, we now compare the results when the firm shifts from dynamic to myopic, while the agency behaves myopically (these correspond to cases MD and MM, respectively):

$$\widehat{P}^{2N}_{MD}(S;Q) = \frac{\phi\beta}{\alpha_A + \beta\phi^2}\left[\sigma - Q + (V^{2N}_{FMD})'(S)\right], \quad \widehat{P}^{2N}_{MM}(S;Q) = \frac{\phi\beta}{\alpha_A + \beta\phi^2}(\sigma - Q),$$
$$\widehat{E}^{2N}_{MD}(S;Q) = \frac{\alpha_A\sigma + \beta\phi^2 Q + \alpha_A(V^{2N}_{FMD})'(S)}{\alpha_A + \beta\phi^2}, \quad \widehat{E}^{2N}_{MM}(S;Q) = \frac{\alpha_A\sigma + \beta\phi^2 Q}{\alpha_A + \beta\phi^2}.$$

Again, the self-regulation mechanism induces a reduction in emissions and monitoring activities when the firm behaves in a dynamic way, although the effect is less pronounced than in the Stackelberg case. The intensity of the firm's self-regulation mechanism under Nash reads:

$$FSR^N = \frac{\alpha_A}{\alpha_A + \beta\phi^2} < FSR^S.$$

5.2 Dynamic versus myopic agency: agency's dynamic incentive

When the agency and the firm play *à la Nash*, the agency never behaves dynamically. Hence, the effect of the dynamic behavior of the agency on the results can only be analyzed in the two-layer Stackelberg scenario. In this scenario, the agency cannot be myopic if the firm is dynamic and, therefore we compare the results when the agency shifts from dynamic to myopic, assuming that the firm is myopic (these correspond to cases DM and MM, respectively). Hence, the firm ignores that the regulatory policy depends on the pollution stock and the main difference between the two cases is that in DM the agency is aware that the standard depends on the pollution stock. Moreover, it anticipates how the inspection probability influences emissions and hence pollution accumulation.

Monitoring efforts in the two cases are given by:

$$\widehat{P}^{2S}_{DM}(S;Q) = \frac{\phi}{\alpha_A + 2\beta\phi^2}\left[\beta(\sigma - Q) - (V^{2S}_{ADM})'(S)\right],$$
$$\widehat{P}^{2S}_{MM}(S;Q) = \frac{\phi\beta}{\alpha_A + 2\beta\phi^2}(\sigma - Q).$$

When the agency is farsighted it knows that current monitoring disincentives current emissions and pollution accumulation. A lower pollution stock narrows future non-compliance and hence future revenues. Thus, *ceteris paribus*, farsightedness induces a reduction in monitoring activities. This effect is captured by the agency's dynamic incentive which appears in $\widehat{P}^{2S}_{DM}(S;Q)$, but not in $\widehat{P}^{2S}_{MM}(S;Q)$. The intensity of this effect is given by:

$$ADI = \frac{\phi}{\alpha_A + 2\beta\phi^2}.$$

Less monitoring leads to higher emissions $\widehat{E}^{2S}_{DM}(S;Q) > \widehat{E}^{2S}_{MM}(S;Q)$. Since the agency has less incentive to monitor, the regulator needs to tighten the emission standard, i.e. the two regulatory instruments are substitutes here: $Q^{2S}_{DM}(S) < Q^{2S}_{MM}(S)$.

5.3 Stackelberg versus Nash: agency's static incentive

A leading agency knows how the inspection probability affects firm's best-response emissions, while this is not the case when the agency and the firm play simultaneously. This gives the agency an incentive to reduce monitoring activities seeking to increase emissions, non-compliance, fines and current revenues. We denote this effect as the agency's static incentive. The gap between the inspection probabilities in (15) and (17) encompasses this static incentive as well as the agency's dynamic incentive explained in Subsection 5.2. To isolate the static incentive, we compare inspection probabilities under Nash and Stackelberg, assuming that both the firm and the agency are myopic: $\widehat{P}^{2S}_{MM}(S;Q)$ versus $\widehat{P}^{2N}_{MM}(S;Q)$. To have a measure of the intensity of the agency's static incentive we compute:

$$ASI = \frac{\widehat{P}^{2N}_{MM}(S;Q)}{\widehat{P}^{2S}_{MM}(S;Q)} = \frac{\alpha_A + 2\beta\phi^2}{\alpha_A + \beta\phi^2} > 1.$$

Less monitoring when the agency has Stackelberg leadership increases emissions with respect to the Nash scenario, i.e., $\widehat{E}^{2N}_{MM}(S;Q) < \widehat{E}^{2S}_{MM}(S;Q)$.

5.4 Centralization versus decentralization

The three effects related to the firm and the local agency being farsighted or myopic, and playing simultaneously or hierarchically, are summarized in Table 2. They help explaining our main result, highlighted in Figure 2: decentralization can be welfare improving even if there is a cost advantage for the regulator (sets Ω^S and Ω^N, where $\varepsilon^S(S_0)$, $\varepsilon^N(S_0) < 1$).

	Agency's static incentive	Agency's dynamic incentive	Firm's self-regulation
Stackelberg	Yes	Yes	Big
Nash	No	No	Small

Table 2: Factors influencing social welfare comparison.

As already stated, a dynamic firm has an incentive to self-regulate its emissions (Subsection 5.1.). This is welfare improving because the firm aligns incentives with those of society. On the other hand, an agency whose objective deviates from that of the regulator has dynamic and static incentives to cut monitoring activities (Subsections 5.2 and 5.3, respectively), with the corresponding negative effect on social welfare.

With all this analysis, we are now ready to explain why the fully dynamic two-layer scenarios can be superior to the one-layer case in social welfare terms. Figure 3 represents the corresponding threshold cost disadvantages/advantages for the regulator ($\varepsilon > 1/\varepsilon < 1$) above which the one-layer scenario is socially preferred, for all the possible scenarios analyzed so far (from fully dynamic analyzed in Section 3 to myopia by the firm or by both the firm and the local agency, analyzed in Subsections 5.1 to 5.3). First note that both $\varepsilon^S_{DD}(S_0)$ and $\varepsilon^N_{MD}(S_0)$ match exactly $\varepsilon^S(S_0)$ and $\varepsilon^N(S_0)$ in Figure 2 (left and right, respectively).

In order to see why $\varepsilon_{MD}^{N}(S_0)$ lies below $\varepsilon_{DD}^{S_0}(S_0)$ (both below $\varepsilon = 1$), it is very useful to start by looking at the two fully myopic cases, $\varepsilon_{MM}^{S}(S_0)$ and $\varepsilon_{MM}^{N}(S_0)$, whose difference is the effect of the agency's static incentive. The difference between the two curves shows the negative effect that the leading agency places on social welfare in the Stackelberg case by reducing monitoring activities seeking to increase current emissions, non-compliance, fines and current revenues. This agency's static incentive (ASI) effect is prominent in the figure and helps to explain the pure difference between Stackelberg and Nash in a myopic context (Subsection 5.3). Note that the agency's dynamic incentive (ADI) slightly contributes to worsen social welfare even more in the Stackelberg scenario, when we consider that the agency anticipates how the inspection probability influences the accumulation of pollution and future revenues (Subsection 5.2). The firm's self regulatory mechanism present when the firm acts dynamically, analyzed in Subsection 5.1, helps to improve social welfare conditions, hence moving in the opposite direction to both ASI and ADI. As analyzed before, the firm's self regulation is more prominent in the Stackelberg scenario, FSR^S, than in the Nash scenario, FSR^N. This effect offsets the two agency's incentives in the two-layer Stackelberg scenario, which can (as in the case in the figure) end up being socially preferred to the one-layer case. The Nash scenario starts from a much better situation than the Stackelberg case, since both ASI and ADI are not present. Hence, the firm's self regulation FSR^N due to the firm acting dynamically can only improve social welfare, resulting, in some circumstances as those of the figure, in $\varepsilon_{MD}^{N}(S_0)$ being even below 1. Although the firm's self-regulation is softer under Nash, the lack of agency's incentives implies that the decentralization is social welfare improving for a greater regulator's cost advantage under this scenario than under Stackelberg, i.e. $\varepsilon_{MD}^{N}(S_0) < \varepsilon_{DD}^{S}(S_0)$.

Figure 3 allows us to illustrate that decentralization can be better-off only in a dynamic setting. In particular, decentralization is not preferred in a static setting, where the regulator is only concerned about the actual flow of emissions (not the pollution stock).

Remark 1. Decentralization in a static game, where there is no pollution accumulation is never welfare improving, assuming that the regulator and the agency have identical monitoring costs.

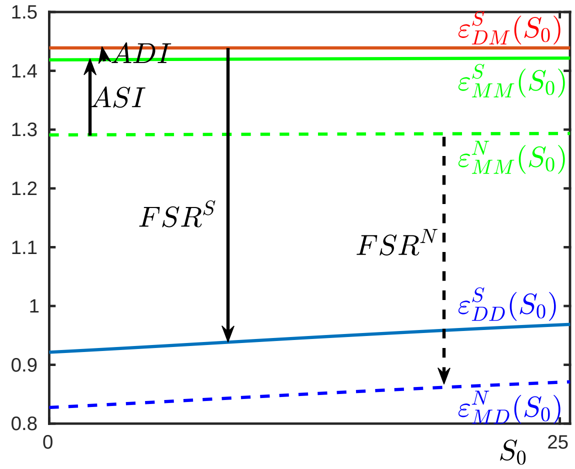

Figure 3: Cost disadvantage for decentralization to be Social Welfare neutral

Proof. See Appendix D. □

6 Extended model: the agency also cares about the environment

In this section, we extend the model assuming that the agency is concerned not only on maximizing revenues coming from fines net of monitoring costs, but also on environmental quality.[16] Thus, we consider that the agency maximizes a weighted combination between own net revenues and the environmental damage suffered by society. The objective function of the agency in (14) now reads:

$$\theta \left[\beta P \phi(E - Q) - \alpha_A \frac{P^2}{2} \right] - (1 - \theta)\gamma \frac{S^2}{2}, \quad \theta \in (0, 1]. \tag{21}$$

The particular case $\theta = 1$ recovers previous analysis. As θ decreases to 0, more weight is assigned to the environmental damage and less to the agency's revenues.

Provided that the environmental damage does not depend on the monitoring effort, the equilibrium strategies in the two-layer Nash scenario do not change under this new specification. Conversely, in the two-layer Stackelberg, the agency anticipates how monitoring

[16] We thank an anonymous reviewer for bringing this extension to our attention.

will affect pollution accumulation, and therefore the environmental damage. In consequence, weighing environmental damage gives the agency an incentive to raise monitoring activities, which in turn, affects the other equilibrium strategies. Thus, the higher the weight given to the environmental damage $1-\theta$, the greater the inspection probability, which allows the regulator to free-ride on this larger monitoring effort and fix a laxer quota. All in all, the firm will reduce emissions slightly the level of non-compliance more strongly.

Recall that social welfare is defined by the addition of production minus the damage from pollution and the monitoring costs of inspection. Notice that given the new definition of the agency's objective function, social welfare no longer corresponds to the sum of regulator and agency's value functions in the two-layer specifications.

When playing *à la Nash* the weights that the agency gives to its own revenues versus environmental damage have no effect on its strategies and hence on social welfare. Thus, Result 2 concerning the comparison between one-layer and two-layer Nash remains unaltered regardless the value of θ. This is not true when playing *à la Stackelberg*. In Figure 4 we present the level curves that illustrate the gap between the one-layer and the two-layer Stackelberg cases in social welfare terms. The horizontal axis depicts the initial pollution stock running from zero to its maximum value compatible with a feasible solution. The vertical axis depicts ε, the cost disadvantage in monitoring activities for regulator ($\varepsilon > 1$) or agency ($\varepsilon < 1$). Graphs moving top-left to bottom-right present the comparison for θ decreasing[17] from 1 to 0.6 (with step 0.1). The top-left graph depicts the case with $\theta = 1$, which corresponds to the previous analysis in Figure 2 (left), presented here to easy the comparison. The charts in the first line, $\theta = \{1, 0.9, 0.8\}$, show that the set Ω^S widens with θ, while the charts in the second line, $\theta = \{0.7, 0.6\}$, show that this set narrows with θ. In consequence, the weight given to the environmental damage by the agency does not have a monotonous effect. Decentralization becomes more attractive when the environmental damage is taken into consideration starting from very low values. However, decentralization becomes less attractive when its weight rises from an already large value.

By reducing θ we observe that the static and dynamic incentives to cut monitoring under

[17]Notice that reducing θ is equivalent to increasing the weight given to the pollution damage, $1-\theta$. We skip values of θ lower or equal than 0.5, because the solution is unfeasible.

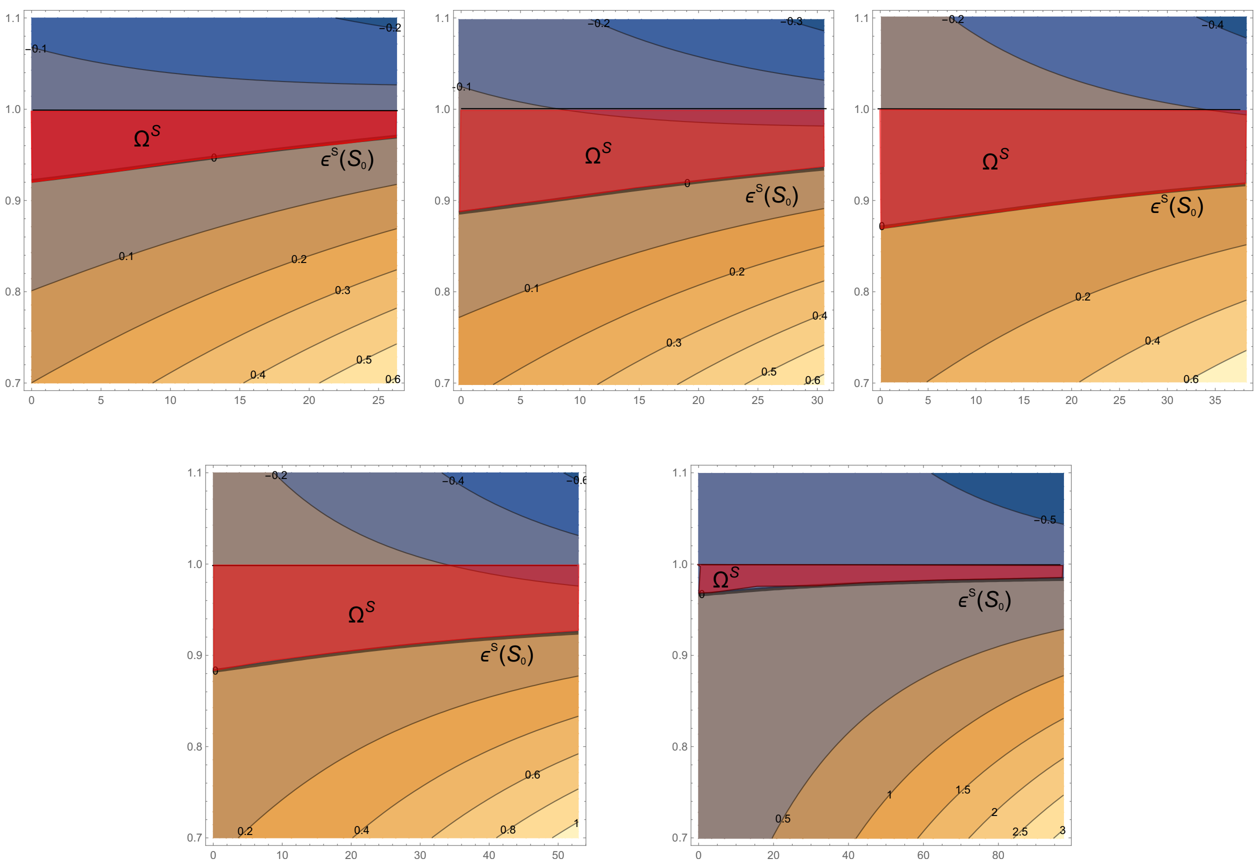

Figure 4: Social welfare: one-layer versus two-layer Stackelberg for different values of θ.

the Stackelberg scenario are offset by the rising willingness to control emissions. Monitoring activities under Stackelberg increase toward their value in the one-layer case, reaching and even overcoming their value under Nash. Correspondingly, Stackelberg emissions decrease towards the one-layer emission even below those under Nash. Starting from $\theta = 1$, reductions in θ approach strategies under Stackelberg to those under Nash. Thus, in the comparison between one-layer and two-layer Stackelberg, the positive effects of less monitoring and more emissions improve with respect to the negative effect of more pollution. However, when θ is already small, further decreases imply strategies too close to those under one-layer. And this makes less and less likely for decentralization to overcome one-layer in social welfare terms, provided that the agency faces higher monitoring costs than the regulator, $\varepsilon < 1$.[18]

7 Conclusions

The design of environmental policy and enforcement is a key issue for both academics and policy-makers, particularly in dynamic settings where damages depend on the accumulation of pollution. A prominent example of such challenges arises in water pollution control, where contaminants accumulate gradually and their effects—such as biodiversity loss or risks to human health—depend critically on stock levels. Understanding how decentralization shapes incentives and enforcement over time is crucial for effective environmental governance.

While the literature has extensively studied the allocation of environmental responsibilities across levels of government, it has largely overlooked dynamic regulatory problems dependent on the stock of accumulated pollution in such hierarchical contexts. By incorporating pollution accumulation and forward-looking behavior into a possibly decentralized enforcement setting, we contribute to filling a relevant gap in the literature.

Specifically, we analyze a regulatory problem in which the environmental policy consists of an emission standard and an inspection probability, both set over time as functions of the pollution stock. The regulator may act as a single authority choosing both instruments or delegate inspection to a local agency. In our decentralized baseline model, the local agency

[18] Alternatively, one could consider that the agency is concerned not only about the environmental damage but also about final output. Under this alternative specification we obtain qualitatively similar results (available upon request).

maximizes fine revenues net of inspection costs, though we later consider the case where it is also concerned about environmental damages.

Our main finding is that the social preference for decentralization is crucially affected by the firm proactively engaging in lower emissions when being farsighted, and hence being aware that its decisions affect the stock of pollution and future regulation. A decentralized agency takes more advantage of this fact than a central regulator by monitoring less, which hence leads to higher emissions. Gains from lower monitoring costs and higher production can overcome the losses from a greater environmental damage, leading to a better situation in social-welfare terms. The preference for decentralization is more likely under Nash than under Stackelberg. The reason is that in the latter, the agency has a static and dynamic incentive to further reduce monitoring effort. For large deviations from the one-layer case gains become weaker and losses stronger. Scenarios where decentralization is superior to the one-layer case are more likely to be those with large monitoring costs, large environmental damages, small fines per unit of violation, and small proportion of the collected fines passed to the local agency to finance its monitoring activity.

While our analysis is theoretical, it addresses real-world institutional arrangements commonly observed in environmental governance. The two-layer framework, in which inspection is delegated to a local agency while standards are set by a higher-level authority, is especially relevant in countries with federal or decentralized administrative structures—such as the United States, the European Union, India, or China—where regional agencies are often tasked with enforcement. Conversely, more centralized systems like Australia typically embody the one-layer case, in which a single authority manages both standard-setting and enforcement. Although systematic evidence on monitoring cost differences across governance layers is limited, case studies (for example, the case of water pollution control in Southern China, as documented by BSR, n.d.) suggest that local enforcement may combine informational advantages with resource constraints. Our model accommodates these differences through a flexible cost parameter (ε) but, importantly, we show that decentralization can outperform centralization even when local inspection is more expensive than centralized enforcement. This suggests that the potential benefits of dynamic incentives can outweigh higher operational costs, reinforcing the relevance of our framework for understanding the trade-offs implicit in real-world institutional design.

Several extensions of our work are possible. One natural extension would be to give entrance to a multiplicity of polluting symmetric firms, both with and without strategic competition. Another interesting direction for our work could be to endogenize the share of the total collected fines from non-compliance transferred from the regulator to the local agency (β), as it is the case in several works that deal with hierarchical settings in the static context (see for example, Jones and Scotchmer, 1990). This is, however, a very complex issue in the dynamic problem we handle, since the assumption on the linear-quadratic structure of the game would be lost, and this would require the development of numerical algorithms to characterize the equilibria of the game outside the linear-quadratic specification. More generally, the need to preserve the linear-quadratic structure of the model is precisely the key methodological limitation of our study. In particular, the quadratic specification of damage and inspection cost functions, as well as the linear formulation of fines are dictated by this requirement. Progress on computational methods for solving differential games with more general functional forms would open the door to relaxing these assumptions and exploring richer regulatory environments.

Appendix

Appendix A: One-layer regulation

The firm's dynamic maximization problem is given in (6). The Hamilton-Jacobi-Bellman (HJB) equation associated with this problem reads:

$$\rho V_F^1(S) = \max_E \left\{ \sigma E - \frac{E^2}{2} - P\phi(E - Q) + (V_F^1)'(S)(E - \delta S) \right\}.$$

The best-response in (7), $\widehat{E}^1(S;P)$, stems from the first-order condition (FOC) for an interior solution of the RHS of the HJB equation.

The regulator's dynamic maximization problem is given in (8). The HJB equation associated with this problem reads:

$$\rho V_R^1(S) = \max_{Q,P} \left\{ \sigma \widehat{E}^1(S;P) - \frac{(\widehat{E}^1(S;P))^2}{2} - \gamma \frac{S^2}{2} - \alpha_L \frac{P^2}{2} + (V_R^1)'(S)(\widehat{E}^1(S;P) - \delta S) \right\}.$$

Because $\widehat{E}^1(S;P)$ does not depend on Q, the maximization problem on the RHS of this equation is also independent of Q. We assume a limit identical to the emissions so that there is full compliance. The FOC for an interior solution of this problem with respect to P is (10). Plugging this expression in (7) one gets the equilibrium emissions and hence the standard in (9) .

The complete characterization of the equilibrium strategies requires determining the value function of the firm and the regulator. Given the linear-quadratic structure of the differential game we conjecture quadratic value functions for the two players:

$$V_F^1(S) = a_{2F}^1 \frac{S^2}{2} + a_{1F}^1 S + a_{0F}^1, \qquad V_R^1(S) = a_{2R}^1 \frac{S^2}{2} + a_{1R}^1 S + a_{0R}^1.$$

Taking these functions into account in the equilibrium strategies (9), and (10), substituting in the HJB equations above and identifying coefficients, one gets a system of six algebraic Riccati equations. This system presents a multiplicity of solutions and given their complexity, to discriminate among them one needs to rely on numerical simulations. We select the stable solution which gives the highest value function to the leader.

Appendix B: Two-layer regulation

Appendix B1: Two-layer Stackelberg

The firm's dynamic maximization problem is identical to the one-layer case and the firm's best response in given in (13). The local agency solves problem (14) by replacing firm's emissions by the best response $\widehat{E}^{2S}(S;P)$. The HJB equation associated with the agency's problem reads:

$$\rho V_A^{2S}(S) = \max_P \left\{ \beta P \phi(\widehat{E}^{2S}(S;P) - Q) - \alpha_A \frac{P^2}{2} + (V_A^{2S})'(S)(\widehat{E}^{2S}(S;P) - \delta S) \right\}.$$

From the FOC one gets the inspection probability as a function of the pollution stock and the standard, given in (15).

The regulator's maximization problem is given in (12), taking into account the agency's and the firm's local best-response functions, $\widehat{P}^{2S}(S;Q)$ and $\widetilde{E}^{2S}(S;Q) = \widehat{E}^{2S}(S;\widehat{P}^{2S}(S;Q))$. The HJB associated with this maximization problem is:

$$\rho V_R^{2S}(S) = \max_Q \left\{ \sigma \widetilde{E}^{2S}(S;Q) - \frac{(\widetilde{E}^{2S}(S;Q))^2}{2} - \gamma \frac{S^2}{2} - \beta \widehat{P}^{2S}(S;Q) \phi(\widetilde{E}^{2S}(S;Q) - Q) + \right.$$
$$\left. (V_R^{2S})'(S)(\widetilde{E}^{2S}(S;Q) - \delta S) \right\}.$$

The first-order condition for this problem gives equilibrium policy $Q^{2S}(S)$, and from it, the equilibrium emissions and inspection probability, $E^{2S}(S)$ and $P^{2S}(S)$.

Appendix B2: Two-layer Nash

The Nash equilibrium of the game played by the firm and the agency is characterized by the solution of the players' maximization problems (6) and (14). This equilibrium satisfies the following HJB equations:

$$\rho V_F^{2N}(S) = \max_E \left\{ \sigma E - \frac{E^2}{2} - P\phi(E-Q) + (V_F^{2N})'(S)(E-\delta S) \right\},$$
$$\rho V_A^{2N}(S) = \max_P \left\{ \beta P\phi(E-Q) - \alpha_A \frac{P^2}{2} + (V_A^{2N})'(S)(E-\delta S) \right\}.$$

From the optimality conditions, the firm and the agent best response (to the emission quota), $\widehat{E}^{2N}(S;Q)$ and $\widehat{P}^{2N}(S;Q)$, are given in (16) and (17).

The regulator's maximization problem is given in (12), taking into account the agency's and the firm's local best-response functions, $\widehat{P}^{2N}(S;Q)$ and $\widehat{E}^{2N}(S;Q)$. The HJB associated with this maximization problem is:

$$\rho V_R^{2N}(S) = \max_Q \left\{ \sigma \widehat{E}^{2N}(S;Q) - \frac{(\widehat{E}^{2N}(S;Q))^2}{2} - \gamma \frac{S^2}{2} - \beta \widehat{P}^{2N}(S;Q)\phi(\widehat{E}^{2N}(S;Q) - Q) + \right.$$
$$\left. (V_R^{2N})'(S)(\widehat{E}^{2N}(S;Q) - \delta S) \right\}.$$

The first-order condition for this problem gives equilibrium policy $Q^{2N}(S)$, and from it, the equilibrium emissions and inspection probability, $E^{2N}(S)$ and $P^{2N}(S)$.

Both in the Stackelberg and in the Nash cases, the final characterization of the value functions, and hence of the equilibrium strategies, follow the same procedure as the one described in the one-layer case.

Appendix C: Sensitivity analysis

As explained in the main text, self-regulation is defined as the reduction in emissions due to a dynamic firm's valuation of the environment. Recall that the firm's best response emissions is:

$$\widehat{E}^i(S;P) = \sigma - \phi P + (V_F^i)'(S), \quad i \in \{2S, 2N\}$$

According to this expression, if the inspection probability was an exogenous constant, then the intensity of self-regulation would be equal to one (one unit increase in the valuation of the environment, $|(V_F^i)'(S)|$, induces a one unit reduction in emissions). However, the inspection probability is not constant, and an increment in the firm's valuation of the environment induces the agency to reduce monitoring. From (15) and (17), these reactions in the Stackelberg and Nash cases read, respectively, as follows:

$$\frac{\beta\phi}{\alpha_A + 2\beta\phi^2}, \quad \frac{\beta\phi}{\alpha_A + \beta\phi^2}.$$

A higher firm's valuation of the environment induces a reduction in emissions and hence in non-compliance, which reduces the agency's marginal income at rate $\beta\phi$ (identical in Stackelberg and Nash). Moreover, the size of the agency's reaction is also inversely dependent on the concavity of the agency's profits.

Hence, the self-regulation terms defined in Subsection 5.1 are given by one minus ϕ times the agency's reaction to the firm's valuation of the environment. A higher monitoring cost, α_A, or a lower share of the fine transferred to the agency, β, smooths the agency's reaction. Similarly, a lower intensity of the fine, ϕ, reduces the product of ϕ times the agency's reaction. In consequence, these three effects enlarge the dynamic firm's self-regulation.

To fully characterize the effect of these parameters when agents play *à la Stackelberg*, we need to study their effect on the agency's dynamic and static incentives. A higher α_A diminishes the dynamic incentive, which reinforces the positive effect of a higher self-regulation, increasing the regulator's cost advantage at which decentralization starts being welfare improving, hence increasing Ω^S. This result also holds for a lower β or ϕ, even when these changes exacerbate the negative dynamic incentive. All in all, these parameter changes make decentralization more favorable, both under Nash or Stackelberg strategic interaction.

<table>
<tr><td></td><td colspan="3">Stackelberg</td><td colspan="2">Nash</td></tr>
<tr><td></td><td>FSR^S</td><td>ADI</td><td>Ω^S</td><td>FSR^N</td><td>Ω^N</td></tr>
<tr><td>$\alpha_A \uparrow$</td><td rowspan="3">$\uparrow$</td><td>$\downarrow$</td><td rowspan="3">$\uparrow$</td><td rowspan="3">$\uparrow$</td><td rowspan="3">$\uparrow$</td></tr>
<tr><td>$\beta \downarrow$</td><td>$\uparrow$</td></tr>
<tr><td>$\phi \downarrow$</td><td>$\uparrow$</td></tr>
</table>

Table 3: Sensitivity analysis wrt main parameters

Finally, we characterize the impact of the environmental problem on the attractiveness of decentralization. As shown in (3) the damage from pollution depends on the stock of pollution and the intensity of environmental damages.

The intensity of environmental damages, γ, does not directly affect the intensities of self-regulation or dynamic and static incentives. The firm values pollution just because it anticipates that higher pollution will imply tougher regulation. In consequence, a higher γ increases the strictness of the anticipated future regulation, hence increasing the firm's valuation of the environment and with it the level of self-regulation, also making decentralization more attractive.

On the other hand, Figure 3 depicts upward-sloping curves, regardless of whether the firm and the agency act myopically or dynamically, *à la Stackelberg* or *à la Nash*. This implies that a higher initial pollution stock requires a higher regulator's cost disadvantage for decentralization to be attractive. This can reflect the fact that if the pollution stock is initially low, strong increments are expected. The firm strongly values pollution because it anticipates that large increases in pollution will imply much tougher future regulation. Conversely, for a large initial pollution stock expected increments are softer.

Summarizing, decentralization is less attractive if the environmental problem is due to an initially high pollution stock, while it is more attractive if it is due to a large intensity of environmental damages.

Appendix D: Static framework

One-layer:

- Firm's (follower's) maximization problem:

$$\max_{E} \left\{ \sigma E - \frac{E^2}{2} - P\phi(E-Q) \right\}.$$

  Its best-reaction function is given by: $\widehat{E}^s(P) = \sigma - \phi P$, where superscript s stands for static framework.

- Regulator's (leader's) maximization problem:

$$\max_{Q,P} \left\{ \sigma E - \frac{E^2}{2} - \gamma \frac{E^2}{2} - \alpha_A \varepsilon \frac{P^2}{2} \right\},$$
$$\text{s.t.: } E = \widehat{E}^s(P).$$

From this maximization problem, the optimal inspection probability is:

$$P^s = \frac{\gamma\sigma\phi}{\alpha_A\varepsilon + (\gamma+1)\phi^2}.$$

And hence emissions and the standard are:

$$E^s = Q^s = \sigma - \frac{\gamma\sigma\phi^2}{\alpha_A\varepsilon + (\gamma+1)\phi^2}.$$

Thus, social welfare in the one-layer case reads:

$$SW^{s1L} = \frac{\sigma^2\left(\alpha_A\varepsilon(1-\gamma) + \phi^2\right)}{2\alpha_A\varepsilon + 2(\gamma+1)\phi^2},$$

where superscript $s1L$ stands for the static one-layer case.

Two-layer:

- Regulator's (leader's) maximization problem:

$$\max_Q \left\{\sigma E - \frac{E^2}{2} - \gamma\frac{E^2}{2} - \beta P\phi(E-Q)\right\}.$$

- The firm's maximization problem mimics the one-layer case.

- The agency's maximization problem:

$$\max_P \left\{\beta P\phi(E-Q) - \alpha_A\frac{P^2}{2}\right\}.$$

Two-layer Stackelberg:

Plugging the firm's best-reaction into the agency's maximization problem, the optimal inspection probability is:

$$\widehat{P}^{s2S}(Q) = \frac{\beta\phi(\sigma - Q)}{\alpha_A + 2\beta\phi^2},$$

where superscript $s2S$ stands for the static two-layer Stackelberg case. Plugging the best-reactions of the agency and the firm into the regulator's maximization problem, one gets:

$$Q^{s2S} = \frac{\alpha_A \sigma(2\beta - \gamma) + \beta\sigma\phi^2(2\beta - \gamma + 1)}{2\alpha_A \beta + \beta\phi^2(2\beta + \gamma + 1)}.$$

And hence,

$$P^{s2S} = \frac{\gamma\sigma\phi}{2\alpha_A + \phi^2(2\beta + \gamma + 1)}, \quad E^{s2S} = \sigma - \frac{\gamma\sigma\phi^2}{2\alpha_A + \phi^2(2\beta + \gamma + 1)}.$$

Leading to the following social welfare:

$$SW^{s2S} = \frac{\sigma^2 \left(4\alpha_A^2(1-\gamma) + \alpha_A\phi^2 \left(8\beta(1-\gamma) + 4 - \gamma^2\right) + \phi^4(4\beta(\beta(1-\gamma)+1) + \gamma + 1)\right)}{2\left(2\alpha_A + \phi^2(2\beta + \gamma + 1)\right)^2}.$$

Two-layer Nash:

The Nash equilibrium between the agency and the firm is:

$$\widehat{E}^{s2N}(Q) = \frac{\alpha_A\sigma + \beta Q\phi^2}{\alpha_A + \beta\phi^2}, \quad \widehat{P}^{s2N}(Q) = \frac{\beta\phi(\sigma - Q)}{\alpha_A + \beta\phi^2},$$

where superscript $s2N$ stands for the static two-layer Nash case. And taking these best-response functions into account the regulator sets the equilibrium standard:‘

$$Q^{s2N} = \frac{\sigma\left(2\alpha_A\beta - \alpha_A\gamma + \beta\phi^2\right)}{2\alpha_A\beta + \beta(\gamma+1)\phi^2}.$$

Hence the equilibrium emissions and inspection probability follow:

$$E^{s2N} = \frac{\sigma\left(2\alpha_A + \phi^2\right)}{2\alpha_A + (\gamma+1)\phi^2}, \quad P^{s2N} = \frac{\gamma\sigma\phi}{2\alpha_A + (\gamma+1)\phi^2}.$$

And the social welfare in this scenario reads:

$$SW^{s2N} = \frac{\sigma^2\left(4\alpha_A^2(1-\gamma) + \alpha_A\left(4-\gamma^2\right)\phi^2 + (\gamma+1)\phi^4\right)}{2\left(2\alpha_A + (\gamma+1)\phi^2\right)^2}.$$

For $\varepsilon = 1$, the gaps between the social welfare in the one-layer versus the two-layer

Stackelberg and Nash are given by the positive expressions:

$$SW^{s1L} - SW^{s2S} = \frac{\gamma^2\sigma^2\phi^2\left(\alpha_A + 2\beta\phi^2\right)^2}{2\left(\alpha_A + (\gamma+1)\phi^2\right)\left(2\alpha_A + \phi^2(2\beta+\gamma+1)\right)^2} > 0,$$

$$SW^{s1L} - SW^{s2N} = \frac{\alpha_A^2\gamma^2\sigma^2\phi^2}{2\left(\alpha_A + (\gamma+1)\phi^2\right)\left(2\alpha_A + (\gamma+1)\phi^2\right)^2} > 0.$$

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