

$$\textcircled{1} \quad \vec{F} = -Kx\vec{u}_x \Rightarrow [F] = [K][x] \Rightarrow [K] = \frac{[F]}{[x]} = \frac{MLT^{-2}}{L} \Rightarrow$$

$$\Rightarrow [K] = MT^{-2} \quad [F] = [-K][x] = [-1][K][x] = [K][x]$$

$$[-K] = [-1][K] = [K]$$

$$\textcircled{2} \quad F = GMm/r^2 \Rightarrow [F] = [G][M][m]/[r^2] \Rightarrow$$

$$\Rightarrow [G] = \frac{[F][r^2]}{[M][m]} = \frac{MLT^{-2}L^2}{M^2} = \frac{L^3T^{-2}}{M} = \boxed{M^{-1}L^3T^{-2}}$$

$$\textcircled{3} \quad h = v^i m^j / a^k \Rightarrow [h] = [v]^i [m]^j [a]^{-k} =$$

$$= L^i T^{-i} M^j L^{-k} T^{2k} = L^{i-k} T^{-i+2k} M^j$$

$$[h] = L \Rightarrow L = L^{i-k} T^{-i+2k} M^j \Rightarrow$$

$$\Rightarrow \left. \begin{array}{l} 1 = i - k \\ 0 = -i + 2k \\ 0 = j \end{array} \right\} \Rightarrow \left. \begin{array}{l} 1 = k \\ i = 2k \end{array} \right\} \Rightarrow \boxed{i = 2}$$

$$\Rightarrow \boxed{j = 0}$$

$$\textcircled{4} \quad \frac{dP}{dz} = c p^\alpha p^\beta g^\gamma \Rightarrow \left[\frac{dP}{dz} \right] = [P]^\alpha [p]^\beta [g]^\gamma \quad [c] = 1$$

$$\left[\frac{dP}{dz} \right] = \frac{[P]}{[dz]} = \frac{[F]/[S]}{L} = \frac{MLT^{-2}/L^2}{L} = \frac{ML^{-1}T^{-2}}{L} = ML^{-2}T^{-2}$$

$$[P] = [F]/[S] = ML^{-1}T^{-2}$$

$$[p] = ML^{-3} \quad [g] = LT^{-2}$$

$$ML^{-2}T^{-2} = (ML^{-1}T^{-2})^\alpha (ML^{-3})^\beta (LT^{-2})^\gamma = M^{\alpha+\beta} L^{-\alpha-3\beta+\gamma} T^{-2\alpha-2\gamma}$$

$$1) \quad 1 = \alpha + \beta$$

$$2) \quad -2 = -\alpha - 3\beta + \gamma$$

$$3) \quad -2 = -2\alpha - 2\gamma \Rightarrow 1 = \alpha + \gamma$$

$$\left. \begin{array}{l} 1 = \alpha + \beta \\ 1 = \alpha + \gamma \end{array} \right\} \Rightarrow \left. \begin{array}{l} \beta = \gamma \\ -2 = -\alpha - 3\beta + \gamma \end{array} \right\} \Rightarrow$$

$$\Rightarrow \left. \begin{array}{l} 1 = \alpha + \beta \\ -2 = -\alpha - 2\beta \end{array} \right\} \Rightarrow -1 = -\beta \Rightarrow \boxed{\beta = 1} \Rightarrow \boxed{\gamma = 1}$$

$$\alpha = 1 - \beta = 0 \Rightarrow \boxed{\alpha = 0}$$

$$\Rightarrow \boxed{\frac{dP}{dz} = c p g}$$

en ecuación real es

$$\frac{dP}{dz} = -\rho g$$

Autor: Iván Gabrila Álvarez

$$15) a^2 = b m v^2 + a \Rightarrow [a^2] = [b m v^2]$$

(12)

$$[a^2] = [a] \quad [a] = [b m v^2]$$

$$[a] = M^\alpha L^\beta T^\gamma$$

$$[a^2] = [a]$$

$$\Rightarrow [a] = 1$$

$$\Rightarrow M^\alpha L^\beta T^\gamma = M^{2\alpha} L^{2\beta} T^{2\gamma} \Rightarrow \alpha = \beta = \gamma = 0 \Rightarrow$$

$$\text{Otra manera: } [a^2] = [a]^2 \Rightarrow [a]^2 = [a] \Rightarrow [a] = 1$$

$$[a] = [b m v^2] \Rightarrow [a] = [b] [m] [v]^2 \Rightarrow 1 = [b] M L^2 T^{-2}$$

$$\Rightarrow [b] = M^{-1} L^{-2} T^2$$

$$16) P = c \rho^x v^y \Rightarrow [P] = [\rho]^x [v]^y$$

$$[P] = [F]/[S] = M L^{-1} T^{-2}$$

$$[\rho] = M L^{-3}$$

$$[v] = L T^{-1}$$

$$[c] = 1$$

$$\Rightarrow M L^{-1} T^{-2} = M^x L^{-3x} T^{-y} \Rightarrow$$

$$\Rightarrow M) 1 = x \Rightarrow x = 1$$

$$L) -1 = -3x + \beta$$

$$T) -2 = -y \Rightarrow y = 2$$

$$\left. \begin{matrix} x=1 \\ y=2 \end{matrix} \right\} \Rightarrow -3x + \beta = -3 + 2 = -1 \Rightarrow \text{La ecuación respecto de } L, -1 = -3x + \beta, \text{ es consistente con las otras dos ecuaciones}$$

$$P = c \rho v^2$$

$$17) T = c \lambda^x g^y m^z \Rightarrow [T] = [c] [\lambda]^x [g]^y [m]^z$$

$$[T] = T$$

$$[c] = 1 \quad [\lambda] = L \quad [g] = L T^{-2} \quad [m] = M$$

$$\Rightarrow T = L^x L^y T^{-2y} M^z \Rightarrow$$

$$M) 0 = z \Rightarrow z = 0$$

$$L) 0 = x + y \Rightarrow x = -y$$

$$T) 1 = -2y \Rightarrow y = -1/2 \Rightarrow x = 1/2$$

$$\text{La ecuación real es } T = 2\pi \sqrt{\frac{L}{g}}$$

Autor: Juan Carlos Álvarez

$$T = c \sqrt{\frac{L}{g}}$$

$\Rightarrow$

$$⑧ F = \eta \frac{\Delta v}{\Delta h} A \Rightarrow \eta = \frac{F \Delta h}{A \Delta v} \Rightarrow [\eta] = \frac{[F] [\Delta h]}{[A] [\Delta v]}$$

$$[\eta] = \frac{MLT^{-2} L}{L^2 LT^{-1}} = \boxed{ML^{-1}T^{-1}}$$

$$⑨ v^2 = \frac{g\lambda}{2\pi} + \frac{2\pi\gamma}{\rho\lambda} \Rightarrow i[v^2] = \left[\frac{g\lambda}{2\pi} \right] ?$$

$$i[v^2] = \left[\frac{2\pi\gamma}{\rho\lambda} \right] ?$$

$$[v^2] = L^2 T^{-2}$$

$$\left[\frac{g\lambda}{2\pi} \right] = [g][\lambda] = LT^{-2} L = L^2 T^{-2}$$

$$\left[\frac{2\pi\gamma}{\rho\lambda} \right] = \frac{[\gamma]}{[\rho][\lambda]} = \frac{[F]/[L]}{[\rho] [\lambda]} = \frac{MLT^{-2}/L}{ML^{-3} L} = \frac{MT^{-2}}{ML^{-2}} = L^2 T^{-2}$$

Los tres términos tienen la misma dimensión $\Rightarrow$ La ecuación es dimensionalmente correcta

$$⑩ R = c U^\alpha \rho^\beta t^\gamma \Rightarrow [R] = [c] [U]^\alpha [\rho]^\beta [t]^\gamma$$

$$\left. \begin{array}{l} [c] = 1 \\ [U] = ML^2 T^{-2} \\ [R] = L \end{array} \right\} \begin{array}{l} [\rho] = ML^{-3} \\ [t] = T \end{array} \Rightarrow$$

$$\Rightarrow L = M^\alpha L^{2\alpha} T^{-2\alpha} M^\beta L^{-3\beta} T^\gamma \Rightarrow$$

$$\Rightarrow \left. \begin{array}{l} \text{M)} \quad 0 = \alpha + \beta \Rightarrow \alpha = -\beta \\ \text{L)} \quad 1 = 2\alpha - 3\beta \end{array} \right\} \Rightarrow 1 = -5\beta \Rightarrow \boxed{\beta = -1/5} \Rightarrow \boxed{\alpha = +1/5}$$

$$\text{T)} \quad 0 = -2\alpha + \gamma$$

$$0 = -2\alpha + \gamma \Rightarrow \gamma = 2\alpha \Rightarrow \boxed{\gamma = +2/5}$$

$$\boxed{R = c U^{1/5} \rho^{-1/5} t^{2/5}}$$

$$⑪ \left[\frac{dx}{dt} \right] = \frac{[dx]}{[dt]} = \frac{L}{T} = \boxed{LT^{-1}}$$

$$[\nabla^2] = \left[\frac{\partial^2}{\partial x^2} \right] = L^{-2} \quad [\nabla^2 \rho] = \left[\frac{\partial^2 \rho}{\partial x^2} \right] = \frac{[\partial^2 \rho]}{[\partial x^2]} = \frac{[\rho]}{L^2}$$

$$\left[\frac{\nabla^2 \rho}{\rho} \right] = \frac{[\nabla^2 \rho]}{[\rho]} = \frac{1}{L^2} = \boxed{L^{-2}}$$

Autor: Iván Cabris Álvarez

$$[\int p dV] = \int [p dV] = [p dV] = [p][dV] = ML^{-3}L^3 = \boxed{M} \quad (1.4)$$

$$p = a e^{-b/r} \Rightarrow \left. \begin{aligned} [p] &= [a][e^{-b/r}] = [a] \\ [p] &= ML^{-3} \end{aligned} \right\} \Rightarrow \boxed{[a] = ML^{-3}}$$

$$[e^{-b/r}] = 1 \Rightarrow [b/r] = 1 \Rightarrow [b] = [r] = \boxed{L}$$

$$p = a \times \ln(br) \Rightarrow \left. \begin{aligned} [p] &= [a][x][\ln br] = [a][x] \\ [p] &= ML^{-3} \quad [x] = L \end{aligned} \right\} \Rightarrow \boxed{[a] = ML^{-4}}$$

$$[\ln br] = 1 \Rightarrow [br] = 1 \Rightarrow \boxed{[b] = L^{-1}}$$

$$p = a \cos(bv^2/l) \Rightarrow [p] = [a][\cos(bv^2/l)] = [a] \Rightarrow \boxed{[a] = ML^{-3}}$$

$$[\cos(bv^2/l)] = 1 \Rightarrow [bv^2/l] = 1 \Rightarrow [b] = \frac{[l]}{[v]^2} = \frac{L}{L^2 T^{-2}} = \boxed{L^{-1} T^2}$$

$$p = a \tan(br) \Rightarrow [p] = [a][\tan(br)] = [a] \Rightarrow \boxed{[a] = ML^{-3}}$$

$$[\tan(br)] = 1 \Rightarrow [br] = 1 \Rightarrow [b] = [r]^{-1} = \boxed{L^{-1}}$$

$$(1.2) \left(P + \frac{n^2 a}{V^2}\right)(V - nb) = nRT \Rightarrow [V] = [nb]$$

$$[P] = \left[\frac{n^2 a}{V^2}\right]$$

$$\left. \begin{aligned} [V] &= L^3 \quad [n] = N \\ [V] &= [nb] = [n][b] \end{aligned} \right\} \Rightarrow L^3 = N[b] \Rightarrow \boxed{[b] = L^3 N^{-1}}$$

$$[P] = [F]/[S] = MLT^{-2}/L^2 = ML^{-1}T^{-2}$$

$$[n] = N \quad [V] = L^3$$

$$\left. \begin{aligned} [P] \left[\frac{n^2 a}{V^2}\right] &= \frac{[n]^2 [a]}{[V]^2} \Rightarrow [a] = \frac{[P][V]^2}{[n]^2} \end{aligned} \right\} \Rightarrow$$

$$\Rightarrow [a] = \frac{ML^{-1}T^{-2}L^6}{N^2} = \boxed{ML^5 T^{-2} N^{-2}}$$

1.13 $S(t) = \sum_{i=1}^3 C_i S(0)^{\alpha_i} t^{\beta_i} v(0)^{\gamma_i} a^{\delta_i}$ $[C_i] = 1$ $i=1-3$ 15

$$[S(t)] = [C_i S(0)^{\alpha_i} t^{\beta_i} v(0)^{\gamma_i} a^{\delta_i}] = [C_i] [S(0)]^{\alpha_i} [t]^{\beta_i} [v(0)]^{\gamma_i} [a]^{\delta_i}$$

$$[S(t)] = L$$

$$[C_i] = 1 \quad [S(0)] = L \quad [t] = T \quad [v(0)] = L T^{-1} \quad [a] = L T^{-2}$$

$$\Rightarrow L = L^{\alpha_i} T^{\beta_i} L^{\gamma_i} T^{-\gamma_i} L^{\delta_i} T^{-2\delta_i} \Rightarrow$$

$$\Rightarrow \begin{cases} 1 = \alpha_i + \gamma_i + \delta_i \\ 0 = \beta_i - \gamma_i - 2\delta_i \end{cases} \Rightarrow \text{Generamos las ecuaciones y} \\ \text{contra incógnitas}$$

Hay varias soluciones que se obtienen eligiendo valores de α_i, β_i o valores de γ_i, δ_i

Elegimos $\alpha_i = 1$ y $\beta_i = 0 \Rightarrow \begin{cases} 1 = \gamma_i + \delta_i \\ 0 = -\gamma_i - 2\delta_i \end{cases} \Rightarrow 1 = 1 - \delta_i \Rightarrow \boxed{\delta_i = 0}$

$$0 = -\gamma_i - 2\delta_i \Rightarrow \boxed{\gamma_i = 0} \Rightarrow \boxed{S(t) = C_1 S(0)}$$

Elegimos $\alpha_i = 0$ y $\beta_i = 1 \Rightarrow \begin{cases} 1 = \gamma_i + \delta_i \\ 0 = 1 - \gamma_i - 2\delta_i \end{cases} \Rightarrow 1 = 1 - \delta_i \Rightarrow \boxed{\delta_i = 0}$

$$1 = \gamma_i + \delta_i \Rightarrow \gamma_i = 1 - \delta_i = \boxed{1} \Rightarrow \boxed{S(t) = C_2 t v(0)}$$

Elegimos $\alpha_i = 0$ y $\gamma_i = 0 \Rightarrow \begin{cases} 1 = \delta_i \\ 0 = \beta_i - 2\delta_i \end{cases} \Rightarrow \boxed{\delta_i = 1} \Rightarrow \boxed{\beta_i = 2}$

$$\Rightarrow \boxed{S(t) = C_3 t^2 a}$$

$$\boxed{S(t) = C_1 S(0) + C_2 t v(0) + C_3 t^2 a}$$

La ecuación del movimiento tiene $C_1 = C_2 = 1$ y $C_3 = 1/2$

1.14	6135	4 cs	1500	2-4 cs
	901	3 cs	145.2	4 cs
	10609	5 cs	145.20	5 cs
	0.01	1 cs	0.0014520	5 cs
			305.1	4 cs
0.000	300.0456	3 cs	30050	4-5 cs
	2.00	3 cs	0.1	1 cs
	10.093	5 cs	0.1000	4 cs
	600	4-3 cs		

Autor: Yoán Abria Álvarez

15) 100.0 ± 0.1 3400 ± 100 0.0005670 ± 0.0000001 16)

4 c.s. 1 c.s. 2 c.s. 1 c.s. 4 c.s. 1 c.s.

0.0003004 ± 0.0000001 0.400 ± 0.001

4 c.s. 1 c.s. 3 c.s. 1 c.s.

16) $24567 \pm 2928 \text{ m} \rightarrow \pm 3000 \rightarrow 25000 \pm 3000 \text{ m}$

$23.463 \pm 0.165 \text{ s} \rightarrow \pm 0.2 \rightarrow 23.5 \pm 0.2 \text{ s}$

$345.20 \pm 3.10 \text{ s} \rightarrow \pm 3 \rightarrow 345 \pm 3 \text{ s}$

$4.3 \pm 0.06 \text{ dm} \rightarrow \pm 0.06 \rightarrow 43.00 \pm 0.06 \text{ dm}$

$0.4672 \pm 0.00482 \text{ m} \rightarrow \pm 0.005 \rightarrow 0.467 \pm 0.005 \text{ m}$

$464.2413 \pm 0.061 \text{ dm} \rightarrow \pm 0.06 \rightarrow 464.24 \pm 0.06 \text{ dm}$

$6.03 \pm 0.0005 \text{ m} \rightarrow \pm 0.0005 \rightarrow 6.0300 \pm 0.0005 \text{ m}$

$46288 \pm 1553 \text{ m} \rightarrow \pm 2000 \rightarrow 46000 \pm 2000 \text{ m}$

$3.248 \pm 0.124 \text{ m} \rightarrow \pm 0.1 \rightarrow 3.2 \pm 0.1 \text{ m}$

$0.018366 \pm 0.00783 \text{ m} \rightarrow \pm 0.008 \rightarrow 0.018 \pm 0.008 \text{ m}$

$0.030 \pm 0.00415 \text{ m} \rightarrow \pm 0.004 \rightarrow 0.030 \pm 0.004 \text{ m}$

$25721 \pm 1520 \text{ m} \rightarrow \pm 2000 \rightarrow 26000 \pm 2000 \text{ m}$

17) $3.15 \text{ m} = 315 \text{ cm}$ $3.25 \text{ Kg} = 3250 \text{ g}$

$1.25 \text{ minutos} = 75 \text{ s}$ $1 \text{ Km} = 1000 \text{ m} = 100000 \text{ cm}$

$1 \text{ m} = 100 \text{ cm} = \frac{100}{2.54} \text{ pulgadas} = 39.37 \text{ pulgadas}$

$1 \text{ Å} = 10^{-10} \text{ m} = 10^{-8} \text{ cm} = \frac{10^{-8}}{2.54} \text{ pulgadas} = 3.937 \cdot 10^{-9} \text{ pulgadas}$

$10^5 \text{ Pa} = 0.1 \text{ MPa}$

$50 \text{ psi} = 50 \frac{\text{libras}}{\text{pulgada}^2} = \frac{50 \cdot 4.45 \text{ N}}{(2.54 \text{ cm})^2} = 34.48 \frac{\text{N}}{\text{cm}^2} = 344875.69 \text{ Pa}$

$68 \times 105 \text{ m}^2 = 7140 \text{ m}^2 = 71.4 \text{ áreas} = 0.714 \text{ has}$

$1 \text{ año-luz} = \frac{300000 \text{ Km}}{\text{s}} \cdot 365.25 \text{ días} \cdot \frac{86400 \text{ s}}{\text{día}} = 9.4673 \cdot 10^{12} \text{ Km}$

$325.15 \text{ has} = 32515 \text{ áreas} = 3251500 \text{ m}^2$

$25.7 \text{ áreas} = 2570 \text{ m}^2$

$35.3 \text{ km}^2 = 35.3 \cdot 10^4 \text{ m}^2 = 35.3 \cdot 10^2 \text{ dm}^2$

1.18 El número π no es una magnitud física y no tiene dimensiones.

1.19 Al decir π radianes, nos estamos refiriendo a un ángulo plano.

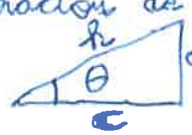
Un ángulo plano de π radianes es una magnitud física sin dimensiones.

Es adimensional porque las dimensiones se cancelan en una división:

$[\theta] = L/L = 1$

1.20 Las razones trigonométricas son adimensionales.

Demostación de que el $\text{sen } \theta$ es adimensional:



$\text{sen } \theta = \frac{o}{h} \Rightarrow [\text{sen } \theta] = \frac{[o]}{[h]} = \frac{L}{L} = 1$

Autor: Iván Cabris Álvarez

(1.7)

(1.21) La ley de Coulomb es: $F = \frac{q_1 q_2}{4\pi \epsilon_0 r^2} \Rightarrow \epsilon_0 = \frac{q_1 q_2}{4\pi r^2 F}$

$$\begin{aligned} [F] &= M L T^{-2} & [q_1] &= [q_2] = Q \\ [r] &= L & [4\pi] &= 1 \end{aligned}$$

$$\Rightarrow [\epsilon_0] = \frac{[Q_1][Q_2]}{[4\pi][r]^2[F]} = \frac{Q^2}{L^2 M L T^{-2}} = \boxed{Q^2 M^{-1} L^{-3} T^2}$$

(1.22) $c = a \epsilon_0^\alpha \mu_0^\beta \Rightarrow [c] = [a] [\epsilon_0]^\alpha [\mu_0]^\beta$
 a es una constante $\Rightarrow [a] = 1$ $[c] = L T^{-1}$

$$[\epsilon_0] = Q^2 M^{-1} L^{-3} T^2 \text{ según el problema 1.21}$$

$$[\mu_0] = M L Q^{-2}$$

$$\Rightarrow L T^{-1} = 1 \cdot (Q^2 M^{-1} L^{-3} T^2)^\alpha (M L Q^{-2})^\beta \Rightarrow$$

$$\Rightarrow \left. \begin{aligned} M^0 &= M^{-\alpha} M^\beta \\ L^1 &= L^{-3\alpha} L^\beta \\ T^{-1} &= T^{2\alpha} \\ Q^0 &= Q^{2\alpha} Q^{-2\beta} \end{aligned} \right\} \Rightarrow \left. \begin{aligned} 0 &= -\alpha + \beta \\ 1 &= -3\alpha + \beta \\ -1 &= 2\alpha \\ 0 &= 2\alpha - 2\beta \end{aligned} \right\} \Rightarrow$$

$0 = -\alpha + \beta$ y $0 = 2\alpha - 2\beta$ son la misma ecuación

$$\Rightarrow \left. \begin{aligned} 0 &= -\alpha + \beta \\ 1 &= -3\alpha + \beta \\ -1 &= 2\alpha \end{aligned} \right\} \Rightarrow \left. \begin{aligned} 0 &= -\alpha + \beta \Rightarrow \alpha = \beta \\ 1 &= -3\alpha + \beta \end{aligned} \right\} \Rightarrow 1 = -2\alpha \Rightarrow$$

$\Rightarrow$ Las ecuaciones $0 = -\alpha + \beta$ y $1 = -3\alpha + \beta$ contienen a la ecuación $-1 = 2\alpha \Rightarrow$ Sólo tenemos dos ecuaciones independientes.

Elegimos las dos siguientes ecuaciones:

$$\left. \begin{aligned} 0 &= -\alpha + \beta \Rightarrow \alpha = \beta \\ 1 &= -3\alpha + \beta \end{aligned} \right\} \Rightarrow 1 = -2\alpha \Rightarrow \boxed{\alpha = -1/2}$$

$$\left. \begin{aligned} \alpha &= -1/2 \\ \alpha &= \beta \end{aligned} \right\} \Rightarrow \boxed{\beta = -1/2} \Rightarrow c = a \epsilon_0^{-1/2} \mu_0^{-1/2} = \boxed{\frac{a}{\sqrt{\mu_0 \epsilon_0}}}$$

Autor: Iván Abria Álvarez

(1.8)

(1.23)

$$e = 100.000 \pm 0.001 \mu\text{m}$$

$$\Delta e = 0.001 \mu\text{m}$$

$$t = 10.3 \pm 0.1 \text{ s}$$

$$\Delta t = 0.1 \text{ s}$$

$$v = \frac{e}{t} \Rightarrow \frac{\partial v}{\partial e} = \frac{1}{t} \quad \frac{\partial v}{\partial t} = -\frac{e}{t^2}$$

$$\Delta v = \left| \frac{\partial v}{\partial e} \right| \Delta e + \left| \frac{\partial v}{\partial t} \right| \Delta t = \left| \frac{1}{t} \right| \Delta e + \left| \frac{-e}{t^2} \right| \Delta t$$

$$= \frac{0.001 \mu\text{m}}{10.3 \text{ s}} + \frac{100 \mu\text{m}}{(10.3 \text{ s})^2} 0.1 \text{ s} = 0.0943566 \mu\text{m/s}$$

$$\Delta v = 0.09 \mu\text{m/s} \text{ según las reglas de errores y redondeos}$$

$$v = \frac{e}{t} = \frac{100}{10.3} \frac{\mu\text{m}}{\text{s}} = 9.7087379 \frac{\mu\text{m}}{\text{s}}$$

$$v = 9.71 \pm 0.09 \mu\text{m/s}$$

(1.24) a) Siemprevé en común la dimensión: Longitud

b) Siemprevé en común dos características: La misma dimensión, temperatura, y el mismo valor numérico, 20. Difieren en las unidades: Grados Celsius y grados Kelvin.

c) Siemprevé en común dos características: La misma dimensión, temperatura, y las mismas unidades, grados Celsius.

(1.25) Para cambiar las unidades de P, V y R se multiplica por un factor de conversión. Estos factores de conversión se escriben en los dos lados de la ecuación $PV = nRT$. El efecto neto es que la forma matemática de, por ejemplo, $P = \frac{nRT}{V}$, no cambia:

$$P = 300 \text{ atm} \quad V = 0.082 \text{ L} \quad T = 300 \text{ K} \quad n = 1 \text{ mol}$$

$$300 \text{ atm} = \frac{1 \text{ mol} \cdot 0.082 \text{ atm} \cdot \text{L} / (\text{mol} \cdot \text{K}) \cdot 300 \text{ K}}{0.082 \text{ L}}$$

$$P = \frac{nRT}{V}$$

Pen Ven Ten men
atm L K mol

$$300 \text{ atm} \cdot \left(\frac{1.013 \cdot 10^5 \text{ Pa}}{\text{atm}} \right) = \frac{1 \text{ mol} \cdot 0.082 \text{ L} \cdot (1.013 \cdot 10^5 \text{ Pa} / (\text{mol} \cdot \text{K})) \cdot 300 \text{ K}}{0.082 \text{ L}}$$

Pen Ven Ten men
atm L K mol

$$a = 1.013 \cdot 10^5 \text{ Pa/atm}$$

a es un factor de conversión
de atm a Pa

$$aP = \frac{nRaT}{V}$$

$$P = \frac{nRT}{V}$$

Autor: Iván Cabris Alvarado

La conversión de Grados Celsius a K y microssa, implica la suma y la resta, respectivamente, de un factor 273.15. Este factor de conversión no se compensa con otro en la ecuación $PV = nRT$ y por tanto, no se puede introducir o usar T en grados Celsius.

(1.26) $PV = nRT$ T en K

$$PV = nR(t_c + 273.15)$$

t_c en Celsius, pero R debe estar en unidades de $\frac{\text{energía}}{\text{mol K}}$
 $\text{atm} \cdot \text{L}$ y $\text{Pa} \cdot \text{m}^3$
 son unidades de energía

(1.27) El radio de un electrón es de unos 10^{-18} m y el radio de un núcleo varía entre 10^{-15} y 10^{-15} m. Por tanto, un núcleo, cualquier núcleo, es más grande que un electrón.

(1.28) En un movimiento circular, $v = \omega r$. En el S.I., las unidades de la velocidad lineal v , de la velocidad angular ω y del radio r son m/s, rad/s y m, respectivamente. Por tanto, a la izquierda de $v = \omega r$ tenemos m/s y a la derecha tenemos rad m/s. Esto no parece consistente.

Lo que falta en $v = \omega r$ y en los razonamientos anteriores son las derivadas $\frac{d}{d\theta} \cos\theta$ y $\frac{d}{d\theta} \sin\theta$. Estas derivadas aparecen al calcular $\vec{v}$ en un movimiento circular.

$$\left. \begin{aligned} \vec{r}(t) &= r \cos\theta(t) \vec{u}_x + r \sin\theta(t) \vec{u}_y \\ \vec{v}(t) &= \frac{d\vec{r}}{dt} \end{aligned} \right\} \Rightarrow \vec{v}(t) = r \frac{d \cos\theta(t)}{dt} \vec{u}_x + r \frac{d \sin\theta(t)}{dt} \vec{u}_y$$

$$\Rightarrow \vec{v}(t) = r \frac{d \cos\theta}{d\theta} \frac{d\theta}{dt} \vec{u}_x + r \frac{d \sin\theta}{d\theta} \frac{d\theta}{dt} \vec{u}_y$$

$$= r \omega \frac{d \cos\theta}{d\theta} \vec{u}_x + r \omega \frac{d \sin\theta}{d\theta} \vec{u}_y$$

Autor: Iván Cabria Álvarez

$$\frac{d \cos\theta}{d\theta} = -\sin\theta (\text{radianes})^{-1} \quad \frac{d \sin\theta}{d\theta} = \cos\theta (\text{radianes})^{-1}$$

Las unidades de $\frac{d \cos\theta}{d\theta}$ y $\frac{d \sin\theta}{d\theta}$ son $(\text{radianes})^{-1}$, porque $d \cos\theta$ y $d \sin\theta$ son adimensionales y $d\theta$ tiene unidades de radianes.

Al calcular el módulo de v , estas derivadas se convierten en la unidad, en un seg :

$$v = r\omega \sqrt{\left(\frac{d \cos \theta}{d\theta}\right)^2 + \left(\frac{d \sin \theta}{d\theta}\right)^2} = r\omega \sqrt{(\sin \theta)^2 + (\cos \theta)^2} = r\omega \cdot 1$$

Sin embargo, el ω tiene unidades, $(\text{rad})^{-1}$. Por tanto, tenemos:

$$v = r\omega \cdot 1(\text{rad})^{-1}$$

A la izquierda tenemos m/s en el S.I., y a la derecha también:

$$\text{m} \frac{\text{rad}}{\text{s}} \frac{1}{\text{rad}} = \frac{\text{m}}{\text{s}}$$

1.29 Unidades de μ_0 en el S.I.: newton/ampere²
 El newton es una unidad de fuerza de corriente
 El ampere es una unidad de intensidad eléctrica } $\Rightarrow$

$$\Rightarrow [\mu_0] = \frac{[F]}{[I]^2}$$

$$\left. \begin{array}{l} [F] = \text{MLT}^{-2} \\ [I] = \frac{\Phi}{T} \end{array} \right\} \Rightarrow [\mu_0] = \frac{\text{MLT}^{-2}}{\Phi^2 \text{T}^{-2}} = \boxed{\text{ML}\Phi^{-2}}$$

1.30 $[e] = \Phi$ $[2\pi] = 1$ $[c] = \text{LT}^{-1}$

$$[h] = [\lambda] = [2\pi] = [2\pi m\lambda] = \text{LMLT}^{-1} = \text{ML}^2\text{T}^{-1}$$

Calculamos la dimensión de k_e :

$$F = k_e q_1 q_2 / r^2 \Rightarrow k_e = \frac{F r^2}{q_1 q_2} \Rightarrow [k_e] = \frac{[F] [r]^2}{[q_1] [q_2]} \Rightarrow$$

$$\Rightarrow [k_e] = \frac{\text{MLT}^{-2} \text{L}^2}{\Phi^2} = \text{ML}^3 \text{T}^{-2} \Phi^{-2}$$

Calculamos la dimensión de α :

$$[\alpha] = \frac{[k_e] [e]^2 [2\pi]}{[h] [c]} = \frac{\text{ML}^3 \text{T}^{-2} \Phi^{-2} \Phi^2 \cdot 1}{\text{ML}^2 \text{T}^{-1} \text{LT}^{-1}} = \boxed{1}$$

α es adimensional

tutor: Iván Cabris Alvarro

$$(1.31) \quad [C_v] = \frac{[E]}{[mT]} = \frac{[Fd]}{[mT]} = \frac{MLT^{-2}L}{M \Theta} = L^2 T^{-2} \Theta^{-1} \quad (1.11)$$

$$[\rho] = ML^{-3} \quad [\lambda] = L \quad [\nu] = LT^{-1}$$

$$[K] = [1.25] [\rho] [C_v] [\lambda] [\nu] = 1 \cdot ML^{-3} L^2 T^{-2} \Theta^{-1} L LT^{-1}$$

$$[K] = MLT^{-3} \Theta^{-1}$$

$$(1.32) \quad \vec{q} = -\kappa \vec{\nabla} T \Rightarrow q_x = -\kappa \frac{\partial T}{\partial x} \Rightarrow$$

$$\Rightarrow [q_x] = [\kappa] \left[\frac{\partial T}{\partial x} \right] \Rightarrow [\kappa] = \frac{[q_x]}{\left[\frac{\partial T}{\partial x} \right]}$$

q_x tiene dimensión de $\frac{\text{calor}}{\text{superficie tiempo}} \Rightarrow [q_x] = \frac{[E]}{L^2 T} \Rightarrow$

$$\Rightarrow [q_x] = \frac{ML^2 T^{-2}}{L^2 T} = MT^{-3}$$

$$\left[\frac{\partial T}{\partial x} \right] = \frac{\Theta}{L}$$

$$[\kappa] = \frac{MT^{-3}}{\Theta L^{-1}} = MLT^{-3} \Theta^{-1}$$

$$(1.33) \quad \nabla^2 \phi = -\rho/\epsilon_0 \Rightarrow [\nabla^2 \phi] = \frac{[\rho]}{[\epsilon_0]} \Rightarrow [\epsilon_0] = \frac{[\rho]}{[\nabla^2 \phi]}$$

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} \Rightarrow [\nabla^2 \phi] = \left[\frac{\partial^2 \phi}{\partial x^2} \right] = \frac{[\phi]}{[dx]^2} \Rightarrow$$

ϕ : Potencial

$$V = E_p = q\phi \Rightarrow [\phi] = \frac{[E_p]}{[q]} = \frac{ML^2 T^{-2}}{Q}$$

$V = E_p$: Energía potencial

$$[dx] = L$$

$$\Rightarrow [\nabla^2 \phi] = \frac{ML^2 T^{-2} Q^{-1}}{L^2} = MT^{-2} Q^{-1}$$

$$[\rho] = \frac{Q}{L^3}$$

Autor: Iván Cabria Álvarez

$$[\epsilon_0] = \frac{Q L^{-3}}{M T^{-2} Q^{-1}} = \boxed{M^{-1} L^{-3} T^2 Q^2}$$

1.12

1.34 $\nabla^2 \phi = 0$ Apparently, tenemos $[\nabla^2 \phi] = 1$ la ecuación en el término de la derecha es cero, porque se trata de un punto $\vec{r}$ del espacio en el que la densidad de carga eléctrica es nula: $\rho(\vec{r}) = 0$. Es un caso particular de la ecuación general $\nabla^2 \phi = -\rho(\vec{r})/\epsilon_0$. El 0 a la derecha tiene unidades y dimensión: Es $0 \frac{\text{culombios}}{\text{m}^3 \text{ unidades de } \epsilon_0}$ en el S.I. Por tanto, es una ecuación dimensionalmente consistente.

1.35 $x = 5t^2 + 3 \Rightarrow [x] = [5][t]^2 = [3] \Rightarrow$
 $[x] = L T^{-1}$
 $[t] = T$

$$\Rightarrow [3] = L T^{-1} \quad 3 \text{ m/s en el S.I.}$$

$$\Rightarrow L T^{-1} = [5] T^2 \Rightarrow [5] = L T^{-3} \quad 5 \text{ m/s}^3 \text{ en el S.I.}$$

1.36 $l = 1.000 \pm 0.001 \text{ m}$ densidad: $\rho = \frac{m}{V} = \frac{m}{l a h}$ 1.13
 $a = 0.500 \pm 0.001 \text{ m}$
 $h = 3.000 \pm 0.001 \text{ m}$ volumen: $V = l a h$
 $m = 2000.0 \pm 0.1 \text{ kg}$ masa: m

$$\Delta \rho = \left| \frac{\partial \rho}{\partial l} \right| \Delta l + \left| \frac{\partial \rho}{\partial a} \right| \Delta a + \left| \frac{\partial \rho}{\partial h} \right| \Delta h + \left| \frac{\partial \rho}{\partial m} \right| \Delta m$$

$$\Delta l = 0.001 \text{ m} \quad \Delta a = 0.001 \text{ m} \quad \Delta h = 0.001 \text{ m} \quad \Delta m = 0.1 \text{ kg}$$

$$\frac{\partial \rho}{\partial l} = - \frac{m}{l^2 a h} = - \frac{m}{l (l a h)} = - \frac{m}{l V} = - \frac{\rho}{l}$$

$$\frac{\partial \rho}{\partial a} = - \frac{m}{l a^2 h} = - \frac{m}{a (l a h)} = - \frac{m}{a V} = - \frac{\rho}{a}$$

$$\frac{\partial \rho}{\partial h} = - \frac{m}{l a h^2} = - \frac{m}{h (l a h)} = - \frac{m}{h V} = - \frac{\rho}{h}$$

$$\frac{\partial \rho}{\partial m} = \frac{1}{l a h} = \frac{m}{m (l a h)} = \frac{m}{m V} = \frac{\rho}{m}$$

$$\Delta \rho = \left| -\frac{\rho}{l} \right| \Delta l + \left| -\frac{\rho}{a} \right| \Delta a + \left| -\frac{\rho}{h} \right| \Delta h + \left| \frac{\rho}{m} \right| \Delta m =$$

$$= \left(\frac{\Delta l}{l} + \frac{\Delta a}{a} + \frac{\Delta h}{h} + \frac{\Delta m}{m} \right) \rho$$

$$\rho = \frac{m}{l a h} = \frac{2000 \text{ kg}}{1 \cdot 0.5 \cdot 3 \text{ m}^3} = 1333.3333 \frac{\text{kg}}{\text{m}^3}$$

$$\Delta \rho = 1333.3333 \frac{\text{kg}}{\text{m}^3} \left(\frac{0.001 \text{ m}}{1 \text{ m}} + \frac{0.001 \text{ m}}{0.5 \text{ m}} + \frac{0.001 \text{ m}}{3 \text{ m}} + \frac{0.1 \text{ kg}}{2000 \text{ kg}} \right)$$

$$= 1333.3333 \frac{\text{kg}}{\text{m}^3} \cdot 0.00338333 = 4.5111 \frac{\text{kg}}{\text{m}^3} \Rightarrow \Delta \rho = 5 \frac{\text{kg}}{\text{m}^3} \Rightarrow$$

$$\Rightarrow \boxed{\rho = 1333 \pm 5 \frac{\text{kg}}{\text{m}^3}}$$

Autor: Juan Carlos Alvarez

$$(1.37) \quad \vec{B} = \frac{\mu_0}{4\pi} \frac{q' \vec{r}' \times (\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3} \Rightarrow B = \frac{\mu_0 q' r' |\sin \alpha|}{4\pi |\vec{r} - \vec{r}'|^2} \quad (1.14)$$

$$[\sin \alpha] = 1 \quad [q'] = Q \quad [r'] = L T^{-1}$$

$$[|\vec{r} - \vec{r}'|^2] = L^2 \quad [4\pi] = 1$$

$$[B] = \frac{[\mu_0] [q'] [r'] [\sin \alpha]}{[4\pi] [|\vec{r} - \vec{r}'|^2]} = [\mu_0] \frac{Q L T^{-1}}{L^2} = [\mu_0] Q L^{-1} T^{-1}$$

$$\vec{F} = q (\vec{r} \times \vec{B}) \Rightarrow F = q r B |\sin \beta|$$

$$[F] = M L T^{-2} \quad [q] = Q \quad [r] = L T^{-1} \quad [\sin \beta] = 1$$

$$[F] = [q] [r] [B] [\sin \beta] \Rightarrow [B] = \frac{[F]}{[q] [r] [\sin \beta]} \Rightarrow$$

$$\Rightarrow [B] = \frac{M L T^{-2}}{Q L T^{-1}} = M T^{-1} Q^{-1}$$

Elementos obtenidos de las ecuaciones:

$$[B] = [\mu_0] Q L^{-1} T^{-1}$$

$$[B] = M T^{-1} Q^{-1}$$

$$[\mu_0] = [B] Q^{-1} L T \quad \Rightarrow$$

$$\Rightarrow [\mu_0] = M T^{-1} Q^{-1} Q^{-1} L T = \boxed{M Q^{-2} L}$$

$$1.38 \quad [B(V,T)] = \frac{[2][h][V]^3}{[c]^2 [e^{hV/k_B T} - 1]}$$

$e^{hV/k_B T} - 1$ y $e^{hV/k_B T} - 1$ deben tener la misma dimensión. $\Rightarrow$

$$\Rightarrow [1] = [e^{hV/k_B T} - 1] \quad \left. \begin{array}{l} \\ [1] = 1 \end{array} \right\} \Rightarrow [e^{hV/k_B T} - 1] = 1$$

$$[h] = [\text{Energía} \times \text{tiempo}] = [E][t] = ML^2 T^{-2} T = ML^2 T^{-1}$$

$$[V] = T^{-1} \quad [c] = LT^{-1}$$

$$[B(V,T)] = \frac{ML^2 T^{-1} T^{-3}}{(LT^{-1})^2} = \boxed{MT^{-2}}$$

$$1.39 \quad T = a p^\alpha \rho^\beta E^\gamma \Rightarrow [T] = [a][p]^\alpha [\rho]^\beta [E]^\gamma$$

$a = \text{cte.} \Rightarrow [a] = 1 \quad [T] = T$

$$[p] = \frac{[F]}{[S]} = \frac{MLT^{-2}}{L^2} = ML^{-1}T^{-2} \quad [\rho] = ML^{-3}$$

$$[E] = [Fz] = ML^2T^{-2}$$

$$\Rightarrow T = (ML^{-1}T^{-2})^\alpha (ML^{-3})^\beta (ML^2T^{-2})^\gamma \Rightarrow$$

$$\Rightarrow T = M^{\alpha+\beta+\gamma} L^{-\alpha-3\beta-2\gamma} T^{-2\alpha-2\gamma} \Rightarrow$$

$$\Rightarrow \left. \begin{array}{l} M) 0 = \alpha + \beta + \gamma \Rightarrow \alpha = -\beta - \gamma \\ L) 0 = -\alpha - 3\beta + 2\gamma \\ T) 1 = -2\alpha - 2\gamma \end{array} \right\} \Rightarrow \begin{array}{l} 0 = \beta + \gamma - 3\beta + 2\gamma \\ \Downarrow \\ 2\beta = 3\gamma \end{array}$$

Autor: Iván Gabrila Álvarez

$$\left. \begin{array}{l} \alpha = -\beta - \gamma \\ 1 = -2\alpha - 2\gamma \end{array} \right\} \Rightarrow 1 = 2\beta + 2\gamma - 2\gamma \Rightarrow 1 = 2\beta \Rightarrow \beta = \frac{1}{2} \quad (1.16)$$

$$\left. \begin{array}{l} 2\beta = 3\gamma \\ \beta = 1/2 \end{array} \right\} \Rightarrow \gamma = \frac{2}{3}\beta = \frac{2}{3} \frac{1}{2} = \frac{1}{3}$$

$$\left. \begin{array}{l} \alpha = -\beta - \gamma \\ \gamma = 1/3 \\ \beta = 1/2 \end{array} \right\} \Rightarrow \alpha = -\frac{1}{2} - \frac{1}{3} = -\frac{3}{6} - \frac{2}{6} = -\frac{5}{6}$$

$$T = a P^{-5/6} \rho^{1/2} E^{1/3}$$

$$(1.40) \quad V_{\text{virus}} = \frac{4\pi}{3} r_{\text{virus}}^3$$

$$V_{\text{atomo}} = \frac{4\pi}{3} r_{\text{at}}^3$$

$$N_{\text{atomo}} = \frac{V_{\text{virus}}}{V_{\text{atomo}}} = \left(\frac{r_{\text{virus}}}{r_{\text{at}}} \right)^3 = \left(\frac{250 \text{ nm}}{0.15 \text{ nm}} \right)^3 \approx 4.6 \cdot 10^9$$

$$1 \text{ Angstrom} = 0.1 \text{ nm} \quad \text{nm: nanómetros} \quad 1 \text{ nm} = 10^{-9} \text{ m}$$

Autor: Juan Carlos Álvarez

$$(1.41) \quad a) \quad P = \left(h + \frac{v^a}{2g} \right) \rho g \Rightarrow \begin{cases} [P] = [h] [\rho]^b [g] \\ [h] = \frac{[v]^a}{[2][g]} \end{cases}$$

$$[h] = L \quad [\rho] = ML^{-3} \quad [g] = LT^{-2}$$

$$\begin{aligned} [P] &= \left[\frac{F}{S} \right] = \left[\frac{ma}{S} \right] = \frac{MLT^{-2}}{L^2} = ML^{-1}T^{-2} \\ [P] &= [h] [\rho]^b [g] \end{aligned} \Rightarrow$$

$$\Rightarrow ML^{-1}T^{-2} = L (ML^{-3})^b LT^{-2} \Rightarrow ML^{-1}T^{-2} = M^b L^{2-3b} T^{-2}$$

$$\Rightarrow \begin{cases} 1 = b \\ -1 = 2 - 3b \\ -2 = -2 \end{cases} \Rightarrow -1 = 2 - 3 \Rightarrow -1 = -1 \Rightarrow \boxed{b=1}$$

y las dos ecuaciones son consistentes entre sí.

$$\begin{aligned} [h] &= L \quad [v] = LT^{-1} \quad [g] = LT^{-2} \quad [2] = 1 \\ [h] &= \frac{[v]^a}{[2][g]} \end{aligned} \Rightarrow$$

$$\Rightarrow L = \frac{(LT^{-1})^a}{1 \cdot LT^{-2}} = L^{a-1} T^{-a+2} \Rightarrow \begin{cases} 1 = a - 1 \Rightarrow a = 2 \\ 0 = -a + 2 \Rightarrow a = 2 \end{cases}$$

$$\Rightarrow \boxed{a=2} \text{ y las dos ecuaciones son consistentes entre sí.}$$

$$b) \quad h = 1.25 \pm 0.03 \text{ m} \quad v = 2.50 \pm 0.05 \text{ m/s}$$

$$\Delta h = 0.03 \text{ m} \quad \Delta v = 0.05 \text{ m/s}$$

$$P = \left(h + \frac{v^2}{2g} \right) \rho g \Rightarrow \begin{cases} \frac{\partial P}{\partial h} = \rho g \\ \frac{\partial P}{\partial v} = v \rho \end{cases} \Rightarrow$$

$$\Delta P = \left| \frac{\partial P}{\partial h} \right| \Delta h + \left| \frac{\partial P}{\partial v} \right| \Delta v$$

$$\Rightarrow \Delta P = \rho g \Delta h + v \rho \Delta v$$

Tutor: Iván Galvira Álvarez

$$\Delta P = 1000 \frac{\text{Kg}}{\text{m}^3} 9.8 \frac{\text{m}}{\text{s}^2} 0.03 \text{ m} + 1000 \frac{\text{Kg}}{\text{m}^3} 2.5 \frac{\text{m}}{\text{s}} 0.05 \frac{\text{m}}{\text{s}}$$

$$= 294 \text{ Pa} + 125 \text{ Pa} = 419 \text{ Pa} \Rightarrow \Delta P = 400 \text{ Pa}$$

$$P = \left(h + \frac{v^2}{2g} \right) \rho g = \left(1.25 \text{ m} + \frac{(2.5 \text{ m/s})^2}{2 \cdot 9.8 \text{ m/s}^2} \right) 1000 \frac{\text{Kg}}{\text{m}^3} 9.8 \frac{\text{m}}{\text{s}^2}$$

$$= 15375 \text{ Pa} \Rightarrow \boxed{P = 15400 \pm 400 \text{ Pa}}$$

a) Otra manera de resolver el apartado a).

$$P = \left(h + \frac{v^a}{2g} \right) \rho^b g \Rightarrow P = h \rho^b g + \frac{v^a \rho^b g}{2g} = h \rho^b g + \frac{v^a \rho^b}{2} \Rightarrow$$

$$\Rightarrow [P] = \frac{[v]^a [\rho]^b}{[2]}$$

$$[2] = 1$$

$$[v] = L T^{-1}$$

$$[\rho] = M L^{-3}$$

$$[P] = \left[\frac{F}{S} \right] = \left[\frac{m a}{S} \right] = \frac{M L T^{-2}}{L^2} = M L^{-1} T^{-2}$$

$$\Rightarrow M L^{-1} T^{-2} = (L T^{-1})^a (M L^{-3})^b \Rightarrow M L^{-1} T^{-2} = L^{a-3b} M^b T^{-a} \Rightarrow$$

$$\Rightarrow M) 1 = b$$

$$\Rightarrow \boxed{b=1}$$

$$L) -1 = a - 3b$$

$$T) -2 = -a \Rightarrow \boxed{a=2}$$

$\Rightarrow -1 = 2 - 3 \Rightarrow -1 = -1 \Rightarrow$ Las tres ecuaciones son consistentes entre sí.

(1.49)

$$\textcircled{1.42} \quad F = ma = m \frac{v(t_f) - v(t_i)}{\Delta t} \quad \left. \begin{array}{l} \Delta t = t_f - t_i \\ v(t_i) = 0 \end{array} \right\} \Rightarrow \Delta t = \frac{m v(t_f)}{F}$$

$$\Delta t = \frac{m}{F} v(t_f) = \frac{0.2503 \text{ kg}}{10.21 \text{ N}} 5.32 \text{ m/s} = 0.13042076 \text{ s}$$

$$\Delta(\Delta t) = \left| \frac{\partial \Delta t}{\partial m} \right| \Delta m + \left| \frac{\partial \Delta t}{\partial F} \right| \Delta F + \left| \frac{\partial \Delta t}{\partial v(t_f)} \right| \Delta v(t_f) =$$

$$= \frac{v(t_f)}{F} \Delta m + \frac{m}{F^2} v(t_f) \Delta F + \frac{m}{F} \Delta v(t_f)$$

$$\Delta m = 0.0001 \text{ kg} \quad \Delta F = 0.02 \text{ N} \quad \Delta v(t_f) = 0.03 \text{ m/s}$$

$$\begin{aligned} \Delta(\Delta t) &= \frac{5.32 \text{ m/s}}{10.21 \text{ N}} 0.0001 \text{ kg} + \frac{0.2503 \text{ kg}}{(10.21 \text{ N})^2} 5.32 \frac{\text{m}}{\text{s}} 0.02 \text{ N} + \\ &+ \frac{0.2503 \text{ kg}}{10.21 \text{ N}} 0.03 \text{ m/s} = 0.0000521058 + 0.0002554765 + \\ &+ 0.0007354554 = 0.0010430377 \Rightarrow \Delta(\Delta t) = 0.001 \text{ s} \end{aligned}$$

$$\Delta t = 0.13042076 \pm 0.001 \text{ s} \Rightarrow \boxed{\Delta t = 0.130 \pm 0.001 \text{ s}}$$

$$\textcircled{1.43} \quad \left. \begin{array}{l} N(t) = N(0)/4 \\ N(t) = N(0)e^{-\lambda t} \end{array} \right\} \Rightarrow \frac{1}{4} = e^{-\lambda t} \Rightarrow \ln\left(\frac{1}{4}\right) = -\lambda t \Rightarrow$$

$$\Rightarrow -\ln 4 = -\lambda t \Rightarrow t = \frac{\ln 4}{\lambda} \quad \left. \begin{array}{l} T_{1/2} = \frac{\ln 2}{\lambda} \Rightarrow \frac{1}{\lambda} = \frac{T_{1/2}}{\ln 2} \end{array} \right\} \Rightarrow t = \frac{\ln 4}{\ln 2} T_{1/2} = 2 \frac{\ln 2}{\ln 2} T_{1/2} \Rightarrow$$

$$\Rightarrow t = 2T_{1/2} = \boxed{11460 \text{ años}}$$

Otra forma de resolver este problema:

La semivida, $T_{1/2}$, es el tiempo que tarda en reducirse a la mitad el n.º de núcleos iniciales. Por tanto, para llegar a $N(0)/4 = N(0)/2/2$, hacen falta dos semividas $\Rightarrow t = 2T_{1/2} = \boxed{11460 \text{ años}}$

Autor: Iván Cabris Álvarez

1.44 $at^2 + bt + c = 0$ $[c] = L$ $[t] = T$ 1.20

Según el análisis dimensional, at^2 , bt , c y el 0 de la derecha tienen la misma dimensión. Por tanto:

$$[at^2] = [c] \Rightarrow [a][t]^2 = [c] \Rightarrow [a] = \frac{[c]}{[t]^2} = \boxed{LT^{-2}}$$

$$[bt] = [c] \Rightarrow [b][t] = [c] \Rightarrow [b] = \frac{[c]}{[t]} = \boxed{LT^{-1}}$$

El cero de la derecha debe tener la misma dimensión que c , es decir, longitud. El cero proviene de pasar todos los términos de una ecuación a la izquierda. Por tanto, se trata de una magnitud física cuya dimensión es longitud, cuyo valor numérico es cero y cuyas unidades son las mismas que tiene c .

$$[0] = [c] \Rightarrow \boxed{[0] = L}$$

1.45 a) $f = h \cdot z \Rightarrow \left. \begin{array}{l} [f] = [h][z] \\ [h] = L \quad [z] = L \end{array} \right\} \Rightarrow [f] = L^2$

$$f = f(\gamma, \cos \theta, \rho, g) \Rightarrow f = a \gamma^\alpha (\cos \theta)^\beta \rho^\delta g^\epsilon \Rightarrow$$

$$\Rightarrow [f] = [a] [\gamma]^\alpha [\cos \theta]^\beta [\rho]^\delta [g]^\epsilon$$

$$[a] = 1 \quad [\gamma] = \frac{[F]}{[L]} = \frac{MLT^{-2}}{L} = MT^{-2}$$

$$[\cos \theta] = 1 \quad [\rho] = ML^{-3} \quad [g] = LT^{-2}$$

$$\Rightarrow L^2 = (MT^{-2})^\alpha 1^\beta (ML^{-3})^\delta (LT^{-2})^\epsilon \Rightarrow$$

$$\Rightarrow L^2 = M^\alpha T^{-2\alpha} M^\delta L^{-3\delta} L^\epsilon T^{-2\epsilon} \Rightarrow$$

$$\Rightarrow L^2 = M^{\alpha+\delta} T^{-2\alpha-2\epsilon} L^{-3\delta+\epsilon} \Rightarrow$$

$$\Rightarrow 0 = \alpha + \delta$$

$$0 = -2\alpha - 2\epsilon$$

$$2 = -3\delta + \epsilon$$

β puede tomar cualquier valor, porque $[\cos \theta] = 1$.

Autor: Iván Cabris Álvarez

$$\begin{aligned}
 0 &= \alpha + \delta \\
 0 &= -2\alpha - 2\varepsilon \Rightarrow 0 = \alpha + \varepsilon \\
 2 &= -3\delta + \varepsilon \\
 2 &= -3\delta + \varepsilon
 \end{aligned}
 \left. \begin{array}{l} \\ \\ \end{array} \right\} \Rightarrow \delta = \varepsilon \left. \begin{array}{l} \\ \\ \end{array} \right\} \Rightarrow 2 = -3\delta + \delta \Rightarrow \\
 \Rightarrow 2 = -2\delta \Rightarrow \delta = -1 \Rightarrow \varepsilon = -1$$

$$\left. \begin{array}{l} 0 = \alpha + \delta \\ \delta = -1 \end{array} \right\} \Rightarrow \alpha = +1$$

$$\Rightarrow \boxed{f = a \frac{\gamma (\cos \theta)^\beta}{e g} \quad \beta \text{ puede tomar cualquier valor}}$$

b) Suponemos $a=1$ y $\beta=1$.

$$\left. \begin{array}{l} f = \frac{\gamma \cos \theta}{e g} \\ f = h r \end{array} \right\} \Rightarrow h r = \frac{\gamma \cos \theta}{e g} \Rightarrow \gamma = \frac{h r e g}{\cos \theta}$$

$$\frac{\partial \gamma}{\partial h} = \frac{r e g}{\cos \theta} \quad \frac{\partial \gamma}{\partial r} = \frac{h e g}{\cos \theta} \quad \frac{\partial \gamma}{\partial e} = \frac{h r g}{\cos \theta}$$

$$\Delta \gamma = \left| \frac{\partial \gamma}{\partial h} \right| \Delta h + \left| \frac{\partial \gamma}{\partial r} \right| \Delta r + \left| \frac{\partial \gamma}{\partial e} \right| \Delta e =$$

$$= \frac{r e g}{\cos \theta} \Delta h + \frac{h e g}{\cos \theta} \Delta r + \frac{h r g}{\cos \theta} \Delta e$$

$$h = 9.400 \text{ cm} \Rightarrow \Delta h = 10^{-3} \text{ cm} = 10^{-5} \text{ m}$$

$$r = 0.615 \text{ cm} \Rightarrow \Delta r = 10^{-3} \text{ cm} = 10^{-5} \text{ m}$$

$$e = 2280 \frac{\text{kg}}{\text{m}^3} \Rightarrow \Delta e = 1 \frac{\text{kg}}{\text{m}^3}$$

$$\Delta \gamma = \frac{0.00615 \text{ m} \cdot 2280 \text{ kg m}^{-3} \cdot 9.8 \text{ m s}^{-2} \cdot 10^{-5} \text{ m}}{\cos 60^\circ} +$$

$$+ \frac{0.094 \text{ m} \cdot 2280 \text{ kg m}^{-3} \cdot 9.8 \text{ m s}^{-2} \cdot 10^{-5} \text{ m}}{\cos 60^\circ} +$$

$$+ \frac{0.094 \text{ m} \cdot 0.00615 \text{ m} \cdot 9.8 \text{ m s}^{-2} \cdot 1 \text{ kg m}^{-3}}{\cos 60^\circ} = 0.0560857 \frac{\text{N}}{\text{m}}$$

Autor: Iván Gabris Álvarez

$$\Delta \gamma = 0.0560857 \frac{N}{m} \Rightarrow \Delta \gamma = 0.06 \frac{N}{m}$$

(1.22)

$$\gamma = \frac{\rho_2 \rho_1}{\cos \theta} = \frac{0.094 \text{ m} \cdot 0.00615 \text{ m} \cdot 2280 \text{ kg m}^{-3} \cdot 9.8 \text{ m s}^{-2}}{\cos 60^\circ} = 25.834133 \frac{N}{m}$$

$$\gamma = 25.83 \pm 0.06 \frac{N}{m}$$

(1.46) a) $f \equiv \frac{d\sigma}{d\Omega}$ $f = f(q_m, q_p, m_p, v_p, K, \sin(\frac{\theta}{2}))$

$$\frac{d\sigma}{d\Omega} = a q_m^\alpha q_p^\beta m_p^\gamma v_p^\delta K^\epsilon \left(\sin\left(\frac{\theta}{2}\right)\right)^\zeta \quad K \equiv 8\pi\epsilon_0$$

$$\left[\frac{d\sigma}{d\Omega}\right] = \frac{[d\sigma]}{[d\Omega]} = \frac{L^2}{1} = L^2$$

$$[a] = 1 \quad \left[\sin\left(\frac{\theta}{2}\right)\right]^\zeta = \left[\sin\left(\frac{\theta}{2}\right)\right]^\zeta = 1^\zeta = 1$$

ζ puede ser cualquier número.

$$[q_m] = \phi \quad [q_p] = \phi \quad [m_p] = M \quad [v_p] = LT^{-1}$$

$$[K] = [8\pi\epsilon_0] = [8\pi][\epsilon_0] = 1 \cdot [\epsilon_0] = [\epsilon_0]$$

$$F = \frac{q^2}{4\pi\epsilon_0 r^2} \Rightarrow [\epsilon_0] = \frac{[q^2]}{[4\pi r^2 F]} = \frac{\phi^2}{1 \cdot [r]^2 [F]} \Rightarrow$$

$$\Rightarrow [\epsilon_0] = \frac{\phi^2}{L^2 M L T^{-2}} = \phi^2 L^{-3} M^{-1} T^2$$

$$[K] = [\epsilon_0] = \phi^2 L^{-3} M^{-1} T^2$$

$$\left[\frac{d\sigma}{d\Omega}\right] = [a] [q_m]^\alpha [q_p]^\beta [m_p]^\gamma [v_p]^\delta [K]^\epsilon \left[\sin\left(\frac{\theta}{2}\right)\right]^\zeta \Rightarrow$$

$$\Rightarrow L^2 = 1 \cdot \phi^\alpha \phi^\beta M^\gamma (LT^{-1})^\delta (\phi^2 L^{-3} M^{-1} T^2)^\epsilon \cdot 1 \Rightarrow$$

Autor: Iván Gabrín Alvarado

$$\Rightarrow L^2 = Q^{\alpha+\beta+2\varepsilon} M^{\gamma-\varepsilon} L^{8-3\varepsilon} T^{-8+2\varepsilon} \Rightarrow$$

$$\Rightarrow L) \quad 2 = 8 - 3\varepsilon$$

$$M) \quad 0 = \gamma - \varepsilon$$

$$T) \quad 0 = -8 + 2\varepsilon$$

$$Q) \quad 0 = \alpha + \beta + 2\varepsilon$$

Suponemos que $\alpha = \beta$. $\Rightarrow$

$$\Rightarrow 2 = 8 - 3\varepsilon$$

$$0 = \gamma - \varepsilon \Rightarrow \gamma = \varepsilon$$

$$0 = -8 + 2\varepsilon$$

$$0 = \alpha + \varepsilon$$

$$\left. \begin{array}{l} 2 = 8 - 3\varepsilon \\ 0 = -8 + 2\varepsilon \end{array} \right\} \Rightarrow 2 = -\varepsilon \Rightarrow \boxed{\varepsilon = -2}$$

$$0 = \alpha + \varepsilon \Rightarrow \alpha = -\varepsilon = +2 \Rightarrow \boxed{\alpha = 2}$$

$$2 = 8 - 3\varepsilon \Rightarrow 8 = 2 + 3\varepsilon = 2 - 6 = -4 \Rightarrow \boxed{8 = -4}$$

$$\gamma = \varepsilon = -2 \Rightarrow \boxed{\gamma = -2}$$

$$\boxed{\frac{d\sigma}{d\Omega} = a \frac{q_m^2 q_p^2}{K^2 m_p^2 v_p^4} \left(\sin\left(\frac{\theta}{2}\right) \right)^5 = a \left(\frac{q_m q_p}{8\pi\epsilon_0 m v_p^2} \right)^2 \left(\sin\left(\frac{\theta}{2}\right) \right)^5}$$

La fórmula exacta es con $a=1$ y $\xi=-4$.

$$b) \quad f \equiv \frac{d\sigma}{d\Omega} = \frac{q_m^2 q_p^2}{K^2 m_p^2 v_p^4} \left(\sin\left(\frac{\theta}{2}\right) \right)^{-4}$$

$$\frac{\partial f}{\partial \theta} = \frac{q_m^2 q_p^2}{K^2 m_p^2 v_p^4} (-4) \left(\sin\left(\frac{\theta}{2}\right) \right)^{-5} \cos\left(\frac{\theta}{2}\right) \frac{1}{2}$$

$$\frac{\partial f}{\partial v_p} = \frac{q_m^2 q_p^2}{K^2 m_p^2} \left(\sin\left(\frac{\theta}{2}\right) \right)^{-4} (-4) v_p^{-5}$$

Autor: Iván Abria Alvarro

$$\Delta f = \left| \frac{\partial f}{\partial \theta} \right| \Delta \theta + \left| \frac{\partial f}{\partial v_p} \right| \Delta v_p =$$

$$= \frac{q_m^2 q_p^2}{K^2 m_p^2 v_p^4 \left(\sin\left(\frac{\theta}{2}\right) \right)^4} 2 \left| \frac{\cos\left(\frac{\theta}{2}\right)}{\sin\left(\frac{\theta}{2}\right)} \right| \Delta \theta + \frac{q_m^2 q_p^2}{K^2 m_p^2 v_p^4 \left(\sin\left(\frac{\theta}{2}\right) \right)^4} \frac{4 \Delta v_p}{v_p} = f \left(2 \left| \frac{\cos\left(\frac{\theta}{2}\right)}{\sin\left(\frac{\theta}{2}\right)} \right| \Delta \theta + \frac{4 \Delta v_p}{v_p} \right)$$

$$\theta = 5 \text{ grados} \quad \sin\left(\frac{\theta}{2}\right) = \sin(2.5 \text{ grados}) = 0.0436193874$$

$$\left(\sin\left(\frac{\theta}{2}\right) \right)^4 = \left(\sin(2.5 \text{ grados}) \right)^4 = 3.62008065 \cdot 10^{-6}$$

$$\frac{\cos\left(\frac{\theta}{2}\right)}{\sin\left(\frac{\theta}{2}\right)} = \frac{\cos(2.5 \text{ grados})}{\sin(2.5 \text{ grados})} = 22.9037655 \text{ radianes}^{-1}$$

$$\Delta \theta = 0.1 \text{ grados} = \frac{0.1 \pi}{180} \text{ radianes} = 0.0017453293 \text{ radianes}$$

$$f(5^\circ, 15000 \text{ km/s}) = \frac{(1.6020 \cdot 10^{-19} \text{ C})^4 2^2 79^2}{\left(8\pi 8.8540 \cdot 10^{-12} \frac{\text{C}^2}{\text{Nm}^2} \right)^2 (6.64 \cdot 10^{-27} \text{ kg})^2 (1.5 \cdot 10^7 \frac{\text{m}}{\text{s}})^4} \frac{1}{3.62008065 \cdot 10^{-6}} \text{ (estereorradián)}^{-1}$$

$$f(5^\circ, 15000 \text{ km/s}) = 4.10947 \cdot 10^{-23} \frac{\text{m}^2}{\text{estereorradián}}$$

$$\Delta f = 4.10947 \cdot 10^{-23} \frac{\text{m}^2}{\text{estereorradián}} \left(2 \cdot 22.9037655 \cdot 0.0017453293 + \frac{4 \cdot 10^3 \text{ m/s}}{1.5 \cdot 10^7 \text{ m/s}} \right) = 4.10947 \cdot 10^{-23} \frac{\text{m}^2}{\text{estereorradián}} (0.079949224 + 2.6 \cdot 10^{-4})$$

$$\Delta f = 3.29644954 \cdot 10^{-24} \frac{\text{m}^2}{\text{estereorradián}} \Rightarrow \Delta f = 3 \cdot 10^{-24} \frac{\text{m}^2}{\text{estereorradián}}$$

$$f = (41 \pm 3) \cdot 10^{-24} \frac{\text{m}^2}{\text{estereorradián}}$$

Autor: Iván Gabriel Álvarez

(1.47) La actividad A es λN y se mide en desintegraciones/unidad de tiempo. (1.25)

$$\lambda = \frac{\ln 2}{T_{1/2}} \quad T_{1/2} = 1.3 \cdot 10^9 \text{ años} = 1.3 \cdot 10^9 \cdot 365.25 \cdot 86400 \text{ segundos}$$
$$= 4.102488 \cdot 10^{16} \text{ segundos}$$

N : número de núcleos de ^{40}K .

M : masa de ^{40}K en el cuerpo humano.

$$N = \frac{M}{m_{^{40}\text{K}}} N_A$$

$$M = 140 \text{ g} \cdot \frac{0.012}{100} = 0.0168 \text{ g}$$

$$N = \frac{0.0168 \text{ g}}{40 \text{ g}} 6.022 \cdot 10^{23} = 2.52924 \cdot 10^{20} \text{ núcleos de } ^{40}\text{K}$$

$$A = \lambda N = \frac{\ln 2}{T_{1/2}} N = \frac{\ln 2}{4.102488 \cdot 10^{16}} 2.52924 \cdot 10^{20} \text{ desint./s}$$

$$A = 4273.3472 \text{ desintegraciones/s} = \frac{4273.3472}{3.7 \cdot 10^{10}} \text{ Ci} =$$
$$= 1.155 \cdot 10^{-7} \text{ Ci} \approx \boxed{0.1155 \text{ } \mu\text{Ci}}$$

(1.48) $T = 2\pi \sqrt{\frac{l}{g}} \Rightarrow \frac{\partial T}{\partial l} = \frac{2\pi}{2\sqrt{g} \sqrt{l}} = 2\pi \sqrt{\frac{l}{g}} \frac{1}{2l}$

$$\frac{\partial T}{\partial g} = - \frac{2\pi \sqrt{l}}{2g \sqrt{g}} = - 2\pi \sqrt{\frac{l}{g}} \frac{1}{2g}$$

$$\Delta T = \left| \frac{\partial T}{\partial l} \right| \Delta l + \left| \frac{\partial T}{\partial g} \right| \Delta g = 2\pi \sqrt{\frac{l}{g}} \frac{\Delta l}{2l} + 2\pi \sqrt{\frac{l}{g}} \frac{\Delta g}{2g}$$
$$= 2\pi \sqrt{\frac{l}{g}} \left(\frac{\Delta l}{2l} + \frac{\Delta g}{2g} \right)$$

$$T = 2\pi \sqrt{\frac{l}{g}} = 2\pi \sqrt{\frac{1.5}{3.7}} \text{ s} = 4.0005954 \text{ s}$$

Autor: Iván Calvira Álvarez

$$\Delta T = 4.000\,59\,54\,s \left(\frac{0.02}{2 \cdot 1.5} + \frac{0.2}{2 \cdot 3.7} \right) =$$

$$= 4.000\,59\,54\,s \cdot 0.033\,6936 = 0.134\,79\,48\,s \Rightarrow \Delta T = 0.1s \Rightarrow$$

$$\Rightarrow \boxed{T = 4.0 \pm 0.1s}$$

1.49 a)

$$T = T(P, \rho, E) = c P^\alpha \rho^\beta E^\gamma \Rightarrow [T] = [c] [P]^\alpha [\rho]^\beta [E]^\gamma$$

$$\left. \begin{aligned} [T] &= T & [c] &= 1 & [P] &= \frac{[F]}{[S]} = \frac{MLT^{-2}}{L^2} = ML^{-1}T^{-2} \\ [\rho] &= ML^{-3} & [E] &= [F][\Delta z] = MLT^{-2}L = ML^2T^{-2} \end{aligned} \right\} \Rightarrow$$

$$\Rightarrow T = (ML^{-1}T^{-2})^\alpha (ML^{-3})^\beta (ML^2T^{-2})^\gamma \Rightarrow$$

$$\Rightarrow L) \quad 0 = -\alpha - 3\beta + 2\gamma$$

$$M) \quad 0 = \alpha + \beta + \gamma$$

$$T) \quad 1 = -2\alpha - 2\gamma$$

$$\left. \begin{aligned} 0 &= \alpha - 3\beta + 2\gamma \\ 1 &= -2\alpha - 2\gamma \end{aligned} \right\} \Rightarrow 1 = -3\alpha - 3\beta \Rightarrow -\frac{1}{3} = \alpha + \beta$$

$$\left. \begin{aligned} 0 &= \alpha + \beta + \gamma \\ -\frac{1}{3} &= \alpha + \beta \end{aligned} \right\} \Rightarrow \boxed{\gamma = +1/3}$$

$$\left. \begin{aligned} 1 &= -2\alpha - 2\gamma \\ \gamma &= +1/3 \end{aligned} \right\} \Rightarrow 1 = -2\alpha - \frac{2}{3} \Rightarrow \boxed{\alpha = -5/6}$$

$$\left. \begin{aligned} -\frac{1}{3} &= \alpha + \beta \\ \alpha &= -5/6 \end{aligned} \right\} \Rightarrow \boxed{\beta = 1/2}$$

$$\boxed{T = c P^{-5/6} \rho^{1/6} E^{2/6}}$$

Autor: Iván Cabris Álvarez

Comprobamos si la solución es correcta o no:

$$\begin{aligned} [C P^{-5/6} e^{3/6} E^{2/6}] &= [C] [P]^{-5/6} [e]^{3/6} [E]^{2/6} = \\ &= \frac{[F]}{[S]^{-5/6}} (ML^{-3})^{3/6} (ML^2T^{-2})^{2/6} = \\ &= (MLT^{-2})^{-5/6} (L^{-2})^{-5/6} M^{3/6} L^{-9/6} M^{2/6} L^{4/6} T^{-4/6} \\ &= (M^{-5} L^{-5} T^{+10} L^{+10} M^3 L^{-9} M^2 L^4 T^{-4})^{1/6} = \\ &= (M^0 L^{-5+10-9+4} T^{10-4})^{1/6} = (M^0 L^0 T^6)^{1/6} = T = [T] \Rightarrow \end{aligned}$$

$\Rightarrow$ La solución es correcta

b) $T = P^{-5/6} e^{3/6} E^{2/6}$

$$\frac{\partial T}{\partial P} = -\frac{5}{6} P^{-5/6-1} e^{3/6} E^{2/6} = -\frac{5}{6} \frac{T}{P}$$

$$\frac{\partial T}{\partial e} = \frac{3}{6} e^{3/6-1} P^{-5/6} E^{2/6} = \frac{3}{6} \frac{T}{e}$$

$$\frac{\partial T}{\partial E} = \frac{2}{6} E^{2/6-1} P^{-5/6} e^{3/6} = \frac{2}{6} \frac{T}{E}$$

$$\begin{aligned} \Delta T &= \left| \frac{\partial T}{\partial P} \right| \Delta P + \left| \frac{\partial T}{\partial e} \right| \Delta e + \left| \frac{\partial T}{\partial E} \right| \Delta E = \frac{5}{6} \frac{T}{P} \Delta P + \frac{3}{6} \frac{T}{e} \Delta e + \\ &+ \frac{2}{6} \frac{T}{E} \Delta E = \frac{T}{6} \left(5 \frac{\Delta P}{P} + 3 \frac{\Delta e}{e} + 2 \frac{\Delta E}{E} \right) \end{aligned}$$

$$T = P^{-5/6} e^{3/6} E^{2/6} = \left(\frac{P^3 E^2}{P^5} \right)^{1/6} = \left(\frac{(1030)^3 (5.5 \cdot 10^6)^2}{(15 \cdot 10^4)^5} \right)^{1/6} s$$

$$= (4.35292072 \cdot 10^{-4})^{1/6} s = 0.27529403 s$$

$$A \equiv 5 \frac{\Delta P}{P} + 3 \frac{\Delta \rho}{\rho} + 2 \frac{\Delta E}{E} = 5 \frac{0.4}{15} + 3 \frac{30}{1030} + 2 \frac{0.2}{5.5} = 0.29343924$$

$$\Delta T = \frac{T}{6} A = \frac{0.27529403}{6} 0.29343924 = 0.01346 \text{ s}$$

$T = 0.27 \pm 0.01 \text{ s}$

1.50 $P = \left(h + \frac{v^2}{2g}\right) \rho g = h \rho g + \frac{v^2}{2} \rho$

$\Delta h = 0.03 \text{ m}$ $\Delta v = 0.05 \text{ m/s}$ $\Delta g = 0.05 \text{ m/s}^2$

$$\Delta P = \left| \frac{\partial P}{\partial h} \right| \Delta h + \left| \frac{\partial P}{\partial v} \right| \Delta v + \left| \frac{\partial P}{\partial g} \right| \Delta g$$

$$\frac{\partial P}{\partial h} = \rho g \quad \frac{\partial P}{\partial v} = v \rho \quad \frac{\partial P}{\partial g} = h \rho$$

$$\Delta P = \rho g \Delta h + v \rho \Delta v + h \rho \Delta g = h \rho g \frac{\Delta h}{h} + h \rho g \frac{\Delta g}{g} + v^2 \rho \frac{\Delta v}{v}$$
$$= h \rho g \left(\frac{\Delta h}{h} + \frac{\Delta g}{g} \right) + v^2 \rho \frac{\Delta v}{v}$$

$$h \rho g = 1.25 \text{ m} \cdot 1000 \frac{\text{kg}}{\text{m}^3} \cdot 9.8 \text{ m/s}^2 = 12250 \text{ Pa}$$

$$\frac{v^2 \rho}{2} = \frac{(2.5 \text{ m/s})^2}{2} \cdot 1000 \frac{\text{kg}}{\text{m}^3} = 3125 \text{ Pa}$$

$$P = h \rho g + \frac{v^2}{2} \rho = 12250 \text{ Pa} + 3125 \text{ Pa} = 15375 \text{ Pa}$$

$$\Delta P = 12250 \text{ Pa} \left(\frac{0.03 \text{ m}}{1.25 \text{ m}} + \frac{0.05 \text{ m/s}^2}{9.8 \text{ m/s}^2} \right) + 6250 \text{ Pa} \frac{0.05 \text{ m/s}}{2.5 \text{ m/s}} = 356.5 \text{ Pa} + 125 \text{ Pa} = 481.5 \text{ Pa} \Rightarrow \Delta P = 500 \text{ Pa}$$

$P = 15400 \pm 500 \text{ Pa}$

Author: João Gabriel Álvares

TEMA 2 CINEMÁTICA

(2.1)

- 2.1) a) Un movimiento lineal tiene radio de curvatura infinito. Puede ser uniforme, uniformemente acelerado o de otro tipo.
- b) Un movimiento uniforme, i.e., con velocidad constante, tiene aceleración tangencial nula. Un movimiento circular, de cualquier tipo, tiene aceleración normal no nula. Por tanto, un movimiento circular uniforme tiene aceleración tangencial nula y normal no nula.
- c) Un movimiento circular uniformemente acelerado tiene aceleración tangencial no nula y normal no nula.
- d) Un movimiento lineal uniforme tiene aceleración tangencial nula y normal nula.
- e) Un movimiento lineal uniformemente acelerado tiene aceleración tangencial no nula y normal nula.
- f) Un movimiento circular de cualquier tipo tiene radio de curvatura finito y constante para todo t .

2.2) $\frac{dx}{dt} = 9t^2 - t$ $\frac{dy}{dt} = 12t + 2\cos 2t$

$$\vec{v} = (9t^2 - t)\vec{u}_x + (12t + 2\cos 2t)\vec{u}_y$$

$$v = \sqrt{(9t^2 - t)^2 + (12t + 2\cos 2t)^2}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = (18t - 1)\vec{u}_x + (12 - 4\sin 2t)\vec{u}_y$$

2.3) $\vec{r} = t^2\vec{u}_x + t\vec{u}_y + \vec{u}_z$

$$\vec{v} = \frac{d\vec{r}}{dt} = 2t\vec{u}_x + \vec{u}_y$$

$$\vec{a} = \frac{d\vec{v}}{dt} = 2\vec{u}_x$$

$$a_n(0) = \frac{v^2(0)}{R(0)} \Rightarrow R(0) = \frac{v^2(0)}{a_n(0)}$$

$$v(t) = \sqrt{4t^2 + 1} \Rightarrow v(0) = 1$$

$$\vec{a}_n(0) = \vec{a}(0) - \vec{a}_t(0) = 2\vec{u}_x - \vec{a}_t(0)$$

$$\vec{a}_t = \frac{dv}{dt} \vec{e} \quad \vec{e} = \frac{\vec{v}}{v} \quad \frac{dv}{dt} = \frac{1}{2}(4t^2 + 1)^{-1/2} 8t = \frac{4t}{v}$$

$$\vec{a}_t = \frac{4t}{v} \vec{v} = \frac{4t}{4t^2 + 1} \vec{v} = \frac{8t^2\vec{u}_x + 4t\vec{u}_y}{4t^2 + 1} \Rightarrow \vec{a}_t(0) = \vec{0} \Rightarrow \vec{a}_n(0) = 2\vec{u}_x$$

$$\Rightarrow a_n(0) = 2 \Rightarrow R(0) = \frac{1^2}{2} = \boxed{\frac{1}{2}}$$

Autor: Iván Cabria Álvarez

2.4 a) $\vec{v} = \frac{d\vec{r}}{dt} = R(1 - \cos t)\vec{u}_x + R \sin t \vec{u}_y$

$$v = \sqrt{R^2(1 - \cos t)^2 + R^2 \sin^2 t} = R\sqrt{1 + \cos^2 t - 2\cos t + \sin^2 t} = R\sqrt{2 - 2\cos t} =$$

$$= \boxed{R\sqrt{2}\sqrt{1 - \cos t}}$$

b) $\frac{\vec{v}}{v} = \frac{\vec{v}}{R\sqrt{2}\sqrt{1 - \cos t}} = \boxed{\frac{(1 - \cos t)\vec{u}_x}{\sqrt{2}\sqrt{1 - \cos t}} + \frac{\sin t \vec{u}_y}{\sqrt{2}\sqrt{1 - \cos t}}}$

c) $\frac{dv}{dt} = R\sqrt{2} \frac{1}{2} (1 - \cos t)^{-1/2} \sin t = \frac{R \sin t}{\sqrt{2}\sqrt{1 - \cos t}}$

$$\vec{a}_t = \frac{dv}{dt} \frac{\vec{v}}{v} = \frac{R \sin t}{\sqrt{2}\sqrt{1 - \cos t}} \frac{(1 - \cos t)\vec{u}_x}{\sqrt{2}\sqrt{1 - \cos t}} + \frac{R \sin t}{\sqrt{2}\sqrt{1 - \cos t}} \frac{\sin t \vec{u}_y}{\sqrt{2}\sqrt{1 - \cos t}}$$

$$= \frac{R \sin t (1 - \cos t)}{2 (1 - \cos t)} \vec{u}_x + \frac{R \sin^2 t}{2 (1 - \cos t)} \vec{u}_y$$

$$\frac{\sin^2 t}{1 - \cos t} = \frac{\sin^2 t}{2 \sin^2 t/2} = \frac{(2 \sin^2 t/2 \cos^2 t/2)^2}{2 \sin^2 t/2} = 2 \cos^2 t/2$$

$$\cos t = \cos^2 t/2 - \sin^2 t/2$$

$$\boxed{\vec{a}_t = \frac{R}{2} \sin t \vec{u}_x + R (\cos^2 t/2) \vec{u}_y}$$

$$\left. \begin{aligned} \vec{a}_m &= \vec{a} - \vec{a}_t \\ \vec{a} &= \frac{d\vec{v}}{dt} = R \sin t \vec{u}_x + R \cos t \vec{u}_y \end{aligned} \right\} \Rightarrow \vec{a}_m = \left(R \sin t - \frac{R}{2} \sin t \right) \vec{u}_x + \left(R \cos t - R (\cos^2 t/2) \right) \vec{u}_y$$

$$\Rightarrow \vec{a}_m = \frac{R}{2} \sin t \vec{u}_x + R (\cos t - (\cos^2 t/2)) \vec{u}_y$$

$$\cos t - (\cos^2 t/2) = \cos^2 t/2 - \sin^2 t/2 - \cos^2 t/2 = -\sin^2 t/2$$

$$\boxed{\vec{a}_m = \frac{R}{2} \sin t \vec{u}_x - R (\sin^2 t/2) \vec{u}_y}$$

2.5 a) $\vec{r} = \frac{d\vec{z}}{dt} \Rightarrow \vec{z}(t) - \vec{z}(0) = \int_0^t \vec{r}(t) dt \Rightarrow$
 $\int_0^t d\vec{z} = \vec{z}(t) - \vec{z}(0)$

$\Rightarrow \vec{z}(t) = \vec{z}(0) + \int_0^t \vec{r}(t) dt = \vec{u}_z + (e^t - 1)\vec{u}_x + \frac{t^3}{3}\vec{u}_y - \frac{t^4}{12}\vec{u}_z$

$\vec{z}(t) = (e^t - 1)\vec{u}_x + \frac{t^3}{3}\vec{u}_y + (1 - \frac{t^4}{12})\vec{u}_z$

b) $\vec{a} = \frac{d\vec{r}}{dt} = e^t\vec{u}_x + 2t\vec{u}_y - t^2\vec{u}_z$

c) $\vec{a}(0) = \vec{u}_x$ $r = \sqrt{e^{2t} + t^4 + t^6/9} \Rightarrow \frac{dr}{dt} = \frac{2e^{2t} + 4t^3 + \frac{6t^5}{9}}{2r}$

$\left. \frac{da}{dt} \right|_0 = \frac{2}{2r(0)} = \frac{2}{2} = 1$ $\vec{r}(0) = \frac{\vec{r}(0)}{r(0)} = \vec{u}_x$

$\vec{a}_t(0) = \left. \frac{dr}{dt} \right|_0 \vec{r}(0) = \vec{u}_x$

$\vec{a}_m(0) = \vec{a}(0) - \vec{a}_t(0) = \vec{0}$

$R(0) = \frac{r^2(0)}{a_m(0)}$

$\Rightarrow R(0) = \infty$

Otra manera: $a_m(0) = \sqrt{a^2(0) - a_t^2(0)}$
 $a^2(0) = 1$ $a_t^2(0) = \left(\left. \frac{dr}{dt} \right|_{t=0} \right)^2 = 1 \Rightarrow$
 $\Rightarrow a_m(0) = 0 \Rightarrow R(0) = \frac{r^2(0)}{a_m(0)} = \infty$

2.6 a) $\vec{a} = \frac{d\vec{r}}{dt} \Rightarrow \int_0^t \vec{a}(t) dt = \int_0^t d\vec{r} \Rightarrow \frac{t^4}{4}\vec{u}_x + \frac{t^2}{2}\vec{u}_y + \frac{t^3}{3}\vec{u}_z = \vec{r}(t) - \vec{r}(0)$

$\Rightarrow \vec{r}(t) = \vec{r}(0) + \frac{t^4}{4}\vec{u}_x + \frac{t^2}{2}\vec{u}_y + \frac{t^3}{3}\vec{u}_z = (2 + \frac{t^4}{4})\vec{u}_x + \frac{t^2}{2}\vec{u}_y + (3 + \frac{t^3}{3})\vec{u}_z$

b) $\vec{r} = \frac{d\vec{z}}{dt} \Rightarrow \int_0^t \vec{r}(t) dt = \int_0^t d\vec{z} \Rightarrow (2t + \frac{t^5}{20})\vec{u}_x + \frac{t^3}{6}\vec{u}_y + (3t + \frac{t^4}{12})\vec{u}_z = \vec{z}(t) - \vec{z}(0)$

$\Rightarrow \vec{z}(t) = \vec{z}(0) + (2t + \frac{t^5}{20})\vec{u}_x + \frac{t^3}{6}\vec{u}_y + (3t + \frac{t^4}{12})\vec{u}_z = (2t + \frac{t^5}{20})\vec{u}_x + (-1 + \frac{t^3}{6})\vec{u}_y + (1 + 3t + \frac{t^4}{12})\vec{u}_z$

c) $\vec{c} = \frac{\vec{r}}{r} = \frac{(2 + \frac{t^4}{4})\vec{u}_x + \frac{t^2}{2}\vec{u}_y + (3 + \frac{t^3}{3})\vec{u}_z}{\sqrt{(2 + \frac{t^4}{4})^2 + \frac{t^4}{4} + (3 + \frac{t^3}{3})^2}}$

(2.4)

2.7 a) $\vec{a} = -10 \vec{u}_y \text{ m/s}^2$

$$\vec{a} = \frac{d\vec{v}}{dt} \Rightarrow \int_0^t \vec{a} dt = \int_0^t d\vec{v} \Rightarrow -10t \vec{u}_y = \vec{v}(t) - \vec{v}(0) \Rightarrow$$

$$\Rightarrow \vec{v}(t) = \vec{v}(0) - 10t \vec{u}_y = \boxed{3\vec{u}_x + (4 - 10t) \vec{u}_y \text{ m/s}}$$

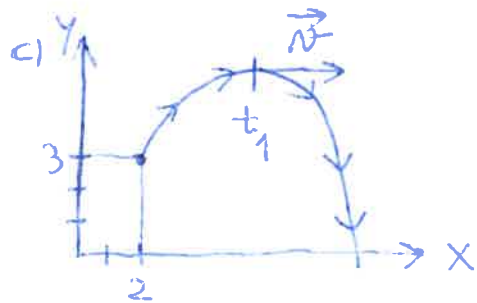
$$\vec{v} = \frac{d\vec{r}}{dt} \Rightarrow \int_0^t \vec{v} dt = \int_0^t d\vec{r} \Rightarrow 3t \vec{u}_x + (4t - 5t^2) \vec{u}_y = \vec{r}(t) - \vec{r}(0) \Rightarrow$$

$$\Rightarrow \vec{r}(t) = \vec{r}(0) + 3t \vec{u}_x + (4t - 5t^2) \vec{u}_y = \boxed{(2 + 3t) \vec{u}_x + (3 + 4t - 5t^2) \vec{u}_y + \vec{u}_z \text{ m}}$$

b) $f(x, y, z) = 0 \quad y = f(x)$

$$\left. \begin{array}{l} x = 2 + 3t \\ y = 3 + 4t - 5t^2 \\ z = 1 \end{array} \right\} \Rightarrow t = \frac{x-2}{3} \Rightarrow y = 3 + \frac{4}{3}(x-2) - \frac{5}{9}(x-2)^2 \Rightarrow$$

$$\Rightarrow y = 3 + \frac{4x}{3} - \frac{8}{3} - \frac{5}{9}x^2 + \frac{20}{9}x - \frac{20}{9} = \boxed{-\frac{17}{9} + \frac{32x}{9} - \frac{5x^2}{9}}$$



El objetivo es calcular $a_t(t_1)$, $a_m(t_1)$ y $R(t_1)$

$$v_y(t_1) = 0$$

$$\left. \begin{array}{l} v_y(t_1) = 0 \\ v_x(t_1) = 4 - 10t_1 \text{ m/s} \end{array} \right\} \Rightarrow t_1 = \frac{4}{10} \text{ s} = 0.4 \text{ s}$$

$$v^2 = 9 + (4 - 10t)^2 = 25 + 100t^2 - 80t \Rightarrow v(t) = \sqrt{100t^2 - 80t + 25} \text{ m/s}$$

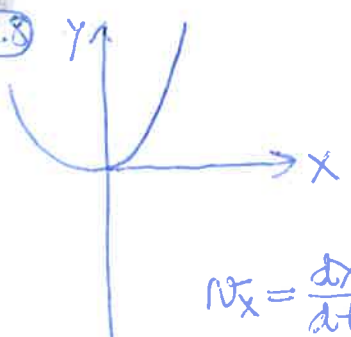
$$\Rightarrow \frac{dv}{dt} = \frac{200t - 80}{2v(t)} = \frac{100t - 40}{v(t)} \text{ m/s}^2$$

$$a_t = \left| \frac{dv}{dt} \right| \Rightarrow a_t(t_1) = \left| \frac{100 \cdot 0.4 - 40}{v(t_1)} \right| = 0 \Rightarrow \boxed{a_t(t_1) = 0 \text{ m/s}^2}$$

$$a_m^2 = a^2 - a_t^2 = 100 - a_t^2(t) \Rightarrow a_m^2(t_1) = 100 \Rightarrow \boxed{a_m(t_1) = 10 \text{ m/s}^2}$$

$$R(t_1) = \frac{v^2(t_1)}{a_m(t_1)} = \frac{25 + 100(0.4)^2 - 80 \cdot 0.4}{10} \text{ m} = \frac{41 - 32}{10} \text{ m} = \boxed{0.9 \text{ m}}$$

Autor: Iván Cabrera Álvarez

2.8 

$$y = \frac{x^2}{2} \text{ m} \quad v_{x0}(t) \equiv v_0 = 2 \text{ m/s}$$

$$x(0) = y(0) = 0$$

a) $v_y(t) = \frac{dy}{dt} = \frac{dy}{dx} \frac{dx}{dt} = \frac{dy}{dx} v_{x0}(t) = \frac{dy}{dx} v_0 = x v_0$

$$v_x = \frac{dx}{dt} \Rightarrow \int_0^t v_x(t) dt = \int_{x(0)}^{x(t)} dx \Rightarrow \int_0^t v_0 dt = \int_{x(0)}^{x(t)} dx \Rightarrow$$

$$\Rightarrow v_0 t = x(t) - x(0) \Rightarrow x(t) = v_0 t \Rightarrow v_y(t) = v_0^2 t$$

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{v_0^2 + v_0^4 t^2} = v_0 \sqrt{1 + v_0^2 t^2}$$

$$v(\sqrt{2} \text{ s}) = v_0 \sqrt{1 + 4(\sqrt{2})^2} = 2 \sqrt{1+8} \frac{\text{m}}{\text{s}} = \boxed{6 \frac{\text{m}}{\text{s}}}$$

b) $a_t = \frac{dv}{dt} = \frac{v_0 2 v_0^2 t}{2 \sqrt{1 + v_0^2 t^2}} = \frac{v_0^3 t}{\sqrt{1 + v_0^2 t^2}} \text{ en } \text{m/s}^2$

$$a_t(\sqrt{2} \text{ s}) = \frac{2^3 \sqrt{2}}{\sqrt{1+4 \cdot 2}} = \boxed{\frac{8}{3} \sqrt{2} \frac{\text{m}}{\text{s}^2}}$$

$a_m^2 = a^2 - a_t^2$ Necesitamos calcular $\vec{a}$

$$\left. \begin{array}{l} a_x = \frac{dv_x}{dt} \\ v_x = v_0 \end{array} \right\} \Rightarrow a_x = 0$$

$$\left. \begin{array}{l} a_y = \frac{dv_y}{dt} \\ v_y = v_0^2 t \end{array} \right\} \Rightarrow a_y = v_0^2$$

$$\vec{a} = v_0^2 \vec{u}_y \Rightarrow a^2(\sqrt{2} \text{ s}) = v_0^4$$

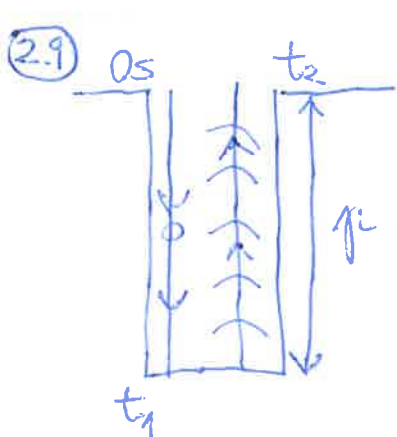
$$\left. \begin{array}{l} a_t(\sqrt{2} \text{ s}) = \frac{8}{3} \sqrt{2} \frac{\text{m}}{\text{s}^2} \Rightarrow a_t^2(\sqrt{2} \text{ s}) = \frac{128}{9} \frac{\text{m}^2}{\text{s}^4} \\ a_m^2 = a^2 - a_t^2 \end{array} \right\} \Rightarrow a_m^2(\sqrt{2} \text{ s}) = v_0^4 - \frac{128}{9} =$$

$$= 16 - \frac{128}{9} = \frac{(144-128)}{9}$$

$$= \frac{16}{9} \frac{\text{m}^2}{\text{s}^4}$$

$$\Rightarrow \boxed{a_m(\sqrt{2} \text{ s}) = \frac{4}{3} \frac{\text{m}}{\text{s}^2}}$$

c) $R = \frac{v^2}{a_m} \Rightarrow R(\sqrt{2} \text{ s}) = \frac{v^2(\sqrt{2} \text{ s})}{a_m(\sqrt{2} \text{ s})} = \frac{6^2 \text{ m}^2/\text{s}^2}{(4/3) \text{ m/s}^2} = \boxed{27 \text{ m}}$



(2.6)

La piedra llega al fondo del pozo en el instante t_1
 El sonido se escucha en el instante t_2
 La piedra se deja caer en el instante $t=0$
 v_s : velocidad del sonido en el aire
 $t_2 = 2\text{ s}$
 $t_2 = t_1 + h/v_s$

La piedra recorre una distancia h en t_1 segundos:

$$S(t) = S(0) + v(0)t + \frac{at^2}{2} = S(0) + \frac{gt^2}{2}$$

$$h = S(t_1) - S(0) = \frac{gt_1^2}{2} \Rightarrow t_2 = t_1 + \frac{g}{2v_s} t_1^2 \Rightarrow$$

$$t_2 \neq \Rightarrow 2 = t_1 + \frac{9.8}{2 \cdot 343} t_1^2 \Rightarrow 0.0142857 t_1^2 + t_1 - 2 = 0 \Rightarrow$$

$$\Rightarrow t_1 = \frac{-1 \pm \sqrt{1 + 8 \cdot 0.0142857}}{2 \cdot 0.0142857} = \begin{cases} 1.95\text{ s} \\ -71.95\text{ s} \end{cases}$$

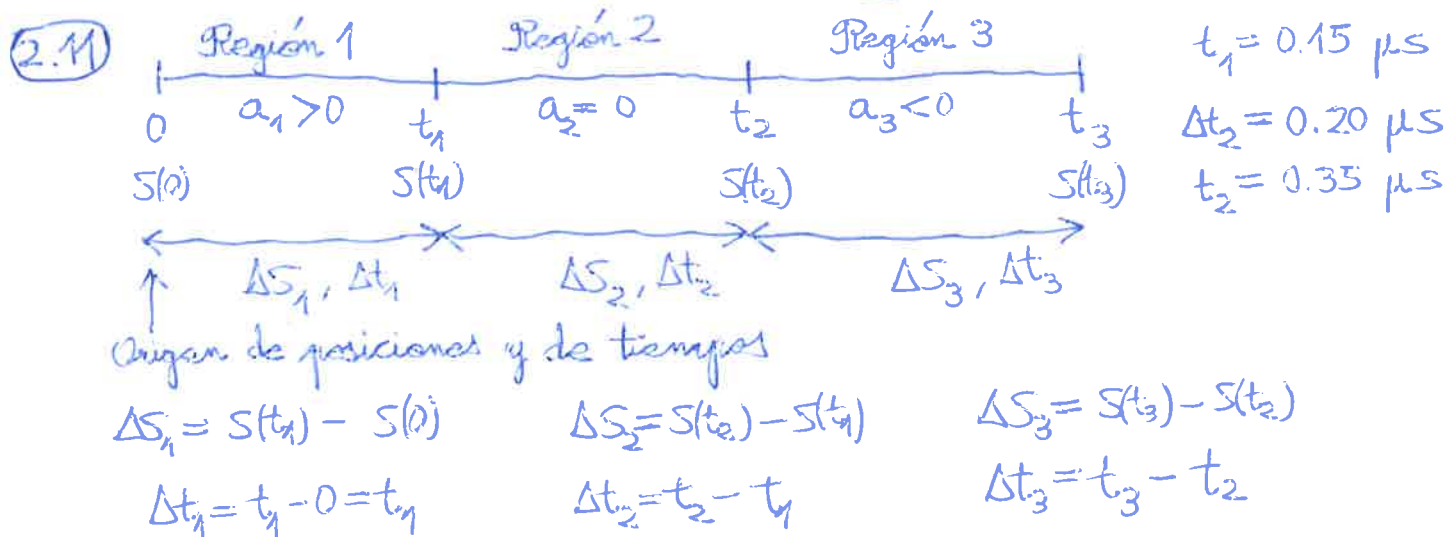
Solo un t_1 positivo tiene sentido físico $\Rightarrow t_1 = 1.95\text{ s}$

$$h = \frac{gt_1^2}{2} = \frac{9.8}{2} (1.95\text{ s})^2 \frac{\text{m}}{\text{s}^2} = \boxed{18.55\text{ m}}$$

(2.10) $v(t) = v(0) + at \Rightarrow v(5) = 5 \frac{\text{m}}{\text{s}} + 5 \frac{\text{m}}{\text{s}^2} \cdot 5\text{ s} = 30 \text{ m/s}$

$$S(t) = S(5) + v(5)(t-5) + \frac{a}{2}(t-5)^2 = S(5) + 30(t-5) \frac{\text{m}}{\text{s}}$$

$$d = S(15) - S(5) = 30(15-5) \frac{\text{m}}{\text{s}} = \boxed{300\text{ m}}$$



$$S(0) = 0 \quad v(0) = 0$$

El problema pide hallar ΔS

$$\Delta S = \Delta S_1 + \Delta S_2 + \Delta S_3$$

Autor: Yoán Cabris Álvarez

En la región 1: $S(t) = S(0) + v(0)t + \frac{a_1 t^2}{2} = \frac{a_1 t^2}{2} \quad 0 \leq t \leq t_1 \quad (2.7)$

$$\Delta S_1 = S(t_1) - S(0) = \frac{a_1 t_1^2}{2} = \frac{5.33 \cdot 10^{12}}{2} \frac{\text{m}}{\text{s}^2} (0.15 \cdot 10^{-6} \text{s})^2 = 0.05996 \text{ m}$$

$$v(t) = v(0) + a_1 t \Rightarrow v(t_1) = v(0) + a_1 t_1 = a_1 t_1 = 5.33 \cdot 10^{12} \frac{\text{m}}{\text{s}^2} \cdot 0.15 \cdot 10^{-6} \text{s}$$

$$v(t_1) = 0.7995 \cdot 10^6 \frac{\text{m}}{\text{s}} \quad S(t_1) = \Delta S_1 \quad 0 \leq t \leq t_1$$

En la región 2: $S(t) = S(t_1) + v(t_1)(t-t_1) + \frac{a_2 (t-t_1)^2}{2} = S(t_1) + v(t_1)(t-t_1) \quad t_1 \leq t \leq t_2$

$$\Delta S_2 = S(t_2) - S(t_1) = v(t_1)(t_2 - t_1) = v(t_1) \Delta t_2 =$$

$$= 0.7995 \cdot 10^6 \frac{\text{m}}{\text{s}} \cdot 0.20 \cdot 10^{-6} \text{s} = 0.1599 \text{ m}$$

$$v(t) = v(t_1) + a_2 (t-t_1) = v(t_1) \Rightarrow v(t_2) = v(t_1) = 0.7995 \cdot 10^6 \frac{\text{m}}{\text{s}} \quad t_1 \leq t \leq t_2$$

En la región 3: $S(t) = S(t_2) + v(t_2)(t-t_2) + \frac{a_3 (t-t_2)^2}{2} \quad t_2 \leq t \leq t_3 \quad a_3 < 0$

$$\Delta S_3 = S(t_3) - S(t_2) = v(t_2)(t_3 - t_2) + \frac{a_3 (t_3 - t_2)^2}{2} = v(t_2) \Delta t_3 + \frac{a_3}{2} (\Delta t_3)^2$$

Necesitamos calcular Δt_3 . Para ello, calculamos $v(t)$ en la región 3:

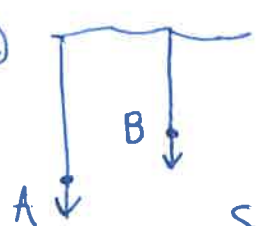
$$\left. \begin{aligned} v(t) &= v(t_2) + a_3 (t-t_2) \quad t_2 \leq t \leq t_3 \\ v(t_3) &= 0 \end{aligned} \right\} \Rightarrow 0 = v(t_2) + a_3 \Delta t_3 \Rightarrow$$

$$\Rightarrow \Delta t_3 = \frac{-v(t_2)}{a_3} = \frac{v(t_2)}{|a_3|} = \frac{0.7995 \cdot 10^6 \text{ m/s}}{2.67 \cdot 10^{13} \text{ m/s}^2} = 0.29944 \cdot 10^{-7} \text{s}$$

$$\Delta S_3 = 0.7995 \cdot 10^6 \frac{\text{m}}{\text{s}} \cdot 0.29944 \cdot 10^{-7} \text{s} - \frac{2.67 \cdot 10^{13}}{2} \frac{\text{m}}{\text{s}^2} (0.29944 \cdot 10^{-7} \text{s})^2$$

$$= 0.02394 \text{ m} - 0.01197 \text{ m} = 0.01197 \text{ m}$$

$$\Delta S = 0.05996 + 0.1599 + 0.01197 \text{ m} = \boxed{0.232 \text{ m}}$$

(2.12)  $t_B > 0$

$$S_A(t) = S_A(0) + v_A(0)(t-0) + \frac{g}{2} (t-0)^2 \quad t \geq 0$$

$$S_B(t) = S_B(0) + v_B(t_B)(t-t_B) + \frac{g}{2} (t-t_B)^2 \quad t \geq t_B$$

$$S_A(0) = S_B(t_B) \quad v_A(0) = v_B(t_B) = 0$$

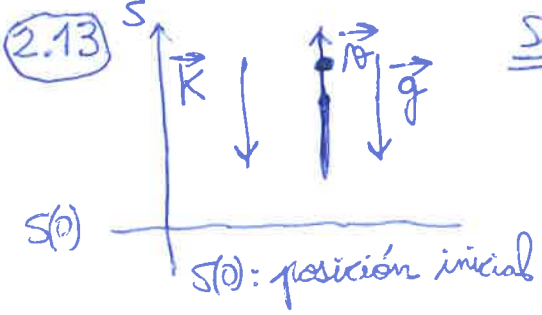
Autor: Iván Cabria Álvarez

$$S_A(t) - S_B(t) = \frac{g}{2} (t-0)^2 - \frac{g}{2} (t-t_B)^2 \quad t \geq t_B$$

$$= \frac{g}{2} (t^2 - t^2 - t_B^2 + 2tt_B) = \boxed{\frac{g}{2} t_B (2t - t_B) \quad t \geq t_B}$$

La distancia entre A y B, $S_A(t) - S_B(t)$, depende de t y aumenta con t

(2.13)

Subida a_s : aceleración durante la subida Δt_s : tiempo de subida t_s : instante en el que el objeto alcanza la altura máxima. Su velocidad es cero en ese instante: $v(t_s) = 0$

$$\Delta t_s = t_s - 0 = t_s \quad a_s > 0 \quad v(0) > 0$$

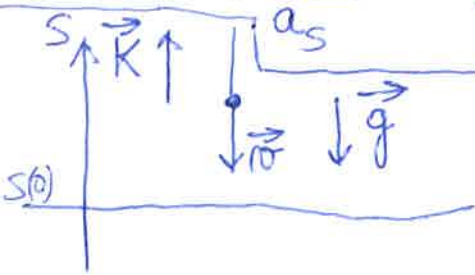
$$a_s = g + K$$

$$\left. \begin{aligned} v(t) &= v(0) - a_s t \\ v(t_s) &= 0 \end{aligned} \right\} \Rightarrow 0 = v(0) - a_s t_s \Rightarrow \Delta t_s = t_s = \frac{v(0)}{a_s}$$

Gamos a calcular $S(t_s)$. Necesitaremos $S(t_s)$ para calcular el tiempo o duración de la bajada.

$$S(t) = S(0) + v(0)t - \frac{a_s}{2}t^2 \Rightarrow S(t_s) = S(0) + v(0)t_s - \frac{a_s}{2}t_s^2 =$$

$$= S(0) + \frac{v^2(0)}{a_s} - \frac{a_s}{2} \frac{v^2(0)}{a_s^2} = S(0) + \frac{v^2(0)}{2a_s}$$

Bajada a_b : aceleración durante la bajada Δt_b : tiempo de bajada t_b : instante en el que el objeto alcanza la posición inicial: $S(t_b) = S(0)$

$$\Delta t_b = t_b - t_s \quad a_b > 0 \quad a_b = |g - K|$$

$$S(t) = S(t_s) + v(t_s)(t - t_s) - \frac{a_b}{2}(t - t_s)^2 = S(t_s) - \frac{a_b}{2}(t - t_s)^2 =$$

$$= S(0) + \frac{v^2(0)}{2a_s} - \frac{a_b}{2}(t - t_s)^2 \Rightarrow S(t_b) = S(0) + \frac{v^2(0)}{2a_s} - \frac{a_b}{2}\Delta t_b^2$$

$$S(t_b) = S(0) \Rightarrow 0 = \frac{v^2(0)}{2a_s} - \frac{a_b}{2}\Delta t_b^2 \Rightarrow \Delta t_b = \frac{v(0)}{\sqrt{a_s a_b}}$$

$$\left. \begin{aligned} a_s &= g + K \\ a_b &= |g - K| \end{aligned} \right\} \Rightarrow \sqrt{a_b} < \sqrt{a_s} \Rightarrow \frac{1}{\sqrt{a_b}} > \frac{1}{\sqrt{a_s}} \Rightarrow \frac{v(0)}{\sqrt{a_s a_b}} > \frac{v(0)}{\sqrt{a_s a_s}} = \frac{v(0)}{a_s}$$

$$\Delta t_s = \frac{v(0)}{a_s}$$

$$\Delta t_b = \frac{v(0)}{\sqrt{a_s a_b}}$$

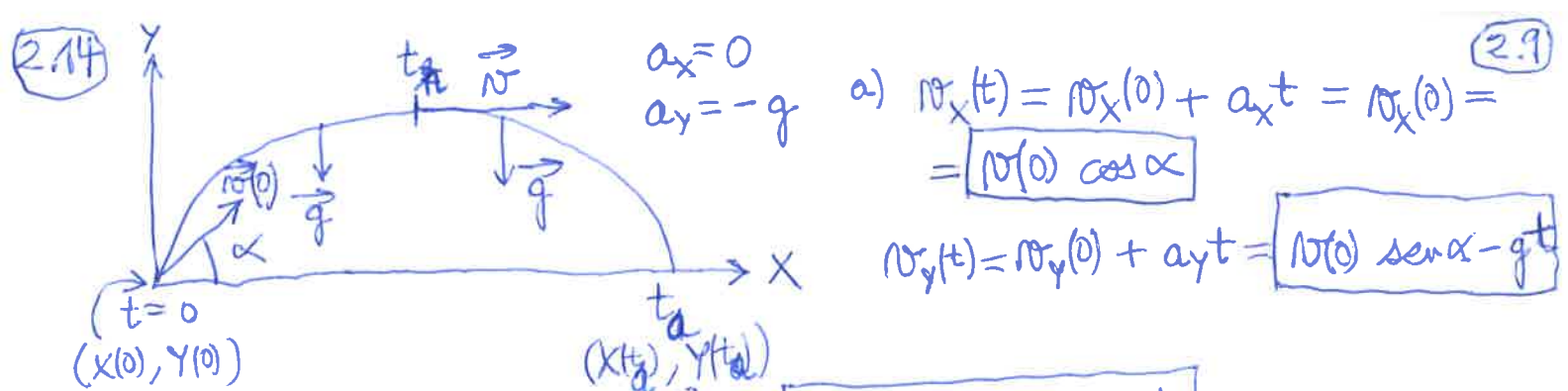
$$\frac{v(0)}{\sqrt{a_s a_b}} > \frac{v(0)}{a_s}$$

Autor: Juan Cabria Alvaro

$\Delta t_b > \Delta t_s$ El tiempo de bajada es mayor que el tiempo de subida

Si no hubiera rozamiento, $K=0 \Rightarrow a_s = a_b \Rightarrow$

$\Rightarrow \Delta t_b = \Delta t_s$ El tiempo de bajada sería igual que el tiempo de subida



b) $x(t) = x(0) + v_x(0)t + \frac{a_x t^2}{2} = x(0) + v(0) \cos \alpha t$

$$y(t) = y(0) + v_y(0)t + \frac{a_y t^2}{2} = y(0) + v(0) \sin \alpha t - \frac{g t^2}{2}$$

c) Alcance máximo: $A = x(t_a) - x(0) \Rightarrow A = v(0) \cos \alpha t_a$
 Necesitamos calcular t_a

En t_a tenemos: $y(0) = y(t_a) \Rightarrow y(0) = y(0) + v(0) \sin \alpha t_a - \frac{g t_a^2}{2} \Rightarrow$
 $\Rightarrow v(0) \sin \alpha t_a = \frac{g}{2} t_a^2 \Rightarrow t_a = \frac{2 v(0) \sin \alpha}{g}$

$$A = v(0) \cos \alpha \frac{2 v(0) \sin \alpha}{g} = \frac{v^2(0)}{g} \sin 2\alpha$$

d) Altura máxima: $h = y(t_a) - y(0)$

t_a es el instante en el que el proyectil alcanza la altura máxima. En ese instante, $v_y(t_a)$ es nula.

$$\left. \begin{aligned} v_y(t_a) &= 0 \Rightarrow \\ v_y(t) &= v(0) \sin \alpha - g t \end{aligned} \right\} \Rightarrow 0 = v(0) \sin \alpha - g t_a \Rightarrow t_a = \frac{v(0) \sin \alpha}{g}$$

$$h = v(0) \sin \alpha t_a - \frac{g}{2} t_a^2 = \frac{v^2(0) \sin^2 \alpha}{g} - \frac{g}{2} \frac{v^2(0) \sin^2 \alpha}{g^2} = \frac{v^2(0) \sin^2 \alpha}{2g}$$

e) $A(\alpha) = A(\beta) \Rightarrow \frac{v^2(0)}{g} \sin 2\alpha = \frac{v^2(0)}{g} \sin 2\beta \Rightarrow$
 $\left. \begin{aligned} \sin(\pi - 2\alpha + 2\pi m) &= \sin 2\alpha \\ m &= 0, \pm 1, \pm 2, \pm 3, \dots \end{aligned} \right\} \Rightarrow$

$$\Rightarrow \pi - 2\alpha + 2\pi m = 2\beta \Rightarrow \alpha + \beta = \frac{\pi}{2} + m\pi$$

Al tratarse de un cañón, $m=0 \Rightarrow \alpha + \beta = \frac{\pi}{2}$ α y β son ángulos complementarios

Es posible conseguir el mismo alcance. Para lograrlo, $\alpha + \beta = \frac{\pi}{2}$

Autor: Iván Cabria Álvarez

(2.15)

(2.10)

a) t_a : tiempo de llegada al suelo

$$t_a = \frac{2v(0) \sin \alpha}{g} \Rightarrow$$

Llegará antes el proyectil que salga con menor velocidad inicial $v(0)$

$$b) t_a = \frac{2v(0) \sin \alpha}{g} \Rightarrow$$

No llegará antes el proyectil con más masa. Llegarán los dos proyectiles al mismo tiempo, porque t_a no depende de la masa y sí depende de $v(0)$ y α

c) h : altura máxima

$$h = \frac{v^2(0) \sin^2 \alpha}{2g}$$

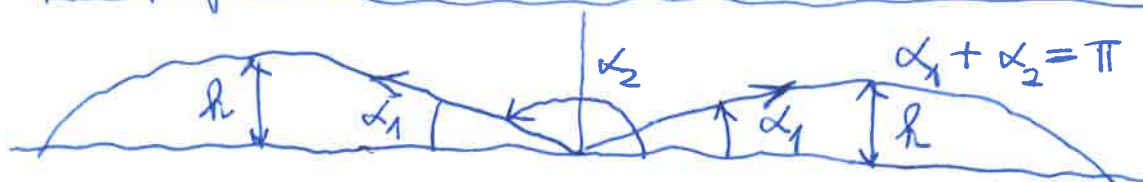
Suponemos que $h(\alpha_1) = h(\alpha_2)$

$$\Rightarrow |\sin \alpha_1| = |\sin \alpha_2| \Rightarrow$$

$$\Rightarrow \pi - \alpha_1 + 2\pi m = \alpha_2 \quad m=0, \pm 1, \pm 2, \dots \Rightarrow \alpha_1 + \alpha_2 = \pi + 2\pi m$$

Al tratarse de un cañón, $0 \leq \alpha_1 \leq \pi$ y $0 \leq \alpha_2 \leq \pi \Rightarrow \alpha_1 + \alpha_2 = \pi \Rightarrow$

$\Rightarrow$ Es posible conseguir la misma altura máxima, pero lanzando el proyectil en el sentido contrario



$$d) h = \frac{v^2(0) \sin^2 \alpha}{2g}$$

$$A = \frac{v^2(0)}{g} \sin 2\alpha$$

$\Rightarrow$ Ninguna de las dos depende de la masa del proyectil

$$e) X(t) = X(0) + v(0) \cos \alpha t$$

$$Y(t) = Y(0) + v(0) \sin \alpha t - \frac{gt^2}{2} \Rightarrow$$

La posición del proyectil en el instante t no depende de la masa del proyectil

Autor: Iván Cabris Alvaro

(2.16)

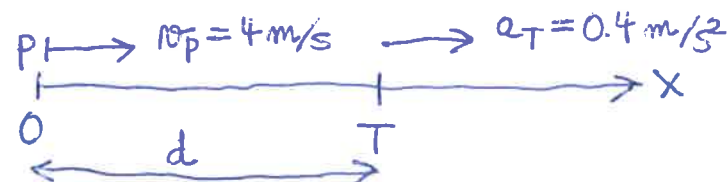


$$v_R = \frac{L}{30} \text{ m/s} \quad v_e = \frac{L}{20} \text{ m/s}$$

Composición de velocidades $\Rightarrow v = v_R + v_e$

$$v = \frac{L}{t} \Rightarrow t = \frac{L}{v} \Rightarrow t = \frac{L}{v_R + v_e} =$$

$$= \frac{L}{\frac{L}{30} + \frac{L}{20}} \text{ s} = \frac{1}{\frac{1}{30} + \frac{1}{20}} \text{ s} = \frac{1}{\frac{20}{600} + \frac{30}{600}} = \frac{600}{50} = \boxed{12 \text{ s}}$$

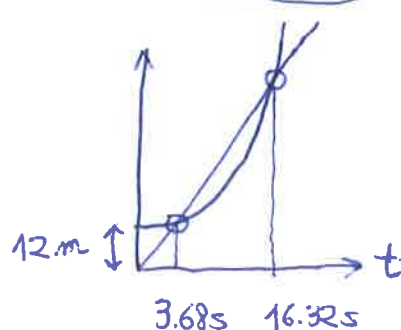
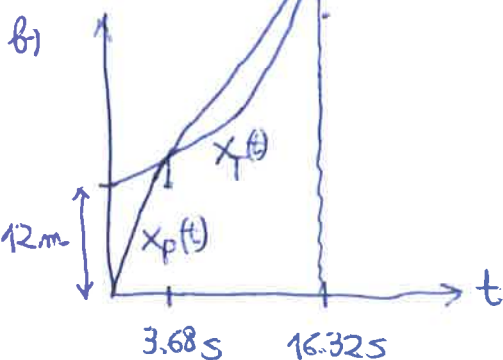
(2.17) 

a) $x_T(0) = d$ $x_P(0) = 0$
 $v_T(0) = 0$ $v_P(t) = v_P(0) = 4 \text{ m/s}$
 $a_T = 0.4 \text{ m/s}^2$ $a_P = 0$

$$\begin{aligned} x_T(t) &= x_T(0) + v_T(0)t + a_T \frac{t^2}{2} = d + a_T \frac{t^2}{2} \\ x_P(t) &= x_P(0) + v_P(0)t + a_P \frac{t^2}{2} = v_P(0)t \end{aligned} \Rightarrow d + a_T \frac{t^2}{2} = v_P(0)t \Rightarrow x_T(t) = x_P(t)$$

$$\Rightarrow a_T \frac{t^2}{2} - v_P(0)t + d = 0 \Rightarrow t = \boxed{3.68 \text{ s}} \text{ para } d = 12 \text{ m}$$

$$\Rightarrow \boxed{16.32 \text{ s}}$$



$x_P(t)$ es una recta
 $x_T(t)$ es una parábola

Autor: Juan Abria Álvarez

c) $x_P(t_c) = x_T(t_c) \Rightarrow d_c + a_T \frac{t_c^2}{2} = v_P(0)t_c \Rightarrow t_c = \frac{v_P(0) \pm \sqrt{v_P^2(0) - 2a_T d_c}}{a_T}$

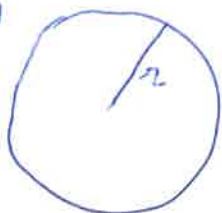
Un solo $t_c \Rightarrow v_P^2(0) - 2a_T d_c = 0 \Rightarrow d_c = \frac{v_P^2(0)}{2a_T} = \boxed{20 \text{ m}}$

d) $d = d_c \Rightarrow t_c = \frac{v_P(0)}{a_T}$

$$v_T(t) = v_T(0) + a_T t = a_T t \Rightarrow v_T(t_c) = a_T \frac{v_P(0)}{a_T} = v_P(0) = \boxed{4 \frac{\text{m}}{\text{s}}}$$

$$v_{T \text{ media}} = (x_T(t_c) - x_T(0)) / t_c = (a_T t_c^2 / 2) / t_c = a_T t_c / 2 = v_P(0) / 2 = \boxed{2 \text{ m/s}}$$

2.18



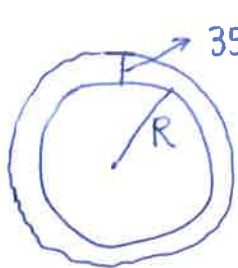
$$a_t = r \alpha = 0.8 \text{ m} \cdot 50 \text{ rad/s}^2 = \boxed{40 \text{ m/s}^2}$$

$$a_m = r \omega = \omega^2 r = (10 \text{ rad/s})^2 \cdot 0.8 \text{ m} = \boxed{80 \text{ m/s}^2}$$

$$a = \sqrt{a_t^2 + a_m^2} = \boxed{89.44 \text{ m/s}^2}$$

2.19

2.19



$$35 \text{ km} = h$$

$$R = 6370 \text{ km}$$

$$r = R + h = 6405 \text{ km} = 6.405 \cdot 10^6 \text{ m}$$

$$v = \frac{1}{T} = \frac{1}{86400 \text{ s}} = \boxed{1.16 \cdot 10^{-5} \text{ s}^{-1}}$$

$$\omega = \frac{2\pi}{T} = \boxed{7.27 \cdot 10^{-5} \text{ rad/s}}$$

$$a_m = \omega r = \omega^2 r = (7.27 \cdot 10^{-5} \text{ rad/s})^2 \cdot 6.405 \cdot 10^6 \text{ m} = \boxed{0.034 \text{ m/s}^2}$$

2.20

$$v(0) = 18000 \text{ km/h} = \frac{18 \cdot 10^6 \text{ m}}{3600 \text{ s}} = 5000 \text{ m/s}$$

$$v(t) = v(0) + at \Rightarrow v(t_1) = v(0) + at_1$$

$$t_1 = 2 \times 3600 \text{ s} = 7200 \text{ s}$$

$$v(t_1) = 0 \text{ km/h}$$

$$v(0) = 5000 \text{ m/s}$$

$$\Rightarrow 0 = 5000 + a \cdot (7200) \Rightarrow$$

$$\Rightarrow a = -\frac{5000}{(7200)} \frac{\text{m}}{\text{s}^2} = -0.694 \text{ m/s}^2 = \boxed{-0.694 \cdot 10^{-3} \text{ km/s}^2}$$

El signo negativo significa que se trata de una desaceleración

$$2.21 \quad S(t) = S(0) + v(0)t + \frac{at^2}{2} \Rightarrow S(5) = S(0) + 0 + 10 \frac{\text{m}}{\text{s}^2} \frac{5^2}{2} \text{ s}^2 = S(0) + 125 \text{ m}$$

$$d(5) = S(5) - S(0) = \boxed{125 \text{ m}}$$

$$2.22 \quad S(t) = S(0) + v(0)t + \frac{at^2}{2} \Rightarrow d_i(t) = S(t) - S(0) = 0 + \frac{g_i t^2}{2} = g_i \frac{t^2}{2} \quad i = \text{Tierra} \text{ o } \text{Luna}$$

$$d_{\text{Tierra}} = 9.8 \frac{\text{m}}{\text{s}^2} \frac{12^2}{2} \text{ s}^2 = \boxed{705.6 \text{ m}}$$

$$d_{\text{Luna}} = 1.63 \frac{\text{m}}{\text{s}^2} \frac{12^2}{2} \text{ s}^2 = \boxed{117.36 \text{ m}}$$

$$2.23 \quad d(t) = S(t) - S(0) = v(0)t + \frac{at^2}{2} \Rightarrow$$

$$= 60 \text{ km} + 5 \text{ km} = \boxed{65 \text{ km}}$$

$$d(1 \text{ h}) = 60 \frac{\text{km}}{\text{h}} 1 \text{ h} + \frac{10}{2} \frac{\text{km}}{\text{h}^2} 1^2 \text{ h}^2 =$$

$$2.24 \quad d(t) = v(0)t + \frac{at^2}{2} \Rightarrow d(1 \text{ h}) = 60 \frac{\text{km}}{\text{h}} 1 \text{ h} - 60 \frac{\text{km}}{\text{h}^2} \frac{1}{2} \text{ h}^2 = 60 \text{ km} - 30 \text{ km} \Rightarrow$$

$$\Rightarrow \boxed{d(1 \text{ h}) = 30 \text{ km}}$$

Autor: Iván Gabris Alvarado

2.25 La velocidad final es $v(t_1)$

$$v(t_1) = v(0) + at_1$$

$$v(0) = 0$$

$$a = 500\,000 \text{ m/s}^2$$

Necesitamos calcular t_1 :

$$\left. \begin{aligned} s(t) &= s(0) + v(0)t + \frac{at^2}{2} = \frac{at^2}{2} \\ s(t_1) &= 1 \text{ m} \end{aligned} \right\} \Rightarrow 1 \text{ m} = 500\,000 \frac{\text{m}}{\text{s}^2} \frac{t_1^2}{2} \Rightarrow$$

$$\Rightarrow t_1 = \sqrt{\frac{2}{500\,000}} \text{ s} = 0.002 \text{ s} \Rightarrow v(t_1) = 500\,000 \frac{\text{m}}{\text{s}^2} 0.002 \text{ s} = \boxed{1000 \frac{\text{m}}{\text{s}}}$$

2.26 Los círculos son $\Delta\theta = 6\pi$ radianes

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{6\pi}{9} \frac{\text{rad}}{\text{s}} = \boxed{\frac{2\pi}{3} \frac{\text{rad}}{\text{s}}}$$

$$2.27 \quad T = \frac{2\pi}{\omega} = \frac{2\pi \text{ rad}}{8.7 \cdot 10^{-4} \text{ rad/s}} = \boxed{7222.05 \text{ s}}$$

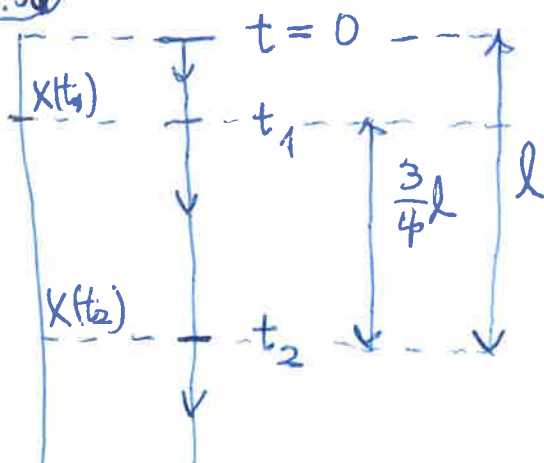
$$2.28 \quad \omega(t) = \omega(0) + \alpha t \Rightarrow \alpha = \frac{\omega(t) - \omega(0)}{t} \Rightarrow \alpha = \frac{\omega(t_1) - \omega(0)}{t_1} = \frac{(5 - 2) \text{ rad/s}}{0.1 \text{ s}} \Rightarrow$$

$$\Rightarrow \alpha = \boxed{30 \text{ rad/s}^2}$$

2.28

$$\left. \begin{aligned} \omega &= 2.1 \frac{\text{rad}}{\text{s}} \\ \omega &= \frac{2\pi}{T} \end{aligned} \right\} \Rightarrow T = \frac{2\pi}{2.1} \text{ s} = \boxed{2.99 \text{ s}}$$

(2.30)



l : espacio recorrido en t_2 segundos

$\frac{3}{4}l$: espacio recorrido entre los instantes t_1 y t_2

$\frac{3}{4}l$: también es el espacio recorrido durante el último segundo

$$\Delta t = t_2 - t_1 = 1 \text{ segundo}$$

$$x(t) = x(0) + v(0)t + \frac{at^2}{2} = \frac{gt^2}{2}$$

$$x(0) = 0 \quad v(0) = 0$$

Autor: Iván Cabria Álvarez

Calculamos t_1 y t_2 :

$$\left. \begin{array}{l} l = X(t_2) \\ \frac{3}{4}l = X(t_2) - X(t_1) \end{array} \right\} \Rightarrow \frac{3}{4}l = l - X(t_1) \Rightarrow X(t_1) = \frac{l}{4}$$

$$\left. \begin{array}{l} X(t) = \frac{gt^2}{2} \Rightarrow X(t_1) = \frac{gt_1^2}{2} \\ X(t_1) = \frac{l}{4} \end{array} \right\} \Rightarrow \frac{l}{4} = \frac{gt_1^2}{2} \Rightarrow t_1^2 = \frac{l}{2g} \Rightarrow t_1 = \sqrt{\frac{l}{2g}}$$

$$\left. \begin{array}{l} X(t) = \frac{gt^2}{2} \Rightarrow X(t_2) = \frac{gt_2^2}{2} \\ X(t_2) = l \end{array} \right\} \Rightarrow l = \frac{gt_2^2}{2} \Rightarrow t_2^2 = \frac{2l}{g} \Rightarrow t_2 = \sqrt{\frac{2l}{g}}$$

Calculamos $(\Delta t)^2$ en función de l/g :

$$\left. \begin{array}{l} \Delta t = t_2 - t_1 \Rightarrow (\Delta t)^2 = t_2^2 + t_1^2 - 2t_1t_2 \\ t_1 = \sqrt{\frac{l}{2g}} \\ t_2 = \sqrt{\frac{2l}{g}} \end{array} \right\} \Rightarrow \left. \begin{array}{l} t_1t_2 = \sqrt{\frac{l}{2g}} \sqrt{\frac{2l}{g}} = \frac{l}{g} \\ t_1^2 = \frac{l}{2g} \\ t_2^2 = \frac{2l}{g} \end{array} \right\} \Rightarrow (\Delta t)^2 = \frac{2l}{g} + \frac{l}{2g} - \frac{2l}{g} \Rightarrow$$

$$\Rightarrow (\Delta t)^2 = \frac{4l}{2g} + \frac{l}{2g} - \frac{4l}{2g} \Rightarrow (\Delta t)^2 = \frac{l}{2g}$$

Sustituimos $2(\Delta t)^2 = \frac{l}{g}$ en t_2 :

$$\left. \begin{array}{l} t_2 = \sqrt{\frac{2l}{g}} \\ 2(\Delta t)^2 = \frac{l}{g} \end{array} \right\} \Rightarrow t_2 = \sqrt{4(\Delta t)^2} = 2\Delta t = \boxed{2s}$$

(2.31)

$$a(t) = \frac{dv}{dt} \Rightarrow dv = a(t) dt \Rightarrow \int_{t_0}^t dv = \int_{t_0}^t a(t) dt \Rightarrow$$

$$\Rightarrow v(t) - v(t_0) = \int_{t_0}^t a(t) dt \Rightarrow v(t) - v(t_0) = \int_{t_0}^t 5t dt \text{ m/s} \Rightarrow$$

$$\Rightarrow v(t) - v(t_0) = \left. \frac{5t^2}{2} \right|_{t_0}^t \text{ m/s}^2 \Rightarrow v(t) - v(t_0) = \frac{5t^2}{2} - \frac{5t_0^2}{2} \text{ m/s}$$

$$\Rightarrow v(t) = v(t_0) + \frac{5}{2} (t^2 - t_0^2) \text{ m/s}$$

$$t_0 = 0 \Rightarrow \boxed{v(t) = v(0) + \frac{5}{2} t^2 \text{ m/s}}$$

(2.32) Parte del reposo, $v(0) = 0$, y acelera con una aceleración igual a $3g$, $a = 3g$, durante 24 horas, $t = 86400 \text{ s}$.

$$v(t) = v(0) + at \Rightarrow v(24 \text{ horas}) = 0 + 3g \cdot 86400 \text{ s} =$$

$$= 3 \cdot 9.8 \frac{\text{m}}{\text{s}^2} \cdot 86400 \text{ s} = \boxed{2\,540\,160 \frac{\text{m}}{\text{s}}}$$

$$\boxed{v(24 \text{ horas}) = 0.0084672 c}$$

$$c = 300\,000 \text{ km/s}$$

$$= 3 \cdot 10^8 \text{ m/s}$$

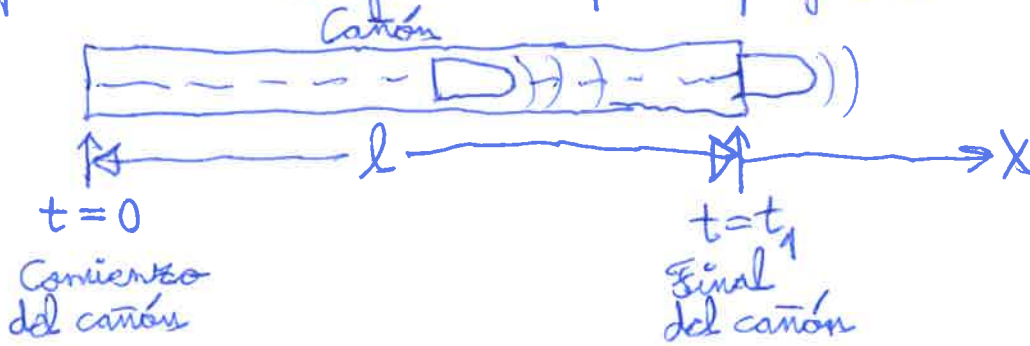
$$s = 4 \text{ años-luz} = c \cdot 4 \text{ años}$$

$$t = \frac{s}{v(24 \text{ horas})} = \frac{c \cdot 4 \text{ años}}{0.0084672 c} = \boxed{472.41 \text{ años}}$$

$$\left. \begin{aligned} (2.33) \quad a(t) &= \frac{dv}{dt} \\ a(t) &= \frac{8}{3} 10^5 - \frac{32}{9} 10^7 t \end{aligned} \right\} \Rightarrow \frac{dv}{dt} = \frac{8}{3} 10^5 - \frac{32}{9} 10^7 t \Rightarrow$$

$$\left. \begin{aligned} \Rightarrow v(t) - v(0) &= \frac{8}{3} 10^5 t - \frac{16}{9} 10^7 t^2 \\ v(0) &= 0 \end{aligned} \right\} \Rightarrow v(t) = \frac{8}{3} 10^5 t - \frac{16}{9} 10^7 t^2$$

t_1 : instante de tiempo en el que el proyectil sale del cañón.



l : longitud del cañón

$$l = X(t_1) - X(0)$$

$$\left. \begin{aligned} v(t_1) &= 1000 \text{ m/s} \\ v(t) &= \frac{8}{3} 10^5 t - \frac{16}{9} 10^7 t^2 \end{aligned} \right\} \Rightarrow 1000 = \frac{8}{3} 10^5 t_1 - \frac{16}{9} 10^7 t_1^2 \Rightarrow$$

$$\Rightarrow \frac{16}{9} 10^7 t_1^2 - \frac{8}{3} 10^5 t_1 + 1000 = 0 \Rightarrow$$

$$\Rightarrow t_1 = \frac{\frac{8}{3} 10^5 \pm \sqrt{\frac{64}{9} 10^{10} - 4 \frac{16}{9} 10^7 10^3}}{\frac{32}{9} 10^7} \quad s = \frac{\frac{8}{3} 10^5}{\frac{32}{9} 10^7} \quad s = \frac{8 \cdot 9}{3 \cdot 8 \cdot 4 \cdot 100} s$$

$$\Rightarrow t_1 = \frac{3^{\cancel{7}}}{\cancel{8} \cdot 4 \cdot 100} s = \frac{3}{4} 10^{-2} s = 0.0075 s = \frac{3}{400} s$$

Calculamos la longitud del cañón: $l = X(t_1) - X(0)$

Calculamos $X = X(t)$:

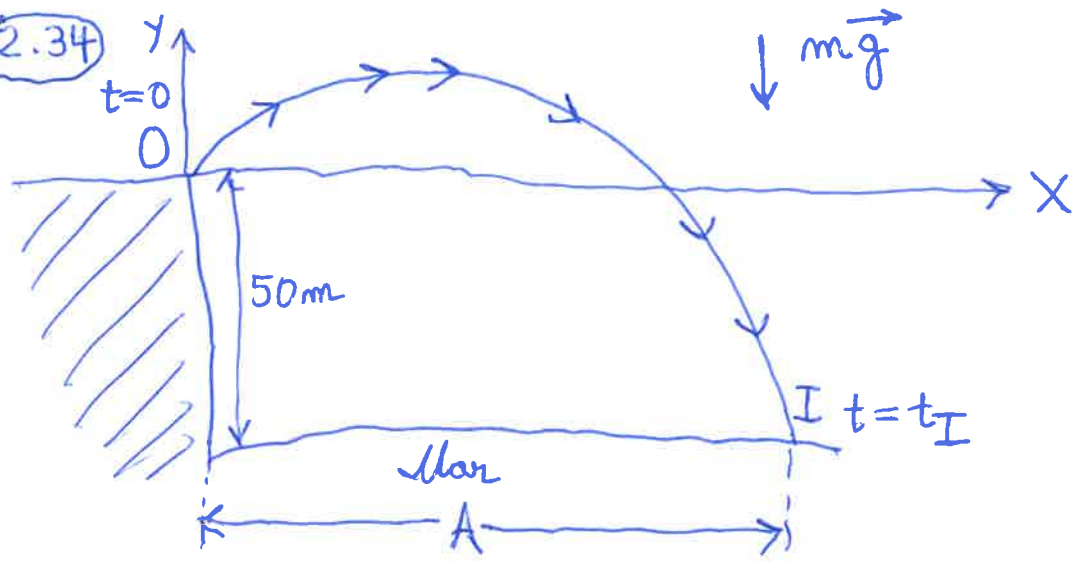
$$\left. \begin{aligned} v(t) &= \frac{dX}{dt} \\ v(t) &= \frac{8}{3} 10^5 t - \frac{16}{9} 10^7 t^2 \end{aligned} \right\} \Rightarrow \frac{dX}{dt} = \frac{8}{3} 10^5 t - \frac{16}{9} 10^7 t^2 \Rightarrow X(t) - X(0) = \frac{8}{6} 10^5 t^2 - \frac{16}{27} 10^7 t^3$$

$$\begin{aligned} l = X(t_1) - X(0) &= \frac{8}{6} 10^5 t_1^2 - \frac{16}{27} 10^7 t_1^3 = \frac{\cancel{4}}{3} 10^5 \frac{9}{4^{\cancel{2}} 10^4} - \frac{16}{27} 10^7 \frac{27}{4^3 10^6} \text{ m} \\ &= \frac{3}{4} 10 - \frac{10}{4} \text{ m} = \frac{20}{4} \text{ m} = \boxed{5 \text{ m}} \end{aligned}$$

Autor: Yoán Cabria Alvarado

2.34

2.17



$t=0$: Instante del lanzamiento.

t_I : Instante de tiempo del impacto o llegada al mar.

$$m\vec{g} = -m g \vec{u}_y \quad x(0)=0 \quad y(0)=0 \quad v(0)=300 \text{ m/s} \quad a_x=0 \quad a_y=-g$$

$$\vec{v}(0) = v(0) \cos 45^\circ \vec{u}_x + v(0) \sin 45^\circ \vec{u}_y = \frac{v(0)}{\sqrt{2}} \vec{u}_x + \frac{v(0)}{\sqrt{2}} \vec{u}_y$$

$$v_x(0) = \frac{v(0)}{\sqrt{2}} \quad v_y(0) = \frac{v(0)}{\sqrt{2}}$$

$$X(t) = X(0) + v_x(0)t + \frac{a_x t^2}{2} = \frac{v(0)}{\sqrt{2}} t$$

$$Y(t) = Y(0) + v_y(0)t + \frac{a_y t^2}{2} = \frac{v(0)}{\sqrt{2}} t - \frac{g t^2}{2}$$

a) Calculamos t_I :

$$Y(t_I) = -50 \text{ m}$$

$$Y(t) = \frac{v(0)t}{\sqrt{2}} - \frac{g t^2}{2} \Rightarrow Y(t_I) = \frac{v(0)}{\sqrt{2}} t_I - \frac{g t_I^2}{2} \Rightarrow$$

$$\Rightarrow \frac{g}{2} t_I^2 - \frac{v(0)}{\sqrt{2}} t_I + Y(t_I) = 0 \Rightarrow t_I = \frac{\frac{v(0)}{\sqrt{2}} \pm \sqrt{\frac{v(0)^2}{2} - 2gY(t_I)}}{g}$$

$$t_I = \frac{\frac{300}{\sqrt{2}} \pm \sqrt{\frac{90000}{2} - 2 \cdot 9.8(-50)}}{9.8} \text{ s} = \begin{cases} 43.52668 \text{ s} \\ -0.2344 \text{ s} \end{cases}$$

t_I debe ser mayor que cero. Por tanto, $t_I = 43.53 \text{ s}$

$$b) A = X(t_I) - X(0) = \frac{v(0)}{\sqrt{2}} t_I = \frac{300}{\sqrt{2}} 43.52668 \text{ m} = 9233.4 \text{ m}$$

Autor: Juan Cabria Alvaro

$$\left. \begin{aligned} (2.35) \quad \vec{r}(t) &= (3t-2)\vec{u}_x + 5\vec{u}_y \\ \vec{r}(t) &= \frac{d\vec{r}}{dt} \end{aligned} \right\} \Rightarrow \frac{d\vec{r}}{dt} = (3t-2)\vec{u}_x + 5\vec{u}_y \Rightarrow (2.18)$$

$$\Rightarrow d\vec{r} = (3t-2)dt\vec{u}_x + 5dt\vec{u}_y \Rightarrow$$

$$\Rightarrow \int_2^t d\vec{r} = \int_2^t (3t-2)dt\vec{u}_x + \int_2^t 5dt\vec{u}_y \Rightarrow$$

$$\Rightarrow \vec{r}(t) - \vec{r}(2) = \left[\frac{3t^2}{2} - 2t \right]_2^t \vec{u}_x + [5t]_2^t \vec{u}_y \Rightarrow$$

$$\Rightarrow \vec{r}(t) = \vec{r}(2) + \left(\frac{3t^2}{2} - 2t - \frac{3 \cdot 2^2}{2} + 2 \cdot 2 \right) \vec{u}_x + (5t - 5 \cdot 2) \vec{u}_y \Rightarrow$$

$$\Rightarrow \vec{r}(t) = \vec{u}_x + 5\vec{u}_y + \left(\frac{3t^2}{2} - 2t - 2 \right) \vec{u}_x + (5t - 10) \vec{u}_y \Rightarrow$$

$$\Rightarrow \vec{r}(t) = \left(\frac{3t^2}{2} - 2t - 1 \right) \vec{u}_x + (5t - 5) \vec{u}_y \Rightarrow$$

$$\left. \begin{aligned} \Rightarrow X &= \frac{3t^2}{2} - 2t - 1 \\ Y &= 5t - 5 \Rightarrow t = \frac{Y}{5} + 1 \end{aligned} \right\} \Rightarrow X = \frac{3}{2} \left(\frac{Y}{5} + 1 \right)^2 - 2 \left(\frac{Y}{5} + 1 \right) - 1 \Rightarrow$$

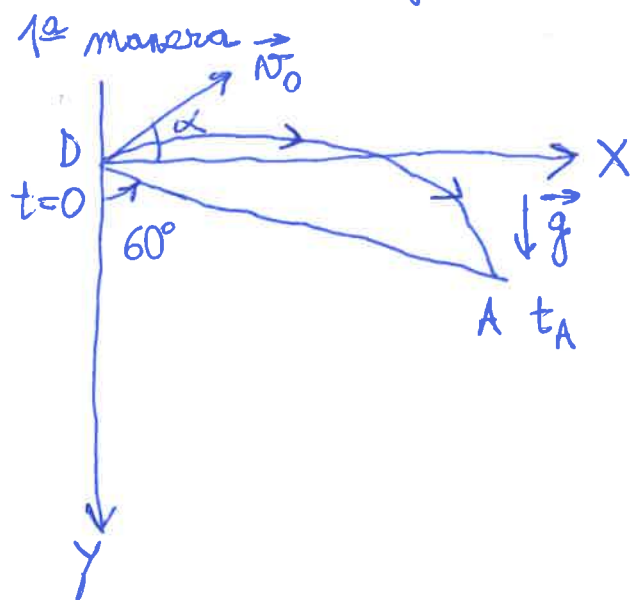
$$\Rightarrow X = \frac{3}{2} \left(\frac{Y^2}{25} + 1 + \frac{2Y}{5} \right) - \frac{2Y}{5} - 2 - 1 \Rightarrow$$

$$\Rightarrow X = \frac{3}{50} Y^2 + \frac{3}{2} + \frac{3Y}{5} - \frac{2Y}{5} - 3 \Rightarrow X = \frac{3Y^2}{50} + \frac{Y}{5} - \frac{3}{2} \Rightarrow$$

$$\Rightarrow 50X = 3Y^2 + 10Y - 75 \Rightarrow \boxed{3Y^2 + 10Y - 75 - 50X = 0}$$

Autor: Yoán Cabria Álvarez

2.36) Vamos a explicar cinco maneras de resolver este problema. (2.11)



$$\vec{v}_0 \equiv \vec{v}(0) = v_0 \cos \alpha \vec{u}_x - v_0 \sin \alpha \vec{u}_y$$

$$v_0 \equiv v(0) = 20 \text{ m/s} \quad \alpha = 10^\circ$$

$$\vec{g} = +g \vec{u}_y \quad g = 9.8 \text{ m/s}^2$$

Punto D: (0,0) $x(0)=0$ $y(0)=0$

Punto A: (X_A, Y_A) $X_A = X(t_A) > 0$ $Y_A = Y(t_A) > 0$

$t=0$: Instante del despegue

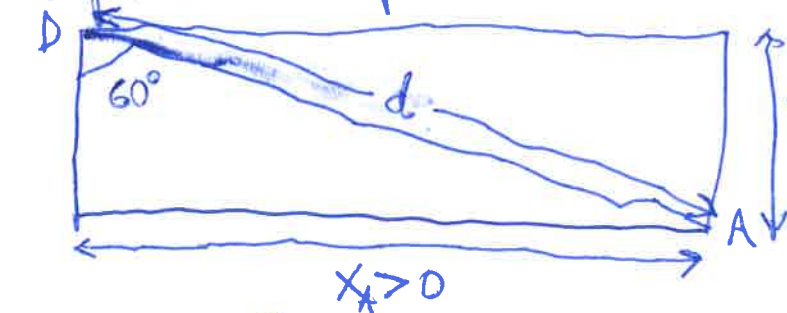
t_A : Instante del aterrizaje $t_A > 0$

$$\Delta t_A = t_A - 0 = t_A$$

$$x(t) = x(0) + v_{x(0)} t$$

$$y(t) = y(0) + v_{y(0)} t + \frac{gt^2}{2} \quad \left. \vphantom{y(t)} \right\} \Rightarrow \quad \begin{aligned} x(t) &= v_0 \cos \alpha t \\ y(t) &= -v_0 \sin \alpha t + \frac{gt^2}{2} \end{aligned}$$

El punto A es el punto de corte de la recta DA con la parábola $r(x(t), y(t))$.



Los puntos de la recta DA cumplen $\cotan 60^\circ = \frac{Y}{X}$

$$\cotan 60^\circ = \frac{Y_A}{X_A}$$

$$X_A = X(t_A) = v_0 \cos \alpha t_A$$

$$Y_A = Y(t_A) = -v_0 \sin \alpha t_A + \frac{gt_A^2}{2}$$

$$\cotan 60^\circ = \frac{-v_0 \sin \alpha t_A + \frac{gt_A^2}{2}}{v_0 \cos \alpha t_A} \quad t_A > 0$$

$$\Rightarrow \cotan 60^\circ = \frac{-v_0 \sin \alpha + \frac{g}{2} t_A}{v_0 \cos \alpha} \Rightarrow v_0 \cos \alpha \cotan 60^\circ = -v_0 \sin \alpha + \frac{g}{2} t_A \Rightarrow$$

$$\Rightarrow t_A = \frac{2v_0}{g} (\cos \alpha \cotan 60^\circ + \sin \alpha) = \frac{2 \cdot 20 \text{ m/s}}{9.8 \text{ m/s}^2} (\cos 10^\circ \cotan 60^\circ + \sin 10^\circ) =$$

$$= \boxed{3.0295 \text{ s}}$$

De la figura anterior se deduce que $\sin 60^\circ = \frac{X_A}{d}$.

$$\sin 60^\circ = \frac{X_A}{d} \Rightarrow d = \frac{X_A}{\sin 60^\circ} = \frac{v_0 \cos \alpha t_A}{\sin 60^\circ} = \frac{20 \text{ m/s} \cos 10^\circ 3.0295 \text{ s}}{\sin 60^\circ} =$$

$$= \boxed{68.9 \text{ m}}$$

Autor: Iván Cabria Álvarez

Otra manera de calcular la distancia d :

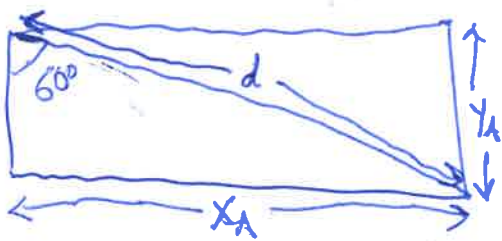
$$X_A = v_0 \cos \alpha t_A = 20 \frac{\text{m}}{\text{s}} \cos 10^\circ 3.0295 \text{ s} = 59.6695 \text{ m}$$

$$Y_A = -v_0 \sin \alpha t_A + \frac{g}{2} t_A^2 = -20 \frac{\text{m}}{\text{s}} \sin 10^\circ 3.0295 \text{ s} + \frac{9.8 \text{ m/s}^2}{2} (3.0295 \text{ s})^2 = 34.4502 \text{ m}$$

$$d = \sqrt{X_A^2 + Y_A^2} = \sqrt{(59.6695 \text{ m})^2 + (34.4502 \text{ m})^2} = \boxed{68.9 \text{ m}}$$

2ª manera Esta manera difiere de la 1ª manera en calcular primero la distancia d y luego el tiempo t_A .

$$\left. \begin{aligned} X_A &= d \sin 60^\circ \\ Y_A &= v_0 \cos \alpha t_A \end{aligned} \right\} \Rightarrow 1 = \frac{d \sin 60^\circ}{v_0 \cos \alpha t_A} \Rightarrow t_A = \frac{d \sin 60^\circ}{v_0 \cos \alpha}$$



$$X_A = d \sin 60^\circ$$

$$Y_A = d \cos 60^\circ$$

$$\left. \begin{aligned} Y_A &= d \cos 60^\circ \\ Y_A &= -v_0 \sin \alpha t_A + \frac{g}{2} t_A^2 \end{aligned} \right\} \Rightarrow d \cos 60^\circ = -v_0 \sin \alpha t_A + \frac{g}{2} t_A^2$$

$$t_A = \frac{d \sin 60^\circ}{v_0 \cos \alpha}$$

$$\left. \begin{aligned} d \cos 60^\circ &= -v_0 \sin \alpha t_A + \frac{g}{2} t_A^2 \\ t_A &= \frac{d \sin 60^\circ}{v_0 \cos \alpha} \end{aligned} \right\} \Rightarrow d \cos 60^\circ = -v_0 \sin \alpha \frac{d \sin 60^\circ}{v_0 \cos \alpha} + \frac{g}{2} \frac{d^2 (\sin 60^\circ)^2}{v_0^2 (\cos \alpha)^2}$$

$$\Rightarrow \cos 60^\circ = -\tan \alpha \sin 60^\circ + \frac{g d (\sin 60^\circ)^2}{2 v_0^2 (\cos \alpha)^2} \Rightarrow$$

$$\Rightarrow \cos 60^\circ + \tan \alpha \sin 60^\circ = d \frac{g}{2 v_0^2} \frac{(\sin 60^\circ)^2}{(\cos \alpha)^2} \Rightarrow$$

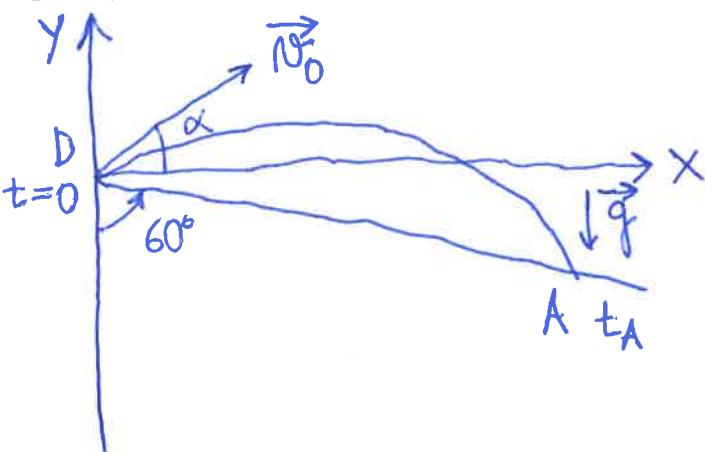
$$\Rightarrow d = \frac{2 v_0^2 (\cos \alpha)^2}{g (\sin 60^\circ)^2} (\cos 60^\circ + \tan \alpha \sin 60^\circ) =$$

$$= \frac{2 (20 \text{ m/s})^2 (\cos 10^\circ)^2}{9.8 \text{ m/s}^2 (\sin 60^\circ)^2} (\cos 60^\circ + \tan 10^\circ \sin 60^\circ) = \boxed{68.9 \text{ m}}$$

$$t_A = \frac{d \sin 60^\circ}{v_0 \cos \alpha} = \frac{68.9 \text{ m} \sin 60^\circ}{20 \text{ m/s} \cos 10^\circ} = \boxed{3.0295 \text{ s}}$$

Autor: Iván Gabriel Alvarado

3ª manera



$$\vec{v}_0 \equiv \vec{v}(0) = v_0 \cos \alpha \vec{u}_x + v_0 \sin \alpha \vec{u}_y \quad (2.21)$$

$$v_0 \equiv v(0) = 20 \text{ m/s} \quad \alpha = 10^\circ$$

$$\vec{g} = -g \vec{u}_y \quad g = 9.8 \text{ m/s}^2$$

$$\text{Punto D: } (0, 0) \quad X(0) = 0 \quad Y(0) = 0$$

$$\text{Punto A: } (X_A, Y_A) \quad X_A = X(t_A) > 0 \quad Y_A = Y(t_A) < 0$$

$t=0$: Instante del despegue

t_A : Instante del aterrizaje $t_A > 0$

$$\Delta t_A = t_A - 0 = t_A$$

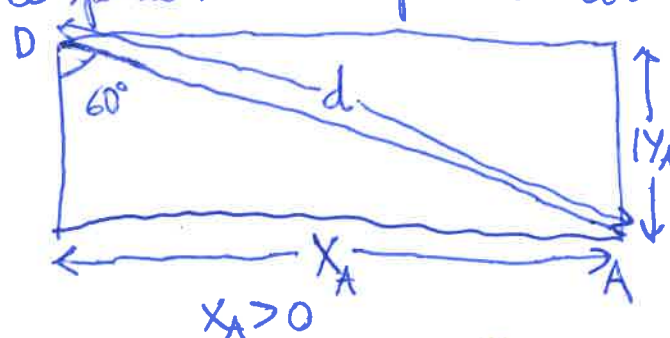
$$X(t) = X(0) + v_{0x}(0)t$$

$$Y(t) = Y(0) + v_{0y}(0)t - \frac{gt^2}{2}$$

$$X(t) = v_0 \cos \alpha t$$

$$Y(t) = v_0 \sin \alpha t - \frac{gt^2}{2}$$

El punto A es el punto de corte de la recta DA con la parábola $(X(t), Y(t))$.



Los puntos de la recta DA cumplen
 $\cotan 60^\circ = \frac{Y}{X}$

$$\cotan 60^\circ = \frac{|Y_A|}{X_A} \Rightarrow \cotan 60^\circ = -\frac{Y_A}{X_A}$$

$$Y_A < 0 \Rightarrow |Y_A| = -Y_A$$

$$X_A = X(t_A) = v_0 \cos \alpha t_A$$

$$Y_A = Y(t_A) = v_0 \sin \alpha t_A - \frac{gt_A^2}{2}$$

$$\Rightarrow \cotan 60^\circ = \frac{-v_0 \sin \alpha t_A + gt_A^2/2}{v_0 \cos \alpha t_A} \Rightarrow \cotan 60^\circ = \frac{-v_0 \sin \alpha + gt_A/2}{v_0 \cos \alpha} \Rightarrow$$

$$t_A > 0$$

$$\Rightarrow v_0 \cos \alpha \cotan 60^\circ = -v_0 \sin \alpha + \frac{g}{2} t_A \Rightarrow t_A = \frac{2v_0}{g} (\cos \alpha \cotan 60^\circ + \sin \alpha) \Rightarrow$$

$$\Rightarrow t_A = \frac{2 \cdot 20 \text{ m/s}}{9.8 \text{ m/s}^2} (\cos 10^\circ \cotan 60^\circ + \sin 10^\circ) = \boxed{3.0295 \text{ s}}$$

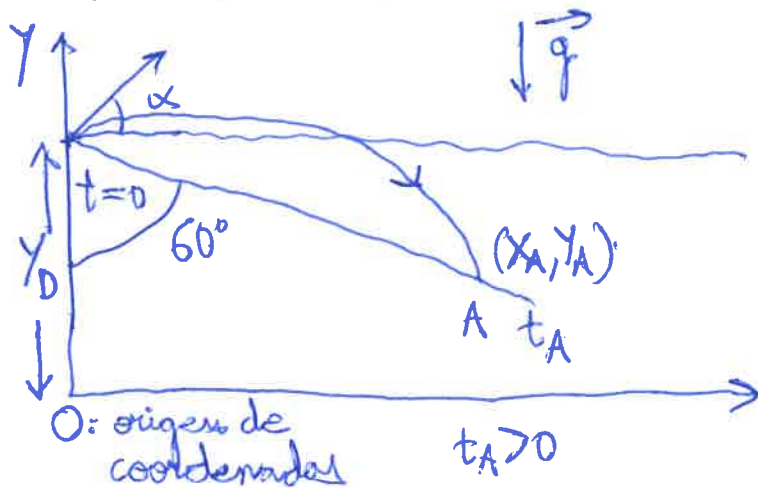
De la figura anterior se deduce que $\sin 60^\circ = \frac{X_A}{d}$.

$$\sin 60^\circ = \frac{X_A}{d} \Rightarrow d = \frac{X_A}{\sin 60^\circ} = \frac{v_0 \cos \alpha t_A}{\sin 60^\circ} = \frac{20 \text{ m/s} \cos 10^\circ 3.0295 \text{ s}}{\sin 60^\circ} =$$

$$= \boxed{68.9 \text{ m}}$$

Autor: Iván Cabria Álvarez

Cuarta manera



$$\vec{r}_0 = \vec{r}(0) = r_0 \cos \alpha \vec{u}_x + r_0 \sin \alpha \vec{u}_y \quad (2.23)$$

$$r_0 = 20 \text{ m/s} \quad \alpha = 10^\circ$$

$$\vec{g} = -g \vec{u}_y \quad g = 9.8 \text{ m/s}^2$$

Punto D: $(0, Y_D)$ $Y_D > Y_A$

$X(0) = 0 \quad Y(0) = Y_D$

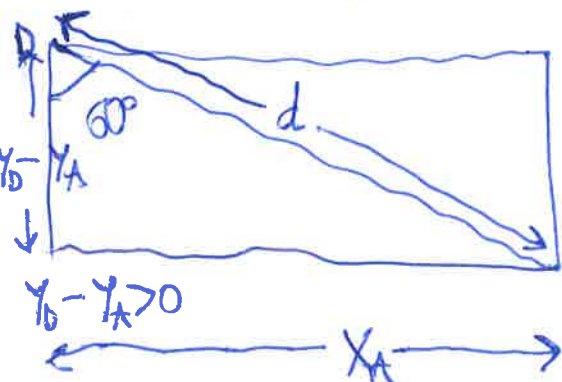
Punto A: (X_A, Y_A)

$X_A = X(t_A) > 0 \quad Y_A = Y(t_A) \leq 0$

$t=0$: Instante del despegue
 t_A : Instante del aterrizaje

$\Delta t_A = t_A - 0 = t_A$

$$\left. \begin{aligned} X(t) &= X(0) + v_{x0} t \\ Y(t) &= Y(0) + v_{y0} t - \frac{gt^2}{2} \end{aligned} \right\} \Rightarrow \begin{aligned} X(t) &= r_0 \cos \alpha t \\ Y(t) &= Y_D + r_0 \sin \alpha t - \frac{gt^2}{2} \end{aligned}$$



$\cotan 60^\circ = \frac{Y_D - Y_A}{X_A}$

$$\left. \begin{aligned} Y_A = Y(t_A) &= Y_D + r_0 \sin \alpha t_A - \frac{g}{2} t_A^2 \Rightarrow \\ \Rightarrow Y_D - Y_A &= -r_0 \sin \alpha t_A + \frac{g}{2} t_A^2 \\ X_A = X(t_A) &= r_0 \cos \alpha t_A \end{aligned} \right\} \Rightarrow$$

$$\Rightarrow \cotan 60^\circ = \frac{(-r_0 \sin \alpha t_A + \frac{g}{2} t_A^2)}{r_0 \cos \alpha t_A} \Rightarrow r_0 \cos \alpha \cotan 60^\circ = -r_0 \sin \alpha + \frac{g}{2} t_A \Rightarrow$$

$$\Rightarrow t_A = \frac{2 r_0}{g} (\cos \alpha \cotan 60^\circ + \sin \alpha) = \frac{2 \cdot 20 \text{ m/s}}{9.8 \text{ m/s}^2} (\cos 10^\circ \cotan 60^\circ + \sin 10^\circ) =$$

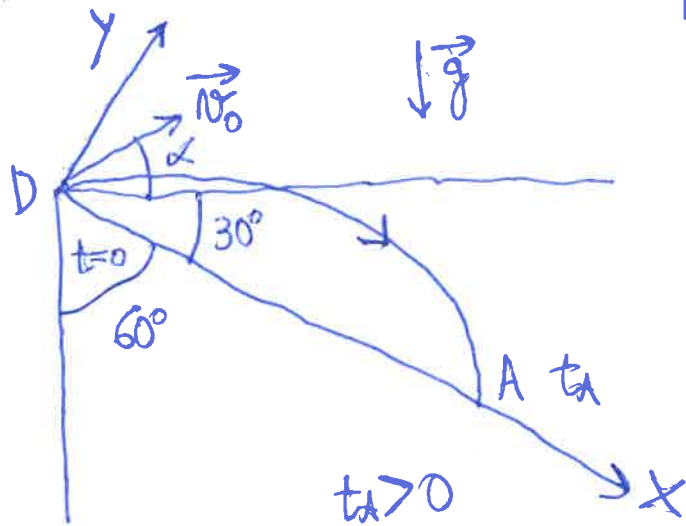
$$= \boxed{3.0294988 \text{ s}}$$

$\sin 60^\circ = \frac{X_A}{d} \Rightarrow d = \frac{X_A}{\sin 60^\circ} = \frac{r_0 \cos \alpha t_A}{\sin 60^\circ} = \frac{20 \text{ m/s} \cos 10^\circ 3.0294988 \text{ s}}{\sin 60^\circ} = \boxed{68.9 \text{ m}}$

$\cos 60^\circ = \frac{Y_D - Y_A}{d} \Rightarrow d = \frac{Y_D - Y_A}{\cos 60^\circ} = \frac{-r_0 \sin \alpha t_A + \frac{g}{2} t_A^2}{\cos 60^\circ} = \frac{-20 \text{ m/s} \sin 10^\circ 3.0294988 \text{ s} + \frac{9.8}{2} (3.0294988 \text{ s})^2}{\cos 60^\circ} = \boxed{68.92 \text{ m}}$

Autor: Iván Cabria Álvarez

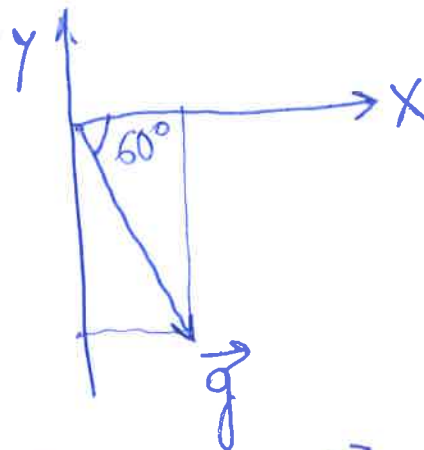
Quinta manera



$$\vec{v}_0 = \vec{v}(0) = v_0 \cos(30^\circ + \alpha) \vec{u}_x + v_0 \sin(30^\circ + \alpha) \vec{u}_y$$

$$v_0 = 20 \text{ m/s} \quad \alpha = 10^\circ$$

(2.23)



Punto D: $(0,0)$ $x(0)=0$ $y(0)=0$

Punto A: (x_A, y_A) $x_A = x(t_A) > 0$

$t=0$: Instante del despegue

t_A : Instante del aterrizaje

$$x(t) = x(0) + v_{x(0)}t + a_x \frac{t^2}{2}$$

$$y(t) = y(0) + v_{y(0)}t + a_y \frac{t^2}{2}$$

$$\vec{g} = +g \sin 30^\circ \vec{u}_x - g \cos 30^\circ \vec{u}_y$$

$$y_A = y(t_A) = 0$$

$$\Delta t_A = t_A - 0 = t_A$$

$$a_x = +g \sin 30^\circ$$

$$a_y = -g \cos 30^\circ$$

$$x(t) = v_0 \cos(30^\circ + \alpha)t + g \sin 30^\circ \frac{t^2}{2}$$

$$y(t) = v_0 \sin(30^\circ + \alpha)t - g \cos 30^\circ \frac{t^2}{2}$$

En el punto A, $y_A = 0$

$$y_A = y(t_A) = v_0 \sin 40^\circ t_A - g \cos 30^\circ \frac{t_A^2}{2}$$

$$\Rightarrow v_0 \sin 40^\circ t_A = g \cos 30^\circ \frac{t_A^2}{2}$$

$$t_A > 0$$

$$\Rightarrow t_A = \frac{2v_0}{g} \frac{\sin 40^\circ}{\cos 30^\circ} = \frac{2 \cdot 20 \text{ m/s}}{9.8 \text{ m/s}^2} \frac{\sin 40^\circ}{\cos 30^\circ} = \boxed{3.0294988 \text{ s}}$$

$$d = x_A - x(0) = x(t_A) - 0 = x(t_A)$$

$$x(t_A) = v_0 \cos 40^\circ t_A + g \sin 30^\circ \frac{t_A^2}{2}$$

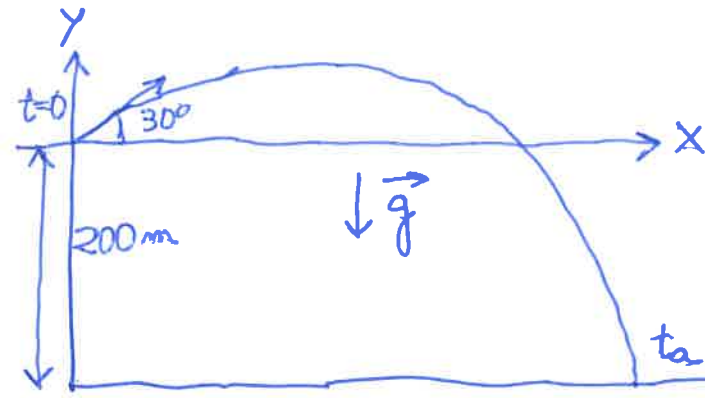
$$\Rightarrow d = v_0 \cos 40^\circ t_A + g \sin 30^\circ \frac{t_A^2}{2}$$

$$\Rightarrow d = 20 \frac{\text{m}}{\text{s}} \cos 40^\circ 3.0294988 \text{ s} + 9.8 \frac{\text{m}}{\text{s}^2} \sin 30^\circ \frac{(3.0294988 \text{ s})^2}{2} = \boxed{68.9 \text{ m}}$$

Autor: Iván Cabria Alvarado

(2.37)

(2.24)



$$E = \frac{m}{2} v^2(0)$$

$$v(0) \Downarrow \sqrt{\frac{2E}{m}} = \sqrt{\frac{2 \cdot 500 \text{ J}}{10 \text{ kg}}} = 10 \text{ m/s}$$

$$\vec{v}(0) = v(0) \cos 30^\circ \vec{u}_x + v(0) \sin 30^\circ \vec{u}_y$$

$$v_x(0) = v(0) \cos 30^\circ \quad y(0) = 0$$

$$v_y(0) = v(0) \sin 30^\circ \quad y(t_a) = -200 \text{ m}$$

$$x(t) = x(0) + v_x(0)t$$

$$\vec{g} = -g \vec{u}_y$$

$$y(t) = y(0) + v_y(0)t - \frac{gt^2}{2} \Rightarrow y(t_a) = y(0) + v_y(0)t_a - \frac{gt_a^2}{2} \Rightarrow$$

$$\Rightarrow \frac{g}{2} t_a^2 - v_y(0)t_a - y(0) + y(t_a) = 0 \Rightarrow$$

$$\Rightarrow t_a = \frac{v_y(0) \pm \sqrt{v_y^2(0) - 4 \frac{g}{2} (y(t_a) - y(0))}}{2 \left(\frac{g}{2} \right)} = \frac{v_y(0) \pm \sqrt{v_y^2(0) + 2g(y(0) - y(t_a))}}{g}$$

$$v_y(0) = v(0) \sin 30^\circ = 10 \text{ m/s} \cdot \frac{1}{2} = 5 \text{ m/s}$$

$$2g(y(0) - y(t_a)) = 2 \cdot 9.8 \text{ m/s}^2 (0 + 200) \text{ m} = 3920 \text{ m}^2/\text{s}^2$$

$$t_a = \frac{5 \pm \sqrt{25 + 3920} \text{ m/s}}{9.8 \text{ m/s}^2} = \begin{cases} 6.9193 \text{ s} \\ -5.8989 \text{ s} \end{cases}$$

$$t_a \text{ debe ser mayor que cero} \Rightarrow t_a = 6.9193 \text{ s}$$

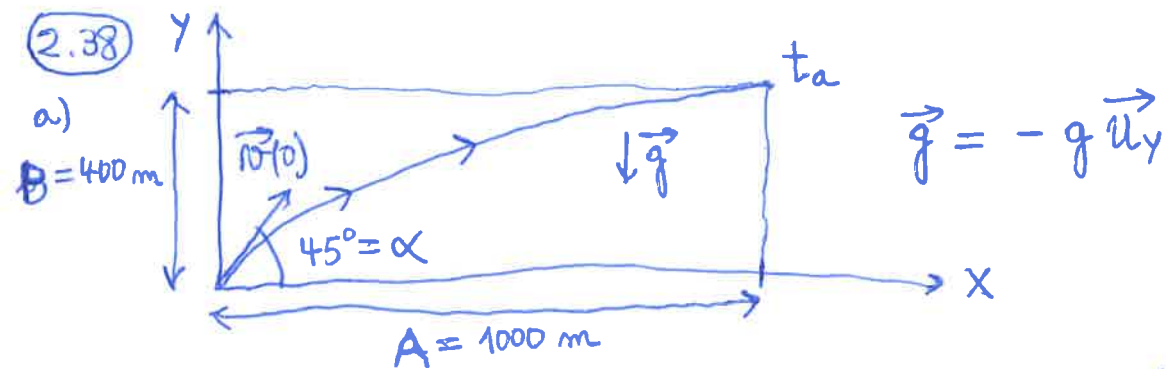
$$\text{Alcance} = x(t_a) - x(0) = v_x(0)t_a = v(0) \cos 30^\circ t_a =$$

$$= 10 \text{ m/s} \cdot \frac{\sqrt{3}}{2} \cdot 6.9193 \text{ s} = \boxed{59.92 \text{ m}}$$

Autor: Iván Gabriel Álvarez

(2.38)

(2.25)



$$\vec{v}(0) = v(0) \cos \alpha \vec{u}_x + v(0) \sin \alpha \vec{u}_y = \frac{v(0)}{\sqrt{2}} \vec{u}_x + \frac{v(0)}{\sqrt{2}} \vec{u}_y$$

$$X(t) = X(0) + v_x(0)t = 0 + \frac{v(0)}{\sqrt{2}} t = \frac{v(0)}{\sqrt{2}} t$$

$$Y(t) = Y(0) + v_y(0)t - \frac{gt^2}{2} = \frac{v(0)}{\sqrt{2}} t - \frac{gt^2}{2}$$

$$A = \frac{v(0)}{\sqrt{2}} t_a \Rightarrow t_a = \frac{\sqrt{2} A}{v(0)}$$

$$B = \frac{v(0)}{\sqrt{2}} t_a - \frac{g}{2} t_a^2 \Rightarrow B = A - \frac{g}{2} \frac{2A^2}{v^2(0)} \Rightarrow$$

$$\Rightarrow B - A = -\frac{g A^2}{v^2(0)} \Rightarrow A - B = \frac{g A^2}{v^2(0)} \Rightarrow v(0) = \sqrt{\frac{g A^2}{A - B}} \Rightarrow$$

$$\Rightarrow v(0) = \sqrt{\frac{9.8 \text{ m/s}^2 \cdot 10^6 \text{ m}^2}{(1000 - 400) \text{ m}}} = \boxed{127.8 \text{ m/s}}$$

b) $A = 2000 \text{ m}$ $B = 2200 \text{ m} \Rightarrow (A - B) < 0 \Rightarrow v(0) \text{ imaginaria} \Rightarrow$

$\Rightarrow$ No es posible alcanzar el punto $(2000, 2200) \text{ m}$.

Otra forma de resolver el apartado b):

$$C = \frac{v(0)}{\sqrt{2}} t_b$$

$$D = \frac{v(0)}{\sqrt{2}} t_b - \frac{g}{2} t_b^2$$

$$\Rightarrow D = C - \frac{g}{2} t_b^2$$

$$C = 2000 \text{ m}$$

$$D = 2200 \text{ m}$$

$$\Rightarrow D > C \Rightarrow$$

$$\Rightarrow C - \frac{g}{2} t_b^2 > C \Rightarrow -\frac{g}{2} t_b^2 > 0 \Rightarrow \text{Esto es inconsistente} \Rightarrow$$

$\Rightarrow$ No es posible alcanzar el punto $(2000, 2200) \text{ m}$.

Autor: Iván Cabris Álvarez

2.39 $|\vec{a}_t(0)| = 1 \text{ m/s}^2$ $|\vec{a}_m(0)| = 3 \text{ m/s}^2$

$\vec{a}_m(0) = 3 \frac{\text{m}}{\text{s}^2} \vec{u}_z$ $\vec{a} = \vec{a}_t + \vec{a}_m \Rightarrow \vec{a}(t) = \vec{a}(0) \quad \forall t \geq 0$

$\vec{v}(0) = 3 \vec{u}_x \frac{\text{m}}{\text{s}}$ $\vec{r}(0) = (\vec{u}_x + \vec{u}_y + \vec{u}_z) \text{ m}$

a) $\vec{v}(t) = ?$

$\vec{r}(t) = ?$ Necesitamos calcular $\vec{a}(t)$.
 $\vec{a}(t) = \vec{a}(0)$ Por tanto,
 calculamos $\vec{a}(0)$, para obtener $\vec{a}(t)$:

$|\vec{a}_t(0)| = 1 \text{ m/s}^2$ $\vec{a}_t(0) \parallel \vec{v}(0) \Rightarrow \vec{a}_t(0) = |\vec{a}_t(0)| \frac{\vec{v}(0)}{|\vec{v}(0)|} \Rightarrow$

$\Rightarrow \vec{a}_t(0) = 1 \frac{\text{m}}{\text{s}^2} \vec{u}_x$

$\vec{a}(0) = \vec{a}_t(0) + \vec{a}_m(0) = 1 \frac{\text{m}}{\text{s}^2} \vec{u}_x + 3 \frac{\text{m}}{\text{s}^2} \vec{u}_z$

Calculamos $\vec{v}(t)$:

$\vec{a} = \frac{d\vec{v}}{dt} \Rightarrow \int_0^t \vec{a}(t) dt = \int_0^t d\vec{v} \Rightarrow \vec{a}(0)t = \vec{v}(t) - \vec{v}(0) \Rightarrow$
 $\vec{v}(t) = \vec{v}(0) + \vec{a}(0)t$

$\Rightarrow \vec{v}(t) = \vec{v}(0) + \vec{a}(0)t = 3 \frac{\text{m}}{\text{s}} \vec{u}_x + t \frac{\text{m}}{\text{s}^2} \vec{u}_x + 3t \frac{\text{m}}{\text{s}^2} \vec{u}_z$

$\Rightarrow \boxed{\vec{v}(t) = (3+t)\vec{u}_x + 3t\vec{u}_z \frac{\text{m}}{\text{s}} \text{ y } t \text{ en segundos}}$

Calculamos $\vec{r}(t)$:

$\vec{v} = \frac{d\vec{r}}{dt} \Rightarrow \int_0^t d\vec{r} = \int_0^t \vec{v}(t) dt \Rightarrow \vec{r}(t) - \vec{r}(0) = \int_0^t [(3+t)\vec{u}_x + 3t\vec{u}_z] dt$

$\Rightarrow \vec{r}(t) - \vec{r}(0) = (3t + \frac{t^2}{2})\vec{u}_x + \frac{3t^2}{2}\vec{u}_z \Rightarrow$

$\Rightarrow \vec{r}(t) = \vec{u}_x + \vec{u}_y + \vec{u}_z + (3t + \frac{t^2}{2})\vec{u}_x + \frac{3t^2}{2}\vec{u}_z \text{ m} \Rightarrow$

$\Rightarrow \boxed{\vec{r}(t) = (1+3t+\frac{t^2}{2})\vec{u}_x + \vec{u}_y + (1+\frac{3t^2}{2})\vec{u}_z \text{ m y } t \text{ en segundos}}$

$$b) \vec{a}_t(t=1s) = ?$$

$$\vec{a}_t = \frac{d\vec{v}}{dt} = \frac{d}{dt} \frac{\vec{r}(t)}{r(t)}$$

$$r(t) = \sqrt{(3+t)^2 + 9t^2} \text{ m/s}$$

$$\frac{d}{dt} = \frac{2(3+t) + 18t}{2\sqrt{(3+t)^2 + 9t^2}} = \frac{3+t+9t}{r(t)} = \frac{3+10t}{r(t)} \text{ m/s}^2$$

$$\vec{a}_t(t) = \frac{(3+10t)}{r^2(t)} \vec{r}(t) \text{ m/s}^2$$

$$r(1) = \sqrt{4^2 + 9} = 5 \text{ m/s} \quad \vec{r}(1) = 4\vec{u}_x + 3\vec{u}_z \text{ m/s} \Rightarrow$$

$$\Rightarrow \vec{a}_t(1) = \frac{13}{25} (4\vec{u}_x + 3\vec{u}_z) \frac{m}{s^2} = \boxed{\frac{52}{25} \vec{u}_x + \frac{39}{25} \vec{u}_z \frac{m}{s^2}}$$

$$\vec{a}_m(t=1s) = ?$$

$$\left. \begin{aligned} \vec{a}(t) &= \vec{a}_m(t) + \vec{a}_t(t) \Rightarrow \vec{a}_m(t) = \vec{a}(t) - \vec{a}_t(t) \\ \vec{a}(t) &= \vec{u}_x + 3\vec{u}_z \text{ m/s}^2 \\ \vec{a}_t(1) &= \frac{52}{25} \vec{u}_x + \frac{39}{25} \vec{u}_z \frac{m}{s^2} \end{aligned} \right\} \Rightarrow$$

$$\Rightarrow \vec{a}_m(1) = \vec{u}_x + 3\vec{u}_z - \frac{52}{25} \vec{u}_x - \frac{39}{25} \vec{u}_z \frac{m}{s^2} \Rightarrow$$

$$\Rightarrow \boxed{\vec{a}_m(1) = -\frac{27}{25} \vec{u}_x + \frac{36}{25} \vec{u}_z \frac{m}{s^2}}$$

$$c) a_m(t) = \frac{r^2(t)}{R(t)} \Rightarrow R(1) = \frac{r^2(1)}{a_m(1)}$$

$$r(1) = \sqrt{(3+1)^2 + 9 \cdot 1^2} = 5 \frac{m}{s} \Rightarrow r^2(1) = 25 \text{ m}^2/\text{s}^2$$

$$a_m(1) = \sqrt{\left(\frac{27}{25}\right)^2 + \left(\frac{36}{25}\right)^2} \frac{m}{s^2} = \frac{45}{25} \frac{m}{s^2} = 1.8 \text{ m/s}^2$$

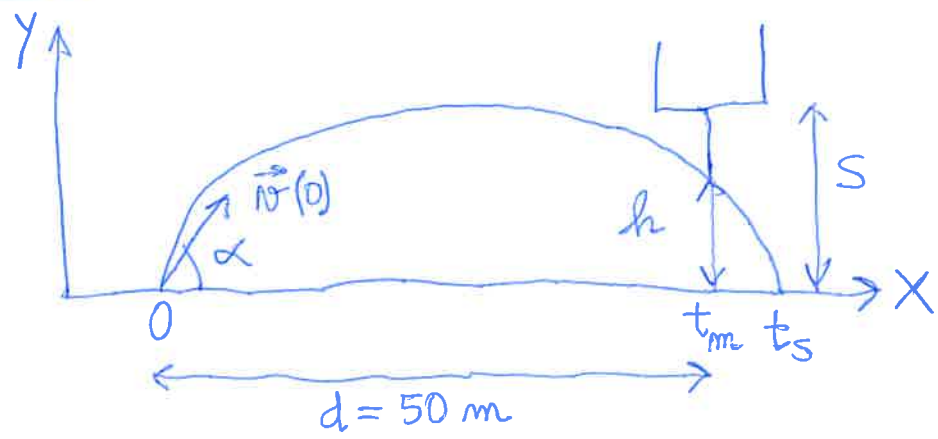
$$R(1) = \frac{25}{1.8} \text{ m} = \boxed{13.89 \text{ m}}$$

Autor: Iván Abria Álvarez

$$v = \omega R \Rightarrow \omega(1) = \frac{v(1)}{R(1)} = \frac{5}{13.89} \frac{\text{rad}}{s} = \frac{9}{25} \frac{\text{rad}}{s} = \boxed{0.36 \frac{\text{rad}}{s}}$$

2.40

2.28



¿ $h > S$?

$$v(0) = 25 \text{ m/s}$$

$$\alpha = 30^\circ$$

$$v_x(0) = v(0) \cos \alpha$$

$$v_y(0) = v(0) \sin \alpha$$

$$h = y(t_m) - y(0) \quad v_x(t) = v_x(0) = \text{cte.}$$

a) ¿ $h > S$? Calculamos h :

$$y(t) = y(0) + v_y(0)t - \frac{g}{2}t^2$$

$$h = y(t_m) - y(0) = v_y(0)t_m - \frac{g}{2}t_m^2$$

Calculamos t_m :

$$d = x(t_m) - x(0)$$

$$x(t) = x(0) + v_x(0)t$$

$$\Rightarrow d = v_x(0)t_m \Rightarrow t_m = \frac{d}{v_x(0)}$$

Calculamos h en función de $v(0)$, α , d y g :

$$\left. \begin{aligned} h &= v_y(0)t_m - \frac{g}{2}t_m^2 \\ t_m &= \frac{d}{v_x(0)} \end{aligned} \right\} \Rightarrow h = \frac{v_y(0)}{v_x(0)} d - \frac{g}{2} \frac{d^2}{v_x^2(0)} \Rightarrow$$

$$v_x(0) = v(0) \cos \alpha$$

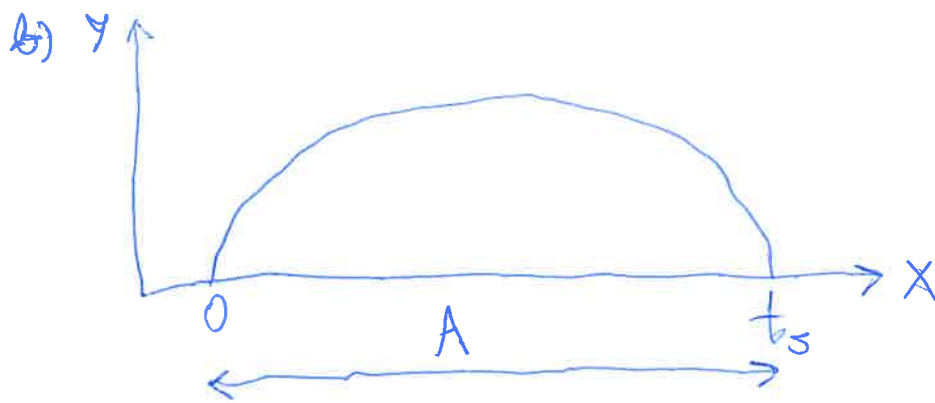
$$v_y(0) = v(0) \sin \alpha$$

$$\Rightarrow h = \frac{\sin \alpha}{\cos \alpha} d - \frac{g d^2}{2 v^2(0) \cos^2 \alpha} = \tan 30^\circ 50 \text{ m}$$

$$- \frac{9.8 \text{ m/s}^2 (50 \text{ m})^2}{2 (25 \text{ m/s})^2 (\cos 30^\circ)^2} = \boxed{2.73 \text{ m}}$$

$$S = 3 \text{ m} \Rightarrow h < S \Rightarrow \boxed{\text{No entra en la meta o portería}}$$

Autor: Iván Gabris Alvarado



$$A = X(t_s) - X(0)$$

Calculamos $X(t_s)$:

$$X(t) = X(0) + v_{x(0)}t \Rightarrow X(t_s) = X(0) + v_{x(0)}t_s$$

En el instante t_s , $Y(t_s) = Y(0)$. Calculamos t_s :

$$Y(t) = Y(0) + v_{y(0)}t - \frac{g}{2}t^2 \Rightarrow Y(t_s) = Y(0) + v_{y(0)}t_s - \frac{g}{2}t_s^2 \quad \left. \begin{array}{l} \\ \\ Y(t_s) = Y(0) \end{array} \right\} \Rightarrow$$

$$\Rightarrow Y(0) = Y(0) + v_{y(0)}t_s - \frac{g}{2}t_s^2 \Rightarrow v_{y(0)}t_s = \frac{g}{2}t_s^2 \quad \left. \begin{array}{l} \\ t_s > 0 \end{array} \right\} \Rightarrow$$

$$\Rightarrow v_{y(0)} = \frac{g}{2}t_s \Rightarrow t_s = \frac{2v_{y(0)}}{g}$$

Introducimos t_s en $X(t_s)$:

$$\left. \begin{array}{l} X(t_s) = X(0) + v_{x(0)}t_s \\ t_s = \frac{2v_{y(0)}}{g} \end{array} \right\} \Rightarrow X(t_s) = X(0) + \frac{2v_{x(0)}v_{y(0)}}{g} \quad \Rightarrow$$

$$A = X(t_s) - X(0)$$

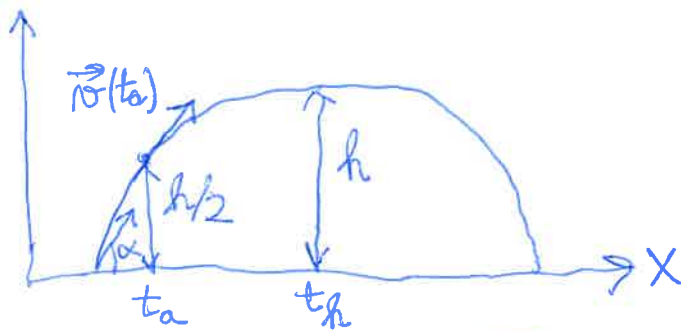
$$\Rightarrow A = \frac{2v_{x(0)}v_{y(0)}}{g} = \frac{2v^2(0)}{g} \sin \alpha \cos \alpha$$

$$A = \frac{2(25 \text{ m/s})^2}{9.8 \text{ m/s}^2} \sin 30^\circ \cos 30^\circ = \boxed{55.23 \text{ m}}$$

(2.41)

$$\vec{r}(0) = r(0) \cos \alpha \vec{u}_x + r(0) \sin \alpha \vec{u}_y$$

(2.30)



$$r(t_a) = \frac{3}{4} r(0) \Rightarrow r^2(t_a) = \frac{9}{16} r^2(0)$$

$$\alpha = ?$$

$$r(t) = \sqrt{r_x^2(t) + r_y^2(t)}$$

Calculamos $r^2(t_a)$ y lo tenemos en cuenta dentro de $r^2(t_a) = \frac{9}{16} r^2(0)$:

$$r_x(t) = r_x(0) = r(0) \cos \alpha \Rightarrow r_x(t_a) = r(0) \cos \alpha$$

$$r_y(t) = r_y(0) - gt \Rightarrow r_y(t_a) = r_y(0) - gt_a = r(0) \sin \alpha - gt_a$$

$$\Rightarrow r(t_a) = \sqrt{r^2(0) \cos^2 \alpha + (r(0) \sin \alpha - gt_a)^2} =$$

$$= \sqrt{r^2(0) \cos^2 \alpha + r^2(0) \sin^2 \alpha + g^2 t_a^2 - 2gt_a r(0) \sin \alpha} =$$

$$= \sqrt{r^2(0) + g^2 t_a^2 - 2gt_a r(0) \sin \alpha} \Rightarrow r^2(t_a) = r^2(0) + g^2 t_a^2 - 2gt_a r(0) \sin \alpha$$

$$r(t_a) = \frac{3}{4} r(0)$$

$$\Rightarrow \frac{9}{16} r^2(0) = r^2(0) + g^2 t_a^2 - 2gt_a r(0) \sin \alpha$$

Calculamos h :

$$h = y(t_h) - y(0)$$

$$y(t) = y(0) + r_y(0)t - \frac{g}{2}t^2 \Rightarrow h = r_y(0)t_h - \frac{g}{2}t_h^2$$

En el instante t_h , $r_y(t_h) = 0$. Calculamos t_h :

$$r_y(t) = r_y(0) - gt \Rightarrow t_h = \frac{r_y(0)}{g}$$

$$r_y(t_h) = 0$$

$$h = r_y(0)t_h - \frac{g}{2}t_h^2 = \frac{r_y^2(0)}{g} - \frac{g}{2} \frac{r_y^2(0)}{g^2} = \frac{2r_y^2(0)}{2g} = \frac{r^2(0) \sin^2 \alpha}{2g}$$

Calculamos $h/2$ en función de t_a :

$$\frac{h}{2} = y(t_a) - y(0)$$

$$y(t) = y(0) + r_y(0)t - \frac{g}{2}t^2 \Rightarrow \frac{h}{2} = r_y(0)t_a - \frac{g}{2}t_a^2 = r(0) \sin \alpha t_a - \frac{g}{2}t_a^2$$

Autor: Iván Gabris Álvarez

Plumas obtenidas h y $h/2$. De las dos ecuaciones, h y $h/2$, obtenemos: (2.31)

$$\left. \begin{aligned} h &= \frac{v^2(0) \sin^2 \alpha}{2g} \\ \frac{h}{2} &= v(0) \sin \alpha t_a - \frac{g}{2} t_a^2 \end{aligned} \right\} \Rightarrow \frac{v^2(0) \sin^2 \alpha}{4g} = v(0) \sin \alpha t_a - \frac{g}{2} t_a^2 \Rightarrow$$

$$\Rightarrow 2g \frac{v^2(0) \sin^2 \alpha}{4g} = 2g t_a v(0) \sin \alpha - g^2 t_a^2 \Rightarrow$$

$$\Rightarrow g^2 t_a^2 - 2g t_a v(0) \sin \alpha = - \frac{v^2(0) \sin^2 \alpha}{2} \Rightarrow$$

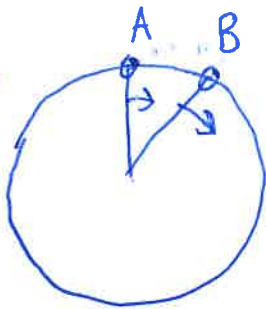
$$\frac{g}{16} v^2(0) = v^2(0) + g^2 t_a^2 - 2g t_a v(0) \sin \alpha$$

$$\Rightarrow \frac{g}{16} v^2(0) = v^2(0) - \frac{v^2(0) \sin^2 \alpha}{2} \Rightarrow \frac{(9-16)}{16} v^2(0) = - \frac{v^2(0)}{2} \sin^2 \alpha \Rightarrow$$

$$\Rightarrow - \frac{7}{16} v^2(0) = - \frac{v^2(0)}{2} \sin^2 \alpha \Rightarrow \frac{14}{16} = \sin^2 \alpha \Rightarrow$$

$$\Rightarrow \sin \alpha = \sqrt{\frac{14}{16}} \Rightarrow \alpha = \arcsin\left(\sqrt{\frac{14}{16}}\right) = \boxed{69.3^\circ}$$

(2.42)



$$T_A = 1800 \text{ s}$$

$$T_B = 24 \cdot 3600 \text{ s} = 48 \cdot 1800 \text{ s} = 48 T_A$$

$$T_A < T_B$$

$$\omega_A = \frac{2\pi}{T_A}$$

$$\omega_B = \frac{2\pi}{T_B}$$

$$T_A < T_B$$

$$\left. \begin{aligned} \omega_A &= \frac{2\pi}{T_A} \\ \omega_B &= \frac{2\pi}{T_B} \\ T_A &< T_B \end{aligned} \right\} \Rightarrow \omega_A > \omega_B$$

$$\left. \begin{aligned} \theta_i(t) &= \theta_i(0) + \omega_i t \\ \theta_A(0) &= \theta_B(0) \\ \theta_A(t_k) &= \theta_B(t_k) \end{aligned} \right\} \begin{aligned} \bar{\omega} &= A \circ B \\ \omega_A &\neq \omega_B \end{aligned}$$

$$\Rightarrow \omega_A t_k = \omega_B t_k + 2\pi k$$

k es un número entero

$$t_K \geq 0$$

$$\omega_A > \omega_B$$

$$\omega_A t_K = \omega_B t_K + 2\pi K$$

$$\Rightarrow K \geq 0$$

K es un número entero

$$\Rightarrow K = 0, 1, 2, 3, \dots$$

$$\omega_A t_K = \omega_B t_K + 2\pi K \Rightarrow (\omega_A - \omega_B) t_K = 2\pi K \Rightarrow t_K = \frac{2\pi K}{\omega_A - \omega_B}$$

Calculamos $\omega_A - \omega_B$:

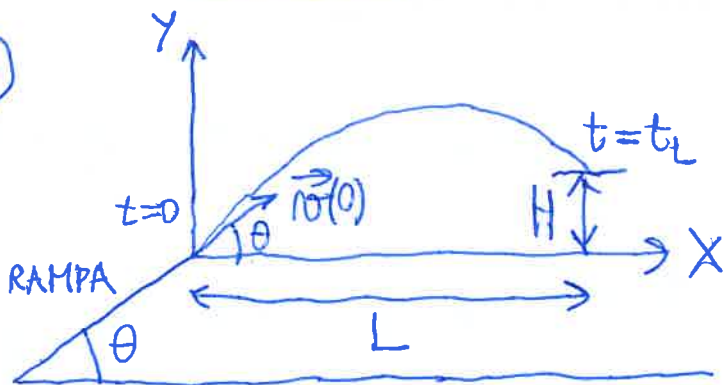
$$\begin{aligned} \omega_A - \omega_B &= \frac{2\pi}{T_A} - \frac{2\pi}{T_B} = \frac{2\pi}{T_A} - \frac{2\pi}{48T_A} = \frac{2\pi}{T_A} \left(1 - \frac{1}{48}\right) = \\ &= \frac{2\pi}{T_A} \frac{47}{48} \end{aligned}$$

$$\begin{aligned} t_K &= \frac{2\pi K T_A 48}{2\pi 47} = K T_A \frac{48}{47} = \boxed{K \frac{48}{47} 1800 \text{ s}} = \boxed{1838.3 K \text{ s}} = \\ &= \boxed{\left(1800 K + \frac{1800}{47} K\right) \text{ s}} \quad \text{con } K = 0, 1, 2, 3, \dots \end{aligned}$$

$$\theta_B(t_K) = \theta_A(t_K) = \omega_A t_K = \frac{2\pi}{T_A} t_K = \frac{2\pi}{T_A} K T_A \frac{48}{47} = \boxed{2\pi K \frac{48}{47} \text{ radianes}}$$

$$\Rightarrow \boxed{\theta_A(t_K) = \frac{2\pi K}{47} \text{ radianes}} \quad \text{con } K = 0, 1, 2, 3, \dots$$

(2.43)



v_{min} es la $v(0)$ tal que

$$X(t_L) - X(0) = L$$

$$Y(t_L) - Y(0) = H$$

$$\vec{v}(0) = v(0) \cos \theta \vec{u}_x + v(0) \sin \theta \vec{u}_y$$

$$v_x(0) = v(0) \cos \theta$$

$$v_y(0) = v(0) \sin \theta$$

$$\begin{aligned}
 X(t) &= X(0) + v_x(0)t \Rightarrow X(t_L) = X(0) + v(0) \cos \theta t_L \Rightarrow \\
 \Rightarrow X(t_L) - X(0) &= v(0) \cos \theta t_L \quad \left. \begin{aligned} &\Rightarrow v(0) \cos \theta t_L = L \Rightarrow t_L = \frac{L}{v(0) \cos \theta} \\ &X(t_L) - X(0) = L \end{aligned} \right\}
 \end{aligned}
 \tag{2.38}$$

$$\begin{aligned}
 Y(t) &= Y(0) + v_y(0)t - g \frac{t^2}{2} \Rightarrow Y(t_L) - Y(0) = v(0) \sin \theta t_L - \frac{g}{2} t_L^2 \\
 Y(t_L) - Y(0) &= H
 \end{aligned}
 \quad \left. \begin{aligned} &\Rightarrow v(0) \sin \theta t_L - \frac{g}{2} t_L^2 = H \end{aligned} \right\}$$

$$\Rightarrow v(0) \sin \theta t_L - \frac{g}{2} t_L^2 = H$$

$$\left. \begin{aligned} t_L &= \frac{L}{v(0) \cos \theta} \\ v(0) \sin \theta t_L - \frac{g}{2} t_L^2 &= H \end{aligned} \right\} \Rightarrow v(0) \frac{\sin \theta L}{v(0) \cos \theta} - \frac{g}{2} \frac{L^2}{v^2(0) \cos^2 \theta} = H \Rightarrow$$

$$\Rightarrow L \tan \theta - \frac{gL^2}{2v^2(0) \cos^2 \theta} = H \Rightarrow L \tan \theta - H = \frac{gL^2}{2v^2(0) \cos^2 \theta} \Rightarrow$$

$$\Rightarrow v^2(0) = \frac{gL^2}{2 \cos^2 \theta (L \tan \theta - H)} \Rightarrow$$

$$\Rightarrow v_{\min} = v(0) = \frac{L}{\cos \theta} \sqrt{\frac{g}{2(L \tan \theta - H)}}$$

$$\begin{aligned}
 \textcircled{2.44} \quad a(t) &= \frac{dv}{dt} \\
 a(t) &= 8\sqrt{10} \cdot 10^4 - \frac{32}{9} 10^7 t \quad \text{en el S.I.} \quad \left. \begin{aligned} &\Rightarrow \frac{dv}{dt} = 8\sqrt{10} \cdot 10^4 - \frac{32}{9} 10^7 t \Rightarrow \\ &\Rightarrow v(t) - v(0) = 8\sqrt{10} \cdot 10^4 t - \frac{16}{9} 10^7 t^2 \end{aligned} \right\} \Rightarrow 900 = 8\sqrt{10} \cdot 10^4 t_1 - \frac{16}{9} 10^7 t_1^2 \\
 v(t_1) &= 900 \text{ m/s} \quad v(0) = 0 \\
 \Rightarrow \frac{16}{9} 10^7 t_1^2 - 8\sqrt{10} \cdot 10^4 t_1 + 900 &= 0 \Rightarrow t_1 = \frac{8\sqrt{10} \cdot 10^4 \pm \sqrt{64 \cdot 10^9 - 4 \cdot \frac{16}{9} 10^7 \cdot 900}}{\frac{32}{9} 10^7} \text{ s}
 \end{aligned}$$

Autor: Iván Gabrín Álvarez

(2.34)

$$64 \cdot 10^9 - 4 \frac{16}{9} 10^7 \cdot 900 = 64 \cdot 10^9 - \frac{64}{9} 10^7 \cdot 10^2 = 0 \Rightarrow$$

$$\Rightarrow t_1 = \frac{8\sqrt{10} \cdot 10^4 \text{ s}}{\frac{32}{9} 10^7} = \frac{8 \cdot 9 \sqrt{10} 10^4 \text{ s}}{8 \cdot 4 \cdot 10^7} = \frac{9\sqrt{10}}{4 \cdot 10^3} \text{ s} = 7.1151 \cdot 10^{-3} \text{ s}$$

$$v = \frac{dx}{dt}$$

$$v = 8\sqrt{10} 10^4 t - \frac{16}{9} 10^7 t^2 \Rightarrow \frac{dx}{dt} = 8\sqrt{10} 10^4 t - \frac{16}{9} 10^7 t^2 \Rightarrow$$

$$\Rightarrow x(t) - x(0) = 8\sqrt{10} 10^4 \frac{t^2}{2} - \frac{16}{9} 10^7 \frac{t^3}{3} \Rightarrow l = 8\sqrt{10} \frac{10^4}{2} t_1^2 - \frac{16}{9} \frac{10^7}{3} t_1^3 \Rightarrow$$

$$l = x(t_1) - x(0)$$

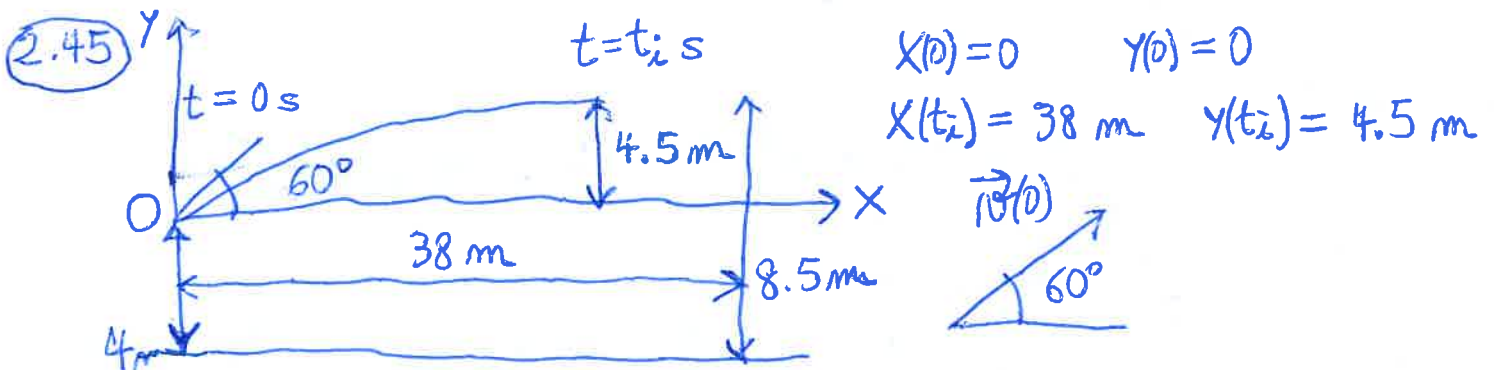
$$\Rightarrow l = 4\sqrt{10} 10^4 t_1^2 - \frac{16}{27} 10^7 t_1^3 \Rightarrow l = 4\sqrt{10} 10^4 \left(\frac{9\sqrt{10}}{4 \cdot 10^3} \right)^2 - \frac{16}{27} 10^7 \left(\frac{9\sqrt{10}}{4 \cdot 10^3} \right)^3$$

m

$$\Rightarrow l = 4\sqrt{10} 10^4 \frac{81 \cdot 10}{16 \cdot 10^6} - \frac{2^4}{3^3} 10^7 \frac{3^6 10\sqrt{10}}{2^6 10^9} \text{ m} =$$

$$= \frac{81 \cdot \sqrt{10} 10^5}{4 \cdot 10^6} - \frac{3^3 10^8 \sqrt{10}}{2^2 10^9} \text{ m} = 81 \frac{\sqrt{10}}{4 \cdot 10} - 27 \frac{\sqrt{10}}{4 \cdot 10} \text{ m} =$$

$$= (81 - 27) \frac{\sqrt{10}}{4 \cdot 10} \text{ m} = \frac{54 \sqrt{10}}{40} \text{ m} = \boxed{4.27 \text{ m}}$$



$$\vec{v}(0) = v(0) \cos 60^\circ \vec{u}_x + v(0) \sin 60^\circ \vec{u}_y$$

$$v_x(0) = v(0) \cos 60^\circ$$

$$v_y(0) = v(0) \sin 60^\circ$$

$$x(t) = x(0) + v_x(0)t$$

$$y(t) = y(0) + v_y(0)t - \frac{g}{2}t^2$$

a) $v(0) = ?$

$$\left. \begin{array}{l} x(t_x) = 38 \text{ m} \quad x(0) = 0 \\ x(t) = x(0) + v(0) \cos 60^\circ t \end{array} \right\} \Rightarrow 38 \text{ m} = v(0) \cos 60^\circ t_x \Rightarrow t_x = \frac{38 \text{ m}}{v(0) \cos 60^\circ}$$

$$\left. \begin{array}{l} y(t_x) = y(0) + v(0) \sin 60^\circ t - \frac{g}{2}t^2 \\ y(t_x) = 4.5 \text{ m} \quad y(0) = 0 \end{array} \right\} \Rightarrow 4.5 \text{ m} = v(0) \sin 60^\circ t_x - \frac{g}{2}t_x^2$$

$$\Rightarrow 4.5 \text{ m} = v(0) \sin 60^\circ \frac{38 \text{ m}}{v(0) \cos 60^\circ} - \frac{g}{2} \left(\frac{38 \text{ m}}{v(0) \cos 60^\circ} \right)^2 \Rightarrow$$

$$\Rightarrow 4.5 = 38 \tan 60^\circ - \frac{g}{2} \frac{38^2}{(\cos 60^\circ)^2} \frac{1}{v(0)^2} \Rightarrow$$

$$\Rightarrow 38 \tan 60^\circ - 4.5 = \frac{g}{2} \frac{38^2}{(\cos 60^\circ)^2} \frac{1}{v(0)^2} \Rightarrow$$

$$\Rightarrow \frac{1}{v(0)^2} = \frac{2(\cos 60^\circ)^2}{g \cdot 38^2} (38 \tan 60^\circ - 4.5) \Rightarrow$$

$$\Rightarrow v(0)^2 = \frac{g \cdot 38^2}{2(\cos 60^\circ)^2 (38 \tan 60^\circ - 4.5)} \Rightarrow$$

$$\Rightarrow v(0) = \sqrt{\frac{9.8 \cdot 38^2}{2 \left(\frac{1}{2}\right)^2 (38\sqrt{3} - 4.5)}} \text{ m/s} = \boxed{21.48 \text{ m/s}}$$

b) $t_x = ?$

Primera manera: En el apartado a) obtuvimos $t_x = \frac{38 \text{ m}}{v(0) \cos 60^\circ} \Rightarrow$

$$\Rightarrow t_x = \frac{38}{21.48 \left(\frac{1}{2}\right)} \text{ s} = \boxed{3.545}$$

Segunda manera: En el apartado a) obtuvimos las dos siguientes ecuaciones:

$$\left. \begin{array}{l} 38 = v(0) \cos 60^\circ t_x \Rightarrow v(0) t_x = \frac{38}{\cos 60^\circ} \\ 4.5 = v(0) \sin 60^\circ t_x - \frac{g}{2}t_x^2 \end{array} \right\} \Rightarrow 4.5 = \frac{38 \sin 60^\circ}{\cos 60^\circ} - \frac{g}{2}t_x^2 \Rightarrow$$

Autor: Iván Cabria Álvarez

$$\Rightarrow \frac{g}{2} t_a^2 = 38 \tan 60^\circ - 4.5 \Rightarrow t_a = \sqrt{\frac{2}{g} (38 \tan 60^\circ - 4.5)} \Rightarrow$$

$$\Rightarrow t_a = \sqrt{\frac{2}{9.8} (38\sqrt{3} - 4.5)} \text{ s} = \boxed{3.54 \text{ s}}$$

$$c) v_x(t) = v_x(0)$$

$$v_y(t) = v_y(0) - gt$$

$$v_x(t) = v_x(0) \Rightarrow v_x(t_a) = v_x(0) = v(0) \cos 60^\circ = \boxed{10.74 \text{ m/s}}$$

$$v_y(t_a) = v_y(0) - gt_a = v(0) \sin 60^\circ - gt_a = 21.48 \frac{\sqrt{3}}{2} - 9.8 \cdot 3.54 \frac{\text{m}}{\text{s}} \\ = \boxed{-16.09 \text{ m/s}}$$

TEMA 3 DINÁMICA

3.1) Una trayectoria curva implica que el vector $\vec{v}$ cambia de dirección con el tiempo. Por tanto, $\frac{d\vec{v}}{dt} \neq 0 \Rightarrow \vec{a}_m = v \frac{d\vec{v}}{dt} \neq 0 \Rightarrow$ Existe una fuerza centrípeta debida a $\vec{a}_m$



3.2) a) $F = mg \Rightarrow m = \frac{F}{g} = \frac{9800 \text{ N}}{9.8 \text{ m/s}^2} = 1000 \text{ Kg}$

$F = m g_{\text{Luna}} = 1000 \text{ Kg} \cdot 1.63 \text{ m/s}^2 = \boxed{1630 \text{ N}}$

b) $v(t) = v(0) + at \Rightarrow v(t_1) = at_1 \quad t_1 = 5 \text{ min} = 300 \text{ s}$

$a = \frac{F}{m} = \frac{100 \text{ N}}{50 \text{ Kg}} = 2 \text{ m/s}^2 \Rightarrow v(t_1) = 2 \frac{\text{m}}{\text{s}^2} 300 \text{ s} = \boxed{600 \text{ m/s}}$

3.3) $g_p = \frac{GM_p}{R_p^2} \Rightarrow M_p = \frac{g_p R_p^2}{G} \Rightarrow$

$\Rightarrow M_p = \frac{7 \text{ m/s}^2 (4 \cdot 10^6 \text{ m})^2}{6.67 \cdot 10^{-11} \text{ Nm}^2 \text{ Kg}^{-2}} = \boxed{16.79 \cdot 10^{23} \text{ Kg}}$

3.4) $T^2 = c R^3 \quad c = \text{constante} \Rightarrow \frac{T_1^2}{T_2^2} = \frac{R_1^3}{R_2^3}$

$T_1 = T_2/8 \Rightarrow \frac{T_2^2/8^2}{T_2^2} = \frac{R_1^3}{R_2^3} \Rightarrow \frac{1}{64} = \left(\frac{R_1}{R_2}\right)^3 \Rightarrow \frac{R_1}{R_2} = \sqrt[3]{\frac{1}{64}} = \sqrt[3]{\frac{1}{2^6}}$

$\Rightarrow \frac{R_1}{R_2} = \frac{1}{2^{6/3}} = \frac{1}{2^2} = \boxed{\frac{1}{4}}$

3.5) a) $\left. \begin{array}{l} R' = 2R_T \\ g = \frac{GM_T}{R_T^2} \\ M' = M_T \end{array} \right\} \Rightarrow g' = \frac{GM'}{R'^2} = \frac{GM_T}{4R_T^2} = \frac{g}{4} \Rightarrow F' = m g' = 70 \text{ Kg} \frac{9.8}{4} \frac{\text{m}}{\text{s}^2} = \boxed{171.5 \text{ N}}$

b) $\left. \begin{array}{l} \rho' = \frac{M'}{V'} = \frac{M'}{\frac{4\pi}{3} R'^3} = \frac{M'}{\frac{4\pi}{3} 8R_T^3} \\ \rho = \frac{M_T}{V} = \frac{M_T}{\frac{4\pi}{3} R_T^3} \end{array} \right\} \Rightarrow \frac{M'}{8R_T^3} = \frac{M_T}{R_T^3} \Rightarrow M' = 8M_T$

Autor: Iván Cabria Álvarez

$$g' = \frac{GM'}{R'^2} = \frac{G 8M_T}{4R_T^2} = 2 \frac{GM_T}{R_T^2} = 2g = 19.6 \text{ m/s}^2$$

$$F = m \cdot g' = 70 \text{ Kg} \cdot 19.6 \text{ m/s}^2 = \boxed{1372 \text{ N}}$$

3.2

$$3.6) g_J = \frac{GM_J}{R_J^2} = \frac{G 318M_T}{R_J^2} = \frac{G 318M_T}{11^2 R_T^2} = \frac{318}{121} g_T = \frac{318}{121} 9.8 \text{ m/s}^2 = \boxed{25.76 \frac{\text{m}}{\text{s}^2}}$$

$$3.7) g = \frac{GM_T}{R_T^2} = \frac{6.67 \cdot 10^{-11} \text{ N m}^2 \text{ Kg}^{-2} \cdot 5.972 \cdot 10^{24} \text{ Kg}}{(6370000 \text{ m})^2} = 0.982 \frac{10^{-11} 10^{24}}{10^{12}} \frac{\text{m}}{\text{s}^2} = \boxed{9.82 \frac{\text{m}}{\text{s}^2}}$$

$$3.8) T^2 = \frac{4\pi^2}{GM_J} r^3 \Rightarrow M_J = \frac{4\pi^2}{GT^2} r^3 = \frac{4\pi^2 (26.4)^3 R_J^3}{GT^2} \Rightarrow \frac{M_J}{R_J^3} = \frac{4\pi^2 (26.4)^3}{GT^2}$$

$$\Rightarrow \rho_J = \frac{M_J}{V_J} = \frac{M_J}{\frac{4\pi}{3} R_J^3} = \frac{1}{\frac{4\pi}{3}} \frac{4\pi^2 (26.4)^3}{GT^2} = \frac{3\pi (26.4)^3}{GT^2} \Rightarrow$$

$$\Rightarrow \rho_J = \frac{3\pi (26.4)^3}{6.67 \cdot 10^{-11} \text{ N m}^2 \text{ Kg}^{-2} (16.69 \cdot 86400 \text{ s})^2} = \boxed{1250.31 \text{ Kg/m}^3}$$

Según los experimentos $\rho = 1.33 \text{ g/cm}^3 = 1330 \text{ Kg/m}^3$

3.9) M_T = Masa del planeta Tierra r_{TL} = radio de la órbita de la Luna
 R_T = Radio del planeta Tierra T = periodo de la órbita de la Luna
 g_T = aceleración de la gravedad en la superficie de la Tierra
 Ley de Kepler, del sistema Tierra-Luna

$$\left. \begin{array}{l} \frac{T^2}{r_{TL}^3} = \frac{4\pi^2}{GM_T} \Rightarrow GM_T = \frac{4\pi^2 r_{TL}^3}{T^2} \\ F = \frac{GM_T m}{R_T^2} \\ F = m g_T \end{array} \right\} \Rightarrow g_T = \frac{GM_T}{R_T^2} \Rightarrow \boxed{g_T = \frac{4\pi^2 r_{TL}^3}{T^2 R_T^2}}$$

$$g_T = \frac{4\pi^2 (0.3844 \cdot 10^9 \text{ m})^3}{(2360592 \text{ s})^2 (6.37 \cdot 10^6 \text{ m})^2} = \boxed{9.9172 \text{ m/s}^2}$$

$$F = \frac{GM_T m}{R_T^2}$$

$F = m g_T$ Se refiere a una masa m situada en la superficie de la Tierra

Autor: Juan Carlos Álvarez

$$3.10 \quad v(t) = v(0) + at \Rightarrow a = \frac{v(t_1) - v(0)}{t} \Rightarrow a = \frac{v(t_1) - v(0)}{t_1} = \frac{(120 - 80) \text{ km/h}}{5 \text{ s}} \quad (3.3)$$

$$= \frac{40/3600 \text{ km/s}}{5 \text{ s}} = 0.002 \frac{\text{km}}{\text{s}^2} = 2.2 \text{ m/s}^2$$

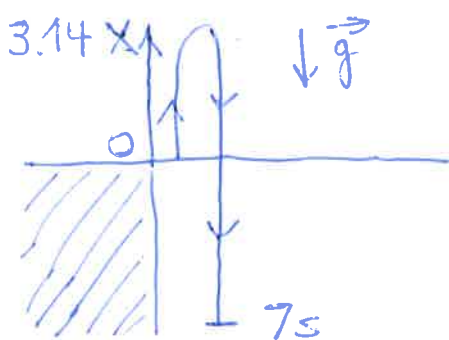
$$F = ma = 1000 \text{ kg} \cdot 2.2 \text{ m/s}^2 = \boxed{2222.2 \text{ N}}$$

$$3.11 \quad F = ma = 200 \text{ kg} \cdot 30 \text{ cm/s}^2 = 200 \text{ kg} \cdot 0.3 \frac{\text{m}}{\text{s}^2} = \boxed{60 \text{ N}}$$

$$3.12 \quad F = ma \Rightarrow m = \frac{F}{a} = \frac{2 \text{ N}}{4 \text{ m/s}^2} = \boxed{0.5 \text{ kg}}$$

$$3.13 \quad a = \frac{F}{m} = \frac{40 \text{ N}}{80 \text{ kg}} = 0.5 \text{ m/s}^2$$

$$s(t) = \frac{at^2}{2} \Rightarrow s(10) = 0.5 \frac{\text{m}}{\text{s}^2} \cdot \frac{10^2}{2} \text{ s}^2 = \boxed{25 \text{ m}}$$



$$X(t) = X(0) + v(0)t + \frac{at^2}{2} = v(0)t - \frac{gt^2}{2}$$

$$X(7) = v(0)7\text{s} - \frac{g}{2}7^2\text{s}^2 = 20 \frac{\text{m}}{\text{s}}7\text{s} - \frac{9.8}{2} \frac{\text{m}}{\text{s}^2}7^2\text{s}^2$$

$$= 140 \text{ m} - 240.1 \text{ m} = \boxed{-100.1 \text{ m}}$$

$$3.15 \text{ a) } a = g_m = \frac{Gm}{r^2} = \frac{6.67 \cdot 10^{-11} \text{ N m}^2 \text{ kg}^{-2} \cdot 10000 \text{ kg}}{(500 \text{ m})^2} = \boxed{2.67 \cdot 10^{-12} \text{ m/s}^2}$$

$$m = 10T = 10000 \text{ kg} \quad M = 10000T$$

$$\text{b) } a = g_m = \frac{Gm}{r^2} \Rightarrow \frac{m}{r^2} = \frac{a}{G} = \frac{10^{-8} \text{ m/s}^2}{6.67 \cdot 10^{-11} \text{ N m}^2 \text{ kg}^{-2}} = \boxed{150 \frac{\text{kg}}{\text{m}^2}}$$

$$3.16 \quad \begin{array}{l} \text{Diagram: A coordinate system with X horizontal and Y vertical. A curved path starts at the origin and curves downwards. A downward arrow is labeled } 10^8 \frac{\text{m}}{\text{s}^2}. \end{array}$$

$$y(0) = 0 \quad v_y(0) = 0 \quad a_y = 10^8 \text{ m/s}^2$$

$$y(t) = y(0) + v_y(0)t + \frac{a_y t^2}{2} = \frac{a_y t^2}{2} \Rightarrow$$

$$y(t_1) = 40 \cdot 10^6 \text{ m}$$

$$\Rightarrow 4 \cdot 10^7 \text{ m} = 10^8 \frac{\text{m}}{\text{s}^2} \frac{t_1^2}{2} \Rightarrow t_1 = \sqrt{\frac{8 \cdot 10^7}{10^8}} \text{ s} = 89442.719 \text{ s}$$

$$1 \text{ año} = 31\,557\,600 \text{ s} \Rightarrow$$

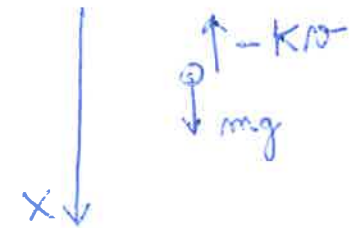
$$\Rightarrow \boxed{t_1 = 2.83 \text{ años}}$$

Autor: Iván Cabria Álvarez

3.17) La velocidad límite es una velocidad constante que se alcanza a partir de un cierto instante de tiempo t_1 . (3.4)

Para $t > t_1$ $v \rightarrow v_2 = \text{cte.} \Rightarrow a = 0$ para $t > t_1 \Rightarrow F_{\text{neto}} = 0$ para $t > t_1$
 $F_{\text{neto}} = m a$

$F_{\text{neto}} = m g - K v$ $F_{\text{neto}} = 0 \Rightarrow m g - K v_2 = 0 \Rightarrow v_2 = \frac{m g}{K}$



Estrictamente, v_2 se alcanza cuando $t \rightarrow \infty$

3.18) $\left. \begin{array}{l} \uparrow -Kv^2 \\ \downarrow mg \end{array} \right\} \begin{array}{l} a) F_{\text{neto}} = m g - K v^2 \quad \forall t \\ F_{\text{neto}} = 0 \quad \text{para } t > t_1 \end{array} \Rightarrow m g - K v_2^2 = 0 \Rightarrow$

$\Rightarrow v_2 = \sqrt{\frac{m g}{K}} = \sqrt{\frac{80 \text{ Kg } 9.8 \text{ m/s}^2}{101.606 \text{ Kg/m}}} = 2.78 \text{ m/s}$

b) $\Delta t = \frac{h_i}{v_2} = \frac{3000 \text{ m}}{2.78 \text{ m/s}} \approx 1080 \text{ s} = 18 \text{ minutos}$

3.19) a) $m g - K v = m \frac{dv}{dt} \Rightarrow \frac{dv}{dt} = g - \frac{K}{m} v$
 $F_{\text{neto}} = m a$

La ecuación $\frac{dv}{dt} = g - \frac{K}{m} v$ se puede resolver de varias maneras.
 Vamos a resolverla mediante un cambio de variable:

$\left. \begin{array}{l} \frac{dv}{dt} = g - \frac{K}{m} v \\ f(t) = g - \frac{K}{m} v(t) \Rightarrow \frac{df}{dt} = -\frac{K}{m} \frac{dv}{dt} \end{array} \right\} \Rightarrow -\frac{m}{K} \frac{df}{dt} = f(t)$

$\Rightarrow \frac{df}{dt} = -\frac{K}{m} f(t) \Rightarrow \frac{df}{f} = -\frac{K}{m} dt \Rightarrow \int_{f(0)}^{f(t)} \frac{df}{f} = -\frac{K}{m} \int_0^t dt$

$\Rightarrow \ln f(t) - \ln f(0) = -\frac{K}{m} t \Rightarrow \ln \left(\frac{f(t)}{f(0)} \right) = -\frac{K}{m} t \Rightarrow \frac{f(t)}{f(0)} = e^{-\frac{K}{m} t}$

$\Rightarrow f(t) = f(0) e^{-\frac{K}{m} t} \Rightarrow g - \frac{K}{m} v(t) = \left(g - \frac{K}{m} v(0) \right) e^{-\frac{K}{m} t} \Rightarrow$

$\Rightarrow \frac{m g}{K} - v(t) = \left(\frac{m g}{K} - v(0) \right) e^{-\frac{K}{m} t} \Rightarrow$

$\Rightarrow v(t) = \frac{m g}{K} + \left(v(0) - \frac{m g}{K} \right) e^{-\frac{K}{m} t}$

Autor: Iván Cabrera Álvarez

b) $v_2 = \lim_{t \rightarrow \infty} v(t) = \lim_{t \rightarrow \infty} \left[\frac{m g}{K} + \left(v(0) - \frac{m g}{K} \right) e^{-\frac{K}{m} t} \right] = \frac{m g}{K}$

(3.5)

(3.20) La tercera ley de Kepler dice: $T^2 = \frac{4\pi^2}{GM_S} r^3 \Rightarrow$
 $\Rightarrow M_S = \frac{4\pi^2 r^3}{GT^2}$
 M_S : masa del Sol
 T : periodo de la órbita de la Tierra
 r : radio de la órbita de la Tierra

$$\rho = \frac{M_S}{V_S} = \frac{M_S}{\frac{4\pi}{3} R_S^3} = \frac{3M_S}{4\pi R_S^3} = \frac{3 \cdot 4\pi^2 r^3}{4\pi R_S^3 GT^2} = \frac{3\pi r^3}{R_S^3 GT^2}$$

$$\rho = \frac{3\pi (1.5 \cdot 10^{11} \text{ m})^3}{(7 \cdot 10^8 \text{ m})^3 \cdot 6.67 \cdot 10^{-11} \text{ N m}^2 \text{ kg}^{-2} (365.25 \cdot 86400 \text{ s})^2} =$$

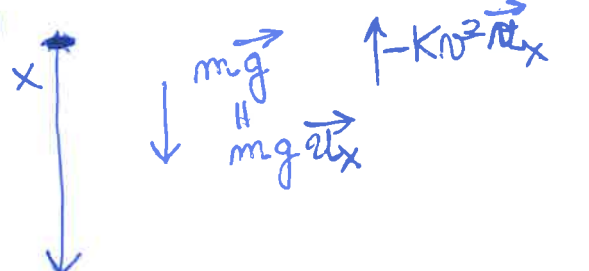
$$= 1396 \frac{\text{kg}^2}{\text{N m}^2 \text{ s}^2} \Rightarrow \rho = 1396 \frac{\text{kg}^2}{\text{kg m s}^{-2} \text{ m}^2 \text{ s}^2} = 1396 \frac{\text{kg}}{\text{m}^3}$$

$$1 \text{ N} = 1 \text{ kg m s}^{-2}$$

$$1 \text{ m}^3 = 10^3 \text{ dm}^3 = 10^3 \text{ L} \Rightarrow$$

$$\rho = \frac{1396}{1000} \frac{\text{kg}}{\text{L}} = 1.396 \frac{\text{kg}}{\text{L}} \approx \boxed{1.4 \frac{\text{kg}}{\text{L}}}$$

La densidad del agua es $1 \frac{\text{kg}}{\text{L}}$

(3.21)  $\vec{F} = m\vec{a} = m \frac{d\vec{v}}{dt} = m \frac{d\vec{v}}{dt} \vec{u}_x$
 $\vec{F} = m\vec{g} - Kx\vec{u}_x = (mg - Kx)\vec{u}_x$

$$\Rightarrow mg - Kx = m \frac{dv}{dt} \Rightarrow \frac{dv}{dt} = g - \frac{K}{m}x = -\frac{K}{m}(x - \frac{mg}{K})$$

$$v_L^2 \equiv \frac{mg}{K} \Rightarrow \frac{dv}{dt} = -\frac{K}{m}(x - v_L^2) \Rightarrow$$

$$\Rightarrow \frac{dv}{(v^2 - v_L^2)} = -\frac{K}{m} dt$$

$$\frac{1}{(v^2 - v_L^2)} = \frac{1}{(v - v_L)(v + v_L)} = \frac{A}{v - v_L} + \frac{B}{v + v_L} \Rightarrow$$

$$\Rightarrow 1 = A(v + v_L) + B(v - v_L) \Rightarrow \begin{matrix} v^0 & 1 = (A + B)v_L \\ v^1 & 0 = A - B \Rightarrow A = -B \end{matrix}$$

Autor: Iván Cabris Álvarez

$$\Rightarrow B = -\frac{1}{2\nu_2} \Rightarrow A = +\frac{1}{2\nu_2}$$

$$\frac{d\nu}{(\nu^2 - \nu_2^2)} = -\frac{K}{m} dt \Rightarrow \int_0^t \frac{d\nu}{(\nu^2 - \nu_2^2)} = -\frac{K}{m} t$$

$$\int_0^t \frac{d\nu}{\nu^2 - \nu_2^2} \stackrel{1}{=} \frac{1}{2\nu_2} \left[\int_0^t \frac{d\nu}{\nu - \nu_2} - \int_0^t \frac{d\nu}{\nu + \nu_2} \right] \stackrel{2}{=} \nu(t) > \nu_2 \quad \forall t \geq 0$$

$$= \frac{1}{2\nu_2} \left[\ln(\nu(t) - \nu_2) \Big|_0^t - \ln(\nu(t) + \nu_2) \Big|_0^t \right] =$$

$$= \frac{1}{2\nu_2} \left[\ln\left(\frac{\nu(t) - \nu_2}{\nu(0) - \nu_2}\right) - \ln\left(\frac{\nu(t) + \nu_2}{\nu(0) + \nu_2}\right) \right] =$$

$$= \frac{1}{2\nu_2} \ln \left(\frac{\nu(t) - \nu_2}{\nu(t) + \nu_2} \cdot \frac{\nu(0) + \nu_2}{\nu(0) - \nu_2} \right)$$

$$a \equiv \frac{\nu(0) + \nu_2}{\nu(0) - \nu_2}$$

$$\int_0^t \frac{d\nu}{(\nu^2 - \nu_2^2)} = -\frac{K}{m} t \Rightarrow \frac{1}{2\nu_2} \ln \left(\frac{\nu(t) - \nu_2}{\nu(t) + \nu_2} a \right) = -\frac{K}{m} t \Rightarrow$$

$$\Rightarrow \left(\frac{\nu(t) - \nu_2}{\nu(t) + \nu_2} \right) a = e^{-\frac{2\nu_2 K}{m} t} \Rightarrow \nu(t) - \nu_2 = (\nu(t) + \nu_2) \frac{e^{-bt}}{a} \Rightarrow$$

$$b \equiv \frac{2\nu_2 K}{m}$$

$$\Rightarrow \nu(t) \left[1 - \frac{e^{-bt}}{a} \right] = \nu_2 \left[1 + \frac{e^{-bt}}{a} \right] \Rightarrow$$

$$\Rightarrow \boxed{\nu(t) = \frac{\nu_2 (a + e^{-bt})}{(a - e^{-bt})}} \quad \text{Si } \nu(t) > \nu_2 \quad \forall t \geq 0 \quad \nu_2 = \lim_{t \rightarrow \infty} \nu(t) = \nu_2 \frac{(a + 0)}{(a - 0)} = \boxed{\nu_2}$$

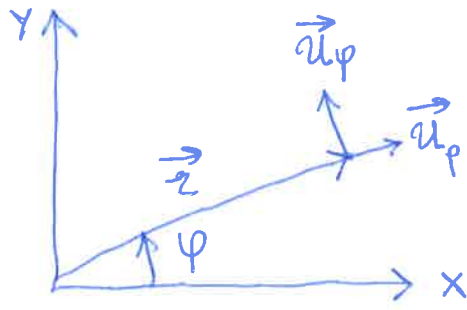
$$= \nu_2 \frac{(\nu(0) + \nu_2) + (\nu(0) - \nu_2) e^{-bt}}{(\nu(0) + \nu_2) - (\nu(0) - \nu_2) e^{-bt}}$$

Autor: Juan Carlos Álvarez

Si $\nu(t) < \nu_2 \quad \forall t \geq 0$ entonces $\int_0^t \frac{d\nu}{\nu - \nu_2} = - \int_0^t \frac{d\nu}{\nu_2 - \nu} = + \ln \left(\frac{\nu_2 - \nu(t)}{\nu_2 - \nu(0)} \right)$
 y se llega a la misma expresión de $\nu(t)$

(3.22)

(3.7)



$$\vec{u}_p = \cos \varphi \vec{u}_x + \sin \varphi \vec{u}_y$$

$$\vec{u}_\varphi = -\sin \varphi \vec{u}_x + \cos \varphi \vec{u}_y \quad \ddot{r} \equiv \frac{d^2 r}{dt^2} \quad \ddot{\varphi} \equiv \frac{d^2 \varphi}{dt^2}$$

$$\dot{r} \equiv \frac{dr}{dt} \quad \dot{\varphi} \equiv \frac{d\varphi}{dt}$$

$$\vec{r} = r \vec{u}_p$$

$$\vec{v} = r \dot{\varphi} \vec{u}_\varphi + \dot{r} \vec{u}_p$$

$$\vec{a} = (\ddot{r} - r \dot{\varphi}^2) \vec{u}_p + (2\dot{r}\dot{\varphi} + r\ddot{\varphi}) \vec{u}_\varphi$$

$$\vec{F} = m \vec{a}$$

$$\vec{F} = -\frac{GMm}{r^3} \vec{r} = -\frac{GMm}{r^2} \vec{u}_p \quad \left. \begin{aligned} & m(\ddot{r} - r \dot{\varphi}^2) = -\frac{GMm}{r^2} \\ & m(2\dot{r}\dot{\varphi} + r\ddot{\varphi}) = 0 \end{aligned} \right\} \Rightarrow$$

$$\Rightarrow \ddot{r} - r \dot{\varphi}^2 = -\frac{K}{r^2} \quad K \equiv GM$$

$$2\dot{r}\dot{\varphi} + r\ddot{\varphi} = 0$$

$$\frac{d\vec{u}_p}{dt} = \dot{\varphi} \vec{u}_\varphi$$

$$\frac{d\vec{u}_\varphi}{dt} = -\dot{\varphi} \vec{u}_p$$

Resolvemos la ecuación $2\dot{r}\dot{\varphi} + r\ddot{\varphi} = 0$

$$\frac{d}{dt}(r^2) = 2r\dot{r}$$

$$2\dot{r}\dot{\varphi} + r\ddot{\varphi} = 0 \Rightarrow 2r\ddot{\varphi} + r^2\ddot{\varphi} = 0 \quad \left. \begin{aligned} & \frac{d}{dt}(r^2)\dot{\varphi} + r^2\frac{d}{dt}(\dot{\varphi}) = 0 \Rightarrow \\ & \Rightarrow \frac{d}{dt}(r^2\dot{\varphi}) = 0 \Rightarrow r^2\dot{\varphi} = C = \text{cte.} \Rightarrow \end{aligned} \right\} \Rightarrow$$

$$\Rightarrow \frac{d}{dt}(r^2\dot{\varphi}) = 0 \Rightarrow r^2\dot{\varphi} = C = \text{cte.} \Rightarrow \boxed{\dot{\varphi} = \frac{C}{r^2}}$$

Resolvemos la ecuación $\ddot{r} - r\dot{\varphi}^2 = -\frac{K}{r^2}$ Hacemos el cambio de variable $s = \frac{1}{r}$

$$\dot{r} = \frac{d}{dt}\left(\frac{1}{s}\right) = -\frac{1}{s^2} \frac{ds}{dt} = -\frac{1}{s^2} \frac{ds}{d\varphi} \frac{d\varphi}{dt} = -\frac{1}{s^2} \frac{ds}{d\varphi} \frac{C}{r^2} = -C \frac{ds}{d\varphi}$$

$$\ddot{r} = \frac{d}{dt}(\dot{r}) = -C \frac{d}{dt}\left(\frac{ds}{d\varphi}\right) = -C \frac{d}{d\varphi}\left(\frac{ds}{d\varphi}\right) \frac{d\varphi}{dt} = -C \frac{d^2 s}{d\varphi^2} \dot{\varphi} =$$

$$= -\frac{C^2}{r^2} \frac{d^2 s}{d\varphi^2} = -C^2 s^2 \frac{d^2 s}{d\varphi^2}$$

$$\ddot{r} - r\dot{\varphi}^2 = -\frac{K}{r^2} \quad s = \frac{1}{r} \quad \ddot{\varphi} = \frac{C}{r^2} = s^2 C$$

$$\Rightarrow -C^2 s^2 \frac{d^2 s}{d\varphi^2} - \frac{1}{s} s^4 C^2 = -s^2 K \Rightarrow \frac{d^2 s}{d\varphi^2} + s = \frac{K}{C^2} \Rightarrow$$

$$\Rightarrow s(\varphi) = A \cos \varphi + B \sin \varphi + \frac{K}{C^2}$$

Autor: Juan Carlos Álvarez

$S(\varphi) = \frac{K}{C^2}$ es una solución de $\frac{d^2 S}{d\varphi^2} + S = \frac{K}{C^2}$

(3.8)

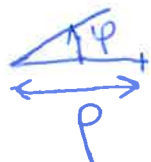
$S(\varphi) = A \cos \varphi + B \sin \varphi$ es solución de $\frac{d^2 S}{d\varphi^2} + S = 0$

La suma de las dos soluciones es la solución general de $\frac{d^2 S}{d\varphi^2} + S = \frac{K}{C^2}$:

$S(\varphi) = A \cos \varphi + B \sin \varphi + \frac{K}{C^2}$

Calculamos las propiedades de A y B:

ρ es máxima para $\varphi=0 \Rightarrow S$ es máxima para $\varphi=0 \Rightarrow$



$\Rightarrow \frac{dS}{d\varphi} \Big|_{\varphi=0} = 0$ y $\frac{d^2 S}{d\varphi^2} \Big|_{\varphi=0} < 0$

$\frac{dS}{d\varphi} = -A \sin \varphi + B \cos \varphi \Rightarrow \left. \begin{array}{l} \frac{dS}{d\varphi} \Big|_{\varphi=0} = B \\ \frac{dS}{d\varphi} \Big|_{\varphi=0} = 0 \end{array} \right\} \Rightarrow \boxed{B=0}$

$\frac{d^2 S}{d\varphi^2} = -A \cos \varphi \Rightarrow \left. \begin{array}{l} \frac{d^2 S}{d\varphi^2} \Big|_{\varphi=0} = -A \\ \frac{d^2 S}{d\varphi^2} \Big|_{\varphi=0} < 0 \end{array} \right\} \Rightarrow \boxed{A > 0}$

$S(\varphi) = A \cos \varphi + \frac{K}{C^2} \Rightarrow \rho = \frac{1}{A \cos \varphi + \frac{K}{C^2}} = \frac{\frac{C^2}{K}}{1 + \frac{AC^2}{K} \cos \varphi}$

La ecuación de una cónica es $\rho = \frac{l}{1 + e \cos \varphi}$

$\Rightarrow$ La trayectoria del planeta es una cónica, con $e = \frac{AC^2}{K}$ y $l = \frac{C^2}{K}$

$A > 0$
 $K \equiv GM > 0$

$e = 0$ circunferencia
 $0 < e < 1$ elipse
 $e = 1$ parábola
 $e > 1$ hipérbola

La trayectoria es una elipse, una parábola o una hipérbola

La trayectoria del planeta es cerrada

$\Rightarrow$ La trayectoria del planeta es una elipse

Autor: Iván Cabria Álvarez

3.23

3.9

$$\vec{r}(0) = r_0 \vec{u}_x$$

$$\vec{F}_2 = -b e^{a r(t)} \vec{u}_x$$

$$r_0 \equiv r(0) \equiv r_x(0)$$

$$m \vec{a} = \vec{F}_{\text{neto}}$$

$$\vec{F}_{\text{neto}} = \vec{F}_2$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{dr_x}{dt} \vec{u}_x = \frac{dr}{dt} \vec{u}_x$$

$$\Rightarrow m \frac{dr}{dt} = -b e^{a r(t)}$$

El movimiento es en una dimensión $\Rightarrow r_x \equiv r$

a) Calculamos t_p :

$$m \frac{dr}{dt} = -b e^{a r(t)} \Rightarrow e^{-a r(t)} dr = -\frac{b}{m} dt \Rightarrow$$

$$\Rightarrow \int_{r(0)}^{r(t_p)} e^{-a r(t)} dr = -\frac{b}{m} \int_0^{t_p} dt$$

t_p : Instante de tiempo en el que la cosa se detiene: $r(t_p) = 0$

$$\left[\frac{e^{-a r(t)}}{(-a)} \right]_{r(0)}^{r(t_p)} = -\frac{b}{m} t_p \Rightarrow \frac{(e^{-a r(t_p)} - e^{-a r(0)})}{(-a)} = -\frac{b}{m} t_p \Rightarrow$$

$$\Rightarrow t_p = \frac{m}{ab} (e^{-a r(t_p)} - e^{-a r(0)}) = \boxed{\frac{m}{ab} (1 - e^{-a r(0)})}$$

b) Calculamos $X(t_p) - X(0)$. Vamos a explicar dos maneras diferentes de calcular $X(t_p) - X(0)$.

Primera manera: Planteando y resolviendo una ecuación diferencial en X y r .

$$r = \frac{dx}{dt} \xrightarrow{\text{Regla de la cadena}} \frac{dx}{dr} \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = r \frac{dr}{dx}$$

$$\left. \begin{aligned} m \frac{dr}{dt} &= -b e^{a r(t)} \\ \Rightarrow m r \frac{dr}{dx} &= -b e^{a r} \end{aligned} \right\} \Rightarrow$$

$$\Rightarrow m r \frac{dr}{dx} = -b e^{a r}$$

Hemos obtenido una ecuación diferencial en X y r .

$$m \cdot v \frac{dv}{dx} = -b e^{a \cdot v} \Rightarrow -\frac{m}{b} v e^{-a \cdot v} dv = dx \Rightarrow$$

$$\Rightarrow \int_{x(0)}^{x(t_p)} dx = -\frac{m}{b} \int_{v(0)}^{v(t_p)} v e^{-a \cdot v} dv \Rightarrow x(t_p) - x(0) = -\frac{m}{b} \int_{v(0)}^{v(t_p)} v e^{-a \cdot v} dv$$

$$\int x e^{-ax} dx = -\frac{x}{a} e^{-ax} - \frac{e^{-ax}}{a^2}$$

⇓

$$\int_{v(0)}^{v(t_p)} v e^{-a \cdot v} dv = \left[-\frac{v}{a} e^{-a \cdot v} - \frac{e^{-a \cdot v}}{a^2} \right]_{v(0)}^{v(t_p)} = -\frac{v(t_p)}{a} e^{-a \cdot v(t_p)} - \frac{e^{-a \cdot v(t_p)}}{a^2}$$

$$+ \frac{v_0}{a} e^{-a \cdot v_0} + \frac{e^{-a \cdot v_0}}{a^2} = -\frac{1}{a^2} + \frac{v_0}{a} e^{-a \cdot v_0} + \frac{e^{-a \cdot v_0}}{a^2}$$

$$x(t_p) - x(0) = -\frac{m}{b a} \left(-\frac{1}{a} + v_0 e^{-a \cdot v_0} + \frac{e^{-a \cdot v_0}}{a} \right)$$

$$x(t_p) - x(0) = \frac{m}{b a^2} (1 - a v_0 e^{-a \cdot v_0} - e^{-a \cdot v_0})$$

Segunda manera: Planteando y resolviendo una ecuación diferencial en X y t .

$$m \frac{dv}{dt} = -b e^{a \cdot v(t)} \Rightarrow \int_{v(0)}^{v(t)} e^{-a \cdot v(t)} dv = -\frac{b}{m} \int_0^t dt \Rightarrow$$

$$\Rightarrow \frac{e^{-a \cdot v(t)} - e^{-a \cdot v_0}}{(-a)} = -\frac{b}{m} t \Rightarrow e^{-a \cdot v(t)} = \frac{ab}{m} t + e^{-a \cdot v_0} \Rightarrow$$

$$\Rightarrow -a \cdot v(t) = \ln \left(\frac{ab}{m} t + e^{-a \cdot v_0} \right) \Rightarrow v(t) = -\frac{1}{a} \ln \left(\frac{ab}{m} t + e^{-a \cdot v_0} \right)$$

¶ Hemos obtenido $v = v(t)$.

$$v(t) = \frac{dx}{dt} \Rightarrow \frac{dx}{dt} = -\frac{1}{a} \ln \left(\frac{ab}{m} t + e^{-a \cdot v_0} \right)$$

¶ Hemos obtenido una ecuación diferencial en X y t .

Autor: Iván Cabria Álvarez

Para escribir menos y simplificar los cálculos, definiremos las siguientes cantidades: $A \equiv \frac{ab}{m}$ $B \equiv e^{-a\tau_0}$ (3.11)

$$\frac{dx}{dt} = -\frac{1}{a} \ln(At + B) \Rightarrow \int_{x(0)}^{x(t_p)} dx = -\frac{1}{a} \int_0^{t_p} \ln(At + B) dt$$

$$\int \ln(AX + B) dX = \left(\frac{AX + B}{A} \right) \ln(AX + B) - \left(\frac{AX + B}{A} \right)$$

$$\Downarrow$$

$$\int_0^{t_p} \ln(At + B) dt = \left[\left(\frac{At + B}{A} \right) \ln(At + B) - \left(\frac{At + B}{A} \right) \right]_0^{t_p} =$$

$$= \left(\frac{At_p + B}{A} \right) \ln(At_p + B) - \left(\frac{At_p + B}{A} \right) - \frac{B}{A} \ln B + \frac{B}{A}$$

$$t_p = \frac{m}{ab} (1 - e^{-a\tau_0}) = \frac{1}{A} (1 - B) \Rightarrow At_p + B = 1 \Rightarrow$$

$$\Rightarrow \int_0^{t_p} \ln(At + B) dt = \frac{1}{A} \ln 1 - \frac{1}{A} - \frac{B}{A} \ln B + \frac{B}{A} =$$

$$= -\frac{1}{A} - \frac{B}{A} \ln B + \frac{B}{A} = \frac{1}{A} (-1 - B \ln B + B)$$

$$\left. \begin{array}{l} \int_{x(0)}^{x(t_p)} dx = X(t_p) - X(0) \\ \int_{x(0)}^{x(t_p)} dx = -\frac{1}{a} \int_0^{t_p} \ln(At + B) dt \end{array} \right\} \Rightarrow$$

$$\Rightarrow X(t_p) - X(0) = -\frac{1}{a} \frac{1}{A} (-1 - B \ln B + B) = \frac{m}{b a^2} (1 + B \ln B - B)$$

$$\ln B = \ln e^{-a\tau_0} = -a\tau_0 \Rightarrow B \ln B = -a\tau_0 e^{-a\tau_0}$$

$$X(t_p) - X(0) = \frac{m}{b a^2} (1 - a\tau_0 e^{-a\tau_0} - e^{-a\tau_0})$$

Autor: Iván Cabria Álvarez

(3.24) M = Masa del planeta Marte z = radio de la órbita de una luna de Marte (3.12)
 R_M = Radio del planeta Marte T = periodo de la órbita de una luna de Marte
 Segunda ley de Kepler: $\frac{T^2}{z^3} = \frac{4\pi^2}{GM}$ g_M = aceleración de la gravedad en la superficie de Marte

Ley de la gravitación universal: $F = \frac{GMm}{R_M^2}$

Segunda ley de la dinámica: $F = mg_M$
 A partir de las tres leyes anteriores se deduce:

$$\left. \begin{array}{l} \frac{T^2}{z^3} = \frac{4\pi^2}{GM} \Rightarrow GM = \frac{4\pi^2 z^3}{T^2} \\ F = \frac{GMm}{R_M^2} \\ F = mg_M \end{array} \right\} \Rightarrow g_M = \frac{GM}{R_M^2} \Rightarrow \boxed{g_M = \frac{4\pi^2 z^3}{T^2 R_M^2}}$$

Usaremos la ecuación de g_M para las dos lunas de Marte, utilizando los valores correspondientes de z y T . R_M tiene el mismo valor para las dos lunas, porque se trata del radio del planeta Marte.

Calculamos g_M usando los datos de Fobos:

$$g_M = \frac{4\pi^2 (9.3770 \cdot 10^6 \text{ m})^3}{(27576 \text{ s})^2 (3.3895 \cdot 10^6 \text{ m})^2} = \boxed{3.7258 \text{ m/s}^2}$$

Calculamos g_M usando los datos de Deimos:

$$g_M = \frac{4\pi^2 (23.46 \cdot 10^6 \text{ m})^3}{(109260 \text{ s})^2 (3.3895 \cdot 10^6 \text{ m})^2} = \boxed{3.7166 \text{ m/s}^2}$$

Autor: Iván Ceballos Álvarez

3.25) M = Masa del planeta Júpiter r = radio de la órbita de una luna (3.13)
 R_J = Radio del planeta Júpiter de Júpiter
 T = periodo de la órbita de una luna de Júpiter

g_J = aceleración de la gravedad en la superficie de Júpiter

De las tres leyes del enunciado del problema, deducimos, de manera similar a la deducción que hicimos en el problema 3.24, lo siguiente:

$$g_J = \frac{4\pi^2 r^3}{T^2 R_J^2}$$

Calculamos g_J usando los datos de Io:

$$g_J = \frac{4\pi^2 (0.4217 \cdot 10^9 \text{ m})^3}{(152853.5 \text{ s})^2 (71.5 \cdot 10^6 \text{ m})^2} = \boxed{24.7861 \text{ m/s}^2}$$

Calculamos g_J usando los datos de Europa:

$$g_J = \frac{4\pi^2 (0.6709 \cdot 10^9 \text{ m})^3}{(306823.68 \text{ s})^2 (71.5 \cdot 10^6 \text{ m})^2} = \boxed{24.7710 \text{ m/s}^2}$$

Calculamos g_J usando los datos de Ganímedes:

$$g_J = \frac{4\pi^2 (1.0704 \cdot 10^9 \text{ m})^3}{(618157.44 \text{ s})^2 (71.5 \cdot 10^6 \text{ m})^2} = \boxed{24.7849 \text{ m/s}^2}$$

Calculamos g_J usando los datos de Calisto:

$$g_J = \frac{4\pi^2 (1.8827 \cdot 10^9 \text{ m})^3}{(1441929.6 \text{ s})^2 (71.5 \cdot 10^6 \text{ m})^2} = \boxed{24.7858 \text{ m/s}^2}$$

Autor: Iván Cabrera Álvarez

3.26) M = Masa del planeta Saturno r = radio de la órbita de una luna de Saturno
 R_S = Radio del planeta Saturno T = periodo de la órbita de una luna de Saturno
 g_S = aceleración de la gravedad en la superficie de Saturno

$$g_S = \frac{4\pi^2 r^3}{T^2 R_S^2}$$

Calculamos g_S usando los datos de Mimas:

$$g_S = \frac{4\pi^2 (0.18552 \cdot 10^9 \text{ m})^3}{(81425 \text{ s})^2 (58.2 \cdot 10^6 \text{ m})^2} = 11.2246 \text{ m/s}^2$$

Calculamos g_S usando los datos de Encelado:

$$g_S = \frac{4\pi^2 (0.238 \cdot 10^9 \text{ m})^3}{(118380 \text{ s})^2 (58.2 \cdot 10^6 \text{ m})^2} = 11.2121 \text{ m/s}^2$$

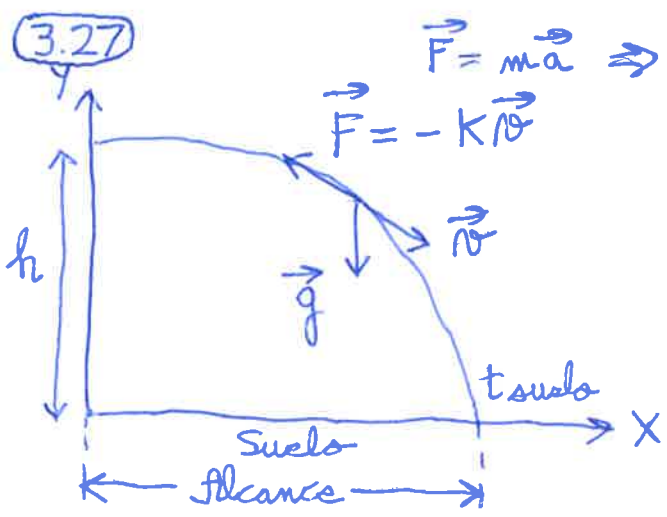
Calculamos g_S usando los datos de Tetis:

$$g_S = \frac{4\pi^2 (0.29462 \cdot 10^9 \text{ m})^3}{(163106 \text{ s})^2 (58.2 \cdot 10^6 \text{ m})^2} = 11.2037 \text{ m/s}^2$$

Calculamos g_S usando los datos de Titán:

$$g_S = \frac{4\pi^2 (1.2219 \cdot 10^9 \text{ m})^3}{(1377687 \text{ s})^2 (58.2 \cdot 10^6 \text{ m})^2} = 11.2026 \text{ m/s}^2$$

Autor: Iván Cabrera Álvarez



$$\vec{F} = m\vec{a} \Rightarrow -kR_x \vec{u}_x - mg \vec{u}_y + kR_y \vec{u}_y =$$

$$= m \frac{dR_x}{dt} \vec{u}_x + m \frac{dR_y}{dt} \vec{u}_y \Rightarrow$$

$$\Rightarrow -mg + kR_y = m \frac{dR_y}{dt}$$

$$-kR_x = m \frac{dR_x}{dt}$$

En el instante de llegada al suelo, la gota ha alcanzado la velocidad límite en el eje Y y el movimiento es uniforme y con velocidad R_l en el eje Y.

$$-mg + kR_l = m \frac{dR_y}{dt} \xrightarrow{t \rightarrow \infty} -mg + kR_l = 0 \Rightarrow R_l = \frac{mg}{k}$$

$$\Delta t_{\text{suelo}} = t_{\text{suelo}} - 0 = t_{\text{suelo}}$$

Movimiento uniforme y con velocidad R_l en el eje Y $\Rightarrow h \approx R_l \Delta t_{\text{suelo}}$

$$\Rightarrow t_{\text{suelo}} \approx \frac{h}{R_l} = \frac{1.4 \text{ m}}{0.7 \text{ m/s}} = \boxed{2 \text{ s}}$$

El alcance de la gota es: $A = X(t_{\text{suelo}}) - X(0)$

Para obtener $X(t)$, tenemos que resolver primero la ecuación $-kR_x = m \frac{dR_x}{dt}$:

$$-kR_x = m \frac{dR_x}{dt} \Rightarrow -\frac{k}{m} dt = \frac{dR_x}{R_x} \Rightarrow -\frac{k}{m} \int_0^t dt = \int_{R_x(0)}^{R_x(t)} \frac{dR_x}{R_x} \Rightarrow$$

$$\Rightarrow -\frac{k}{m} t = \ln\left(\frac{R_x(t)}{R_x(0)}\right) \Rightarrow e^{-\frac{k}{m}t} = \frac{R_x(t)}{R_x(0)} \Rightarrow R_x(t) = R_x(0) e^{-\frac{k}{m}t}$$

En segundo lugar, tenemos que resolver la ecuación $R_x(t) = \frac{dX}{dt}$:

$$R_x(t) = R_x(0) e^{-\frac{k}{m}t} \Rightarrow R_x(0) e^{-\frac{k}{m}t} = \frac{dX}{dt} \Rightarrow R_x(0) e^{-\frac{k}{m}t} dt = dX \Rightarrow$$

$$\Rightarrow \int_0^t R_x(0) e^{-\frac{k}{m}t} dt = \int_{X(0)}^{X(t)} dX \Rightarrow \frac{R_x(0) e^{-\frac{k}{m}t}}{(-\frac{k}{m})} \Big|_0^t = X(t) - X(0) \Rightarrow$$

Autor: Iván Cabris Álvarez

$$\Rightarrow -\frac{m R_x(0)}{k} e^{-\frac{k}{m}t} \Big|_0^t = X(t) - X(0) \Rightarrow -\frac{m}{k} R_x(0) (e^{-\frac{k}{m}t} - 1) = X(t) - X(0) \Rightarrow$$

$$\Rightarrow X(t) = X(0) + \frac{m}{k} R_x(0) - \frac{m}{k} R_x(0) e^{-\frac{k}{m}t}$$

$$\left. \begin{aligned} \frac{k}{m} &= \frac{kg}{mg} = \frac{g}{R_l} \\ \frac{m}{k} &= \frac{mg}{kg} = \frac{R_l}{g} \end{aligned} \right\} \Rightarrow$$

$$\Rightarrow X(t) = X(0) + \frac{v_{x0} v_{x0}}{g} - \frac{v_{x0} v_{x0}}{g} e^{-\frac{gt}{v_{x0}}}$$

$$\text{Altura} = X(t_{\text{subida}}) - X(0) = \frac{v_{x0} v_{x0}}{g} - \frac{v_{x0} v_{x0}}{g} e^{-\frac{gt_{\text{subida}}}{v_{x0}}}$$

Calculamos $e^{-\frac{gt_{\text{subida}}}{v_{x0}}}$:

$$t_{\text{subida}} \simeq \frac{h}{v_{x0}} \Rightarrow \frac{gt_{\text{subida}}}{v_{x0}} \simeq \frac{gh}{v_{x0}^2} = \frac{9.8 \text{ m/s}^2 \cdot 1.4 \text{ m}}{(0.7 \text{ m/s})^2} = 28$$

$$e^{-\frac{gt_{\text{subida}}}{v_{x0}}} \simeq e^{-28} = \frac{1}{e^{28}} \simeq 0$$

$$\text{Altura} \simeq \frac{v_{x0} v_{x0}}{g} = \frac{0.7 \text{ m/s} \cdot 14 \text{ m/s}}{9.8 \text{ m/s}^2} = \boxed{1 \text{ m}}$$

$$\begin{aligned} \text{3.28) } \vec{L} &= \vec{r} \times \vec{p} = r \vec{u}_r \times m (\dot{r} \vec{u}_r + r \dot{\varphi} \vec{u}_\varphi) = m r [\vec{u}_r \times \dot{r} \vec{u}_r + \vec{u}_r \times r \dot{\varphi} \vec{u}_\varphi] \\ &= m r [0 + r \dot{\varphi} \vec{u}_z] = m r^2 \dot{\varphi} \vec{u}_z \end{aligned}$$

$$\vec{u}_r \times \vec{u}_\varphi = \vec{u}_z$$

Por otra parte:

$$\left. \begin{aligned} \vec{F} &= m \vec{a} \\ \vec{F} &= -\frac{GMm}{r^2} \vec{u}_r \end{aligned} \right\} \Rightarrow -\frac{GM}{r^2} \vec{u}_r = \vec{a}$$

$$\vec{a} = (\ddot{r} - r \dot{\varphi}^2) \vec{u}_r + (r \ddot{\varphi} + 2 \dot{r} \dot{\varphi}) \vec{u}_\varphi$$

$$\Rightarrow -\frac{GM}{r^2} \vec{u}_r = (\ddot{r} - r \dot{\varphi}^2) \vec{u}_r + (r \ddot{\varphi} + 2 \dot{r} \dot{\varphi}) \vec{u}_\varphi \Rightarrow 0 = r \ddot{\varphi} + 2 \dot{r} \dot{\varphi} \Rightarrow$$

$$\Rightarrow 0 = r^2 \ddot{\varphi} + 2 r \dot{r} \dot{\varphi} \Rightarrow 0 = \frac{d}{dt} (r^2 \dot{\varphi}) \Rightarrow r^2 \dot{\varphi} = C$$

$C = \text{constante}$

$$\vec{L} = m r^2 \dot{\varphi} \vec{u}_z$$

$m = \text{constante}$

$r^2 \dot{\varphi} = \text{constante}$

$\Rightarrow$

$$\vec{L} = D \vec{u}_z$$

$D = m r^2 \dot{\varphi}$ es constante

Autor: Iván Cabeza Álvarez

Otra forma de calcular $\vec{r} \times \vec{p}$:

$$\begin{aligned} \vec{r} \times \vec{p} &= \vec{r} \times m \vec{v} = m \begin{vmatrix} \vec{u}_r & \vec{u}_\varphi & \vec{u}_z \\ r & 0 & 0 \\ \dot{r} & r \dot{\varphi} & 0 \end{vmatrix} = m \left[\vec{u}_\varphi \begin{vmatrix} 0 & 0 \\ r \dot{\varphi} & 0 \end{vmatrix} - \vec{u}_r \begin{vmatrix} r & 0 \\ \dot{r} & 0 \end{vmatrix} + \vec{u}_z \begin{vmatrix} r & 0 \\ \dot{r} & r \dot{\varphi} \end{vmatrix} \right] = \\ &= m r^2 \dot{\varphi} \vec{u}_z \end{aligned}$$

(3.17)

$$(3.29) \rho_T = \frac{M_T}{V_T} = \frac{M_T}{\frac{4\pi}{3} R_T^3} = \frac{3 M_T}{4\pi R_T^3}$$

 M_T : Masa de la Tierra. R_T : Radio de la Tierra.

Tercera ley de Kepler aplicada al sistema Tierra-Luna:

$$\frac{4\pi^2}{GM_T} = \frac{T_{LT}^2}{r_{LT}^3}$$

 $\Downarrow$ T_{LT} : Período de la órbita de la Luna alrededor de la Tierra. r_{LT} : Distancia del centro de la Tierra al centro de la Luna.

$$M_T = \frac{4\pi^2 r_{LT}^3}{G T_{LT}^2} = \frac{4\pi^2 (384\,400\,000 \text{ m})^3}{6.67 \cdot 10^{-11} \text{ Nm}^2 \text{ kg}^{-2} (2360592 \text{ s})^2} = 6.0331 \cdot 10^{24} \text{ kg}$$

Calculamos R_T :

$$g = \frac{GM_T}{R_T^2} \Rightarrow R_T = \sqrt{\frac{GM_T}{g}} = \sqrt{\frac{6.67 \cdot 10^{-11} \text{ Nm}^2 \text{ kg}^{-2} \cdot 6.0331 \cdot 10^{24} \text{ kg}}{9.8 \text{ m/s}^2}}$$

$$R_T = 6407977.35 \text{ m}$$

$$\rho_T = \frac{3M_T}{4\pi R_T^3} = \frac{3 \cdot 6.0331 \cdot 10^{24} \text{ kg}}{4\pi (6407977.35 \text{ m})^3} = 5473.82 \frac{\text{kg}}{\text{m}^3} \approx 5.5 \frac{\text{kg}}{\text{L}}$$

$$(3.30) P = \rho g_p h$$

 $g_p = ?$

$$\rho = 13.59 \frac{\text{g}}{\text{cm}^3} = 13590 \frac{\text{kg}}{\text{m}^3}$$

 ρ : densidad del mercurio. g_p : aceleración de la gravedad en la superficie del planeta. h : altura del líquido en el tubo.

$$h = 0.8 \text{ m}$$

Calculamos g_p usando la tercera ley de Kepler y la gravedad en la superficie del planeta. r_{sp} : distancia media del satélite natural (s) al planeta (p). T : período de la órbita del satélite natural alrededor del planeta.

$$\frac{T^2}{r_{sp}^3} = \frac{4\pi^2}{GM_{\text{planeta}}}$$

$$K = \frac{T^2}{r_{sp}^3} = \frac{(1382400 \text{ s})^2}{(1.22 \cdot 10^9 \text{ m})^3} = 1.0524 \cdot 10^{-15} \frac{\text{s}^2}{\text{m}^3}$$

$$g_p = \frac{GM_{\text{planeta}}}{R_{\text{planeta}}^2} \Rightarrow g_p = \frac{4\pi^2}{K R_{\text{planeta}}^2}$$

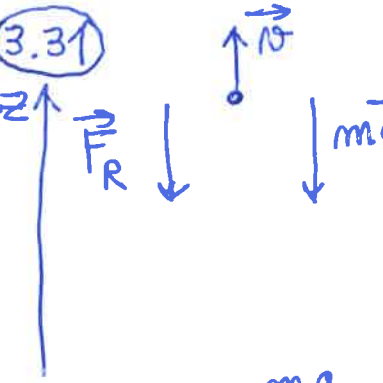
$$K = \frac{4\pi^2}{GM_{\text{planeta}}}$$

Autor: Iván Cabrera Álvarez

$$g_p = \frac{4\pi^2}{K R_{\text{planeta}}^2} = \frac{4\pi^2}{1.0524 \cdot 10^{15} \frac{\text{S}^2}{\text{m}^3} (6 \cdot 10^7 \text{m})^2} = 10.42 \frac{\text{m}}{\text{s}^2} \quad (3.18)$$

$$P = \rho g_p h = 13590 \frac{\text{kg}}{\text{m}^3} 10.42 \frac{\text{m}}{\text{s}^2} 0.8 \text{m} = \boxed{113286 \text{ Pa}}$$

(3.31)



$$\vec{F}_R = -Kv \vec{u}_z \quad \vec{g} = -g \vec{u}_z$$

$$\left. \begin{aligned} \vec{F} &= m\vec{a} \\ \vec{F} &= \vec{F}_R + m\vec{g} \end{aligned} \right\} \Rightarrow -Kv - mg = m \frac{dv}{dt} \Rightarrow$$

$$\Rightarrow v + \frac{mg}{K} = -\frac{m}{K} \frac{dv}{dt} \quad \left. \begin{aligned} v &+ v_{\ell} = -\frac{m}{K} \frac{dv}{dt} \Rightarrow \\ v_{\ell} &\equiv \frac{mg}{K} \end{aligned} \right\}$$

$$\Rightarrow v + v_{\ell} = -\frac{v_{\ell}}{g} \frac{dv}{dt} \Rightarrow -\frac{g}{v_{\ell}} dt = \frac{dv}{v + v_{\ell}} \Rightarrow$$

$$\Rightarrow -\frac{g}{v_{\ell}} \int_0^t dt = \int_{v(0)}^{v(t)} \frac{dv}{v + v_{\ell}} \Rightarrow -\frac{g}{v_{\ell}} t = \ln \left(\frac{v(t) + v_{\ell}}{v(0) + v_{\ell}} \right)$$

$$v(t_a) = 0 \Rightarrow -\frac{g}{v_{\ell}} t_a = \ln \left(\frac{0 + v_{\ell}}{v(0) + v_{\ell}} \right) \Rightarrow t_a = \frac{v_{\ell}}{g} \ln \left(\frac{v(0) + v_{\ell}}{v_{\ell}} \right)$$

$$v_{\ell} = \frac{mg}{K} = 30 \text{ m/s}$$

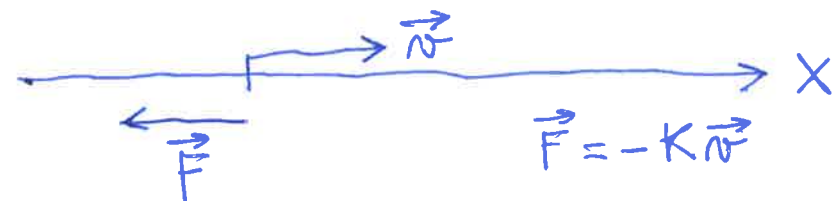
$$v(0) = 10 \text{ m/s}$$

$$g = 9.8 \text{ m/s}^2$$

$$t_a = \frac{30 \text{ m/s}}{9.8 \text{ m/s}^2} \ln \left(\frac{40}{30} \right) = \boxed{0.88 \text{ s}}$$

Autor: Iván Gabris Álvarez

(3.32)



(3.19)

$$\begin{aligned}
 a) \vec{F} &= m\vec{a} \Rightarrow -Kv_x = m \frac{dv_x}{dt} \Rightarrow -\frac{K}{m} dt = \frac{dv_x}{v_x} \Rightarrow \\
 \Rightarrow -\frac{K}{m} \int_0^t dt &= \int_{v_x(0)}^{v_x(t)} \frac{dv_x}{v_x} \Rightarrow -\frac{K}{m} t = \ln \left(\frac{v_x(t)}{v_x(0)} \right) \Rightarrow \\
 \Rightarrow e^{-\frac{K}{m}t} &= \frac{v_x(t)}{v_x(0)} \Rightarrow \boxed{v_x(t) = v_x(0) e^{-\frac{K}{m}t}}
 \end{aligned}$$

$$\begin{aligned}
 b) \left. \begin{aligned} v_x(t) &= \frac{dx}{dt} \\ v_x(t) &= v_x(0) e^{-\frac{K}{m}t} \end{aligned} \right\} \Rightarrow \frac{dx}{dt} = v_x(0) e^{-\frac{K}{m}t} \Rightarrow dx = v_x(0) e^{-\frac{K}{m}t} dt \Rightarrow
 \end{aligned}$$

$$\Rightarrow \int_{x(0)}^{x(t)} dx = v_x(0) \int_0^t e^{-\frac{K}{m}t} dt \Rightarrow x(t) - x(0) = \frac{v_x(0)}{\left(-\frac{K}{m}\right)} (e^{-\frac{K}{m}t} - 1) \Rightarrow$$

$$\Rightarrow x(t) = x(0) - \frac{m v_x(0)}{K} (e^{-\frac{K}{m}t} - 1) \Rightarrow \boxed{x(t) = x(0) + \frac{m v_x(0)}{K} (1 - e^{-\frac{K}{m}t})}$$

(3.33)



$$g_a = 0.01 \text{ m/s}^2$$

$$\rho = 5 \text{ g/cm}^3 = 0.005 \text{ kg/cm}^3 = 5000 \text{ kg/m}^3$$

$$g_a = \frac{GM_a}{R_a^2}$$

$$\rho = \frac{M_a}{V_a} = \frac{M_a}{\frac{4\pi}{3} R_a^3} \Rightarrow M_a = \frac{4\pi}{3} R_a^3 \rho$$

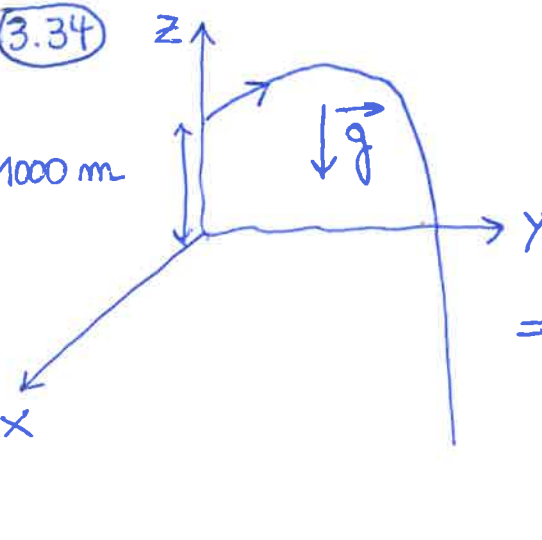
$$\Rightarrow g_a = \frac{G}{R_a^2} \frac{4\pi}{3} R_a^3 \rho \Rightarrow$$

$$\Rightarrow g_a = \frac{G 4\pi \rho}{3} R_a \Rightarrow R_a = \frac{3g_a}{G 4\pi \rho} = \frac{3 \cdot 0.01 \text{ m/s}^2}{6.67 \cdot 10^{-11} \text{ Nm}^2 \text{ kg}^{-2} 4\pi 5000 \text{ kg/m}^3} =$$

$$= \boxed{7158.39 \text{ m}}$$

Autor: Iván Gabrila Álvarez

(3.34)



1000 m

$$\left. \begin{aligned} \vec{F} &= m\vec{a} \\ \vec{F} &= -mg\vec{u}_z \end{aligned} \right\} \Rightarrow -mg\vec{u}_z = m\frac{d\vec{v}}{dt}$$

$$\frac{d\vec{v}}{dt} = \frac{dv_x}{dt}\vec{u}_x + \frac{dv_y}{dt}\vec{u}_y + \frac{dv_z}{dt}\vec{u}_z$$

$$\left. \begin{aligned} \Rightarrow \frac{dv_x}{dt} &= 0 \\ \frac{dv_y}{dt} &= 0 \\ \frac{dv_z}{dt} &= -g \end{aligned} \right\} \Rightarrow \begin{aligned} v_x(t) &= v_x(0) = 0 \\ v_y(t) &= v_y(0) = 0 \\ v_z(t) &= v_z(0) - gt \end{aligned}$$

(3.20)

$$\left. \begin{aligned} v_x &= \frac{dx}{dt} \\ v_x(t) &= v_x(0) \end{aligned} \right\} \Rightarrow x(t) = x(0) + v_x(0)t$$

$$\left. \begin{aligned} v_y(t) &= \frac{dy}{dt} \\ v_y(t) &= v_y(0) \end{aligned} \right\} \Rightarrow y(t) = y(0) + v_y(0)t$$

$$\left. \begin{aligned} v_z(t) &= \frac{dz}{dt} \\ v_z(t) &= v_z(0) - gt \end{aligned} \right\} \Rightarrow v_z(0) - gt = \frac{dz}{dt} \Rightarrow z(t) = z(0) + v_z(0)t - \frac{gt^2}{2}$$

$x(0) = y(0) = 0$ $z(0) = 1000 \text{ m}$ $v_x(0) = v_y(0) = v_z(0) = \frac{v(0)}{\sqrt{3}} = \frac{100}{\sqrt{3}} \text{ m/s}$

t_s : Instante de tiempo en el que llega al suelo.

$$z(t_s) = 0 \Rightarrow 0 = z(0) + v_z(0)t_s - \frac{g}{2}t_s^2 \Rightarrow$$

$$\Rightarrow t_s = \frac{-v_z(0) \pm \sqrt{v_z^2(0) - 4(-g/2)z(0)}}{2(-g/2)} = \frac{v_z(0) \pm \sqrt{v_z^2(0) + 2gz(0)}}{g} =$$

$$= \frac{\frac{100}{\sqrt{3}} \pm \sqrt{\frac{10000}{3} + 19600}}{9.8} = \begin{cases} -9.56 \text{ s} \\ 21.34 \text{ s} \end{cases}$$

t_s debe ser $> 0 \Rightarrow$

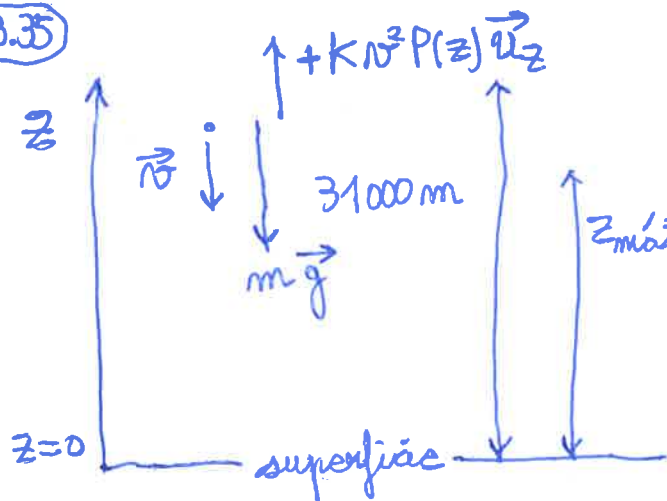
$$\Rightarrow \boxed{t_s = 21.34 \text{ s}}$$

$$x(t_s) = v_x(0)t_s = \frac{v(0)}{\sqrt{3}}t_s = \frac{100}{\sqrt{3}} \frac{\text{m}}{\text{s}} 21.34 \text{ s} = \boxed{1232.07 \text{ m}}$$

$$y(t_s) = v_y(0)t_s = \frac{v(0)}{\sqrt{3}}t_s = \frac{100}{\sqrt{3}} \frac{\text{m}}{\text{s}} 21.34 \text{ s} = \boxed{1232.07 \text{ m}}$$

Autor: Iván Cabria Álvarez

(3.35)



$$m = 80\text{ Kg} \quad \vec{r} = -\alpha\vec{u}_z \quad (3.21)$$

$$\vec{g} = -g\vec{u}_z$$

$$\vec{F}_2 = +K\alpha^2 P(z)\vec{u}_z$$

$$\vec{F}_{\text{neto}} = m\vec{g} + \vec{F}_2$$

$$\alpha_{\text{máx}} = \alpha(z_{\text{máx}}) = \alpha(t_{\text{máx}})$$

$$\left. \begin{aligned} m\vec{a} &= \vec{F}_{\text{neto}} \\ \vec{F}_{\text{neto}} &= m\vec{g} + \vec{F}_2 \\ \vec{a} &= \frac{d\alpha}{dt}\vec{u}_z \end{aligned} \right\} \Rightarrow m \frac{d\alpha}{dt} = -mg + K\alpha^2 P(z)$$

Calculamos $\alpha_{\text{máx}}$. Hay dos maneras de calcular $\alpha_{\text{máx}}$.

Primera manera:

$$\left. \begin{aligned} \frac{d\alpha}{dt} &= \frac{d\alpha}{dz} \frac{dz}{dt} = \frac{d\alpha}{dz} \alpha \\ m \frac{d\alpha}{dt} &= -mg + K\alpha^2 P(z) \end{aligned} \right\} \Rightarrow m \frac{d\alpha}{dz} \alpha = -mg + K\alpha^2 P(z) \Rightarrow \left. \begin{aligned} \alpha(z) \text{ será máxima donde } \frac{d\alpha}{dz} \text{ sea cero} &\Rightarrow \frac{d\alpha}{dz} \Big|_{z_{\text{máx}}} = 0 \end{aligned} \right\} \Rightarrow$$

$$\Rightarrow 0 = -mg + K\alpha_{\text{máx}}^2 P(z_{\text{máx}}) \Rightarrow \alpha_{\text{máx}} = \sqrt{\frac{mg}{K P(z_{\text{máx}})}}$$

Segunda manera:

$$\left. \begin{aligned} \alpha(t) \text{ será máxima cuando } \frac{d\alpha}{dt} \text{ sea cero} &\Rightarrow \frac{d\alpha}{dt} \Big|_{t_{\text{máx}}} = 0 \\ m \frac{d\alpha}{dt} &= -mg + K\alpha^2 P(z) \end{aligned} \right\} \Rightarrow$$

$$\Rightarrow 0 = -mg + K\alpha_{\text{máx}}^2 P(z_{\text{máx}}) \Rightarrow \alpha_{\text{máx}} = \sqrt{\frac{mg}{K P(z_{\text{máx}})}}$$

Autor: Iván Cabria Alvaros

Calculamos $P(z_{\text{máx}})$:

3.22

$$e(z) \propto P(z) \Rightarrow \frac{e(0)}{e(z)} = \frac{P(0)}{P(z)} \text{ y } e(z) = b P(z) \forall z \Rightarrow$$

$$\Rightarrow b = \frac{e(0)}{P(0)}$$

$$\left. \begin{aligned} \frac{dp}{dz} &= -e(z)g \\ e(z) &= b P(z) \end{aligned} \right\} \Rightarrow \frac{dp}{dz} = -b g P(z) \Rightarrow \frac{dp}{p} = -b g dz \Rightarrow$$

$$\Rightarrow \int_{P(0)}^{P(z)} \frac{dp}{p} = -b g \int_0^z dz \Rightarrow \ln\left(\frac{P(z)}{P(0)}\right) = -b g z \Rightarrow$$

$$\Rightarrow \frac{P(z)}{P(0)} = e^{-b g z} \Rightarrow P(z) = P(0) e^{-b g z}$$

$$v_{\text{máx}} = \sqrt{\frac{mg}{K P(z_{\text{máx}})}} = \sqrt{\frac{mg}{K P(0) e^{-b g z_{\text{máx}}}}}$$

$$b g z_{\text{máx}} = \frac{e(0) g z_{\text{máx}}}{P(0)} = \frac{1.3 \text{ Kg m}^{-3} 9.8 \text{ m s}^{-2} 27000 \text{ m}}{101300 \text{ Pa}}$$

$$1 \text{ Pa} = 1 \text{ N m}^{-2} = 1 \text{ Kg m s}^{-2} \text{ m}^{-2} = 1 \text{ Kg m}^{-1} \text{ s}^{-2}$$

$$b g z_{\text{máx}} = \frac{1.3 \text{ Kg m}^{-3} 9.8 \text{ m s}^{-2} 27000 \text{ m}}{101300 \text{ Kg m}^{-1} \text{ s}^{-2}} = 3.3956565$$

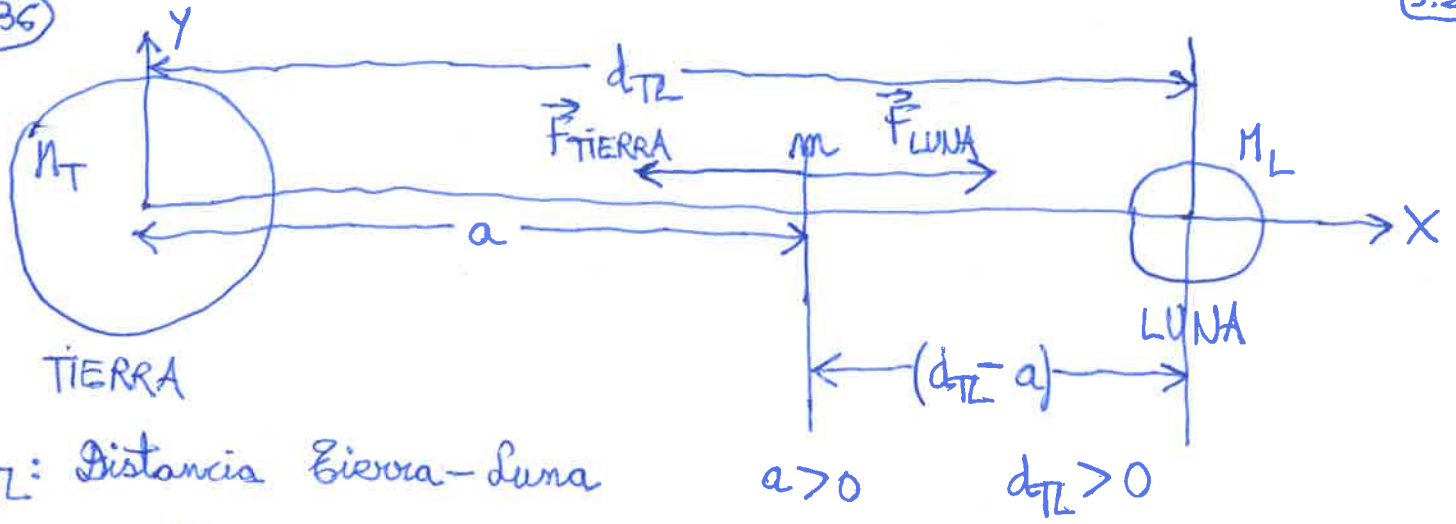
$$e^{-b g z_{\text{máx}}} = 0.033518543$$

$$v_{\text{máx}} = \sqrt{\frac{80 \text{ Kg } 9.8 \text{ m/s}^2}{0.3 \text{ Kg m}^{-1} 0.033518543}} = \boxed{279.2 \text{ m/s}}$$

Autor: Iván Cabria Álvarez

(3.36)

(3.23)



d_{TL} : Distancia Tierra-Luna

$a > 0$

$d_{TL} > 0$

$d_{TL} - a > 0$

$$\vec{F}_{TIERRA} = - \frac{GM_T m}{|a|^2} \vec{u}_x$$

$$\vec{F}_{LUNA} = + \frac{GM_L m}{|d_{TL} - a|^2} \vec{u}_x$$

$$\vec{F}_{TIERRA} + \vec{F}_{LUNA} = \vec{0} \Rightarrow - \frac{GM_T m}{|a|^2} + \frac{GM_L m}{|d_{TL} - a|^2} = 0 \Rightarrow$$

$$\Rightarrow - \frac{M_T}{|a|^2} + \frac{M_L}{|d_{TL} - a|^2} = 0 \Rightarrow \frac{M_T}{M_L} = \frac{|a|^2}{|d_{TL} - a|^2} \Rightarrow$$

$$\Rightarrow \left. \begin{array}{l} \frac{|a|}{|d_{TL} - a|} = \pm \sqrt{\frac{M_T}{M_L}} \\ |a| > 0 \quad |d_{TL} - a| > 0 \end{array} \right\} \Rightarrow \left. \begin{array}{l} \frac{|a|}{|d_{TL} - a|} = \sqrt{\frac{M_T}{M_L}} \\ a > 0 \Rightarrow |a| = a \\ d_{TL} - a > 0 \Rightarrow |d_{TL} - a| = d_{TL} - a \end{array} \right\} \Rightarrow$$

$$\Rightarrow \frac{a}{d_{TL} - a} = \sqrt{\frac{M_T}{M_L}} \Rightarrow a = d_{TL} \sqrt{\frac{M_T}{M_L}} - a \sqrt{\frac{M_T}{M_L}} \Rightarrow$$

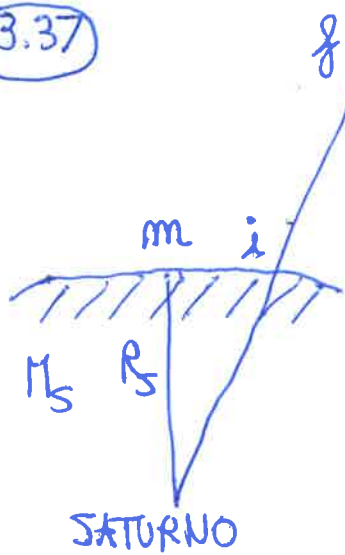
$$\Rightarrow a \left(1 + \sqrt{\frac{M_T}{M_L}} \right) = d_{TL} \sqrt{\frac{M_T}{M_L}} \Rightarrow a = \frac{d_{TL} \sqrt{\frac{M_T}{M_L}}}{1 + \sqrt{\frac{M_T}{M_L}}}$$

$$\sqrt{\frac{M_T}{M_L}} = \sqrt{\frac{5.9720 \cdot 10^{24} \text{ Kg}}{7.3490 \cdot 10^{22} \text{ Kg}}} = 9.0145858$$

$$a = \frac{384\,000 \text{ km} \cdot 9.0145858}{1 + 9.0145858} = \boxed{345655.95 \text{ km}}$$

Autor: Iván Cabria Álvarez

(3.37)


 M_S : Masa de Saturno

 R_S : Radio de Saturno (3.34)

$$E_i = E_{ci} + E_{pi} = \frac{m}{2} v_i^2 - \frac{GM_S m}{R_S}$$

$$E_f = E_{cf} + E_{pf} = \frac{m}{2} v_f^2 - \frac{GM_S m}{R_{oo}}$$

$$v_f = 0 \quad R_{oo} \rightarrow \infty \Rightarrow \frac{-GM_S m}{R_{oo}} = 0$$

$$E_i = E_f$$

 $\Rightarrow$

$$\Rightarrow \frac{m}{2} v_i^2 - \frac{GM_S m}{R_S} = 0 \Rightarrow v_i = \sqrt{\frac{2GM_S}{R_S}}$$

$$T_{REA} = 4.5180 \cdot 86400 \text{ s} = 390355.2 \text{ s}$$

$$\frac{T_{REA}^2}{\tau_{REA}^3} = \frac{4\pi^2}{GM_S} \Rightarrow GM_S = \frac{4\pi^2 \tau_{REA}^3}{T_{REA}^2} = \frac{4\pi^2 (5.27 \cdot 10^8 \text{ m})^3}{(390355.2 \text{ s})^2} \Rightarrow$$

$$\Rightarrow GM_S = 3.7920289 \cdot 10^{16} \frac{\text{m}^3}{\text{s}^2}$$

$$v_i = \sqrt{\frac{2GM_S}{R_S}} = \sqrt{\frac{2 \cdot 3.7920289 \cdot 10^{16} \text{ m}^3 \text{ s}^{-2}}{5.82 \cdot 10^7 \text{ m}}} = 36098.51 \text{ m/s} \Rightarrow$$

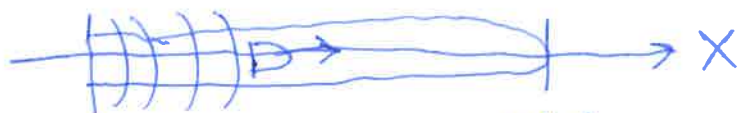
$$\Rightarrow \boxed{v_i \approx 36.1 \frac{\text{km}}{\text{s}}}$$

Según algunos experimentos, $v_i = 35.5 \text{ km/s}$.

Autor: Iván Cabria Álvarez

(3.38)

(3.25)



$$X(0)$$

$$t=0s$$

$$v(0)=0 \text{ m/s}$$

$$X(t_s)$$

$$t_s=0.003s$$

$$v(t_s)=1000 \text{ m/s}$$

$$L = X(t_s) - X(0) = 6 \text{ m}$$

Play das incógnitas: c y e

$$\left. \begin{array}{l} a(t) = \frac{dv}{dt} \\ a(t) = c + et \end{array} \right\} \Rightarrow (c + et) = \frac{dv}{dt} \Rightarrow \int_0^t (c + et) dt = \int_{v(0)}^{v(t)} dv \Rightarrow$$

$$\Rightarrow ct + e \frac{t^2}{2} \Big|_0^t = v(t) - v(0) \Rightarrow v(t) = ct + \frac{et^2}{2} \quad 0 \leq t \leq t_s$$

$$\left. \begin{array}{l} v(t) = \frac{dX}{dt} \\ v(t) = ct + \frac{et^2}{2} \end{array} \right\} \Rightarrow ct + \frac{et^2}{2} = \frac{dX}{dt} \Rightarrow \int_0^t (ct + \frac{et^2}{2}) dt = \int_{X(0)}^{X(t)} dX \Rightarrow$$

$$\Rightarrow \frac{ct^2}{2} + \frac{et^3}{6} \Big|_0^t = X(t) - X(0) \Rightarrow X(t) - X(0) = \frac{ct^2}{2} + \frac{et^3}{6}$$

Play das incógnitas y das ecuaciones.

$$\left. \begin{array}{l} L = X(t_s) - X(0) \\ X(t) - X(0) = \frac{ct^2}{2} + \frac{et^3}{6} \end{array} \right\} \Rightarrow L = \frac{ct_s^2}{2} + \frac{et_s^3}{6}$$

Las dos ecuaciones son:

$$L = \frac{ct_s^2}{2} + \frac{et_s^3}{6}$$

$$v(t_s) = ct_s + \frac{et_s^2}{2}$$

Calculamos la incógnita e :

$$v(t_s) = ct_s + \frac{et_s^2}{2} \Rightarrow ct_s = v(t_s) - \frac{et_s^2}{2}$$

Autor: Iván Cabrera Álvarez

$$\left. \begin{aligned} ct_s &= r(t_s) - \frac{et_s^3}{2} \\ L &= \frac{ct_s^3}{2} + \frac{et_s^3}{6} \end{aligned} \right\} \Rightarrow L = \frac{t_s}{2} (r(t_s) - \frac{et_s^3}{2}) + \frac{et_s^3}{6} \Rightarrow \\
 \Rightarrow L - \frac{t_s r(t_s)}{2} = -\frac{et_s^3}{4} + \frac{et_s^3}{6} \Rightarrow L - \frac{t_s r(t_s)}{2} = \frac{et_s^3}{2} \left(\frac{1}{3} - \frac{1}{2} \right) \\
 \Rightarrow L - \frac{t_s r(t_s)}{2} = -\frac{et_s^3}{12} \Rightarrow e = \frac{12}{t_s^3} \left(\frac{t_s r(t_s)}{2} - L \right) \Rightarrow \\
 \Rightarrow e = \frac{12}{(0.003 \text{ s})^3} \left(\frac{0.003 \text{ s } 1000 \text{ m/s}}{2} - 6 \text{ m} \right) = \frac{12}{27 \cdot 10^{-9} \text{ s}^3} (1.5 - 6) \text{ m} \Rightarrow \\
 \Rightarrow \boxed{e = -2 \cdot 10^9 \text{ m/s}^3}$$

Calculamos la incógnita c:

$$\left. \begin{aligned} ct_s &= r(t_s) - \frac{et_s^2}{2} \\ e &= \frac{12}{t_s^3} \left(\frac{t_s r(t_s)}{2} - L \right) \end{aligned} \right\} \Rightarrow ct_s = r(t_s) - \frac{t_s^3}{2} \frac{12}{t_s^3} \left(\frac{t_s r(t_s)}{2} - L \right) \Rightarrow \\
 \Rightarrow c = \frac{r(t_s)}{t_s} - \frac{6}{t_s^2} \left(\frac{t_s r(t_s)}{2} - L \right) = \frac{1000 \text{ m/s}}{0.003 \text{ s}} - \\
 - \frac{6}{(0.003 \text{ s})^2} \left(\frac{0.003 \text{ s } 1000 \text{ m/s}}{2} - 6 \text{ m} \right) = \frac{10^6}{3} \frac{\text{m}}{\text{s}^2} - \\
 - \frac{6}{9 \cdot 10^{-6} \text{ s}^2} (1.5 - 6) \text{ m} = \frac{10^6}{3} \frac{\text{m}}{\text{s}^2} + \frac{6 \cdot 4.5}{9} 10^6 \frac{\text{m}}{\text{s}^2} = \\
 = \frac{10^6}{3} \frac{\text{m}}{\text{s}^2} + 3 \cdot 10^6 \frac{\text{m}}{\text{s}^2} = \left(\frac{1}{3} + 3 \right) 10^6 \frac{\text{m}}{\text{s}^2} = \frac{10}{3} 10^6 \frac{\text{m}}{\text{s}^2} = \boxed{0.3 \cdot 10^7 \frac{\text{m}}{\text{s}^2}}$$

Autor: Iván Cabria Álvarez

(3.27)

Comprobación de que $a(t) = 0.3 \cdot 10^7 - 2 \cdot 10^9 t$ en m/s^2 :

$$\left. \begin{aligned} a(t) &= \frac{dv}{dt} \\ a(t) &= 0.3 \cdot 10^7 - 2 \cdot 10^9 t \end{aligned} \right\} \Rightarrow \int_0^t (0.3 \cdot 10^7 - 2 \cdot 10^9 t) dt = \int_{v(0)}^{v(t)} dv \Rightarrow$$

$$\Rightarrow 0.3 \cdot 10^7 t - 10^9 t^2 \Big|_0^t = v(t) - v(0) \Big|_{v(0)=0} \Rightarrow v(t) = 0.3 \cdot 10^7 t - 10^9 t^2 \Rightarrow$$

$$\Rightarrow v(t_s) = (0.3 \cdot 10^7 \cdot 0.003 - 10^9 (0.003)^2) m/s =$$

$$= \left(\frac{1}{3} \cdot 10^7 \cdot 3 \cdot 10^{-3} - 10^9 \cdot 9 \cdot 10^{-6} \right) m/s = (10^4 - 9 \cdot 10^3) m/s = 1000 m/s$$

El valor de $v(t_s)$ que hemos obtenido es el correcto.

$$\left. \begin{aligned} v(t) &= \frac{dx}{dt} \\ v(t) &= 0.3 \cdot 10^7 t - 10^9 t^2 \end{aligned} \right\} \Rightarrow \int_0^t (0.3 \cdot 10^7 t - 10^9 t^2) dt = \int_{x(0)}^{x(t)} dx \Rightarrow$$

$$\Rightarrow x(t) - x(0) = 0.3 \cdot \frac{10^7 t^2}{2} - \frac{10^9 t^3}{3} \Big|_0^t \Rightarrow x(t) - x(0) = \frac{0.3}{2} \cdot 10^7 t^2 - \frac{10^9}{3} t^3 \Rightarrow$$

$$\Rightarrow L = x(t_s) - x(0) = \frac{0.3}{2} \cdot 10^7 t_s^2 - \frac{10^9}{3} t_s^3 =$$

$$= \left(\frac{1}{6} \cdot 10^7 (3 \cdot 10^{-3})^2 - \frac{10^9}{3} (3 \cdot 10^{-3})^3 \right) m = \left(\frac{9}{6} \cdot 10 - 9 \right) m =$$

$$= (15 - 9) m = 6 m$$

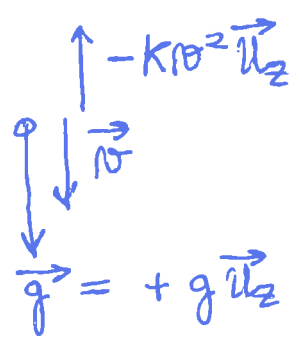
El valor de L que hemos obtenido es el correcto.

Autor: Iván Cabria Álvarez

3.39

3.28

z



$$\left. \begin{aligned} \vec{F} &= +m g \vec{u}_z - K r^2 \vec{u}_z \\ \vec{F} &= m \vec{a} = m \frac{dr}{dt} \vec{u}_z \end{aligned} \right\} \Rightarrow$$

$$m g - K r^2(0) = m g > 0 \quad r(0) = 0$$

$$m g - K^2 r^2 = 0 \quad r(t) < r_2 \Rightarrow m g - K r^2(t) > 0 \quad \forall t > 0$$

$$\Rightarrow m \frac{dr}{dt} = m g - K r^2 \Rightarrow \int_{r(0)}^{r(t)} \frac{dr}{\left(g - \frac{K}{m} r^2\right)} = \int_0^t dt \Rightarrow$$

$$\frac{1}{g - \frac{K}{m} r^2} = \frac{1}{2\sqrt{g}} \left(\frac{1}{\sqrt{g} - \sqrt{\frac{K}{m}} r} + \frac{1}{\sqrt{g} + \sqrt{\frac{K}{m}} r} \right)$$

$$\Rightarrow \frac{1}{2\sqrt{g}} \int_{r(0)}^{r(t)} \frac{dr}{\sqrt{g} - \sqrt{\frac{K}{m}} r} + \frac{1}{2\sqrt{g}} \int_{r(0)}^{r(t)} \frac{dr}{\sqrt{g} + \sqrt{\frac{K}{m}} r} = t$$

$$m g - K r^2(t) > 0 \quad \forall t > 0 \Rightarrow g - \frac{K}{m} r^2(t) > 0 \Rightarrow$$

$$\Rightarrow \left(\sqrt{g} - \sqrt{\frac{K}{m}} r(t)\right) \left(\sqrt{g} + \sqrt{\frac{K}{m}} r(t)\right) > 0 \Rightarrow \sqrt{g} - \sqrt{\frac{K}{m}} r(t) > 0 \quad \forall t > 0$$

$$\Rightarrow \text{Es correcto calcular } \int_{r(0)}^{r(t)} \frac{dr}{\sqrt{g} - \sqrt{\frac{K}{m}} r} \text{ como}$$

$$\ln\left(\sqrt{g} - \sqrt{\frac{K}{m}} r\right) \left(-\sqrt{\frac{m}{K}}\right) \Big|_{r(0)}^{r(t)}$$

$$\frac{1}{2\sqrt{g}} \left(-\sqrt{\frac{m}{K}}\right) \ln\left(\frac{\sqrt{g} - \sqrt{\frac{K}{m}} r(t)}{\sqrt{g} - \sqrt{\frac{K}{m}} r(0)}\right) + \frac{1}{2\sqrt{g}} \left(\sqrt{\frac{m}{K}}\right) \ln\left(\frac{\sqrt{g} + \sqrt{\frac{K}{m}} r(t)}{\sqrt{g} + \sqrt{\frac{K}{m}} r(0)}\right) = t \Rightarrow$$

$$\Rightarrow -\frac{1}{2} \sqrt{\frac{m}{gK}} \ln\left(\frac{\sqrt{g} - \sqrt{\frac{K}{m}} r(t)}{\sqrt{g} - \sqrt{\frac{K}{m}} r(0)}\right) + \frac{1}{2} \sqrt{\frac{m}{gK}} \ln\left(\frac{\sqrt{g} + \sqrt{\frac{K}{m}} r(t)}{\sqrt{g} + \sqrt{\frac{K}{m}} r(0)}\right) = t \Rightarrow$$

$$\Rightarrow \frac{1}{2} \sqrt{\frac{m}{gK}} \ln\left(\frac{\sqrt{g}}{\sqrt{g} - \sqrt{\frac{K}{m}} r(t)} \frac{\sqrt{g} + \sqrt{\frac{K}{m}} r(t)}{\sqrt{g}}\right) = t \Rightarrow$$

tutor: Juan Cabria Alvarado

$$\Rightarrow \left. \begin{aligned} \frac{\sqrt{g} + \sqrt{\frac{K}{m}} v(t)}{\sqrt{g} - \sqrt{\frac{K}{m}} v(t)} &= e^{\sqrt{\frac{gK}{m}} 2t} \\ f(t) &\equiv e^{\sqrt{\frac{gK}{m}} 2t} \end{aligned} \right\} \Rightarrow$$

$$\Rightarrow \sqrt{g} + \sqrt{\frac{K}{m}} v(t) = f(t) \sqrt{g} - f(t) \sqrt{\frac{K}{m}} v(t) \Rightarrow$$

$$\Rightarrow \sqrt{\frac{K}{m}} v(t) + f(t) \sqrt{\frac{K}{m}} v(t) = f(t) \sqrt{g} - \sqrt{g} \Rightarrow$$

$$\Rightarrow \sqrt{\frac{K}{m}} v(t) (1 + f(t)) = \sqrt{g} (f(t) - 1) \Rightarrow$$

$$\Rightarrow \boxed{v(t) = \sqrt{\frac{mg}{K}} \frac{f(t) - 1}{1 + f(t)} \quad f(t) \equiv e^{\sqrt{\frac{gK}{m}} 2t}$$

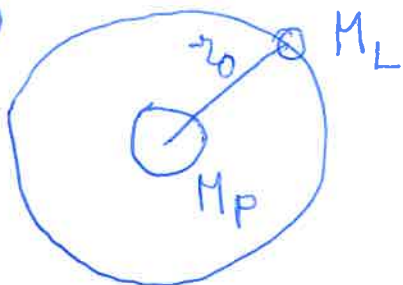
$$\sqrt{\frac{gK}{m}} 2 = \sqrt{\frac{9.8 \text{ m/s}^2 \cdot 100 \text{ kg/m}}{60 \text{ kg}}} 2 = 8.0829 \text{ s}^{-1}$$

$$v_\infty = \lim_{t \rightarrow \infty} v(t) = \sqrt{\frac{mg}{K}} \lim_{t \rightarrow \infty} \frac{f(t) - 1}{1 + f(t)} = \sqrt{\frac{mg}{K}}$$

$$v_\infty = \sqrt{\frac{60 \text{ kg} \cdot 9.8 \text{ m/s}^2}{100 \text{ kg/m}}} = \boxed{2.4249 \text{ m/s}}$$

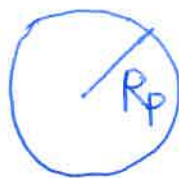
$$b) v_\infty = \frac{S}{\Delta t} \Rightarrow \Delta t = \frac{S}{v_\infty} = \frac{2000 \text{ m}}{2.4248 \text{ m/s}} = \boxed{824.7861 \text{ s}}$$

(3.40)



$$M_L = 7.3 \cdot 10^{22} \text{ Kg}$$

$$L_0 = 2.8 \cdot 10^{34} \text{ Kg m}^2/\text{s}$$



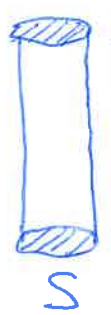
$$\rho = 13.59 \frac{\text{g}}{\text{cm}^3} = 13590 \frac{\text{Kg}}{\text{m}^3}$$

$$h = 800 \text{ mm} = 0.8 \text{ m}$$

$$r_0 = 500000 \text{ Km} = 5 \cdot 10^8 \text{ m}$$

$$R_P = 7000 \text{ Km} = 7 \cdot 10^6 \text{ m}$$

(3.30)



← Columna de mercurio

h V : volumen de la columna de mercurio

S : superficie de la base de la columna de mercurio

$$V = S h$$

$$p = \frac{F_{\text{mer}}}{S} = \frac{m_{\text{mer}} g}{S} = \frac{\rho V g}{S} = \rho h g$$

$$g = \frac{G M_P}{R_P^2}$$

Calculamos $G M_P$ mediante la tercera ley de Kepler:

T_0 : periodo de la órbita de la luna

$$\frac{T_0^2}{r_0^3} = \frac{4\pi^2}{G M_P} \Rightarrow G M_P = \frac{4\pi^2 r_0^3}{T_0^2}$$

Hay varias maneras de continuar la resolución del problema. Vamos a explicar dos maneras.

1ª manera. Calcularemos la expresión de g en función de otras magnitudes físicas y obtendremos sólo el valor de g en m/s^2 y no los valores numéricos de las otras magnitudes.

Calculamos T_0 . Hay dos maneras de calcular T_0 :

a) Mediante la segunda ley de Kepler:

$$\left. \begin{aligned} \sigma &= \frac{L_0}{2M_L} & \sigma &= \text{de.} \\ \sigma &= \frac{A_0}{T_0} = \frac{\pi r_0^2}{T_0} \end{aligned} \right\} \Rightarrow \frac{L_0}{2M_L} = \frac{\pi r_0^2}{T_0} \Rightarrow T_0 = \frac{2\pi r_0^2 M_L}{L_0}$$

A_0 : área de la órbita de la luna

Autor: Juan Cabria Álvarez

b) Mediante las definiciones de momento angular y velocidad angular: (3.31)

$$\left. \begin{aligned} L_0 &= r_0 p_L = r_0 M_L v_L \\ v_L &= \omega_0 r_0 \end{aligned} \right\} \Rightarrow L_0 = M_L \omega_0 r_0^2 \Rightarrow \omega_0 = \frac{2\pi}{T_0}$$

$$\Rightarrow L_0 = \frac{2\pi M_L r_0^2}{T_0} \Rightarrow T_0 = \frac{2\pi M_L r_0^2}{L_0}$$

Calculamos una expresión de g y luego calcularemos su valor numérico en m/s^2 :

$$\left. \begin{aligned} GM_p &= \frac{4\pi^2 r_0^3}{T_0^2} \\ T_0 &= \frac{2\pi M_L r_0^2}{L_0} \\ g &= \frac{GM_p}{R_p^2} \end{aligned} \right\} \Rightarrow GM_p = \frac{4\pi^2 r_0^3 L_0^2}{4\pi^2 M_L^2 r_0^4} = \frac{L_0^2}{r_0 M_L^2}$$

$$\Rightarrow g = \frac{L_0^2}{r_0 M_L^2 R_p^2} = \frac{(2.8 \cdot 10^{34} \text{ kg m}^2/\text{s})^2}{5 \cdot 10^8 \text{ m} (7.3 \cdot 10^{22} \text{ kg})^2 (7 \cdot 10^6 \text{ m})^2} = 6.004879 \cdot 10^{-4} \frac{10^{68}}{10^{8+44+12}} \frac{\text{m}}{\text{s}^2} = 6.004879 \text{ m/s}^2$$

$$P = \rho h g = 13590 \frac{\text{kg}}{\text{m}^3} 0.8 \text{ m} 6.004879 \frac{\text{m}}{\text{s}^2} = \boxed{65285 \text{ Pa}}$$

2ª manera: Calcularemos los valores numéricos de algunas magnitudes físicas en el S.I. y al final calcularemos g en m/s^2 .

Calculamos T_0 de dos maneras:

a) Mediante la segunda ley de Kepler:

$$\sigma = \frac{L_0}{2M_L} = \frac{2.8 \cdot 10^{34} \text{ kg m}^2/\text{s}}{2 \cdot 7.3 \cdot 10^{22} \text{ kg}} = 1.9178 \cdot 10^{11} \frac{\text{m}^2}{\text{s}}$$

$$\sigma = \frac{\pi r_0^2}{T_0} \Rightarrow T_0 = \frac{\pi r_0^2}{\sigma} = \frac{\pi (5 \cdot 10^8 \text{ m})^2}{1.9178 \cdot 10^{11} \text{ m}^2/\text{s}} = 4.0953 \cdot 10^6 \text{ s}$$

Autor: Iván Cabris Alvarro

b) Mediante las definiciones de momento angular y velocidad angular: 3.32

$$L_0 = r_0 p_L = r_0 M_L \omega_L \Rightarrow \omega_L = \frac{L_0}{r_0 M_L} = \frac{2.8 \cdot 10^{34} \text{ kg m}^2/\text{s}}{5 \cdot 10^8 \text{ m} \cdot 7.3 \cdot 10^{22} \text{ kg}} = 767.1233 \text{ m/s}$$

$$\omega_L = \omega_0 r_0 \Rightarrow \omega_0 = \frac{\omega_L}{r_0} = \frac{767.1233}{5 \cdot 10^8} \frac{\text{rad}}{\text{s}} = 1.5342 \cdot 10^{-6} \frac{\text{rad}}{\text{s}}$$

$$T_0 = \frac{2\pi}{\omega_0} = \frac{2\pi}{1.5342 \cdot 10^{-6}} \text{ s} = 4.0953 \cdot 10^6 \text{ s}$$

$$GM_p = \frac{4\pi^2 r_0^3}{T_0^2} = \frac{4\pi^2 (5 \cdot 10^8 \text{ m})^3}{(4.0953 \cdot 10^6 \text{ s})^2} = 2.9424 \cdot 10^{14} \frac{\text{m}^3}{\text{s}^2}$$

$$g = \frac{GM_p}{R_p^2} = \frac{2.9424 \cdot 10^{14} \text{ m}^3/\text{s}^2}{(7 \cdot 10^6 \text{ m})^2} = 6.004898 \text{ m/s}^2$$

$$P = \rho h g = 13590 \frac{\text{kg}}{\text{m}^3} \cdot 0.8 \text{ m} \cdot 6.004898 \frac{\text{m}}{\text{s}^2} = \boxed{65285 \text{ Pa}}$$

3.41

TEMA 4 TRABAJO Y ENERGÍA

$$(4.1) W = -F \Delta z \cos \theta = -60 \text{ N } 10 \text{ m } \cos 60^\circ = \boxed{-300 \text{ J}}$$

(4.2) Este problema se puede resolver de tres maneras.

Calculamos la aceleración: $a = \frac{F}{m} = \frac{600 \text{ N}}{1000 \text{ kg}} = 0.6 \text{ m/s}^2$

$$S(0) = 0 \quad v(0) = 0$$

1ª manera:

$$S(t) = S(0) + v(0)t + \frac{at^2}{2} \quad \left. \begin{array}{l} S(0) = 0 \\ v(0) = 0 \end{array} \right\} \Rightarrow S(t) = \frac{at^2}{2} \Rightarrow t = \sqrt{\frac{2S(t)}{a}}$$

$$\left. \begin{array}{l} S(t_1) = 100 \text{ m} \\ t = \sqrt{\frac{2S(t)}{a}} \end{array} \right\} \Rightarrow t_1 = \sqrt{\frac{2 \cdot 100 \text{ m}}{0.6 \text{ m/s}^2}} = 18.257 \text{ s}$$

$$\left. \begin{array}{l} v(t) = v(0) + at \\ v(0) = 0 \end{array} \right\} \Rightarrow v(t) = at \Rightarrow v(t_1) = at_1$$

$$\Rightarrow v(t_1) = 0.6 \text{ m/s}^2 \cdot 18.257 \text{ s} = \boxed{10.95 \text{ m/s}}$$

2ª manera:

$$F \Delta S = T_f - T_i$$

$$\Delta S = S(t_1) - S(0) = 100 \text{ metros}$$

$$T_f = \frac{m}{2} v_f^2 = \frac{m}{2} v^2(t_1)$$

$$T_i = \frac{m}{2} v_i^2 = \frac{m}{2} v^2(0) = 0$$

$$m = 1000 \text{ kg} \quad F = 600 \text{ N}$$

$$\Rightarrow \boxed{v(t_1) = 10.95 \text{ m/s}}$$

3ª manera:

$$\left. \begin{array}{l} v^2(t) = v^2(0) + 2a(S(t) - S(0)) \\ v(0) = 0 \quad S(0) = 0 \end{array} \right\} \Rightarrow v(t) = \sqrt{2aS(t)} \quad \left. \begin{array}{l} S(t_1) = 100 \text{ m} \end{array} \right\} \Rightarrow$$

$$\Rightarrow v(t_1) = \sqrt{2 \cdot 0.6 \text{ m/s}^2 \cdot 100 \text{ m}} = \boxed{10.95 \text{ m/s}}$$

Autor: Iván Cobria Álvarez

(4.2)

$$(4.3) \quad T = \frac{m}{2} v^2 = \frac{65 \text{ kg}}{2} (30 \text{ m/s})^2 = \boxed{29250 \text{ J}}$$

(4.4) Este problema se puede resolver de tres maneras.

Calculamos la aceleración: $a = \frac{F}{m} = \frac{6000 \text{ N}}{0.1 \text{ kg}} = 60\,000 \text{ m/s}^2$

$S(0) = 0 \quad v(0) = 0$

1ª manera:

$$\left. \begin{aligned} S(t) &= S(0) + v(0)t + \frac{at^2}{2} \\ S(0) &= 0 \quad v(0) = 0 \end{aligned} \right\} \Rightarrow S(t) = \frac{at^2}{2} \Rightarrow t = \sqrt{\frac{2S(t)}{a}}$$

$$\left. \begin{aligned} v(t) &= v(0) + at \\ v(0) &= 0 \end{aligned} \right\} \Rightarrow v(t) = at$$

$$\Rightarrow v(t_1) = a \sqrt{\frac{2S(t_1)}{a}} = \sqrt{2aS(t_1)} \Rightarrow v(t_1) = \sqrt{2 \cdot 60\,000 \text{ m/s}^2 \cdot 0.1 \text{ m}} = \boxed{109.54 \text{ m/s}}$$

$S(t_1) = 0.1 \text{ m}$

2ª manera:

$$\left. \begin{aligned} F \Delta S &= T_f - T_i \\ \Delta S &= S(t) - S(0) \\ T_f &= \frac{m}{2} v_f^2 = \frac{m}{2} v^2(t) \\ T_i &= \frac{m}{2} v_i^2 = \frac{m}{2} v^2(0) \\ v(0) &= 0 \end{aligned} \right\} \Rightarrow F(S(t) - S(0)) = \frac{m}{2} v^2(t) \Rightarrow$$

$$\Rightarrow v(t) = \sqrt{\frac{2FS(t)}{m}} \Rightarrow v(t_1) = \sqrt{\frac{2 \cdot 6000 \text{ N} \cdot 0.1 \text{ m}}{0.1 \text{ kg}}} = \boxed{109.54 \text{ m/s}}$$

$S(t_1) = 0.1 \text{ m}$

3ª manera:

$$\left. \begin{aligned} v^2(t) &= v^2(0) + 2a(S(t) - S(0)) \\ v(0) &= 0 \quad S(0) = 0 \end{aligned} \right\} \Rightarrow v(t) = \sqrt{2aS(t)} \Rightarrow$$

$S(t_1) = 0.1 \text{ m}$

$$\Rightarrow v(t_1) = \sqrt{2 \cdot 60\,000 \text{ m/s}^2 \cdot 0.1 \text{ m}} = \boxed{109.54 \text{ m/s}}$$

Autor: Yoán Cabria Álvarez

(4.3)

(4.5) Este problema se puede resolver de tres maneras.

1ª manera:

$$\left. \begin{aligned} S(t) &= S(0) + v(0)t + \frac{gt^2}{2} \\ S(0) &= 0 \quad v(0) = 0 \end{aligned} \right\} \Rightarrow S(t) = \frac{gt^2}{2} \Rightarrow t = \sqrt{\frac{2S(t)}{g}}$$

$$\left. \begin{aligned} S(t_1) &= 1000 \text{ m} \\ t &= \sqrt{\frac{2S(t)}{g}} \end{aligned} \right\} \Rightarrow t_1 = \sqrt{\frac{2 \cdot 1000 \text{ m}}{9.8 \text{ m/s}^2}} = 14.2857 \text{ s}$$

$$\left. \begin{aligned} v(t) &= v(0) + gt \\ v(0) &= 0 \end{aligned} \right\} \Rightarrow v(t) = gt \Rightarrow v(t_1) = 9.8 \text{ m/s}^2 \cdot 14.2857 \text{ s} = \boxed{140 \text{ m/s}}$$

2ª manera:

$$\begin{aligned} E &= T + V \\ E_i &= E_f \quad T_i = 0 \quad V_i = mgh \quad T_f = \frac{m}{2}v_f^2 \quad V_f = 0 \end{aligned} \Rightarrow mgh = \frac{m}{2}v_f^2 \Rightarrow$$

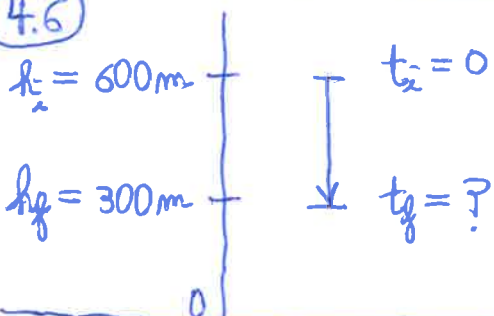
$$\Rightarrow 2gh = v_f^2 \Rightarrow v_f = \sqrt{2gh} = \sqrt{2 \cdot 9.8 \frac{\text{m}}{\text{s}^2} \cdot 1000 \text{ m}} = \boxed{140 \text{ m/s}}$$

3ª manera:

$$\left. \begin{aligned} v^2(t) &= v^2(0) + 2a(S(t) - S(0)) \\ v(0) &= 0 \quad a = g \quad S(0) = 0 \quad v_f = v(t_1) \quad S(t_1) = h \end{aligned} \right\} \Rightarrow v_f^2 = 2gh \Rightarrow$$

$$\Rightarrow v_f = \sqrt{2gh} = \sqrt{2 \cdot 9.8 \frac{\text{m}}{\text{s}^2} \cdot 1000 \text{ m}} = \boxed{140 \text{ m/s}}$$

(4.6)



$$\left. \begin{aligned} E_i &= E_f \\ E_i &= T_i + V_i = 0 + mgh_i \\ E_f &= T_f + V_f = \frac{m}{2}v_f^2 + mgh_f \end{aligned} \right\} \Rightarrow$$

$$\Rightarrow gh_i = \frac{v_f^2}{2} + gh_f \Rightarrow v_f = \sqrt{2g(h_i - h_f)} = \sqrt{2 \cdot 9.8 \frac{\text{m}}{\text{s}^2} (600 - 300) \text{ m}} \Rightarrow$$

$$\Rightarrow \boxed{v_f = 76.68 \text{ m/s}}$$

$$(4.7) \quad v_i = 36 \text{ km/h} = 36000 \text{ m} / 3600 \text{ s} = 10 \text{ m/s} \quad h_i = 10 \text{ m} \quad h_f = 0 \text{ m}$$

$$\left. \begin{aligned} E_i &= E_f \\ E_i &= T_i + V_i = \frac{m}{2}v_i^2 + mgh_i \\ E_f &= T_f + V_f = \frac{m}{2}v_f^2 + mgh_f \end{aligned} \right\} \Rightarrow \frac{m}{2}v_i^2 + mgh_i = \frac{m}{2}v_f^2 \Rightarrow$$

Autor: Yván Cabrera Alvarado

(4.4)

$$\Rightarrow v_f^2 = v_i^2 + 2gh_i \Rightarrow v_f = \sqrt{v_i^2 + 2gh_i} = \sqrt{(10 \text{ m/s})^2 + 2 \cdot 9.8 \frac{\text{m}}{\text{s}^2} \cdot 10 \text{ m}} \Rightarrow$$

$$\Rightarrow \boxed{v_f = 17.20 \text{ m/s}}$$

$$(4.8) \quad h_i = 100 \text{ m} \quad h_f = 0 \text{ m} \quad v_i = 0$$

$$\left. \begin{aligned} E_i &= E_f \\ E_i &= T_i + V_i = \frac{m}{2} v_i^2 + mgh_i = mgh_i \\ E_f &= T_f + V_f = \frac{m}{2} v_f^2 + mgh_f = \frac{m}{2} v_f^2 \end{aligned} \right\} \Rightarrow T_f = \frac{m}{2} v_i^2 + mgh_i \left. \vphantom{\begin{aligned} E_i &= E_f \\ E_i &= T_i + V_i = \frac{m}{2} v_i^2 + mgh_i = mgh_i \\ E_f &= T_f + V_f = \frac{m}{2} v_f^2 + mgh_f = \frac{m}{2} v_f^2 \end{aligned}} \right\} \Rightarrow$$

$$v_i = 0$$

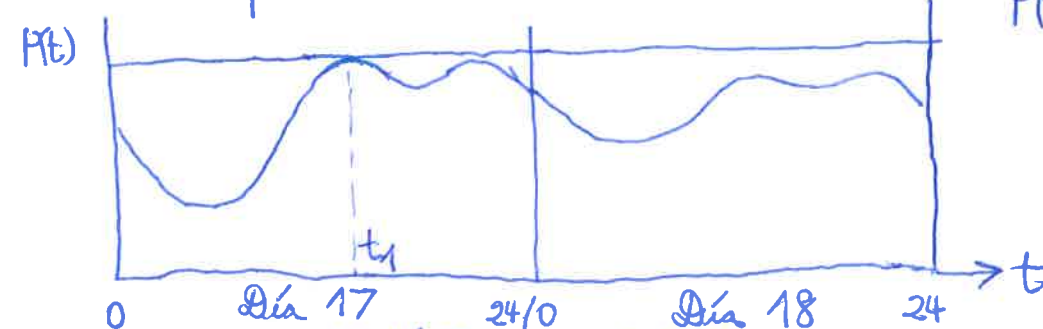
$$\Rightarrow T_f = mgh_i \Rightarrow T_f = 50 \text{ kg} \cdot 9.8 \frac{\text{m}}{\text{s}^2} \cdot 100 \text{ m} = \boxed{49000 \text{ J}}$$

$$(4.9) \quad V = mgh = 70 \text{ kg} \cdot 9.8 \frac{\text{m}}{\text{s}^2} \cdot 9000 \text{ m} = \boxed{6174000 \text{ J}}$$

$$(4.10) \quad T = \frac{m}{2} v^2 = \frac{1.3 \cdot 10^8 \text{ kg}}{2} \left(7800 \frac{\text{m}}{\text{s}} \right)^2 = 3.9546 \cdot 10^{15} \text{ J} = 3954.6 \text{ TJ} =$$

$$= \frac{3954.6 \text{ TJ}}{4.18 \text{ TJ/KTNT}} = 946.0766 \text{ KTNT} = \boxed{59.13 \text{ bombas de Hiroshima}}$$

(4.11) No es una contradicción. La demanda instantánea de potencia tuvo su máximo el día 17, pero la demanda diaria de energía fue menor el día 17 que el día 18.



$$P(t_1) = 44.9 \text{ GW}$$

$$W(\text{día 17}) = \int_{0 \text{ día 17}}^{24 \text{ día 17}} P(t) dt < \int_{0 \text{ día 18}}^{24 \text{ día 18}} P(t) dt = W(\text{día 18})$$

$$(4.12) \quad W = P \Delta t = 5000 \text{ W} \cdot 3600 \text{ s} = \boxed{1.8 \cdot 10^7 \text{ J}}$$

$$(4.13) \quad v(t) = v(0) + at \Rightarrow a = \frac{v(t) - v(0)}{t} = \frac{(100 - 88) \text{ m/s}}{30 \text{ s}} = 0.4 \text{ m/s}^2$$

$$F = ma = 1000 \text{ kg} \cdot 0.4 \text{ m/s}^2 = 400 \text{ N}$$

$$S(t) = S(0) + v(0)t + \frac{at^2}{2} \Rightarrow \Delta z = 88 \text{ m/s} \cdot 30 \text{ s} + \frac{0.4 \text{ m/s}^2 \cdot (30 \text{ s})^2}{2} = 2820 \text{ m}$$

$$\Delta z = S(30) - S(0)$$

$$|W| = F \Delta z = 400 \text{ N} \cdot 2820 \text{ m} = 1.128 \cdot 10^6 \text{ J} \quad P = \frac{|W|}{\Delta t} = \frac{1.128 \cdot 10^6 \text{ J}}{30 \text{ s}} = \boxed{37600 \text{ W}}$$

Autor: Iván Cabris Alvarado

(4.14) $a = \frac{v(t) - v(0)}{t} = \frac{(7-5) \text{ m/s}}{2 \text{ s}} = 1 \text{ m/s}^2$

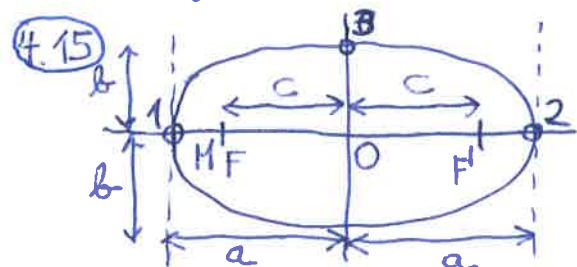
(4.5)

$F = m \cdot a = 60 \text{ kg} \cdot 1 \text{ m/s}^2 = 60 \text{ N}$

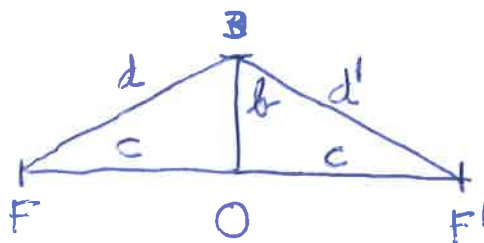
$\Delta z = S(2) - S(0) = v(0) 2 \text{ s} + \frac{a}{2} 2^2 \text{ s}^2 = 5 \frac{\text{m}}{\text{s}} 2 \text{ s} + \frac{1}{2} \text{ m/s}^2 2^2 \text{ s}^2 = 12 \text{ m}$

$|W| = F \Delta z = 60 \text{ N} \cdot 12 \text{ m} = 720 \text{ J}$

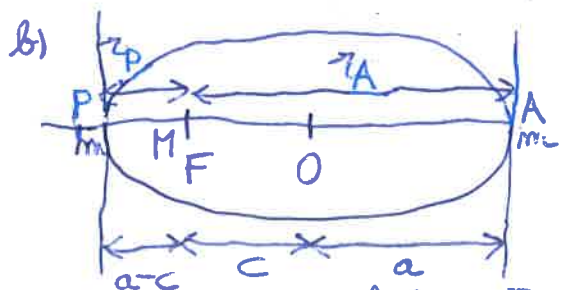
$P = \frac{|W|}{\Delta t} = \frac{720 \text{ J}}{2 \text{ s}} = \boxed{360 \text{ W}}$



a)



$d^2 = b^2 + c^2$ en el punto B
 $d = d'$ en el punto B
 $d + d' = 2a$ } $\Rightarrow \boxed{a^2 = b^2 + c^2}$
 $\Rightarrow d = a$



$r_p = a - c$
 $r_A = a + c$ } $\Rightarrow \frac{1}{r_p} - \frac{1}{r_A} = \frac{1}{a-c} - \frac{1}{a+c} =$

$= \frac{a+c - (a-c)}{(a-c)(a+c)} = \boxed{\frac{2c}{(a-c)(a+c)}}$

El Sol está en el foco F y tiene masa M

c) $\frac{1}{r_p^2} - \frac{1}{r_A^2} = \frac{1}{(a-c)^2} - \frac{1}{(a+c)^2} = \frac{(a+c)^2 - (a-c)^2}{(a-c)^2 (a+c)^2} = \frac{a^2 + c^2 + 2ac - a^2 - c^2 + 2ac}{(a-c)^2 (a+c)^2}$

$= \boxed{\frac{4ac}{(a-c)^2 (a+c)^2}}$

(4.16) $\vec{L}_P = \vec{L}_A$
 $\vec{L}_i = \vec{r}_i \times \vec{p}_i = r_i m_i v_i \vec{u}_z \quad i=A, P$ } $\Rightarrow r_P v_P = r_A v_A \Rightarrow v_P^2 = \frac{r_A^2 v_A^2}{r_P^2}$

$T_P + V_P = T_A + V_A \Rightarrow \frac{m}{2} v_P^2 - \frac{GMm}{r_P} = \frac{m}{2} v_A^2 - \frac{GMm}{r_A} \Rightarrow \frac{v_P^2}{2} - \frac{v_A^2}{2} = GM \left(\frac{1}{r_P} - \frac{1}{r_A} \right)$

$\Rightarrow \frac{r_A^2 v_A^2}{2 r_P^2} - \frac{v_A^2}{2} = GM \left(\frac{1}{r_P} - \frac{1}{r_A} \right) \Rightarrow \frac{r_A^2 v_A^2}{2 r_P^2} - \frac{r_A^2 v_A^2}{2 r_A^2} = GM \left(\frac{1}{r_P} - \frac{1}{r_A} \right) \Rightarrow$

$\Rightarrow r_A^2 v_A^2 \left(\frac{1}{r_P^2} - \frac{1}{r_A^2} \right) = 2GM \left(\frac{1}{r_P} - \frac{1}{r_A} \right)$

Autor: Iván Gabrila Filiberto

$$\frac{z_A^2 v_A^2}{A} \frac{4ac}{(a-c)^2 (a+c)^2} = 2GM \frac{2c}{(a-c)(a+c)} \Rightarrow \frac{z_A^2 v_A^2}{A} \frac{a}{(a-c)^2 (a+c)^2} = \frac{GM}{(a-c)(a+c)} \quad (4.6)$$

$$\Rightarrow \frac{z_A^2 v_A^2}{A} = \frac{GM (a-c)^2 (a+c)^2}{a (a-c)(a+c)} = \boxed{\frac{GM (a-c)(a+c)}{a}}$$

$$b) \quad \left. \begin{aligned} \sigma &= \frac{\text{área elipse}}{T} = \frac{\pi ab}{T} \\ \sigma &= \frac{|\vec{T}_A|}{2m} = \frac{m \frac{z_A v_A}{A}}{2m} = \frac{z_A v_A}{2A} \\ \sigma &= \text{cte.} \end{aligned} \right\} \Rightarrow$$

$$\Rightarrow \frac{\pi ab}{T} = \frac{z_A v_A}{2A} \Rightarrow T = \frac{2\pi ab}{z_A v_A} \Rightarrow \boxed{T^2 = \frac{4\pi^2 a^2 b^2}{z_A^2 v_A^2}}$$

$$c) \quad \left. \begin{aligned} \frac{z_A^2 v_A^2}{A} &= \frac{GM (a-c)(a+c)}{a} \\ T^2 &= \frac{4\pi^2 a^2 b^2}{z_A^2 v_A^2} \end{aligned} \right\} \Rightarrow T^2 = \frac{4\pi^2 a^2 b^2 a}{GM (a-c)(a+c)} \Rightarrow$$

$$a^2 = b^2 + c^2 \Rightarrow a^2 - c^2 = b^2 \Rightarrow (a-c)(a+c) = b^2$$

$$\Rightarrow T^2 = \frac{4\pi^2 a^3 b^2}{GM b^2} = \boxed{\frac{4\pi^2 a^3}{GM}}$$

$$(4.17) \quad \left. \begin{aligned} \vec{F} &= -\vec{\nabla} V \\ \vec{\nabla} V &= \frac{\partial V}{\partial x} \vec{u}_x + \frac{\partial V}{\partial y} \vec{u}_y + \frac{\partial V}{\partial z} \vec{u}_z \\ \vec{r} &= x\vec{u}_x + y\vec{u}_y + z\vec{u}_z \\ \vec{F} &= -\frac{GMm}{r^2} \frac{\vec{r}}{r} \end{aligned} \right\} \Rightarrow \begin{aligned} \frac{GMm}{r^3} x &= \frac{\partial V}{\partial x} \\ \frac{GMm}{r^3} y &= \frac{\partial V}{\partial y} \\ \frac{GMm}{r^3} z &= \frac{\partial V}{\partial z} \end{aligned} \quad r = \sqrt{x^2 + y^2 + z^2}$$

$$f(x, y, z) = \frac{1}{\sqrt{x^2 + y^2 + z^2}} \Rightarrow \frac{\partial f}{\partial x} = -\frac{1}{(x^2 + y^2 + z^2)^{3/2}} x = -\frac{x}{r^3}$$

Comparando $\frac{\partial f}{\partial x}$ con $\frac{\partial V}{\partial x}$ deducimos, integrando con respecto a x:

$$V(x, y, z) = -\frac{GMm}{r} + g(y, z)$$

Integramos también $\frac{\partial V}{\partial y}$ y $\frac{\partial V}{\partial z}$, y obtenemos, respectivamente:

$$V(x, y, z) = -\frac{GMm}{r} + h(x, z)$$

$$V(x, y, z) = -\frac{GMm}{r} + i(x, y)$$

Autor: Iván Cabrera Álvarez

Las tres ecuaciones deben ser iguales. Por tanto:

(4.7)

$$g(r,z) = h(x,z) = i(x,y) \Rightarrow g = h = i = \text{cte.} = C$$

$$V(x,y,z) = -\frac{GMm}{r} + C$$

$$V_{\infty} = \lim_{r \rightarrow \infty} V = C \quad \text{Se suele hacer } C = V_{\infty} = 0$$

$$V(x,y,z) = -\frac{GMm}{r}$$

(4.18) $V = -\frac{GM_T m}{r}$ procede de la ley de gravitación universal y es la correcta.

$V = mgh$ es la energía potencial de una masa m situada a una altura h sobre la superficie de la Tierra. Es una aproximación válida para alturas h pequeñas.

$$\left. \begin{array}{l} r = R_T + h \\ V = -\frac{GM_T m}{r} \end{array} \right\} \Rightarrow V = -\frac{GM_T m}{r} = -\frac{GM_T m}{R_T + h} = -\frac{GM_T m}{R_T} \frac{1}{\left(1 + \frac{h}{R_T}\right)}$$

$$\frac{1}{1+x} \approx 1-x \quad \text{si } x \ll 1 \quad x = \frac{h}{R_T}$$

$$\text{Si } h \ll R_T, \text{ entonces } \frac{1}{1 + \frac{h}{R_T}} \approx 1 - \frac{h}{R_T}$$

$$\left. \begin{array}{l} V = -\frac{GM_T m}{R_T} \frac{1}{\left(1 + \frac{h}{R_T}\right)} \approx -\frac{GM_T m}{R_T} + \frac{GM_T m h}{R_T^2} \\ F = \frac{GM_T m}{R_T^2} \\ F = mg \end{array} \right\} \begin{array}{l} \text{en la superficie} \\ \text{de la Tierra} \end{array} \Rightarrow g = \frac{GM_T}{R_T^2}$$

$$\Rightarrow V \approx -\frac{GM_T m}{R_T} + mgh$$

La diferencia de energía potencial entre dos alturas h_1 y h_2 es

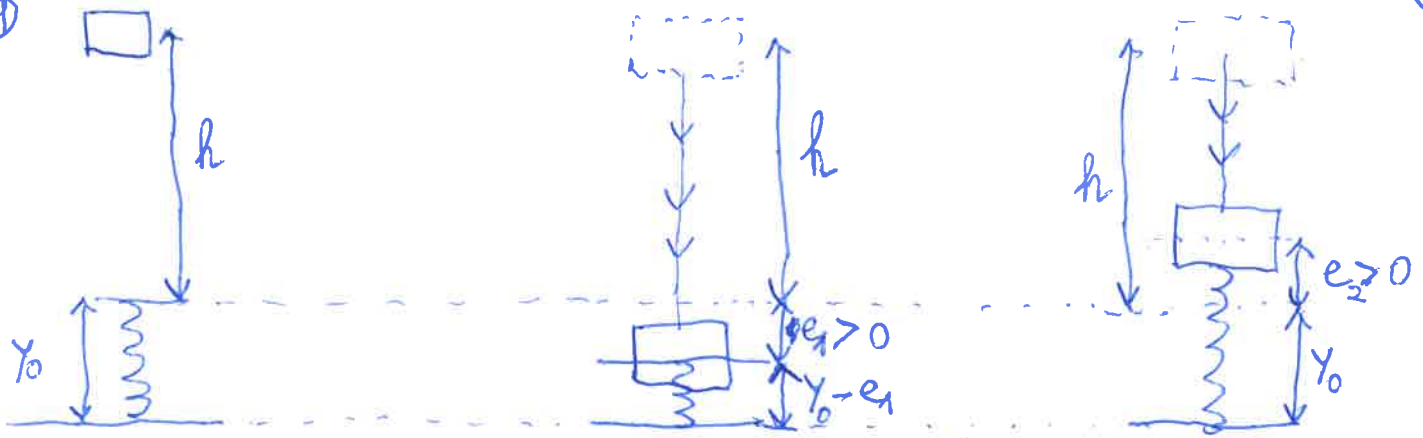
$$\Delta V = V(h_2) - V(h_1) \approx -\frac{GM_T m}{R_T} + mgh_2 + \frac{GM_T m}{R_T} - mgh_1 = mg(h_2 - h_1)$$

Si usamos $V(h) = mgh$, obtenemos la misma diferencia de energía potencial ΔV .

Autor: Iván Cabria Álvarez

4.19

4.8



$$E_i = mg(h + Y_0)$$

$$E_f = mg(Y_0 - e_1) + \frac{K}{2} e_1^2$$

$$E_f = mg(Y_0 - e) + \frac{K}{2} e^2$$

$$E_f = mg(Y_0 + e_2) + \frac{K}{2} e_2^2$$

$$E_i = E_f \Rightarrow mg(h + Y_0) = mg(Y_0 - e) + \frac{K}{2} e^2 \Rightarrow$$

con e mayor o menor que cero

$$e = \begin{cases} e_1 > 0 \\ -e_2 < 0 \end{cases}$$

$$\Rightarrow mgh = -mge + \frac{K}{2} e^2 \Rightarrow e^2 - \frac{mgh}{K} e - \frac{mgh^2}{K} = 0$$

$$\Rightarrow e = \frac{\frac{2mg}{K} \pm \sqrt{\frac{4m^2g^2}{K^2} + \frac{8mgh}{K}}}{2} = \frac{mg}{K} \pm \sqrt{\frac{m^2g^2}{K^2} + \frac{2mgh}{K}}$$

$$= \frac{mg}{K} \pm \sqrt{\frac{m^2g^2}{K^2} + \frac{m^2g^2}{K^2} \frac{2hK}{mg}} = \frac{mg}{K} \pm \frac{mg}{K} \sqrt{1 + \frac{2hK}{mg}}$$

$$\Rightarrow \boxed{e_1 = \frac{mg}{K} + \frac{mg}{K} \sqrt{1 + \frac{2hK}{mg}}} \quad \text{y} \quad e = \begin{cases} e_a = \frac{mg}{K} + \frac{mg}{K} \sqrt{1 + \frac{2hK}{mg}} > 0 \\ e_b = \frac{mg}{K} - \frac{mg}{K} \sqrt{1 + \frac{2hK}{mg}} < 0 \end{cases}$$

$$e_2 = \left| \frac{mg}{K} - \frac{mg}{K} \sqrt{1 + \frac{2hK}{mg}} \right|$$

$$e_1 = e_a \quad e_2 = -e_b$$

$$\sqrt{1 + \frac{2hK}{mg}} > \sqrt{1} = 1 \Rightarrow \frac{mg}{K} < \frac{mg}{K} \sqrt{1 + \frac{2hK}{mg}} \Rightarrow$$

$$\Rightarrow e_b < 0 \Rightarrow e_2 = -e_b$$

$$\frac{mg}{K} < \frac{mg}{K} \sqrt{1 + \frac{2hK}{mg}} \Rightarrow e_2 < e_1 \Rightarrow$$

e_1 es la máxima elongación posible

(4.20) $\vec{F} = -K(x\vec{u}_x + y\vec{u}_y + z\vec{u}_z)$ (4.9)

$$\vec{\nabla}V = \frac{\partial V}{\partial x}\vec{u}_x + \frac{\partial V}{\partial y}\vec{u}_y + \frac{\partial V}{\partial z}\vec{u}_z$$

$$\vec{F} = -\vec{\nabla}V$$

$$\left. \begin{aligned} -KX &= -\frac{\partial V}{\partial x} \\ -KY &= -\frac{\partial V}{\partial y} \\ -KZ &= -\frac{\partial V}{\partial z} \end{aligned} \right\} \Rightarrow$$

$$\left. \begin{aligned} V(x,y,z) &= \frac{KX^2}{2} + f(y,z) \\ V(x,y,z) &= \frac{KY^2}{2} + g(x,z) \\ V(x,y,z) &= \frac{KZ^2}{2} + h(x,y) \end{aligned} \right\} \Rightarrow \boxed{V(x,y,z) = \frac{K}{2}(X^2 + Y^2 + Z^2) + D}$$

$D = \text{constante}$

$$f(y,z) = \frac{K}{2}Y^2 + \frac{K}{2}Z^2 + D \quad h(x,y) = \frac{K}{2}X^2 + \frac{K}{2}Y^2 + D$$

$$g(x,z) = \frac{K}{2}X^2 + \frac{K}{2}Z^2 + D$$

(4.21) 1ª manera: Suponemos que es conservativo y que, por tanto, se cumple $\vec{F} = -\vec{\nabla}V$.

$$\vec{F} = -\frac{3zX^2}{2}\vec{u}_x - XY\vec{u}_y - \frac{X^3}{2}\vec{u}_z$$

$$\vec{\nabla}V = -\frac{\partial V}{\partial x}\vec{u}_x - \frac{\partial V}{\partial y}\vec{u}_y - \frac{\partial V}{\partial z}\vec{u}_z$$

$$\left. \begin{aligned} -\frac{3zX^2}{2} &= -\frac{\partial V}{\partial x} \\ -XY &= -\frac{\partial V}{\partial y} \\ -\frac{X^3}{2} &= -\frac{\partial V}{\partial z} \end{aligned} \right\} \Rightarrow$$

$$V(x,y,z) = +\frac{z}{2}X^3 + f(y,z) \quad (1)$$

$$\Rightarrow V(x,y,z) = +\frac{XY^2}{2} + g(x,z) \quad (2)$$

$$V(x,y,z) = +\frac{X^3z}{2} + h(x,y) \quad (3)$$

Para que se cumpla $\vec{F} = -\vec{\nabla}V$, las tres ecuaciones deben ser iguales.

Hacemos tres comparaciones, para obtener $f(y,z)$, $g(x,z)$ y $h(x,y)$:

Comparamos (1) con (2) $\Rightarrow g(x,z) = \frac{zX^3}{2}$ y $f(y,z)$ tendría que ser igual a $\frac{XY^2}{2}$, pero no es posible $\Rightarrow$ es inconsistente.

Comparamos (1) con (3) $\Rightarrow f(y,z) = h(x,y)$, pero esto es inconsistente.

Comparamos (2) con (3) $\Rightarrow g(x,z) = \frac{zX^3}{2}$ y $h(x,y) = \frac{XY^2}{2}$.

Las inconsistencias que hemos encontrada significan que $\vec{F} \neq -\vec{\nabla}V$ y, por tanto, $\vec{F}$ no es una fuerza conservativa.

Autor: Iván Cabria Álvarez

2ª manera: Calculamos $\vec{\nabla} \times \vec{F}$.

$$\vec{\nabla} \times \vec{F} = \begin{vmatrix} \vec{u}_x & \vec{u}_y & \vec{u}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix} = \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z} \right) \vec{u}_x - \left(\frac{\partial F_z}{\partial x} - \frac{\partial F_x}{\partial z} \right) \vec{u}_y + \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} \right) \vec{u}_z$$

Si la fuerza es conservativa, entonces $\vec{\nabla} \times \vec{F} = 0$.

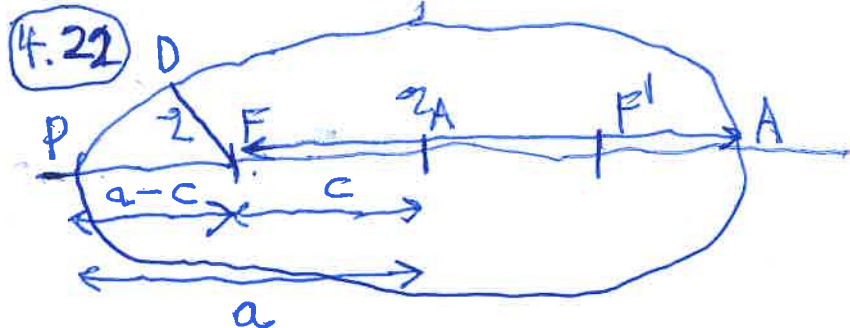
Si $\vec{\nabla} \times \vec{F} \neq 0$, entonces la fuerza no es conservativa.

$$\vec{F} = -\frac{3zx^3}{2} \vec{u}_x - xy \vec{u}_y - \frac{x^3}{2} \vec{u}_z \Rightarrow \left. \begin{aligned} F_x &= -\frac{3zx^3}{2} \\ F_y &= -xy \\ F_z &= -\frac{x^3}{2} \end{aligned} \right\} \Rightarrow$$

$$\Rightarrow \left. \begin{aligned} \frac{\partial F_x}{\partial y} &= 0 & \frac{\partial F_x}{\partial z} &= -\frac{3x^3}{2} \\ \frac{\partial F_y}{\partial x} &= -y & \frac{\partial F_y}{\partial z} &= 0 \\ \frac{\partial F_z}{\partial x} &= -\frac{3x^2}{2} & \frac{\partial F_z}{\partial y} &= 0 \end{aligned} \right\} \Rightarrow \left. \begin{aligned} \frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z} &= 0 - 0 = 0 \\ \frac{\partial F_z}{\partial x} - \frac{\partial F_x}{\partial z} &= -\frac{3x^2}{2} + \frac{3x^2}{2} = 0 \\ \frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} &= -y - 0 = -y \end{aligned} \right\}$$

$$\Rightarrow \vec{\nabla} \times \vec{F} = 0\vec{u}_x - 0\vec{u}_y - y\vec{u}_z = -y\vec{u}_z \Rightarrow \vec{\nabla} \times \vec{F} \neq 0 \Rightarrow$$

$\Rightarrow \vec{F}$ no es conservativa



4.11

Calculamos la velocidad en el punto D de la elipse. La distancia entre D y el foco es r . v es la velocidad en el punto D. La distancia entre A y el foco es r_A .

La energía es constante $\Rightarrow E_D = E_A$

E_D : energía en el punto D. $E_D = E(r)$

E_A : energía en el punto A, el apelio. $E_A = E(r_A)$

$$E(r) = \frac{1}{2} m v^2 - \frac{GMm}{r}$$

$$\Rightarrow \frac{1}{2} m v^2 - \frac{GMm}{r} = E(r_A)$$

Calculamos $E(r_A)$:

$$r_A^2 v_A^2 = \frac{GM(a-c)(a+c)}{a} \Rightarrow v_A^2 = \frac{GM(a-c)(a+c)}{a r_A^2}$$

$$E(r_A) = \frac{m}{2} v_A^2 - \frac{GMm}{r_A}$$

$$\Rightarrow E(r_A) = \frac{1}{2} m \frac{GM(a-c)(a+c)}{a r_A^2} - \frac{GMm}{r_A}$$

$$r_A = a + c$$

$$\Rightarrow E(r_A) = \frac{GMm(a-c)(a+c)}{2a(a+c)(a+c)} - \frac{GMm}{(a+c)} = \frac{GMm}{(a+c)} \left(\frac{(a-c)}{2a} - 1 \right) \Rightarrow$$

$$\Rightarrow E(r_A) = \frac{GMm}{(a+c)2a} (a-c-2a) = \frac{GMm}{(a+c)2a} (-c-a) \Rightarrow$$

$$\Rightarrow E(r_A) = - \frac{GMm}{2a}$$

$$\frac{1}{2} m v^2 - \frac{GMm}{r} = E(r_A)$$

$$\Rightarrow \frac{m}{2} v^2 = GMm \left(\frac{1}{r} - \frac{1}{2a} \right) \Rightarrow$$

$$v = \sqrt{2GM \left(\frac{1}{r} - \frac{1}{2a} \right)}$$

Autor: Yocán Cabria Álvarez

(4.23)

(4.12)

$$\left. \begin{aligned} a) \vec{F} &= -a(x-y)\vec{u}_x - a(y-x)\vec{u}_y - bz\vec{u}_z \\ \vec{F} &= -\vec{\nabla}V = -\frac{\partial V}{\partial x}\vec{u}_x - \frac{\partial V}{\partial y}\vec{u}_y - \frac{\partial V}{\partial z}\vec{u}_z \end{aligned} \right\} \Rightarrow$$

$$\Rightarrow \left. \begin{aligned} -a(x-y) &= -\frac{\partial V}{\partial x} \\ -a(y-x) &= -\frac{\partial V}{\partial y} \\ -bz &= -\frac{\partial V}{\partial z} \end{aligned} \right\} \Rightarrow \begin{aligned} V &= \frac{a(x-y)^2}{2} + f(y,z) & (1) \\ V &= \frac{a(y-x)^2}{2} + g(x,z) & (2) \end{aligned}$$

$$V = \frac{bz^2}{2} + h(x,y) \quad (3)$$

De las ecuaciones (1) y (2), deducimos:

$$V = \frac{a(x-y)^2}{2} + i(z) \quad i(z) = f(y,z) = g(x,z)$$

De las ecuaciones (3) y (4), deducimos:

$$V = \frac{a(x-y)^2}{2} + \frac{bz^2}{2} + C$$

$$\begin{aligned} h(x,y) &= \frac{a(x-y)^2}{2} + C \\ i(z) &= \frac{bz^2}{2} \end{aligned}$$

C no depende de x, y, z

Se puede escribir $\vec{F} = -\vec{\nabla}V \Rightarrow \boxed{\vec{F} \text{ es conservativo}}$

Comprobamos que la energía potencial $V(x,y,z)$ obtenida, da el vector $\vec{F}$ del enunciado:

$$F_x = -\frac{\partial V}{\partial x} = -\frac{2a(x-y)}{2} = -a(x-y)$$

$$F_y = -\frac{\partial V}{\partial y} = -\frac{2a(x-y)(-1)}{2} = a(y-x) \Rightarrow \text{Esta } \vec{F} \text{ coincide con la } \vec{F} \text{ del enunciado.}$$

$$F_z = -\frac{\partial V}{\partial z} = -\frac{2bz}{2} = -bz$$

b) Calculamos el rotacional de $\vec{F}$:

(4.13)

$$\vec{\nabla} \times \vec{F} = \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z} \right) \vec{u}_x - \left(\frac{\partial F_z}{\partial x} - \frac{\partial F_x}{\partial z} \right) \vec{u}_y + \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} \right) \vec{u}_z$$

$$F_x = -a(x-y)$$

$$F_y = -a(y-x)$$

$$F_z = -bz$$

$\Downarrow$

$\Downarrow$

$\Downarrow$

$$\frac{\partial F_x}{\partial y} = a$$

$$\frac{\partial F_y}{\partial x} = a$$

$$\frac{\partial F_z}{\partial x} = 0$$

$$\frac{\partial F_x}{\partial z} = 0$$

$$\frac{\partial F_y}{\partial z} = 0$$

$$\frac{\partial F_z}{\partial y} = 0$$

$$\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z} = 0 - 0 = 0$$

$$\frac{\partial F_z}{\partial x} - \frac{\partial F_x}{\partial z} = 0 - 0 = 0$$

$$\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} = a - a = 0$$

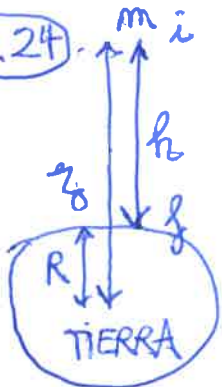
$$\Rightarrow \vec{\nabla} \times \vec{F} = \vec{0} \quad \forall \vec{r}$$

$\Downarrow$

$\vec{F}$ es conservativa

(4.24)

a)



$$z_0 = R + h$$

$$h = 10\,000 \text{ km}$$

$$R = 6370 \text{ km}$$

$$\Rightarrow \frac{h}{R} > 1 \Rightarrow \begin{cases} V = -\frac{GMm}{z_0} \\ V \neq mgh \end{cases}$$

$$E_i = -\frac{GMm}{z_0}$$

$$E_f = \frac{m}{2} v^2 - \frac{GMm}{R}$$

$$E_i = E_f$$

$$\Rightarrow -\frac{GMm}{z_0} = \frac{m}{2} v^2 - \frac{GMm}{R} \Rightarrow$$

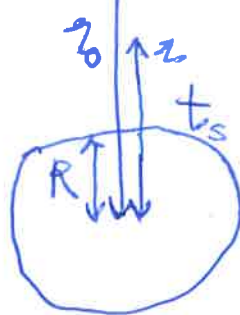
$$\Rightarrow \frac{v^2}{2} = \frac{GM}{R} - \frac{GM}{z_0} \Rightarrow v = \sqrt{2GM \left(\frac{1}{R} - \frac{1}{z_0} \right)}$$

$$v = \sqrt{2 \cdot 6.67 \cdot 10^{-11} \frac{\text{m}^3}{\text{kg} \cdot \text{s}^2} \cdot 5.9720 \cdot 10^{24} \text{ kg} \left(\frac{1}{6.37 \cdot 10^6 \text{ m}} - \frac{1}{1.637 \cdot 10^7 \text{ m}} \right)}$$

$$= 8741 \text{ m/s}$$

Autor: Iván Cabria Álvarez

$$b) \quad v = -\frac{dz}{dt} \Rightarrow dt = -\frac{dz}{v} \Rightarrow \Delta t_s = \int_{t_0}^{t_s} dt = \int_{z_0}^R \left\{ -\frac{dz}{v} \right\} \Rightarrow$$



$$v(z) = \sqrt{2GM \left(\frac{1}{z} - \frac{1}{z_0} \right)}$$

$$z(t_s) = R$$

$$\Rightarrow \Delta t_s = \frac{-1}{\sqrt{2GM}} \int_{z_0}^R \frac{dz}{\sqrt{\frac{1}{z} - \frac{1}{z_0}}} = \left\{ X = \frac{z}{z_0} \right. \\ \left. dx = \frac{dz}{z_0} \right\} =$$

$$= \frac{-1}{\sqrt{2GM}} \int_1^{R/z_0} \frac{z_0 dx}{\sqrt{\frac{1}{x z_0} - \frac{1}{z_0}}} = \frac{-z_0 \sqrt{z_0}}{\sqrt{2GM}} \int_1^{R/z_0} \frac{dx}{\sqrt{\frac{1}{x} - 1}} \Rightarrow$$

$$\frac{1}{\sqrt{\frac{1}{x} - 1}} = \frac{\sqrt{x}}{\sqrt{1-x}} = \sqrt{\frac{x}{1-x}}$$

$$\Rightarrow \Delta t_s = \frac{-z_0 \sqrt{z_0}}{\sqrt{2GM}} \int_1^{R/z_0} \sqrt{\frac{x}{1-x}} dx = \frac{z_0 \sqrt{z_0}}{\sqrt{2GM}} \left[\arctan \left(\sqrt{\frac{x}{1-x}} \right) + \sqrt{x-x^2} \right]_1^{R/z_0}$$

$$= \frac{z_0 \sqrt{z_0}}{\sqrt{2GM}} \left[\arctan \left(\sqrt{\frac{R/z_0}{1-R/z_0}} \right) + \sqrt{\frac{R}{z_0} - \frac{R^2}{z_0^2}} + \arctan(\infty) - \sqrt{1-1} \right]$$

$$= \frac{z_0 \sqrt{z_0}}{\sqrt{2GM}} \left[\arctan \left(\sqrt{\frac{R/z_0}{1-R/z_0}} \right) + \sqrt{\frac{R}{z_0} - \frac{R^2}{z_0^2}} + \frac{\pi}{2} \right]$$

$$z_0 = R + R = 6370 \text{ km} + 10000 \text{ km} = 16370 \text{ km} = 1.637 \cdot 10^4 \text{ km}$$

$$z_0 = 1.6370 \cdot 10^4 \text{ km} = 1.6370 \cdot 10^7 \text{ m}$$

$$R = 6370 \text{ km} = 6.37 \cdot 10^3 \text{ km} = 6.37 \cdot 10^6 \text{ m}$$

$$f = \frac{R}{z_0} = \frac{6.37 \cdot 10^6 \text{ m}}{1.6370 \cdot 10^7 \text{ m}} = 0.38912645 \quad \arctan \left(\sqrt{\frac{R/z_0}{1-R/z_0}} \right) = \arctan \sqrt{\frac{f}{1-f}} =$$

$$= 0.67359526 \text{ radianes}$$

$$\sqrt{\frac{R}{z_0} - \frac{R^2}{z_0^2}} = \sqrt{f - f^2} = 0.48755211$$

$$\Delta t_s = \frac{1.6370 \cdot 10^7 \text{ m} \cdot \sqrt{1.6370 \cdot 10^7 \text{ m}}}{\sqrt{2 \cdot 6.67 \cdot 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2} \cdot 5.9720 \cdot 10^{24} \text{ kg}}} \left[0.67359526 + 0.48755211 + \frac{\pi}{2} \right]$$

$$\boxed{\Delta t_s = 3249.4 \text{ s}}$$

Autor: Iván Abria Alvaro

(4.25)

$$\vec{v} = v \vec{u}_x$$



$$\vec{F}_2 = -K v^3 \frac{\vec{v}}{v} = -K v^3 \vec{u}_x$$

$$\left. \begin{array}{l} m\vec{a} = \vec{F}_{\text{net}} \\ \vec{F}_{\text{net}} = \vec{F}_2 \end{array} \right\} \Rightarrow m \frac{dv}{dt} \vec{u}_x = -K v^3 \vec{u}_x \Rightarrow$$

(4.15)

$$\Rightarrow m \frac{dv}{dt} = -K v^3 \Rightarrow \frac{dv}{v^3} = -\frac{K}{m} dt \Rightarrow \int_{v(0)}^{v(t)} \frac{dv}{v^3} = -\frac{K}{m} \int_0^t dt \Rightarrow$$

$$\Rightarrow \frac{v^{-2}}{(-2)} \Big|_{v(0)}^{v(t)} = -\frac{K}{m} t \Rightarrow \frac{1}{v^2} \Big|_{v(0)}^{v(t)} = \frac{2Kt}{m} \Rightarrow$$

$$\Rightarrow \frac{1}{v^2(t)} - \frac{1}{v^2(0)} = \frac{2Kt}{m} \Rightarrow v^2(t) = \frac{1}{\frac{1}{v^2(0)} + \frac{2Kt}{m}} \Rightarrow$$

$$\Rightarrow v(t) = \frac{1}{\sqrt{\frac{1}{v^2(0)} + \frac{2Kt}{m}}} = \boxed{\frac{v(0)}{\sqrt{1 + \frac{2Kv^2(0)t}{m}}}}$$

De maneras de calcular $\Delta = X(10s) - X(0)$ y $v|_{0 \rightarrow 10s}$.

Primera manera:

$$m \frac{dv}{dt} = -K v^3$$

$$\frac{dv}{dt} = \frac{dv}{dX} \frac{dX}{dt} = \frac{dv}{dX} v$$

$$\Rightarrow \frac{dv}{v^2} = -\frac{K}{m} dX \Rightarrow$$

$$\left. \begin{array}{l} m \frac{dv}{dt} = -K v^3 \\ \frac{dv}{dt} = \frac{dv}{dX} \frac{dX}{dt} = \frac{dv}{dX} v \end{array} \right\} \Rightarrow m \frac{dv}{dX} v = -K v^3 \Rightarrow m \frac{dv}{dX} = -K v^2 \Rightarrow$$

$$\Rightarrow \int_{v(0)=v(X(0))}^{v(t)=v(X(t))} \frac{dv}{v^2} = -\frac{K}{m} \int_{X(0)}^{X(t)} dX \Rightarrow$$

$$\Rightarrow -\frac{1}{v} \Big|_{v(X(0))}^{v(X(t))} = -\frac{K}{m} (X(t) - X(0)) \Rightarrow \frac{1}{v} \Big|_{v(0)}^{v(t)} = \frac{K}{m} (X(t) - X(0))$$

$$\Rightarrow \frac{1}{v(t)} - \frac{1}{v(0)} = \frac{K}{m} (X(t) - X(0)) \Rightarrow$$

$$\Rightarrow X(t) - X(0) = \frac{m}{K} \left(\frac{1}{v(t)} - \frac{1}{v(0)} \right)$$

Autor: Juan Cabria Flores

$$\Delta = X(10s) - X(0) = \frac{m}{K} \left(\frac{1}{v(10s)} - \frac{1}{v(0)} \right)$$

$$v(0) = 72 \frac{\text{km}}{\text{h}} = \frac{72000 \text{ m}}{3600 \text{ s}} = 20 \text{ m/s}$$

$$v(10s) = \frac{v(0)}{\sqrt{1 + \frac{2K v^2(0) 10s}{m}}} = \frac{20 \text{ m/s}}{\sqrt{1 + \frac{2 \cdot 0.05 \frac{\text{kg} \cdot \text{s}}{\text{m}^2} (20 \frac{\text{m}}{\text{s}})^2 10s}{225 \text{ kg}}}}$$

$$= \frac{20 \text{ m/s}}{\sqrt{1 + 1.7}} = 12 \text{ m/s}$$

$$\Delta = \frac{225 \text{ kg}}{0.05 \text{ kg} \cdot \text{s} / \text{m}^2} \left(\frac{1}{12 \text{ m/s}} - \frac{1}{20 \text{ m/s}} \right) = \boxed{150 \text{ m}}$$

$$W = - \int_i^f \vec{F} \cdot d\vec{r} = - \int_i^f m \frac{d\vec{v}}{dt} \cdot d\vec{r} = - \int_i^f m d\vec{v} \cdot \frac{d\vec{r}}{dt} =$$

$$= - \int_i^f m d\vec{v} \cdot \vec{v} = - m \frac{v^2}{2} \Big|_i^f = - m \frac{v_f^2}{2} + m \frac{v_i^2}{2}$$

$$W_{0 \rightarrow 10s} = \frac{m}{2} (v^2(0) - v^2(10s)) = \frac{225 \text{ kg}}{2} \left((20 \frac{\text{m}}{\text{s}})^2 - (12 \frac{\text{m}}{\text{s}})^2 \right)$$

$$= \boxed{+ 28.800 \text{ J}}$$

Segunda manera:

$$v(t) = \frac{dX}{dt} \Rightarrow \int_{X(0)}^{X(t)} dX = \int_0^t v(t) dt$$

$$v(t) = \frac{v(0)}{\sqrt{1 + \frac{2K v^2(0) t}{m}}} \Rightarrow \int_{X(0)}^{X(t)} dX = v(0) \int_0^t \frac{dt}{\sqrt{1 + \frac{2K v^2(0) t}{m}}}$$

$$a \equiv \frac{2K v^2(0)}{m} = \frac{2 \cdot 0.05 \text{ kg} \cdot \text{s} / \text{m}^2 (20 \text{ m/s})^2}{225 \text{ kg}} = 0.17 \text{ s}^{-1}$$

$$\int_{X(0)}^{X(t)} dX = v(0) \int_0^t \frac{dt}{\sqrt{1 + at}} = v(0) \frac{2\sqrt{1+at}}{a} \Big|_0^t = \frac{2v(0)}{a} (\sqrt{1+at} - 1)$$

Autor: Juan Cabria Álvarez

$$\int_{x(0)}^{x(t)} dx = x(t) - x(0) \Rightarrow x(t) - x(0) = \frac{2v(0)}{a} (\sqrt{1+at} - 1)$$

$$\Delta = x(10s) - x(0) = \frac{2v(0)}{a} (\sqrt{1+a10s} - 1) =$$

$$= \frac{2 \cdot 20 \text{ m/s}}{0.17 \text{ s}^{-1}} (\sqrt{1+1.7} - 1) = \boxed{150 \text{ m}}$$

$$W = - \int_i^f \vec{F} \cdot d\vec{z} = + \int_i^f K v^3 \vec{u}_x \cdot d\vec{z} = \int_i^f K v^3 dx \quad \Rightarrow$$

$$v(t) = \frac{dx}{dt} \Rightarrow dx = v(t) dt$$

$$\Rightarrow W = \int_i^f K v^4 dt \quad \Rightarrow W = \int_i^f \frac{K v^4(0)}{(1+at)^2} dt = - \frac{K v^4(0)}{(1+at)a} \Big|_i^f$$

$$v(t) = \frac{v(0)}{\sqrt{1+at}}$$

$$W_{0 \rightarrow 10s} = - \frac{K v^4(0)}{(1+at)a} \Big|_0^{10s} = - \frac{K v^4(0)}{a} \left(\frac{1}{1+a10s} - 1 \right)$$

$$= \frac{-0.05 \text{ kg} \cdot \text{m}^{-2} (20 \text{ m/s})^4}{0.17 \text{ s}^{-1}} \left(\frac{1}{1+1.7} - 1 \right) = \boxed{+28800 \text{ J}}$$

Autor: Juan Cabria Álvarez

(4.26) $F_x = \frac{2x^3 \cos z}{3}$ $F_y = \frac{ze^y}{3}$ $F_z = \frac{e^y}{3} - \frac{x^4 \sin z}{6}$

(4.18)

a) $\frac{\partial F_z}{\partial y} = \frac{e^y}{3}$ $\frac{\partial F_y}{\partial z} = \frac{e^y}{3} \Rightarrow \frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z} = 0$

$\frac{\partial F_z}{\partial x} = -\frac{4x^3 \sin z}{6} = -\frac{2x^3}{3} \sin z$ $\frac{\partial F_x}{\partial z} = -\frac{2x^3 \sin z}{3} \Rightarrow \frac{\partial F_z}{\partial x} - \frac{\partial F_x}{\partial z} = 0$

$\frac{\partial F_y}{\partial x} = 0$

$\frac{\partial F_x}{\partial y} = 0 \Rightarrow \frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} = 0$

$\vec{\nabla} \times \vec{F} = \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z} \right) \vec{u}_x - \left(\frac{\partial F_z}{\partial x} - \frac{\partial F_x}{\partial z} \right) \vec{u}_y + \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} \right) \vec{u}_z =$
 $= 0 \vec{u}_x - 0 \vec{u}_y + 0 \vec{u}_z = \vec{0} \Rightarrow \boxed{\text{La fuerza es conservativa}}$
 $V(x, y, z)$

b)

$\vec{F} = -\vec{\nabla} V \Rightarrow$

$$\left. \begin{aligned} \frac{2x^3 \cos z}{3} &= -\frac{\partial V}{\partial x} \\ \frac{ze^y}{3} &= -\frac{\partial V}{\partial y} \\ \frac{e^y}{3} - \frac{x^4 \sin z}{6} &= -\frac{\partial V}{\partial z} \end{aligned} \right\} \Rightarrow$$

$\Rightarrow V(x, y, z) = -\frac{2x^4}{12} \cos z + f(y, z)$

$V(x, y, z) = -\frac{ze^y}{3} + g(x, z)$

$V(x, y, z) = -\frac{ze^y}{3} - \frac{x^4 \cos z}{6} + h(x, y)$

$\Rightarrow V(x, y, z) = -\frac{x^4 \cos z}{6} - \frac{ze^y}{3} + C \Rightarrow \boxed{\text{La fuerza es conservativa}}$

$f(y, z) = -\frac{ze^y}{3} + C$

$g(x, z) = -\frac{x^4 \cos z}{6} + C$

$h(x, y) = C$

Autor: Juan Cabria Alvarez

Comprobamos que la energía potencial obtenida es la correcta.

Si es la energía potencial correcta, entonces sus derivadas parciales, con el signo menos, deberán ser iguales a las componentes de la fuerza.

$$V(x, y, z) = -\frac{x^4 \cos z}{6} - \frac{ze^y}{3} + C \Rightarrow$$

$$\Rightarrow -\frac{\partial V}{\partial x} = +\frac{4x^3 \cos z}{6} = +\frac{2x^3 \cos z}{3}$$

$$-\frac{\partial V}{\partial y} = +\frac{ze^y}{3}$$

$$-\frac{\partial V}{\partial z} = -\frac{x^4 \sin z}{6} + \frac{e^y}{3}$$

$$F_x = \frac{2x^3 \cos z}{3}$$

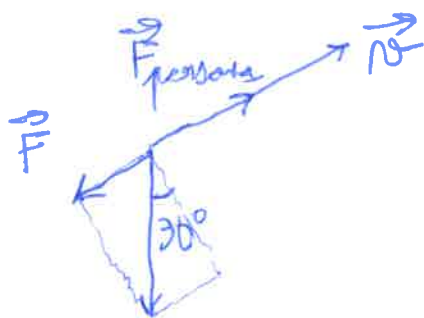
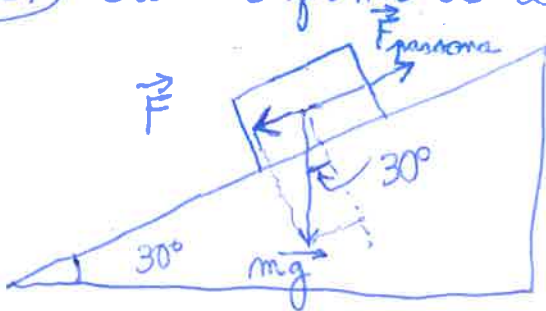
$$F_y = \frac{ze^y}{3}$$

$$F_z = \frac{e^y}{3} - \frac{x^4 \sin z}{6}$$

$$\Rightarrow -\frac{\partial V}{\partial x} = F_x \quad -\frac{\partial V}{\partial y} = F_y \quad -\frac{\partial V}{\partial z} = F_z \Rightarrow$$

$\Rightarrow$ La energía potencial obtenida es la correcta.

(4.27) Primera forma de resolver este problema:



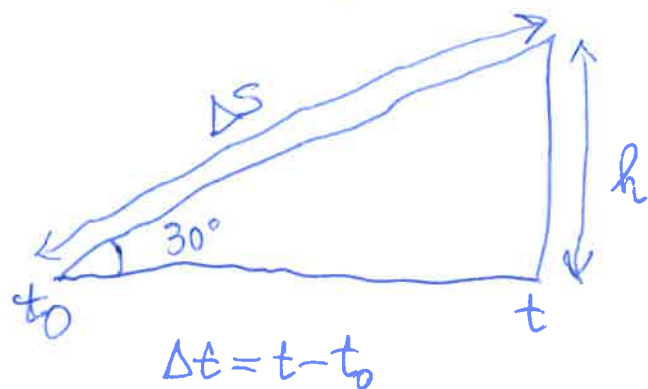
$$\left. \begin{array}{l} F = mg \sin 30^\circ \\ v = \text{cte.} \Rightarrow F_{\text{net}} = 0 \\ F_{\text{net}} = F_{\text{persona}} - F \end{array} \right\} \Rightarrow F_{\text{persona}} = mg \sin 30^\circ$$

$$\begin{aligned} P &= |\vec{F}_{\text{persona}} \vec{v}| = F_{\text{persona}} v = \\ &= mg \sin 30^\circ v = \\ &= 55 \text{ Kg} \cdot 10 \text{ m/s}^2 \sin 30^\circ \cdot 2 \text{ m/s} = \\ &= \boxed{550 \text{ W}} \end{aligned}$$

Segunda forma de resolver este problema:

(4.20)

$$\left. \begin{array}{l} E_i = T_i + V_i \\ E_f = T_f + V_f \\ v = \text{cte.} \Rightarrow T_i = T_f \\ W = \Delta E = E_f - E_i \end{array} \right\} \begin{array}{l} V_i = mgh_i \\ V_f = mgh_f \end{array} \Rightarrow W = mg(h_f - h_i) = mgh$$



$$\left. \begin{array}{l} \Delta S = v \Delta t \\ h = \Delta S \sin 30^\circ \end{array} \right\} \Rightarrow h = v \Delta t \sin 30^\circ$$

$$P = \frac{|W|}{\Delta t} = \frac{mgh}{\Delta t} = mg v \sin 30^\circ = 55 \text{ Kg} \cdot 10 \text{ m/s}^2 \sin 30^\circ \cdot 2 \text{ m/s} = \boxed{550 \text{ W}}$$

(4.28) Primera forma de resolver este problema:

$$\left. \begin{array}{l} v = \text{terminal} \Rightarrow v = \text{cte.} \Rightarrow F_{\text{net}} = 0 \\ F_{\text{net}} = mg - F_z \end{array} \right\} \Rightarrow F_z = mg$$

$\vec{v} = v \hat{z}$
 $\vec{F}_z = F_z \hat{z}$
 $\vec{mg} = -mg \hat{z}$

$$P = |\vec{F}_z \cdot \vec{v}| = F_z v_z = mg v_z = 50 \text{ Kg} \cdot 9.8 \text{ m/s}^2 \cdot 75 \text{ m/s} = \boxed{36750 \text{ W}}$$

Segunda forma de resolver este problema:

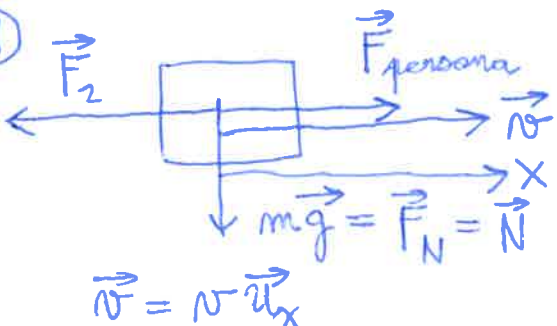
$$\left. \begin{array}{l} v = \text{cte.} \Rightarrow T_i = T_f \\ W = E_f - E_i \\ E_i = mgh_i \\ E_f = mgh_f \end{array} \right\} \Rightarrow W = mg(h_f - h_i) = mgh$$

$\hat{i}$
 Δh
 $\hat{j}$

$$P = \frac{|W|}{\Delta t} = \frac{mgh}{\Delta t} = mg v_z = \boxed{36750 \text{ W}}$$

Autor: Joaquín Cabria Álvarez

4.29



$\vec{F}_2$
 $\vec{F}_{\text{persona}}$
 $\vec{v}$
 $m\vec{g} = \vec{F}_N = \vec{N}$
 $\vec{v} = v\vec{u}_x$

$v = \text{cte.} \Rightarrow \vec{F}_{\text{net}} = \vec{0}$
 $\vec{F}_{\text{net}} = \vec{F}_{\text{persona}} + \vec{F}_2$
 $\vec{F}_2 = -\mu_d |\vec{F}_N| \vec{u}_x$
 $\vec{F}_{\text{persona}} = F_{\text{persona}} \vec{u}_x$

$\Rightarrow F_{\text{persona}} \vec{u}_x - \mu_d |\vec{F}_N| \vec{u}_x = \vec{0} \Rightarrow F_{\text{persona}} = \mu_d |\vec{F}_N|$
 $|\vec{F}_N| = mg$

$\Rightarrow F_{\text{persona}} = \mu_d mg$
 $P_{\text{persona}} = |\vec{F}_{\text{persona}} \vec{v}| = F_{\text{persona}} v \Rightarrow P_{\text{persona}} = \mu_d mg v \Rightarrow$
 $\Rightarrow P_{\text{persona}} = 0.2 \cdot 20 \text{ Kg} \cdot 9.8 \text{ m/s}^2 \cdot 1.5 \text{ m/s} = \boxed{58.8 \text{ W}}$

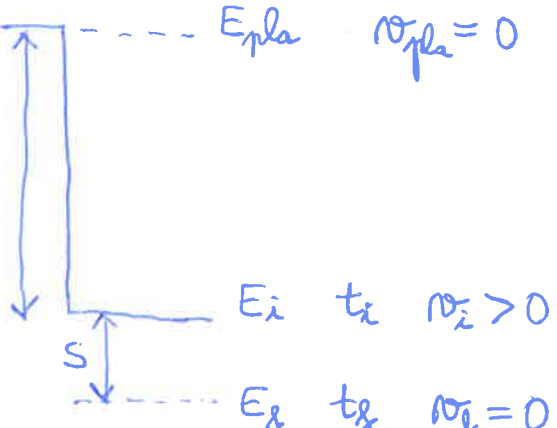
4.30



$W = \Delta E = E_f - E_i = mgh$
 $P = \frac{|W|}{\Delta t} \Rightarrow \Delta t = \frac{|W|}{P} \Rightarrow \Delta t = \frac{mgh}{P} \Rightarrow$

$\Rightarrow \Delta t = \frac{100 \text{ Kg} \cdot 9.8 \text{ m/s}^2 \cdot 10 \text{ m}}{2000 \text{ W}} = \boxed{4.9 \text{ s}}$

4.31



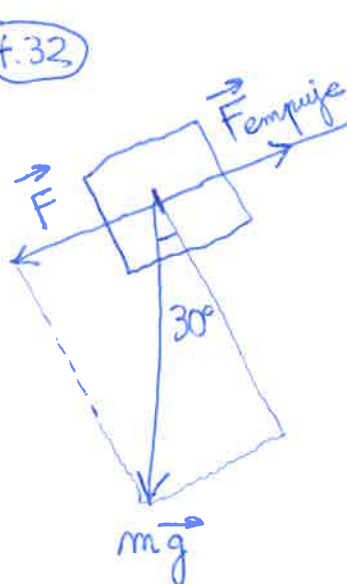
$E_{\text{pla}} \quad v_{\text{pla}} = 0$
 $E_i \quad t_i \quad v_i > 0$
 $E_f \quad t_f \quad v_f = 0$

$\Delta t = t_f - t_i = 0.4 \text{ s}$
 $E_{\text{pla}} = T_{\text{pla}} + V_{\text{pla}} = mg(h+s)$
 $E_i = E_{\text{pla}}$
 $E_f = T_f + V_f = 0 + 0$
 $\Delta E = E_f - E_i$

$\Rightarrow |\Delta E| = mg(h+s)$
 $P = \frac{|W|}{\Delta t} = \frac{|\Delta E|}{\Delta t} \Rightarrow P = \frac{mg(h+s)}{\Delta t} = \frac{70 \text{ Kg} \cdot 9.8 \text{ m/s}^2 \cdot 21 \text{ m}}{0.4 \text{ s}} =$
 $= \boxed{36015 \text{ W}}$

Autor: Iván Cabria Álvarez

(4.32)

1 metro en 4 segundos $\Rightarrow v = 0.25 \text{ m/s}$

(4.22)

$$v = \text{cte.} \Rightarrow \vec{F}_{\text{meta}} = \vec{0}$$

$$\vec{F}_{\text{meta}} = \vec{F}_{\text{empuje}} + \vec{F}$$

$$\vec{F}_{\text{empuje}} = F_{\text{empuje}} \vec{u}_x$$

$$\vec{F} = mg \sin 30^\circ$$

$$\Rightarrow F_{\text{empuje}} = mg \sin 30^\circ$$

$$\vec{v} = v \vec{u}_x$$

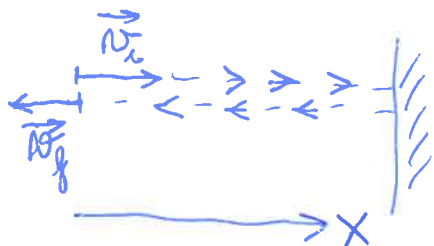
$$\vec{F}_{\text{empuje}} = F_{\text{empuje}} \vec{u}_x$$

$$\Rightarrow \vec{F}_{\text{empuje}} \cdot \vec{v} = F_{\text{empuje}} v$$

$$P_{\text{empuje}} = |\vec{F}_{\text{empuje}} \cdot \vec{v}| = F_{\text{empuje}} v = mg \sin 30^\circ v = 5 \text{ kg} \cdot 9.8 \text{ m/s}^2 \sin 30^\circ \cdot 0.25 \frac{\text{m}}{\text{s}}$$

$$= \boxed{6.1250 \text{ W}}$$

(4.33)



$$\vec{v}_i = v \vec{u}_x$$

$$v = 20 \text{ m/s}$$

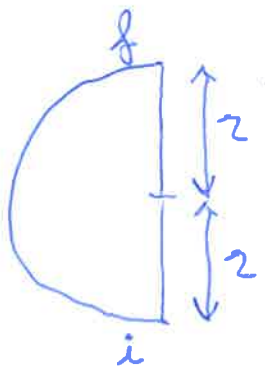
$$\vec{v}_f = -v \vec{u}_x$$

$$v_i = v_f = v$$

$$W = \Delta E = E_f - E_i = \frac{m v_f^2}{2} - \frac{m v_i^2}{2} = \boxed{0 \text{ J}}$$

(4.34)

$$z = 10 \text{ m}$$



$$P = \frac{|W|}{\Delta t} = \frac{|\Delta E|}{\Delta t} = \frac{|E_f - E_i|}{\Delta t} = mg \frac{\Delta S}{\Delta t} =$$

$$\Delta S = 2z$$

$$= 95 \text{ kg} \cdot 9.8 \text{ m/s}^2 \frac{20 \text{ m}}{30 \text{ s}} = \boxed{620.67 \text{ W}}$$

$$\Delta t = t_f - t_i = 30 \text{ s}$$

(4.35)

$$P = \frac{|\Delta E_{0-60}|}{\Delta t_{0-60}} \Rightarrow$$

$$\Delta t_{0-60} = \frac{|\Delta E_{0-60}|}{P} = \frac{\left| \frac{m}{2} v_{60}^2 - \frac{m}{2} v_0^2 \right|}{P} =$$

$$= \frac{m v_{60}^2}{2P}$$

$$v_0 = 0 \text{ m/s}$$

$$v_{60} = 60 \text{ m/s}$$

Δt_{0-60} : tiempo que tarda en acelerar desde 0 m/s hasta 60 m/s.

Autor: Iván Cabria Álvarez

Primera forma de resolver este problema: Calculando el valor numérico (4.23) de la potencia P en vatios.

$$v_{45} = 45 \text{ m/s}$$

$$\Delta t_{0-45} = 4.5 \text{ s}$$

$$P = \frac{|\Delta E_{0-45}|}{\Delta t_{0-45}} = \frac{m(v_{45}^2 - v_0^2)}{2 \Delta t_{0-45}} = \frac{1500 \text{ kg} (45 \text{ m/s})^2}{2 \cdot 4.5 \text{ s}} = 337.5 \text{ W}$$

$$\Delta t_{0-60} = \frac{m v_{60}^2}{2P} = \frac{1500 \text{ kg} (60 \text{ m/s})^2}{2 \cdot 337.5 \text{ W}} = \boxed{8 \text{ s}}$$

Segunda forma de resolver este problema: Sin calcular el valor numérico de la potencia P en vatios.

$$P = \frac{|\Delta E_{0-45}|}{\Delta t_{0-45}} = \frac{m v_{45}^2}{2 \Delta t_{0-45}} \quad \Rightarrow \quad \Delta t_{0-60} = \frac{m v_{60}^2 2 \Delta t_{0-45}}{2 m v_{45}^2} \Rightarrow$$

$$\Delta t_{0-60} = \frac{m v_{60}^2}{2P}$$

$$\Rightarrow \Delta t_{0-60} = \left(\frac{v_{60}}{v_{45}} \right)^2 \Delta t_{0-45} = \left(\frac{60}{45} \right)^2 4.5 \text{ s} = \left(\frac{4}{3} \right)^2 \frac{9}{2} \text{ s} =$$

$$= \frac{16}{9} \frac{9}{2} \text{ s} = \boxed{8 \text{ s}}$$

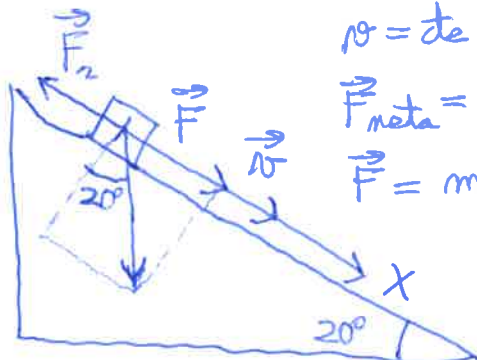
(4.36) $P = \frac{|W|}{\Delta t}$

$$W = E_f - E_i = \frac{m}{2} v^2 - 0$$

$$\Delta t = t - 0$$

$$\Rightarrow P = \frac{m v^2}{2t} \Rightarrow \boxed{v = \sqrt{\frac{2P}{m}} \sqrt{t}}$$

(4.37)



$$\left. \begin{array}{l} v = \text{cte.} \Rightarrow \vec{F}_{\text{meta}} = \vec{0} \\ \vec{F}_{\text{meta}} = \vec{F}_2 + \vec{F} \\ \vec{F} = m g \sin 20^\circ \vec{u}_x \\ \vec{F}_2 = -F_2 \vec{u}_x \\ \vec{v} = v \vec{u}_x \end{array} \right\} \Rightarrow F_2 = m g \sin 20^\circ$$

$$P_2 = |\vec{F}_2 \cdot \vec{v}| = m g \sin 20^\circ v = 80 \text{ kg} \cdot 9.8 \text{ m/s}^2 \sin 20^\circ 3 \text{ m/s} = \boxed{804.43 \text{ W}}$$

tutor: Yván Cabria Álvarez

4.38 $\vec{F} = (e^{yz}, e^{xy}, e^{yz}xy)$ newtons

4.24

a) $\vec{F} = -\vec{\nabla}V \Rightarrow \begin{cases} e^{yz} = -\frac{\partial V}{\partial x} \\ e^{xy} = -\frac{\partial V}{\partial y} \\ e^{yz}xy = -\frac{\partial V}{\partial z} \end{cases} \Rightarrow \begin{cases} V = -xe^{yz} + f(y,z) & (1) \\ V = -\frac{e^{xy}}{x} + g(x,z) & (2) \\ V = -xe^{yz} + h(x,y) & (3) \end{cases}$

De las ecuaciones (1) y (3) deducimos: $f(y,z) = h(x,y) \Rightarrow$
 $\Rightarrow f(y) = h(y)$ Las funciones f y h sólo dependen de y .

Consideramos las ecuaciones (1) y (2):

$$\left. \begin{aligned} V &= -xe^{yz} + f(y) \\ V &= -\frac{e^{xy}}{x} + g(x,z) \end{aligned} \right\} \Rightarrow -xe^{yz} + f(y) = -\frac{e^{xy}}{x} + g(x,z) \quad (4)$$

La ecuación (4) es inconsistente $\Rightarrow \vec{F} \neq -\vec{\nabla}V \Rightarrow \boxed{\vec{F} \text{ no es conservativa.}}$

b) $F_x = e^{yz} \quad F_y = e^{xy} \quad F_z = e^{yz}xy$

$$\frac{\partial F_x}{\partial y} = ze^{yz} \quad \frac{\partial F_y}{\partial x} = ye^{xy} \Rightarrow$$

$$\Rightarrow \frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} = ye^{xy} - ze^{yz} \neq 0 \quad \forall (x,y,z) \Rightarrow$$

$$\Rightarrow \vec{\nabla} \times \vec{F} \neq \vec{0} \quad \forall (x,y,z) \Rightarrow \boxed{\vec{F} \text{ no es conservativa.}}$$

Autor: Iván Cabria Álvarez

4.39 a) $m \frac{dv}{dt} \vec{u}_x = -K \sqrt{r} \vec{u}_x \Rightarrow m \frac{dr}{dt} = -K \sqrt{r} \Rightarrow$
 $-K \sqrt{r} \vec{u}_x$ $\xrightarrow{\quad \quad \quad}$ $\vec{r}$ $\xrightarrow{\quad \quad \quad}$ x
 $t \leq t_{\text{máx}}$ $t \leq t_{\text{máx}}$

$$\Rightarrow \frac{dr}{\sqrt{r}} = -\frac{K}{m} dt \Rightarrow \int_{r(0)}^{r(t)} \frac{dr}{\sqrt{r}} = -\frac{K}{m} \int_0^t dt \Rightarrow$$

$$\Rightarrow 2 \sqrt{r} \Big|_{r(0)}^{r(t)} = -\frac{K}{m} t \Rightarrow \sqrt{r(t)} - \sqrt{r(0)} = -\frac{K}{2m} t \Rightarrow$$

$$\Rightarrow r(t) = \left(\sqrt{r(0)} - \frac{K}{2m} t \right)^2 \quad t \leq t_{\text{máx}}$$

En $t_{\text{máx}}$ la velocidad es cero y para $t > t_{\text{máx}}$,

$$m \frac{dr}{dt} = 0 \Rightarrow \left. \begin{array}{l} r(t) = r(t_{\text{máx}}) \\ r(t_{\text{máx}}) = 0 \end{array} \right\} \Rightarrow r(t) = 0 \quad t > t_{\text{máx}}$$

$$r(t) = \begin{cases} \left(\sqrt{r(0)} - \frac{K}{2m} t \right)^2 & t \leq t_{\text{máx}} \\ 0 & t > t_{\text{máx}} \end{cases}$$

$$r(t_{\text{máx}}) = 0 \Rightarrow 0 = \sqrt{r(0)} - \frac{K}{2m} t_{\text{máx}} \Rightarrow$$

$$\Rightarrow t_{\text{máx}} = \frac{2m \sqrt{r(0)}}{K} = \boxed{12649.11 \text{ s}}$$

b) $\Delta \equiv X(4 \text{ horas}) - X(0)$

$$4 \text{ horas} = 4 \cdot 3600 \text{ s} = 14400 \text{ s} \Rightarrow 4 \text{ horas} > t_{\text{máx}} \Rightarrow X(4 \text{ horas}) = X(t_{\text{máx}}) \Rightarrow$$

$$\Rightarrow \Delta = X(t_{\text{máx}}) - X(0)$$

Primera forma de calcular Δ :

Buscamos una ecuación en x y r .

$$\left. \begin{array}{l} m \frac{dr}{dt} = -K \sqrt{r} \quad t \leq t_{\text{máx}} \\ \frac{dr}{dt} = \frac{dr}{dx} \frac{dx}{dt} = \frac{dr}{dx} v \end{array} \right\} \Rightarrow m \frac{dr}{dx} v = -K \sqrt{r} \Rightarrow$$

Autor: Iván Cabria Álvarez

$$\Rightarrow -\frac{m}{K} d\nu \sqrt{\nu} = dX \Rightarrow \int_0^t dX = -\frac{m}{K} \int_{\nu(0)}^{\nu(t)} \sqrt{\nu} d\nu \Rightarrow$$

$$\Rightarrow X(t) - X(0) = -\frac{m}{K} \frac{2}{3} \nu^{3/2} \Big|_{\nu(0)}^{\nu(t)} \Rightarrow$$

$$\Rightarrow X(t) - X(0) = -\frac{2m}{3K} (\nu^{3/2}(t) - \nu^{3/2}(0)) = \frac{2m}{3K} (\nu^{3/2}(0) - \nu^{3/2}(t)) \Bigg\} \Rightarrow$$

$$\Delta = X(t_{\text{máx}}) - X(0)$$

$$\Rightarrow \Delta = \frac{2m}{3K} (\nu^{3/2}(0) - \nu^{3/2}(t_{\text{máx}})) \Bigg\} \Rightarrow \Delta = \frac{2m}{3K} \nu^{3/2}(0) \Rightarrow$$

$$\nu(t_{\text{máx}}) = 0$$

$$\Rightarrow \Delta = \frac{2 \cdot 100 \text{ kg} \cdot 5 \sqrt{5}}{3 \cdot 0.05 \text{ kg} \sqrt{\text{m}_0}} \left(10 \frac{\text{m}}{\text{s}}\right)^{3/2} = \boxed{42163.70 \text{ m}}$$

Segunda forma de calcular Δ :

Buscamos una ecuación en X y t .

$$\nu(t) = \frac{dX}{dt} \Rightarrow dX = \nu(t) dt \Rightarrow \int_{X(0)}^{X(t)} dX = \int_0^t \nu(t) dt \quad t \leq t_{\text{máx}}$$

$$\nu(t) = \left(\sqrt{\nu(0)} - \frac{K}{2m} t\right)^2 = \nu(0) + \frac{K^2}{4m^2} t^2 - \frac{K\sqrt{\nu(0)}}{m} t = a_0 + a_1 t + a_2 t^2$$

$$\int_{X(0)}^{X(t)} dX = \int_0^t \nu(t) dt \Rightarrow X(t) - X(0) = \int_0^t (a_0 + a_1 t + a_2 t^2) dt \Rightarrow$$

$$\Rightarrow X(t) - X(0) = a_0 t + \frac{a_1 t^2}{2} + \frac{a_2 t^3}{3}$$

$$a_0 = \nu(0) = 10 \text{ m/s} \quad a_1 = -\frac{K\sqrt{\nu(0)}}{m} = -\frac{0.05\sqrt{10}}{100} \frac{\text{m}}{\text{s}^2} =$$

$$= -0.0015811388 \text{ m/s}^2$$

$$a_2 = \frac{K^2}{4m^2} = \frac{(0.05)^2}{4(100)^2} \frac{\text{m}}{\text{s}^3} = 6.25 \cdot 10^{-8} \text{ m/s}^3$$

$$\Delta = X(t_{\text{máx}}) - X(0) = 10 \cdot 12649.11 - 1.5811 \cdot 10^{-3} \frac{(12649.11)^2}{2} + 6.25 \cdot 10^{-8} \frac{(12649.11)^3}{3} \text{ m} = \boxed{42163.70 \text{ m}}$$

Autor: Iván Cabrera Álvarez

$$c) \left. \begin{aligned} W &= - \int_i^f \vec{F} \cdot d\vec{r} \\ \vec{F} &= \vec{0} \text{ si } t > t_{\max} \\ t &= 4 \text{ horas} > t_{\max} \end{aligned} \right\} \Rightarrow W_{0 \rightarrow 4 \text{ horas}} = W_{0 \rightarrow t_{\max}}$$

$$W_{0 \rightarrow t_{\max}} = - \int_{t=0, r(0)}^{t=t_{\max}, r(t_{\max})} \vec{F} \cdot d\vec{r}$$

Primera forma de calcular $W_{0 \rightarrow t_{\max}}$:

$$W_{0 \rightarrow t_{\max}} = - \int_{0, r(0)}^{t_{\max}, r(t_{\max})} \vec{F} \cdot d\vec{r} = - \int_{0, r(0)}^{t_{\max}, r(t_{\max})} m \vec{v} \cdot d\vec{v} = - \frac{m}{2} \vec{v}^2 \Big|_{r(0)}^{r(t_{\max})} \Rightarrow$$

$$\Rightarrow W_{0 \rightarrow t_{\max}} = - \frac{m}{2} (v^2(t_{\max}) - v^2(0)) \Bigg\} \Rightarrow$$

$$v(t_{\max}) = 0$$

$$\Rightarrow W_{0 \rightarrow t_{\max}} = \frac{m}{2} v^2(0) = \frac{100}{2} 10^2 \text{ J} = 5000 \text{ J}$$

$$W_{0 \rightarrow 4 \text{ horas}} = W_{0 \rightarrow t_{\max}} = \boxed{5000 \text{ J}}$$

Segunda forma de calcular $W_{0 \rightarrow t_{\max}}$: Integrando con respecto al tiempo.

$$W_{0 \rightarrow t_{\max}} = - \int_{0, r(0)}^{t_{\max}, r(t_{\max})} \vec{F} \cdot d\vec{r} = + \int_{0, r(0)}^{t_{\max}, r(t_{\max})} K \sqrt{r} dx \Bigg\} \Rightarrow$$

$$r(t) = \frac{dx}{dt} \Rightarrow dx = v(t) dt$$

$$\Rightarrow W_{0 \rightarrow t_{\max}} = \int_0^{t_{\max}} K v^{3/2} dt$$

$$K v^{3/2}(t) = K \left(\sqrt{v(0)} - \frac{K}{2m} t \right)^3 = b_0 + b_1 t + b_2 t^2 + b_3 t^3$$

Autor: Yreán Cabria Álvarez

$$\begin{aligned}
 K v^{3/2}(t) &= K \left(v(0) + \frac{K^2}{4m^2} t^2 - \frac{K \sqrt{v(0)}}{m} t \right) \left(\sqrt{v(0)} - \frac{Kt}{2m} \right) \quad (4.28) \\
 &= K \left(v^{3/2}(0) - \frac{v(0)K}{2m} t + \frac{K^2}{4m^2} \sqrt{v(0)} t^2 - \frac{K^3}{8m^3} t^3 \right. \\
 &\quad \left. - \frac{2K v(0)}{2m} t + \frac{2K^2}{4m^2} \sqrt{v(0)} t^2 \right) = \\
 &= K v^{3/2}(0) - \frac{3K^2 v(0)}{2m} t + \frac{3K^3}{4m^2} \sqrt{v(0)} t^2 - \frac{K^4}{8m^3} t^3 \Rightarrow
 \end{aligned}$$

$$\Rightarrow b_0 = K v^{3/2}(0) = 0.05 \cdot 10^{3/2} \frac{\text{J}}{\text{s}} = 1.58113883 \frac{\text{J}}{\text{s}}$$

$$b_1 = -\frac{3K^2 v(0)}{2m} = -\frac{3(0.05)^2 \cdot 10}{2 \cdot 100} \frac{\text{J}}{\text{s}^2}$$

$$b_2 = \frac{3}{4} \frac{K^3}{m^2} \sqrt{v(0)} = \frac{3}{4} \frac{(0.05)^3 \sqrt{10}}{(100)^2} \frac{\text{J}}{\text{s}^3} =$$

$$= 2.964635 \cdot 10^{-8} \frac{\text{J}}{\text{s}^3}$$

$$b_3 = -\frac{K^4}{8m^3} = -\frac{(0.05)^4 \frac{\text{J}}{\text{s}^4}}{8(100)^3} = -7.8125 \cdot 10^{-13} \frac{\text{J}}{\text{s}^4}$$

$$W_{0 \rightarrow t_{\max}} = \int_0^{t_{\max}} K v^{3/2}(t) dt = \int_0^{t_{\max}} (b_0 + b_1 t + b_2 t^2 + b_3 t^3) dt =$$

$$= b_0 t_{\max} + b_1 \frac{t_{\max}^2}{2} + b_2 \frac{t_{\max}^3}{3} + b_3 \frac{t_{\max}^4}{4} = 5000 \text{ J}$$

$$W_{0 \rightarrow 4 \text{ hours}} = W_{0 \rightarrow t_{\max}} = \boxed{5000 \text{ J}}$$

Autor: Yoán Cabria Álvarez

d) Primera forma de calcular las unidades:

(4.29)

$$ma = -K\sqrt{r} \Rightarrow [K] = \frac{[m][a]}{[\sqrt{r}]} = \frac{MLT^{-2}}{(LT^{-1})^{1/2}} \Rightarrow$$

$$\Rightarrow [K] = MLT^{-2} L^{-1/2} T^{+1/2} = ML^{1/2} T^{-3/2} \Rightarrow$$

$\Rightarrow$ Las unidades de K en el S.I. son $Kg \sqrt{m} s^{-3/2}$.

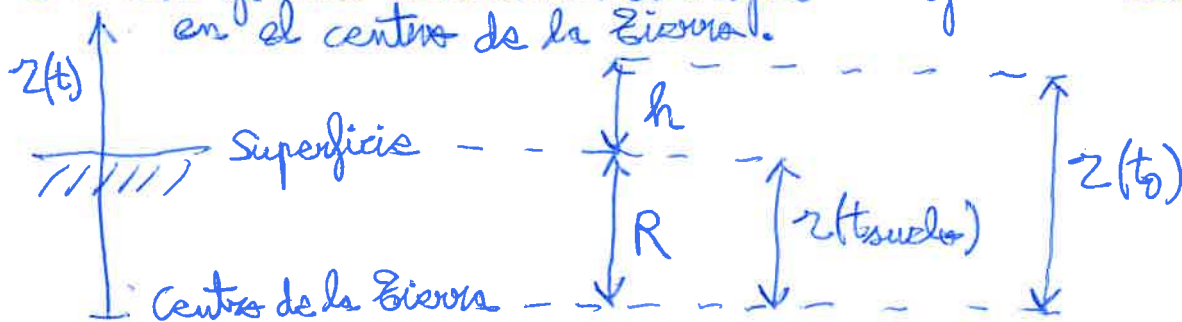
Segunda forma de calcular las unidades:

$$F = -K\sqrt{r} \Rightarrow |K| = \left| \frac{F}{\sqrt{r}} \right| \Rightarrow$$

$\Rightarrow$ Las unidades de K en el S.I. son $\frac{N\sqrt{s}}{\sqrt{m}}$.

(4.40) Vamos a explicar tres formas de resolver este problema.

Primera forma: Colocamos el origen del eje de la coordenada z en el centro de la Tierra.



R : Radio de la Tierra

h : Altura inicial del objeto respecto de la superficie de la Tierra.

$z(t)$: Distancia entre el objeto y el centro de la Tierra en el instante t de tiempo.

$z(t_0)$: Distancia entre el objeto y el centro de la Tierra en el instante inicial de tiempo t_0 . El objeto está a h metros de altura sobre la superficie en ese instante t_0 .

$z(t_{suelo})$: Distancia entre el objeto y el centro de la Tierra en el instante en el que el objeto llega al suelo, t_{suelo} .

$$z(t_0) = R + h \quad z(t_{suelo}) = R$$

$$z(t_{suelo}) \leq z(t) \leq z(t_0) \quad t_0 \leq t \leq t_{suelo}$$

Autor: Yoón Cabria Álvarez

El objeto cae desde $z(t_0)$ hacia el suelo. Por lo tanto,

$$dz = z(t+dt) - z(t) = z(t) - z(t) = 0 \Rightarrow dz < 0$$

$$\left. \begin{array}{l} v(t) > 0 \\ dz < 0 \\ dt > 0 \end{array} \right\} \Rightarrow v(t) = - \frac{dz}{dt} \Rightarrow dt = - \frac{dz}{v(t)} \Rightarrow \Delta t_{\text{suelo}} = \int_{t_0}^{t_{\text{suelo}}} dt \Rightarrow \Delta t_{\text{suelo}} = - \int_{z(t_0)}^{z(t_{\text{suelo}})} \frac{dz}{v(z)}$$

Calculamos v en función de z ; $v(z)$:

$$\begin{array}{c} E_k = E_f \\ \uparrow \quad \uparrow \\ t_0 \quad t \end{array} \Rightarrow - \frac{GMm}{z(t_0)} = - \frac{GMm}{z} + \frac{mv^2}{2} \Rightarrow$$

$$\Rightarrow v^2 = \frac{2GMm}{m} \left(\frac{1}{z} - \frac{1}{z(t_0)} \right) \Rightarrow v = \sqrt{2GM \left(\frac{1}{z} - \frac{1}{z(t_0)} \right)}$$

$$\begin{aligned} \Delta t_{\text{suelo}} &= - \int_{z(t_0)}^{z(t_{\text{suelo}})} \frac{dz}{\sqrt{2GM \left(\frac{1}{z} - \frac{1}{z(t_0)} \right)}} = - \frac{1}{\sqrt{2GM}} \int_{z(t_0)}^R \frac{dz}{\sqrt{\frac{1}{z} - \frac{1}{z(t_0)}}} \\ &= - \frac{1}{\sqrt{2GM}} z(t_0) \sqrt{z(t_0)} \left[\arctan \left(\sqrt{\frac{z/z(t_0)}{1 - z/z(t_0)}} \right) - \sqrt{\frac{z}{z(t_0)} - \frac{z^2}{z^2(t_0)}} \right] \Big|_{z(t_0)}^R \\ &= - \frac{z(t_0) \sqrt{z(t_0)}}{\sqrt{2GM}} \left[\arctan \left(\sqrt{\frac{R/z(t_0)}{1 - R/z(t_0)}} \right) - \arctan(\infty) - \sqrt{\frac{R}{z(t_0)} - \frac{R^2}{z^2(t_0)}} \right] \end{aligned}$$

Definimos f , para simplificar los cálculos, como: $f \equiv \frac{R}{z(t_0)}$

$$\Delta t_{\text{suelo}} = \frac{z(t_0) \sqrt{z(t_0)}}{\sqrt{2GM}} \left[- \arctan \left(\sqrt{\frac{f}{1-f}} \right) + \frac{\pi}{2} + \sqrt{f - f^2} \right]$$

$$z(t_0) = R + h = 1.1370 \cdot 10^7 \text{ m}$$

$$f = \frac{R}{z(t_0)} = \frac{6.37 \cdot 10^6 \text{ m}}{1.1370 \cdot 10^7 \text{ m}} = 0.5602462$$

Autor: Iván Gabrín Álvarez

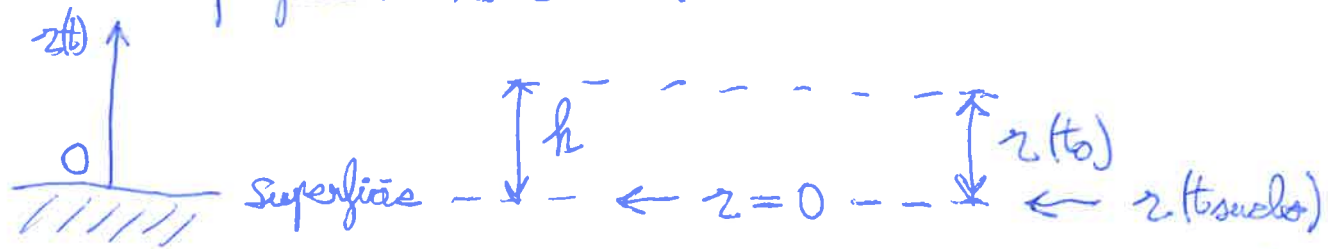
$$\arctan\left(\sqrt{\frac{f}{1-f}}\right) = 0.8457911 \text{ radianes}$$

$$\sqrt{f-f^2} = 0.4963571$$

$$\Delta t_{\text{suelo}} = \frac{(1.1370 \cdot 10^7 \text{ m})^{3/2} [-0.8457911 + \frac{\pi}{2} + 0.4963571]}{\sqrt{2 \cdot 6.67 \cdot 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2} \cdot 5.9720 \cdot 10^{24} \text{ kg}}}$$

$\Delta t_{\text{suelo}} = 1659 \text{ s}$

Segunda forma: Colocamos el origen del eje de la coordenada z en la superficie de la Tierra.



$$z(t_0) = h \quad z(t_{\text{suelo}}) = 0 \quad 0 \leq z(t) \leq z(t_0) \quad t_0 \leq t \leq t_{\text{suelo}}$$

El objeto cae desde $z(t_0)$ hacia la superficie. Por lo tanto,
 $z(t+dt) < z(t) \Rightarrow dz = z(t+dt) - z(t) < 0$

$$\left. \begin{array}{l} v(t) > 0 \\ dz < 0 \\ dt > 0 \end{array} \right\} \Rightarrow v(t) = - \frac{dz}{dt} \Rightarrow dt = - \frac{dz}{v(t)} \Rightarrow \Delta t_{\text{suelo}} = \int_{t_0}^{t_{\text{suelo}}} dt = - \int_{z(t_0)}^{z(t_{\text{suelo}})} \frac{dz}{v(t)} = - \int_h^0 \frac{dz}{v} = \int_0^h \frac{dz}{v}$$

$$E_k = E_p \Rightarrow - \frac{GMm}{R+h} = - \frac{GMm}{(R+z(t))} + \frac{m}{2} v^2(t) \Rightarrow$$

$$\Rightarrow v(z(t)) = \sqrt{2GM \left(\frac{1}{R+z(t)} - \frac{1}{R+h} \right)}$$

Autor: Iván Cabria Alvarado

$$\Delta t_{\text{suelo}} = \int_0^h \frac{dz}{v(z)} = \int_0^h \frac{dz}{\sqrt{2GM} \sqrt{\frac{1}{R+z} - \frac{1}{R+h}}} = \left[\begin{matrix} x=R+z \\ dx=dz \end{matrix} \right] = \int_R^{R+h} \frac{dx}{\sqrt{2GM} \sqrt{\frac{1}{x} - \frac{1}{R+h}}} \quad a \equiv R+h$$

$$\Delta t_{\text{suelo}} = \frac{1}{\sqrt{2GM}} \int_R^a \frac{dx}{\sqrt{\frac{1}{x} - \frac{1}{a}}} = \frac{a^{3/2}}{\sqrt{2GM}} \left[\arctan\left(\sqrt{\frac{x/a}{1-x/a}}\right) - \sqrt{\frac{x}{a} - \frac{x^2}{a^2}} \right] \Bigg|_{x=R}^{x=a} = \frac{a^{3/2}}{\sqrt{2GM}} \left[\arctan(\infty) - \arctan\left(\sqrt{\frac{R/a}{1-R/a}}\right) + \sqrt{\frac{R}{a} - \frac{R^2}{a^2}} \right]$$

$$f \equiv \frac{R}{a} = \frac{R}{R+h} = \frac{6.37 \cdot 10^6 \text{ m}}{1.1370 \cdot 10^7 \text{ m}} = 0.5602462$$

$$\Delta t_{\text{suelo}} = \frac{a^{3/2}}{\sqrt{2GM}} \left[\frac{\pi}{2} - \arctan\left(\sqrt{\frac{f}{1-f}}\right) + \sqrt{f-f^2} \right]$$

$$\arctan\left(\sqrt{\frac{f}{1-f}}\right) = 0.8457911 \text{ radianes}$$

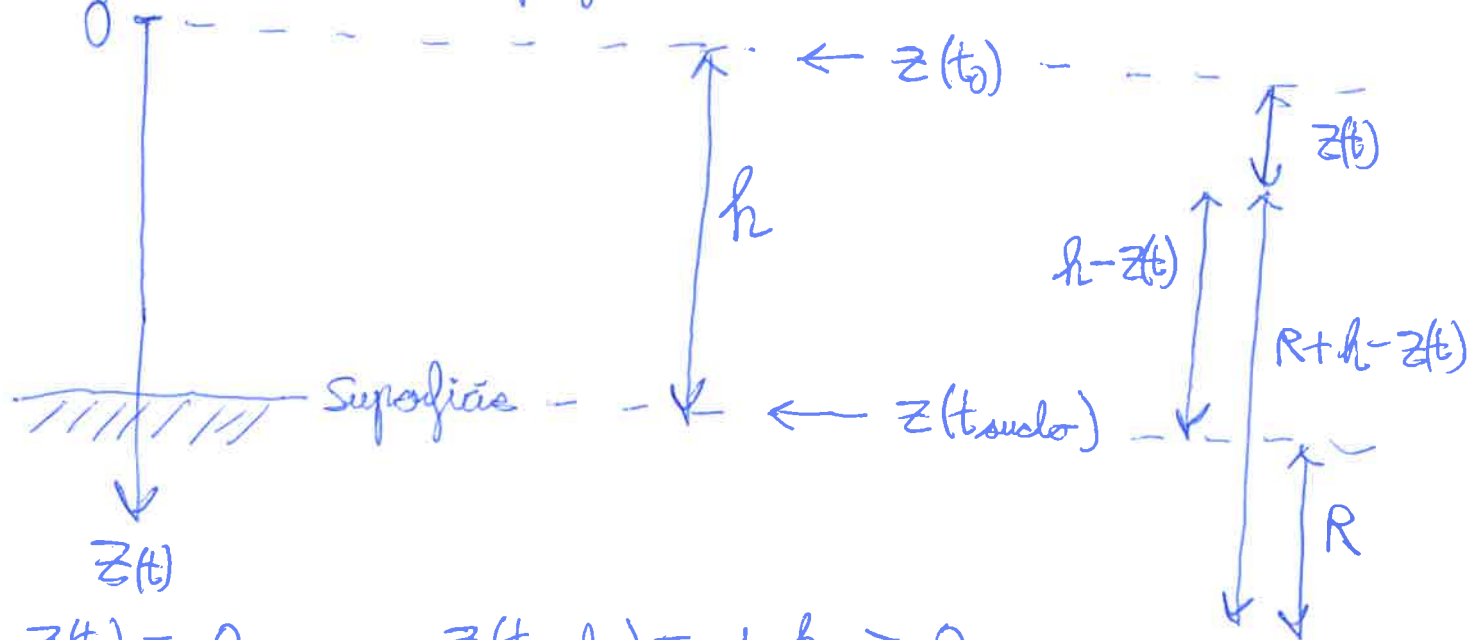
$$\sqrt{f-f^2} = 0.4963571$$

$$\Delta t_{\text{suelo}} = \frac{(1.1370 \cdot 10^7 \text{ m})^{3/2} \left[\frac{\pi}{2} - 0.8457911 + 0.4963571 \right]}{\sqrt{2 \cdot 6.67 \cdot 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2} \cdot 5.9720 \cdot 10^{24} \text{ kg}}}$$

$$\boxed{\Delta t_{\text{suelo}} = 1659 \text{ s}}$$

Autor: Yoán Cabrera Álvarez

Esencia forma: Colocando el origen del eje de la coordenada z en el lugar donde está inicialmente el objeto y orientando el eje z hacia la superficie.



$$z(t_0) = 0 \quad z(t_{\text{suelo}}) = +h > 0$$

$$0 \leq z(t) \leq z(t_{\text{suelo}}) \quad t_0 \leq t \leq t_{\text{suelo}}$$

El objeto cae desde $z(t_0)$ hacia la superficie. Por lo tanto,

$$z(t+dt) > z(t) \Rightarrow dz = z(t+dt) - z(t) > 0$$

$$\left. \begin{array}{l} \frac{dz}{dt} > 0 \\ dt > 0 \end{array} \right\} \Rightarrow v(t) = \frac{dz}{dt} \Rightarrow dt = \frac{dz}{v(t)} \quad \Rightarrow \Delta t_{\text{suelo}} = \int_{z(t_0)}^{z(t_{\text{suelo}})} \frac{dz}{v(z)} \Rightarrow$$

$$\Rightarrow \Delta t_{\text{suelo}} = \int_0^h \frac{dz}{v(z)}$$

Calculamos $v(z)$:

El objeto se encuentra a una distancia del centro de la Tierra igual a $R+h$ en el instante t_0 .

El objeto se encuentra a una distancia del centro de la Tierra igual a $R+h-z(t)$ en el instante t .

$$\left. \begin{array}{l} E_i = E(t_0) \\ E_f = E_f \end{array} \right\} \Rightarrow -\frac{GMm}{R+h} = -\frac{GM}{(R+h-z)} + \frac{m}{2}v^2(t) \Rightarrow$$

Autor: Juan Cabria Alvarado

$$\Rightarrow v(z) = \sqrt{2GM} \sqrt{\frac{1}{R+h-z} - \frac{1}{R+h}}$$

$$\Delta t_{\text{suelo}} = \int_0^h \frac{dz}{v(z)} = \frac{1}{\sqrt{2GM}} \int_0^h \frac{dz}{\sqrt{\frac{1}{R+h-z} - \frac{1}{R+h}}} \quad a \equiv R+h$$

$$\begin{aligned} \Delta t_{\text{suelo}} &= \frac{1}{\sqrt{2GM}} \int_0^h \frac{dz}{\sqrt{\frac{1}{a-z} - \frac{1}{a}}} = \left\{ \begin{array}{l} x = a - z \\ dx = -dz \end{array} \right\} = \\ &= -\frac{1}{\sqrt{2GM}} \int_{a-h}^{a-h} \frac{dx}{\sqrt{\frac{1}{x} - \frac{1}{a}}} = \frac{1}{\sqrt{2GM}} \int_{a-h}^a \frac{dx}{\sqrt{\frac{1}{x} - \frac{1}{a}}} = \\ &= \frac{a^{3/2}}{\sqrt{2GM}} \left[\arctan\left(\sqrt{\frac{x/a}{1-x/a}}\right) - \sqrt{\frac{x}{a} - \frac{x^3}{a^2}} \right] \Bigg|_{x=a-h}^{x=a} \end{aligned}$$

$$a-h = R+h-h = R \quad f \equiv R/a$$

$$\Delta t_{\text{suelo}} = \frac{a^{3/2}}{\sqrt{2GM}} \left[\arctan(\infty) - \arctan\left(\sqrt{\frac{f}{1-f}}\right) + \sqrt{f-f^2} \right]$$

$$f \equiv \frac{R}{a} = \frac{R}{R+h} = \frac{6.37 \cdot 10^6 \text{ m}}{1.1370 \cdot 10^7 \text{ m}} = 0.5602462$$

$$\arctan\left(\sqrt{\frac{f}{1-f}}\right) = 0.8457911 \text{ radianes}$$

$$\sqrt{f-f^2} = 0.4963571$$

$$\Delta t_{\text{suelo}} = \frac{(1.1370 \cdot 10^7 \text{ m})^{3/2} \left[\frac{\pi}{2} - 0.8457911 + 0.4963571 \right]}{\sqrt{2 \cdot 6.67 \cdot 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2} \cdot 5.9720 \cdot 10^{24} \text{ kg}}}$$

$$\boxed{\Delta t_{\text{suelo}} = 1659 \text{ s}}$$

Autor: Iván Cabris Álvarez

$$(4.41) \quad \vec{F} = -10(x-y)\vec{u}_x - (10(y-x) + 6(y-z))\vec{u}_y - 6(z-y)\vec{u}_z \quad (4.35)$$

$$a) \quad \vec{F} = -\vec{\nabla} V \Rightarrow -10(x-y) = -\frac{\partial V}{\partial x}$$

$$-(10(y-x) + 6(y-z)) = -\frac{\partial V}{\partial y}$$

$$-6(z-y) = -\frac{\partial V}{\partial z}$$

Hay varias maneras de calcular $V = V(x, y, z)$ a partir de las ecuaciones anteriores. Vamos a explicar dos maneras.

1ª manera:

$$\frac{\partial V}{\partial x} = 10(x-y) \Rightarrow V = \frac{10}{2}(x-y)^2 + f(y, z)$$

$$\frac{\partial V}{\partial y} = 10(y-x) + 6(y-z) \Rightarrow V = \frac{10}{2}(y-x)^2 + \frac{6}{2}(y-z)^2 + g(x, z)$$

$$\frac{\partial V}{\partial z} = 6(z-y) \Rightarrow V = \frac{6}{2}(z-y)^2 + h(x, y)$$

$$\Rightarrow f(y, z) = \frac{6}{2}(y-z)^2 + C, \quad g(x, z) = C, \quad h(x, y) = \frac{10}{2}(x-y)^2 + C$$

$$\text{y } \boxed{V(x, y, z) = \frac{10}{2}(x-y)^2 + \frac{6}{2}(z-y)^2 + C} \Rightarrow \boxed{\vec{F} \text{ es conservativo}}$$

2ª manera:

$$\frac{\partial V}{\partial x} = 10(x-y) \Rightarrow V = \frac{10}{2}x^2 - 10xy + f(y, z)$$

$$\frac{\partial V}{\partial y} = 10(y-x) + 6(y-z) \Rightarrow V = \frac{10}{2}y^2 - 10xy + \frac{6y^2}{2} - 6zy + g(x, z)$$

$$\frac{\partial V}{\partial z} = 6(z-y) \Rightarrow V = \frac{6}{2}z^2 - 6yz + h(x, y)$$

$$\Rightarrow f(y, z) = \frac{10}{2}y^2 + \frac{6}{2}y^2 - 6zy + \frac{6}{2}z^2 + C$$

$$g(x, z) = \frac{10}{2}x^2 + \frac{6}{2}z^2 + C \quad \text{Autor: Iván Cabrera Álvarez}$$

$$h(x, y) = \frac{10}{2}x^2 - 10xy + \frac{10}{2}y^2 + \frac{6}{2}y^2 + C$$

$$V(x, y, z) = \frac{10}{2}x^2 - 10xy + \frac{10}{2}y^2 + \frac{6}{2}y^2 - 6zy + \frac{6}{2}z^2 + C \Rightarrow$$

$\Rightarrow V(x,y,z) = \frac{10}{2}(x-y)^2 + \frac{6}{2}(y-z)^2 + C \Rightarrow \vec{F} \text{ es conservativa}$

b) Si el rotacional de la fuerza, $\vec{\nabla} \times \vec{F}$, es $\vec{0}$, entonces la fuerza será conservativa.

Calculamos las derivadas parciales que se necesitan para calcular $\vec{\nabla} \times \vec{F}$.

$F_x = -10(x-y) \quad F_y = -(10(y-x) + 6(y-z)) \quad F_z = -6(z-y)$

$\frac{\partial F_x}{\partial y} = 10$	$\frac{\partial F_y}{\partial x} = 10 \Rightarrow \frac{\partial F_x}{\partial y} - \frac{\partial F_y}{\partial x} = 0$	} $\Rightarrow$
$\frac{\partial F_x}{\partial z} = 0$	$\frac{\partial F_z}{\partial x} = 0 \Rightarrow \frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x} = 0$	
$\frac{\partial F_y}{\partial z} = 6$	$\frac{\partial F_z}{\partial y} = 6 \Rightarrow \frac{\partial F_y}{\partial z} - \frac{\partial F_z}{\partial y} = 0$	

$\Rightarrow \vec{\nabla} \times \vec{F} = 0\vec{u}_x + 0\vec{u}_y + 0\vec{u}_z = \vec{0} \Rightarrow \vec{F} \text{ es conservativa}$

c) El trabajo se puede calcular de dos maneras.

1ª manera: $W = - \int_C^f \vec{F} \cdot d\vec{r} \Rightarrow W = + \int_i^f \vec{\nabla} V \cdot d\vec{r} = \int_i^f dV = V_f - V_i$
 $\vec{F}$ es conservativa } la integral no depende del camino C

$W = V_f - V_i = V(1,5,0) - V(0,5,0) = 5 \cdot (1-5)^2 + 3 \cdot (5-0)^2 - 5 \cdot (0-5)^2 - 3 \cdot (5-0)^2 = 5 \cdot 4^2 - 5 \cdot 5^2 J = 5(16-25) J = -5 \cdot 9 J = -45 J$

2ª manera: Escribiendo en cuenta el camino C: $y=5 \Rightarrow dy=0, z=0 \Rightarrow dz=0$
 x varía entre 0 y 1 $\Rightarrow dx \neq 0$

$W = - \int_C^f \vec{F} \cdot d\vec{r} = - \int_{(0,5,0)}^{(1,5,0)} F_x dx = - \int_0^1 F_x(x,5,0) dx = \int_0^1 10(x-5) dx = \left[\frac{10}{2}x^2 - 50x \right]_0^1 = 5 - 50 J = -45 J$

$$\left. \begin{array}{l} (4.42) \quad E_i = -\frac{GMm}{R} + \frac{m}{2}v^2 \\ E_f = 0 \quad E_i = E_f \end{array} \right\} \Rightarrow \frac{v^2}{2} = \frac{GM}{R} \Rightarrow v = \sqrt{\frac{2GM}{R}} \quad (4.37)$$

Calculamos GM mediante la tercera ley de Kepler:

$$\frac{T_0^2}{r_0^3} = \frac{4\pi^2}{GM} \Rightarrow GM = \frac{4\pi^2 r_0^3}{T_0^2}$$

A partir de aquí, hay tres maneras diferentes de resolver este problema.

Primera manera: Calculamos v en función de r_0 , g y T_0 . Después sustituimos los valores numéricos de r_0 , g y T_0 en $v = v(r_0, g, T_0)$.

$$g = \frac{GM}{R^2} \Rightarrow R = \sqrt{\frac{GM}{g}} \Rightarrow \sqrt{R} = \left(\frac{GM}{g}\right)^{1/4}$$

$$\left. \begin{array}{l} v = \sqrt{\frac{2GM}{R}} \\ \sqrt{R} = \left(\frac{GM}{g}\right)^{1/4} \end{array} \right\} \Rightarrow v = \frac{\sqrt{2GM} g^{1/4}}{(GM)^{1/4}} = \sqrt{2} (GM)^{1/4} g^{1/4}$$

$$\left. \begin{array}{l} v = \sqrt{2} (GM)^{1/4} g^{1/4} \\ GM = \frac{4\pi^2 r_0^3}{T_0^2} \end{array} \right\} \Rightarrow v = \sqrt{2} \left(\frac{4\pi^2 r_0^3 g}{T_0^2} \right)^{1/4}$$

$$r_0 = 670900 \text{ km} = 6.7090 \cdot 10^8 \text{ m}$$

$$T_0 = 3.5512 \text{ días} = 306823.68 \text{ s}$$

$$g = 24.78 \text{ m/s}^2$$

$$v = \sqrt{2} \left(\frac{4\pi^2 (6.7090 \cdot 10^8 \text{ m})^3 24.78 \text{ m/s}^2}{(306823.68 \text{ s})^2} \right)^{1/4} =$$

$$= 59522.23 \text{ m/s} = \boxed{59.5 \text{ km/s}}$$

Autor: Iván Cabrera Álvarez

Segunda manera: Hemos calculado ν en función de GM y R . (4.38)
 Calculamos los valores numéricos de GM y R y los sustituimos en $\nu = \nu(GM, R)$.

$$GM = \frac{4\pi^2 z_0^3}{T_0^2} = \frac{4\pi^2 (6.7090 \cdot 10^8 \text{ m})^3}{(306823.68 \text{ s})^2} = 1.266355 \cdot 10^{17} \text{ m}^3/\text{s}^2$$

$$R = \sqrt{\frac{GM}{g}} = \sqrt{\frac{1.266355 \cdot 10^{17} \text{ m}^3/\text{s}^2}{24.78 \text{ m/s}^2}} = 71486993.2 \text{ m}$$

$$\begin{aligned} \nu &= \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2 \cdot 1.266355 \cdot 10^{17} \text{ m}^3/\text{s}^2}{71486993.2 \text{ m}}} = 59522.23 \text{ m/s} = \\ &= \boxed{59.5 \text{ km/s}} \end{aligned}$$

Tercera manera: Calculamos ν en función de g y R . Calculamos el valor numérico de R y sustituimos los valores numéricos de g y R en $\nu = \nu(g, R)$.

$$\left. \begin{aligned} \nu &= \sqrt{\frac{2GM}{R}} \\ g &= \frac{GM}{R^2} \Rightarrow GM = gR^2 \end{aligned} \right\} \Rightarrow \nu = \sqrt{\frac{2gR^2}{R}} = \sqrt{2gR}$$

$$GM = \frac{4\pi^2 z_0^3}{T_0^2} = 1.266355 \cdot 10^{17} \text{ m}^3/\text{s}^2$$

$$R = \sqrt{\frac{GM}{g}} = 71486993.2 \text{ m}$$

$$\begin{aligned} \nu &= \sqrt{2gR} = \sqrt{2 \cdot 24.78 \text{ m/s}^2 \cdot 71486993.2 \text{ m}} = 59522 \text{ m/s} = \\ &= \boxed{59.5 \text{ km/s}} \end{aligned}$$

4.43

$z_0 = R + h = 6370 + 8000 \text{ km} = 1.4370 \cdot 10^7 \text{ m}$ (4.39)

$t = 0$ initial $z = 0$
 $t = t_s$ final $0 < t < t_s$
 $0 < z < h$

$\Delta t_s = \int_0^{t_s} dt$

$v = \frac{dz}{dt} \Rightarrow dt = \frac{dz}{v(z)}$

$\Rightarrow \Delta t_s = \int_{z(0)}^{z(t_s)} \frac{dz}{v(z)}$

$z(t_s) = h = 8000 \text{ km} = 8 \cdot 10^6 \text{ m}$

$z(0) = 0$

Calculamos $v(z)$:

$E_k = - \frac{GMm}{z_0}$

$E_g = \frac{m}{2} v^2 - \frac{GMm}{h-z}$

$\Rightarrow - \frac{GMm}{z_0} = \frac{m}{2} v^2 - \frac{GMm}{h-z} \Rightarrow$

$E_k = E_g$

$\Rightarrow \frac{v^2}{2} = \frac{GM}{h-z} - \frac{GM}{z_0} \Rightarrow v(z) = \sqrt{2GM \left(\frac{1}{h-z} - \frac{1}{z_0} \right)}$

$\Delta t_s = \int_{z(0)}^{z(t_s)} \frac{dz}{v(z)} = \int_0^h \frac{dz}{\sqrt{2GM \left(\frac{1}{h-z} - \frac{1}{z_0} \right)}} = \left[s = h-z \right] =$

$= \int_h^0 \frac{(-ds)}{\sqrt{2GM \left(\frac{1}{s} - \frac{1}{z_0} \right)}} = \int_0^h \frac{ds}{\sqrt{2GM \left(\frac{1}{s} - \frac{1}{z_0} \right)}} = \left[x = \frac{s}{z_0} \right] =$

$= \int_0^{h/z_0} \frac{z_0 dx}{\sqrt{2GM} \sqrt{\frac{1}{x} - 1}} = \frac{z_0 \sqrt{z_0}}{\sqrt{2GM}} \int_0^{h/z_0} \frac{dx}{\sqrt{\frac{1}{x} - 1}} = \frac{z_0 \sqrt{z_0}}{\sqrt{2GM}} J$

Autor: Iván Cabria Álvarez

$$J \equiv \int_0^{h/z_0} \frac{dx}{\sqrt{\frac{1}{x} - 1}} = \left[\arctan\left(\sqrt{\frac{x}{1-x}}\right) - \sqrt{x-x^2} \right]_0^{h/z_0}$$

(4.40)

$$J = \arctan\left(\sqrt{\frac{h/z_0}{1-h/z_0}}\right) - \sqrt{\frac{h}{z_0} - \frac{h^2}{z_0^2}} - \arctan(0) + \sqrt{0-0^2}$$

$$\frac{h}{z_0} = \frac{8000}{14370} = 0.5567153$$

$$\arctan\left(\sqrt{\frac{h/z_0}{1-h/z_0}}\right) = 48.256558 \text{ grados} = 0.8422358 \text{ radianes}$$

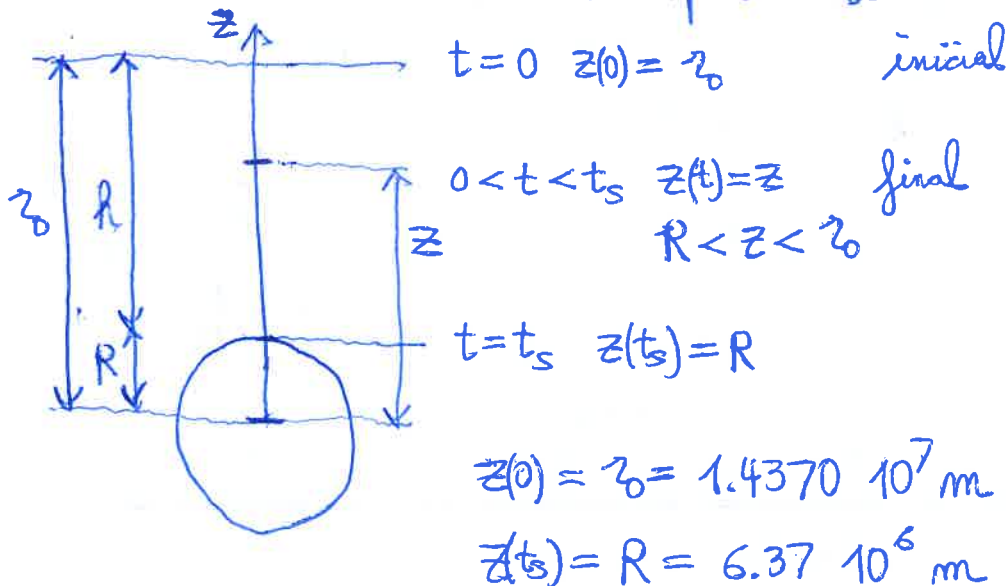
$$\sqrt{\frac{h}{z_0} - \frac{h^2}{z_0^2}} = 0.4967728$$

$$J = 1.3390087$$

$$\Delta t_s = \frac{z_0 \sqrt{z_0}}{\sqrt{2GM}} J = \frac{(1.4370 \cdot 10^7 \text{ m})^{3/2} \cdot 1.3390087}{\sqrt{2 \cdot 6.67 \cdot 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2} \cdot 5.9720 \cdot 10^{24} \text{ kg}}} =$$

$$= \frac{5.4473484 \cdot 10^{10} \cdot 1.3390087}{28225251} \text{ s} = \boxed{2584.2 \text{ s}}$$

Otra manera de resolver este problema:



Autor: Yoda Cabris Alvares

$$\left. \begin{aligned} \Delta t_s &= \int_0^{t_s} dt \\ v &= -\frac{dz}{dt} \Rightarrow dt = -\frac{dz}{v(z)} \end{aligned} \right\} \Rightarrow \Delta t_s = - \int_{z(0)}^{z(t_s)} \frac{dz}{v(z)}$$

Calculamos $v(z)$:

$$\left. \begin{aligned} E_x &= -\frac{GMm}{r_0} \\ E_f &= \frac{m}{2} v^2(z) - \frac{GMm}{z} \\ E_x &= E_f \end{aligned} \right\} \Rightarrow -\frac{GMm}{r_0} = \frac{m}{2} v^2(z) - \frac{GMm}{z} \Rightarrow$$

$$\Rightarrow \frac{v^2(z)}{2} = \frac{GM}{z} - \frac{GM}{r_0} \Rightarrow v(z) = \sqrt{2GM \left(\frac{1}{z} - \frac{1}{r_0} \right)}$$

$$\Delta t_s = - \int_{z(0)}^{z(t_s)} \frac{dz}{v(z)} = - \int_{r_0}^R \frac{dz}{\sqrt{2GM \left(\frac{1}{z} - \frac{1}{r_0} \right)}} = \left\{ \begin{aligned} x &= \frac{z}{r_0} \\ dx &= \frac{dz}{r_0} \end{aligned} \right\} =$$
$$= - \int_1^{R/r_0} \frac{r_0 dx}{\sqrt{2GM} \sqrt{\frac{1}{x} - \frac{1}{r_0}}} = - \frac{r_0 \sqrt{r_0}}{\sqrt{2GM}} \int_1^{R/r_0} \frac{dx}{\sqrt{\frac{1}{x} - 1}} =$$

$$= \frac{r_0 \sqrt{r_0}}{\sqrt{2GM}} \int_{R/r_0}^1 \frac{dx}{\sqrt{\frac{1}{x} - 1}} = \frac{r_0 \sqrt{r_0}}{\sqrt{2GM}} J$$

$$J \equiv \int_{R/r_0}^1 \frac{dx}{\sqrt{\frac{1}{x} - 1}} = \left[\arctan \left(\sqrt{\frac{x}{1-x}} \right) - \sqrt{x-x^2} \right]_{R/r_0}^1$$

$$J = \arctan(\infty) - \arctan \left(\sqrt{\frac{R/r_0}{1-R/r_0}} \right) - \sqrt{1-1} + \sqrt{\frac{R}{r_0} - \left(\frac{R}{r_0} \right)^2}$$
$$= \frac{\pi}{2} - \arctan \left(\sqrt{\frac{f}{1-f}} \right) + \sqrt{f-f^2}$$

$$f \equiv \frac{R}{r_0} = \frac{6370}{14370} = 0.4432846$$

Autor: Iván Cabris Alvarado

$$\sqrt{f - f^2} = 0.4967729$$

$$\arctan\left(\sqrt{\frac{f}{1-f}}\right) = 41.743438 \text{ grados} = 0.7285604 \text{ radianes}$$

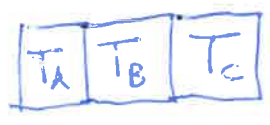
$$\Delta t_s = \frac{2\sqrt{20}}{\sqrt{2GM}} J = \frac{(1.4370 \cdot 10^7 \text{ m})^{3/2} \left[\frac{\pi}{2} - 0.7285604 + 0.4967729 \right]}{\sqrt{2 \cdot 6.67 \cdot 10^{-11} \text{ m}^3 \text{kg}^{-1} \text{s}^{-2} \cdot 5.9720 \cdot 10^{24} \text{ kg}}} =$$

$$= \frac{5.4473484 \cdot 10^{10} \cdot 1.3390088}{28225251} \text{ s} = \boxed{2584.2 \text{ s}}$$

TEMA 5 CALOR Y TEMPERATURA

(5.1) a) La pared es diatérmica. Por tanto, conduce el calor y los dos sistemas alcanzarán la misma temperatura si permanecen en contacto durante un tiempo prolongado.

b) No hay razón para que deban estar en equilibrio térmico con un tercer objeto.



$T_A = T_B$ A y B están en equilibrio térmico
 T_C puede ser igual a T_A o diferente de T_A

(5.2)



$l_1(0)$: longitud de la barra 1 a 0°C
 t_c : temperatura en $^\circ\text{C}$

$$l_2(t_c) = l_2(0)(1 + \alpha_2 t_c)$$
$$l_1(t_c) = l_1(0)(1 + \alpha_1 t_c)$$

$l_2(0)$: longitud de la barra 2 a 0°C

$$\Delta l = l_1(t_c) - l_2(t_c) = l_1(0) - l_2(0) + l_1(0)\alpha_1 t_c - l_2(0)\alpha_2 t_c$$

Δl no depende de la temperatura $\Rightarrow l_1(t_c) - l_2(t_c) = l_1(0) - l_2(0) \Rightarrow$

$$\Rightarrow l_1(0)\alpha_1 t_c - l_2(0)\alpha_2 t_c = 0 \Rightarrow l_1(0)\alpha_1 t_c = l_2(0)\alpha_2 t_c \Rightarrow \frac{l_1(0)}{l_2(0)} = \frac{\alpha_2}{\alpha_1}$$

$$z = \frac{l_1(t_c)}{l_2(t_c)} = \frac{l_1(0)}{l_2(0)} \frac{1 + \alpha_1 t_c}{1 + \alpha_2 t_c} = \frac{\alpha_2}{\alpha_1} \frac{1 + \alpha_1 t_c}{1 + \alpha_2 t_c}$$

$l_1(0) > l_2(0)$.

$$\Rightarrow \alpha_2 > \alpha_1 \Rightarrow \alpha_2 = 1.7 \cdot 10^{-5} \text{ } ^\circ\text{C}^{-1}$$
$$\alpha_1 = 0.8 \cdot 10^{-5} \text{ } ^\circ\text{C}^{-1}$$

$$z(300^\circ\text{C}) = \frac{1.7 \cdot 10^{-5} \text{ } (^\circ\text{C})^{-1}}{0.8 \cdot 10^{-5} \text{ } (^\circ\text{C})^{-1}} \frac{1 + 0.8 \cdot 10^{-5} 300}{1 + 1.7 \cdot 10^{-5} 300} = \boxed{2.119}$$

(5.3)

$$V_{\text{mercurio}}(0^\circ\text{C}) = \frac{m_{\text{mercurio}}(0)}{\rho_{\text{mercurio}}(0)}$$

$$V_{\text{metal}}(0^\circ\text{C}) = \frac{m_{\text{metal}}(0)}{\rho_{\text{metal}}(0)}$$

Para que la dilatación aparente sea nula: $V_{\text{vaso}}(t^\circ\text{C}) = V_{\text{mercurio}}(t^\circ\text{C}) + V_{\text{metal}}(t^\circ\text{C})$
 t : temperatura en $^\circ\text{C}$
entre 0°C y $t^\circ\text{C}$

$$\Rightarrow V_{\text{vaso}}(0)(1 + K_{\text{v}} t) = V_{\text{mercurio}}(0)(1 + K_{\text{mercurio}} t) + V_{\text{metal}}(0)(1 + K_{\text{metal}} t)$$
$$V_{\text{vaso}}(0) = V_{\text{mercurio}}(0) + V_{\text{metal}}(0)$$

Autor: Iván Cabris Álvarez

$$\Rightarrow V_{\text{vaso}}(0) K_{\text{ro}} = V_{\text{mercurio}}(0) K_{\text{mercurio}} + V_{\text{metal}}(0) K_{\text{metal}}$$

$$V_{\text{vaso}}(0) = V_{\text{mercurio}}(0) + V_{\text{metal}}(0) = \frac{m_{\text{mercurio}}(0)}{\rho_{\text{mercurio}}(0)} + \frac{m_{\text{metal}}(0)}{\rho_{\text{metal}}(0)} \Rightarrow$$

$$\Rightarrow \frac{m_{\text{mercurio}}(0)}{\rho_{\text{mercurio}}(0)} K_{\text{ro}} + \frac{m_{\text{metal}}(0)}{\rho_{\text{metal}}(0)} K_{\text{ro}} = \frac{m_{\text{mercurio}}(0)}{\rho_{\text{mercurio}}(0)} K_{\text{mercurio}} +$$

$$+ \frac{m_{\text{metal}}(0)}{\rho_{\text{metal}}(0)} K_{\text{metal}} \Rightarrow \frac{m_{\text{mercurio}}(0)}{\rho_{\text{mercurio}}(0)} (K_{\text{ro}} - K_{\text{mercurio}}) = \frac{m_{\text{metal}}(0)}{\rho_{\text{metal}}(0)} (K_{\text{metal}} - K_{\text{ro}})$$

$$\Rightarrow \boxed{\frac{m_{\text{mercurio}}(0)}{m_{\text{metal}}(0)} = \frac{\rho_{\text{mercurio}}(0)}{\rho_{\text{metal}}(0)} \frac{K_{\text{metal}} - K_{\text{ro}}}{K_{\text{ro}} - K_{\text{mercurio}}}}$$

$$\textcircled{5.4} \quad Q = Q_1 + Q_2 + Q_3$$

Q_1 : calor para fundir el hielo

Q_2 : calor para calentar el agua desde 0°C hasta 100°C

Q_3 : calor para evaporar 5 g de agua

$$Q_1 = m L_f = 20 \text{ g } 80 \frac{\text{cal}}{\text{g}} = 1600 \text{ cal} = 1600 \cdot 4.18 \text{ J} = 6688 \text{ J}$$

$$Q_2 = m c_{\text{agua}} \Delta T = 20 \text{ g } 1 \frac{\text{cal}}{\text{g } ^\circ\text{C}} (100 - 0)^\circ\text{C} = 2000 \text{ cal} = 8360 \text{ J}$$

$$Q_3 = m_{\text{vapor agua}} L_v = 5 \text{ g } 540 \frac{\text{cal}}{\text{g}} = 2700 \text{ cal} = 2700 \cdot 4.18 \text{ J} = 11286 \text{ J}$$

$$\boxed{Q = 26334 \text{ J}}$$

$$\textcircled{5.5} \quad t = \frac{W_{\text{horno}}}{P_{\text{horno}}} \quad \text{Enememos que calcular } W_{\text{horno}} \text{ y } P_{\text{horno}}$$

$$P_{\text{horno}} = I_{\text{solar}} A_{\text{espejo}} 0.4 = 100 \frac{\text{W}}{\text{m}^2} \pi (0.3)^2 \text{ m}^2 0.4 = 11.31 \text{ W}$$

$$W_{\text{horno}} = Q_1 + Q_2 \quad (0.5 \text{ Kg de agua})$$

Q_1 : calor para calentar el agua desde 20°C hasta 100°C

Q_2 : calor para evaporar 0.5 Kg de agua

Autor: Yoán Abria Álvarez

$$Q_1 = m C_{\text{agua}} \Delta T = 0.5 \text{ kg} \cdot 1 \frac{\text{cal}}{\text{g}^\circ\text{C}} (100 - 20)^\circ\text{C} = 500 \text{ g} \cdot 1 \frac{\text{cal}}{\text{g}^\circ\text{C}} 80^\circ\text{C} \quad (5.3)$$

$$= 40\,000 \text{ cal} = 167\,200 \text{ J}$$

$$Q_2 = m L_v = 500 \text{ g} \cdot 540 \frac{\text{cal}}{\text{g}} = 270\,000 \text{ cal} = 1\,128\,600 \text{ J}$$

$$W_{\text{barro}} = 129\,580 \text{ J} \quad t = \frac{129\,580 \text{ J}}{11.31 \text{ W}} = 11\,457 \text{ s} = \boxed{31.83 \text{ horas}}$$

$$1 \text{ W} = 1 \frac{\text{J}}{\text{s}}$$

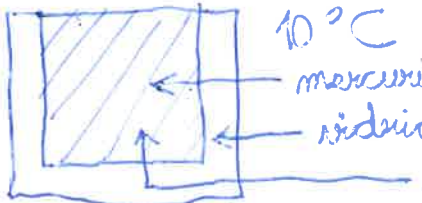
(5.6) El calor necesario para calentar una masa m es $Q = m C_e \Delta T$.
El calor específico C_e del agua es mayor que el calor específico del barro.
Por tanto:

$$\left. \begin{array}{l} Q_{\text{agua}} = m C_{\text{agua}} \Delta T \\ Q_{\text{barro}} = m C_{\text{barro}} \Delta T \\ C_{\text{agua}} > C_{\text{barro}} \end{array} \right\} \Rightarrow Q_{\text{agua}} > Q_{\text{barro}} \Rightarrow$$

Esta más energía
calentar un grado Celsius
100 kg de agua.

$$Q_{\text{agua}} = m C_{\text{agua}} \Delta T = 10^5 \text{ g} \cdot 1 \frac{\text{cal}}{\text{g}^\circ\text{C}} 1^\circ\text{C} = \boxed{100\,000 \text{ calorías}}$$

$$Q_{\text{barro}} = m C_{\text{barro}} \Delta T = 10^5 \text{ g} \cdot 0.22 \frac{\text{cal}}{\text{g}^\circ\text{C}} 1^\circ\text{C} = \boxed{22\,000 \text{ calorías}}$$

(5.7)  $V_{\text{mercurio}}(10^\circ\text{C}) = V_{\text{vaso}}(10^\circ\text{C}) = 60 \text{ cm}^3$
volumen de mercurio a 10°C = volumen vaso de vidrio a 10°C

Hay que encontrar: $V_{\text{mercurio}}(30^\circ\text{C}) - V_{\text{vaso}}(30^\circ\text{C})$

$$V_a(t) = V_a(0^\circ\text{C})(1 + K_a t)$$

$$\frac{V_{\text{mercurio}}(30^\circ\text{C})}{V_{\text{mercurio}}(10^\circ\text{C})} = \frac{1 + K_{\text{mercurio}} 30^\circ\text{C}}{1 + K_{\text{mercurio}} 10^\circ\text{C}} = \frac{1 + 1.8 \cdot 10^{-4} 30}{1 + 1.8 \cdot 10^{-4} 10} = 1.0035935 \Rightarrow$$

$$\Rightarrow V_{\text{mercurio}}(30^\circ\text{C}) = 60.2156 \text{ cm}^3$$

$$V_{\text{vidrio}} \equiv V_{\text{vaso}}_{\text{vidrio}}$$

$$\frac{V_{\text{vidrio}}(30^\circ\text{C})}{V_{\text{vidrio}}(10^\circ\text{C})} = \frac{1 + K_{\text{vidrio}} 30^\circ\text{C}}{1 + K_{\text{vidrio}} 10^\circ\text{C}} = 1.0005399 \Rightarrow V_{\text{vaso}}(30^\circ\text{C}) = 60.0324 \text{ cm}^3$$

tutor: Iván Gabría Álvarez

$$\boxed{V_{\text{mercurio}}(30^\circ\text{C}) - V_{\text{vaso}}(30^\circ\text{C}) = 0.1832 \text{ cm}^3}$$

(5.4)

$$(5.8) \quad t_c = \frac{5}{9} (t_F - 32) = \frac{5}{9} (86 - 32) = 30^\circ\text{C}$$

La longitud del péndulo aumenta con la temperatura: $l(30^\circ\text{C}) > l(15^\circ\text{C})$

$$T = 2\pi \sqrt{\frac{l}{g}} \Rightarrow T(30^\circ\text{C}) = 2\pi \sqrt{\frac{l(30^\circ\text{C})}{g}} > 2\pi \sqrt{\frac{l(15^\circ\text{C})}{g}} = T(15^\circ\text{C})$$

Según el enunciado del problema:

$$15^\circ\text{C} \quad 86400 \text{ s} = n T(15^\circ\text{C})$$

$$30^\circ\text{C} \quad 86415 \text{ s} = n T(30^\circ\text{C})$$

n : número de periodos efectuados por el péndulo en un día

$$\left. \begin{aligned} \frac{T(30^\circ\text{C})}{T(15^\circ\text{C})} &= \frac{86415}{86400} & \frac{T(30^\circ\text{C})}{T(15^\circ\text{C})} &= \frac{\sqrt{l(30^\circ\text{C})}}{\sqrt{l(15^\circ\text{C})}} \\ l(t) &= l(0^\circ\text{C}) (1 + \alpha t) \Rightarrow \frac{T(30^\circ\text{C})}{T(15^\circ\text{C})} = \sqrt{\frac{1 + \alpha 30}{1 + \alpha 15}} \end{aligned} \right\} \Rightarrow$$

$$\Rightarrow \left(\frac{86415}{86400} \right)^2 = \frac{1 + \alpha 30}{1 + \alpha 15} \quad a \equiv \left(\frac{86415}{86400} \right)^2 \Rightarrow$$

$$\Rightarrow a = \frac{1 + \alpha 30}{1 + \alpha 15} \Rightarrow a(1 + \alpha 15) = 1 + \alpha 30 \Rightarrow a - 1 = \alpha 30 - a\alpha 15$$

$$\Rightarrow \alpha = \frac{a - 1}{30 - a15} = \frac{3.472524 \cdot 10^{-4}}{14.994791} = \boxed{2.316 \cdot 10^{-5} (\text{C})^{-1}}$$

Autor: Iván Gabría Álvarez

(5.9) (a) Q_1 : calor absorbido para calentar el recipiente metálico desde -20°C hasta 0°C . (5.5)

Q_2 : calor absorbido para calentar el hielo desde -20°C hasta 0°C .

Q_3 : calor cedido por los 15 Kg de agua al pasar de 100°C a 0°C .

Q_X : calor absorbido para fundir X gramos de hielo.
Esta es la hipótesis: Se funden X gramos de hielo. $0 < X < 100\,000\text{ g}$

$$Q_1 > 0 \quad Q_2 > 0 \quad Q_3 < 0 \quad Q_X > 0 \quad T_{\text{final}} = 0^\circ\text{C}$$

$$Q_1 = m_{\text{metal}} C_{\text{metal}} (T_{\text{final}} - T_{\text{inicial metal}}) = 273.15\text{ K}$$

$$Q_2 = m_{\text{hielo}} C_{\text{hielo}} (T_{\text{final}} - T_{\text{inicial hielo}})$$

$$Q_3 = m_{\text{agua}} C_{\text{agua}} (T_{\text{final}} - T_{\text{inicial agua}})$$

$$Q_X = X L_f$$

$$Q_1 + Q_2 + Q_3 + Q_X = 0$$

$$T_{\text{inicial metal}} = -20^\circ\text{C} \quad T_{\text{inicial hielo}} = -20^\circ\text{C} \quad T_{\text{inicial agua}} = +100^\circ\text{C}$$

$$Q_1 = 30\,000\text{ g} \cdot 0.2 \frac{\text{cal}}{\text{g}^\circ\text{C}} \cdot 20^\circ\text{C} = +120\,000\text{ cal}$$

$$Q_2 = 100\,000\text{ g} \cdot 0.5 \frac{\text{cal}}{\text{g}^\circ\text{C}} \cdot 20^\circ\text{C} = +1\,000\,000\text{ cal}$$

$$Q_3 = 15\,000\text{ g} \cdot 1 \frac{\text{cal}}{\text{g}^\circ\text{C}} \cdot (0 - 100)^\circ\text{C} = -1\,500\,000\text{ cal}$$

$$Q_X = X \cdot 80 \text{ cal/g} = X \cdot 80 \text{ cal}$$

tutor: Juan Cabria Álvarez

$$120\,000 + 1\,000\,000 - 1\,500\,000 + X \cdot 80 = 0 \Rightarrow X = \frac{380\,000}{80} \Rightarrow$$

$$\Rightarrow X = 4750\text{ g} < 100\,000\text{ g de hielo} \Rightarrow \text{La hipótesis es correcta.}$$

$100 - 4.75\text{ Kg} =$	95.25 Kg de hielo a 0°C
$15 + 4.75\text{ Kg} =$	19.75 Kg de agua a 0°C

(5.9) (b) Q_1 : calor absorbido para calentar el recipiente desde 0°C hasta $+25^\circ\text{C}$ (5.6)

Q_2 : calor absorbido para fundir los 95.25 kg de hielo

Q_3 : calor absorbido para calentar 115 kg de agua desde 0°C hasta $+25^\circ\text{C}$

Q_4 : calor cedido al condensar Y gramos de vapor de agua

Q_5 : calor cedido al enfriar Y gramos de agua desde $+100^\circ\text{C}$ hasta $+25^\circ\text{C}$

$$Q_1 > 0 \quad Q_2 > 0 \quad Q_3 > 0 \quad Q_4 < 0 \quad Q_5 < 0 \quad Y > 0$$

$$Q_1 + Q_2 + Q_3 + Q_4 + Q_5 = 0$$

$$Q_1 = 30\,000\text{ g} \cdot 0.2 \frac{\text{cal}}{\text{g}^\circ\text{C}} \cdot 25^\circ\text{C} = +150\,000\text{ cal}$$

$$Q_2 = 95\,250\text{ g} \cdot L_f = 95\,250\text{ g} \cdot 80 \frac{\text{cal}}{\text{g}} = +7\,620\,000\text{ cal}$$

$$Q_3 = 115\,000\text{ g} \cdot 1 \frac{\text{cal}}{\text{g}^\circ\text{C}} \cdot 25^\circ\text{C} = +2\,875\,000\text{ cal}$$

$$Q_4 = -Y L_v = -Y \cdot 540 \frac{\text{cal}}{\text{g}} = -Y \cdot 540\text{ cal} < 0$$

$$Q_5 = -Y \cdot 1 \frac{\text{cal}}{\text{g}^\circ\text{C}} \cdot 75^\circ\text{C} = -75Y\text{ cal} < 0$$

$$Q_5 = Y \cdot c_{\text{agua}} (T_{\text{final}} - T_{\text{inicial}}) = Y \cdot 1 \frac{\text{cal}}{\text{g}^\circ\text{C}} (25 - 100)^\circ\text{C} = -75Y\text{ cal}$$

$$150\,000 + 7\,620\,000 + 2\,875\,000 - Y \cdot 540 - 75Y = 0 \Rightarrow$$

$$\Rightarrow Y = \frac{10\,645\,000}{615} = \boxed{17\,309\text{ g de vapor de agua}}$$

Autor: Juan Abria Alvaro

(5.7)

(5.10) a) Suponemos que se funde todo el hielo y que $T_{\text{final}} > 0^\circ\text{C}$.

Q_1 : calor absorbido para calentar el hielo desde 268 K hasta 273.15 K.

Q_2 : calor absorbido para fundir el hielo.

Q_3 : calor absorbido para calentar el agua, que antes era hielo, desde 273.15 K hasta T_{final} K.

Q_4 : calor cedido para enfriar el agua desde 368 K hasta T_{final} K.

$$Q_1 > 0 \quad Q_2 > 0 \quad Q_3 > 0 \quad Q_4 < 0 \quad T_{\text{final}} < 368 \text{ K}$$

$$Q_1 + Q_2 + Q_3 + Q_4 = 0$$

$$Q_1 = 1000 \text{ g} \cdot 0.5 \frac{\text{cal}}{\text{g K}} (273.15 - 268) \text{ K} = +2575 \text{ cal} > 0$$

$$Q_2 = 1000 \text{ g} \cdot 80 \frac{\text{cal}}{\text{g}} = +80\,000 \text{ cal} > 0$$

$$Q_3 = 1000 \text{ g} \cdot 1 \frac{\text{cal}}{\text{g K}} (T_{\text{final}} - 273.15) \text{ K} = 1000 (T_{\text{final}} - 273.15) \text{ cal} > 0$$

$$Q_4 = 1000 \text{ g} \cdot 1 \frac{\text{cal}}{\text{g K}} (T_{\text{final}} - 368) \text{ K} < 0$$

$$Q_1 + Q_2 + Q_3 + Q_4 = 0 \Rightarrow 82575 + 1000 (T_{\text{final}} - 273.15) + 1000 (T_{\text{final}} - 368) = 0$$

$$\Rightarrow 558575 = 2000 T_{\text{final}} \Rightarrow \boxed{T_{\text{final}} = 279.29 \text{ K} = +6.14^\circ\text{C}} \\ \boxed{2 \text{ Kg de agua}}$$

$$T_{\text{final}} > 0^\circ\text{C} = 273.15 \text{ K} \Rightarrow \text{La hipótesis es correcta.}$$

Autor: Iván Cabria Álvarez

(5.10) (b) Suponemos que los 2 kg de hielo se funden y ^{que la mezcla alcanza una} temperatura final $T_f > 273.15 \text{ K} = 0^\circ\text{C}$. (5.8)

Q_1 : calor absorbido para calentar 2 kg de hielo desde 268 K hasta 273.15 K.

Q_2 : calor absorbido para fundir 2 kg de hielo.

Q_3 : calor absorbido para calentar 2 kg de agua, que antes era hielo, desde 273.15 K hasta T_f K.

Q_4 : calor cedido para enfriar 1 kg de agua desde 368 K hasta T_f K.

$$Q_1 > 0 \quad Q_2 > 0 \quad Q_3 > 0 \quad Q_4 < 0 \quad T_f < 368 \text{ K}$$

$$Q_1 = 2000 \text{ g} \cdot 0.5 \frac{\text{cal}}{\text{g K}} (273.15 - 268) \text{ K} = +5150 \text{ cal} > 0$$

$$Q_2 = 2000 \text{ g} \cdot 80 \frac{\text{cal}}{\text{g}} = +160\,000 \text{ cal} > 0$$

$$Q_3 = 2000 \text{ g} \cdot 1 \frac{\text{cal}}{\text{g K}} (T_f - 273.15) \text{ K} = 2000 (T_f - 273.15) \text{ cal}$$

$$Q_4 = 1000 \text{ g} \cdot 1 \frac{\text{cal}}{\text{g K}} (T_f - 368) \text{ K} = 1000 (T_f - 368) \text{ cal} < 0$$

$$Q_1 + Q_2 + Q_3 + Q_4 = 0 \Rightarrow 165150 + 2000 (T_f - 273.15) + 1000 (T_f - 368) = 0 \Rightarrow$$

$$\Rightarrow 749150 = 3000 T_f \Rightarrow T_f = 249.72 \text{ K} < 273.15 \text{ K}$$

¶ Hemos obtenido un valor de T_f inferior a 273.15 K, en contradicción con la hipótesis de $T_f > 273.15 \text{ K}$ y de que todo el hielo se funde.

¶ Planteamos otra hipótesis. Suponemos que se funden X gramos de hielo y que, por tanto, $T_f = 0^\circ\text{C} = 273.15 \text{ K}$. $0 < X < 2000 \text{ g}$

$$Q_1 = 2000 \text{ g} \cdot 0.5 \frac{\text{cal}}{\text{g K}} (273.15 - 268) \text{ K} = +5150 \text{ cal} > 0$$

$$Q_2 = X \cdot 80 \frac{\text{cal}}{\text{g}} = 80X \text{ cal} > 0$$

$$Q_3 \text{ no existe.} \quad Q_4 = 1000 (T_f - 368) \text{ cal} = -94850 \text{ cal} < 0$$

$$Q_1 + Q_2 + Q_4 = 0 \Rightarrow 5150 + 80X - 94850 = 0 \Rightarrow$$

$$\Rightarrow X = 1121.25 \text{ g} < 2000 \text{ g} \Rightarrow \text{La hipótesis es correcta.}$$

$1000 \text{ g} + 1121.25 \text{ g} \simeq$	$T_f = 273.15 \text{ K}$
$2000 \text{ g} - 1121.25 \text{ g} \simeq$	2.12 Kg de agua
	0.88 Kg de hielo

Autor: Iván Cabria Álvarez

$$(5.11) \quad Q = m c_e \Delta T \Rightarrow c_e = \frac{Q}{m \Delta T} = \frac{25000 \text{ cal}}{600 \text{ g } 10^\circ \text{C}} = \boxed{4.17 \frac{\text{cal}}{\text{g}^\circ \text{C}}}$$

$$C_e = m \cdot c_e = \frac{Q}{\Delta T} = \frac{25000 \text{ cal}}{10^\circ \text{C}} = \boxed{2500 \frac{\text{cal}}{^\circ \text{C}}}$$

(5.12) Q_1 : calor para calentar el trozo de plata desde 22°C hasta 961°C

Q_2 : calor para fundir el trozo de plata Ag : Símbolo químico de la plata

$$Q_1 = m c_{\text{Ag}} \Delta T = 10 \text{ g } 0.056 \frac{\text{cal}}{\text{g}^\circ \text{C}} (961 - 22)^\circ \text{C} = 525.84 \text{ cal}$$

$$Q_2 = m L_{\text{Ag}} = 10 \text{ g } 21.1 \frac{\text{cal}}{\text{g}} = 211 \text{ cal}$$

$$Q = Q_1 + Q_2 = \boxed{736.84 \text{ cal}}$$

$$(5.13) \text{ a) } T = m t_F + m$$

$$\left. \begin{array}{l} 273.15 = m 32 + m \\ 373.15 = m 212 + m \end{array} \right\} \Rightarrow 100 = m 180 \Rightarrow m = \frac{100}{180} = \frac{5}{9}$$

$$m = 273.15 - m 32 = 273.15 - \frac{5}{9} 32 = 255.37\hat{2}$$

$$\boxed{T = \frac{5}{9} t_F + 255.37\hat{2}}$$

$$T = \frac{5}{9} t_F + 273.15 - \frac{5}{9} 32 = \boxed{\frac{5}{9} (t_F - 32) + 273.15}$$

Autor: Iván Cabria Álvarez

$$T = 0K \Rightarrow 0 = \frac{5}{9} t_F + 255.372 \Rightarrow t_F = -255.372 \frac{9}{5} = \boxed{-459.67^\circ F}$$

$$c) t_F = -500^\circ F \Rightarrow T = \frac{5}{9} (-500) + 255.372 = \underline{\underline{-22.41 K}} \quad \times$$

$\times$: No existen temperaturas Kelvin negativas $\Rightarrow$

$\Rightarrow$ No es posible una temperatura de $-500^\circ F$

(5.14) a) El calor del agua se transmite primero a las zonas internas del vidrio y más tarde a las zonas externas. El calor del agua produce una dilatación del vidrio. Las zonas internas se dilatan antes que las zonas externas. Si las zonas internas se dilatan ^y mucho más rápido que las zonas externas, entonces el vidrio se romperá. ^{más}

b) Si el vaso de vidrio es delgado, entonces es posible que la dilatación llegue a las zonas externas sin que se rompa el vaso. Es más probable que las zonas internas y externas se dilaten sin romper el vaso.

Si el vaso de vidrio es grueso, entonces es más probable que las zonas internas se dilaten más y mucho antes que las zonas externas, produciéndose la ruptura.

(5.15) Al soplar, quitamos las moléculas que están sobre la superficie de la sopa, vaciando de materia el espacio que hay sobre la superficie. Las moléculas más calientes de la sopa no chocan ahora con las moléculas que estaban encima de la superficie de la sopa y por tanto, escapan fácilmente y más rápido de la sopa. Cuanto más rápido (más caliente) sea una molécula, más probable será que escape. Esto produce que escapen más moléculas rápidas (calientes) que antes de soplar y que en la sopa se queden las moléculas lentas (frías). Por tanto, la sopa se enfría.

Autor: Iván Abria Alvaro

$$\left. \begin{array}{l} (5.16) \quad t_F = -4^\circ\text{F} \\ t_C = \frac{5}{9} (t_F - 32) \end{array} \right\} \Rightarrow t_C = \frac{5}{9} (-4 - 32)^\circ\text{C} = -\frac{5}{9} 36^\circ\text{C} = -20^\circ\text{C} \quad (5.17)$$

$$\left. \begin{array}{l} t_F = +212^\circ\text{F} \\ t_C = \frac{5}{9} (t_F - 32) \end{array} \right\} \Rightarrow t_C = \frac{5}{9} (212 - 32)^\circ\text{C} = \frac{5}{9} 180^\circ\text{C} = \frac{5}{9} 9 \cdot 20^\circ\text{C} = 100^\circ\text{C}$$

$$m_{\text{hielo}} = 100 \text{ Kg} = 100\,000 \text{ g}$$

$$m_{\text{agua}} = 1 \text{ Kg} = 1000 \text{ g}$$

Suponemos que la temperatura final, T_f , es 0°C . Hay varias escenarios en los que $T_f = 0^\circ\text{C}$. Esos escenarios son combinaciones de los estados ^{finales} de la masa de hielo inicial y de la masa de agua inicial.

Estados finales del hielo: No se funde el hielo, se funde todo o parte del hielo.

Estados finales del agua: No se congela el agua, se congela todo o parte del agua.

^{o en lugar de} Antes de considerar todos los escenarios, vamos a calcular algunos calores y a comparar esos calores. Suponemos que $T_f = 0^\circ\text{C}$, ^{que el hielo no se funde y que el kilo de agua se congela.}

Q_1 : calor absorbido para calentar los 100 Kg de hielo desde -20°C hasta 0°C .

Q_2 : calor cedido para enfriar el kilo de agua desde 100°C hasta 0°C .

Q_3 : calor cedido para congelar el kilo de agua.

$$\begin{array}{ccc} Q_1 > 0 & Q_2 < 0 & Q_3 < 0 \end{array}$$

$$Q_1 = m_{\text{hielo}} C_{\text{hielo}} (T_f - T_{\text{inicial}}^{\text{hielo}}) = 100\,000 \text{ g} \cdot 0.5 \frac{\text{cal}}{\text{g}^\circ\text{C}} (0 - (-20))^\circ\text{C} = 1\,000\,000 \text{ cal}$$

$$Q_2 = m_{\text{agua}} C_{\text{agua}} (T_f - T_{\text{inicial}}^{\text{agua}}) = 1000 \text{ g} \cdot 1 \frac{\text{cal}}{\text{g}^\circ\text{C}} (0 - 100)^\circ\text{C} = -100\,000 \text{ cal}$$

$$Q_3 = - m_{\text{agua}} L_f = -1000 \text{ g} \cdot 80 \frac{\text{cal}}{\text{g}} = -80\,000 \text{ cal}$$

^{que se convierte en hielo}

Observamos que la suma de Q_1 , Q_2 y Q_3 es bastante mayor que cero:

$Q_1 + Q_2 + Q_3 = 820\,000 \text{ cal}$. La suma de los calores absorbidos, Q_1 , es muy superior a la suma de los ^{módulos de los} calores cedidos:

$$Q_1 \gg |Q_2| + |Q_3|$$

Tutor: Iván Cabris Álvarez

$$1\,000\,000 \gg |-100\,000| + |-80\,000| = 180\,000$$

Esto significa que el calor cedido, $Q_2 + Q_3$, no es suficiente para calentar el hielo hasta 0°C .

Es necesario ceder más calor y absorber menos calor. Por tanto, $T_f < 0^\circ\text{C}$.

Suponemos que $T_f < 0^\circ\text{C}$.

Q_1 debe ser redefinido y recalculado. Q_2 y Q_3 tienen el mismo significado y valor numérico. Queremos que defina un nuevo calor, Q_4 .

Q_1 : calor absorbido para calentar los 100 kg de hielo desde -20°C hasta $T_f^\circ\text{C}$.

Q_4 : calor cedido para enfriar el kilo de hielo, que antes era agua, desde 0°C hasta $T_f^\circ\text{C}$.

La temperatura final de la mezcla debe estar entre -20°C y 0°C .

T_f está en $^\circ\text{C}$.

$$Q_1 > 0 \quad Q_2 < 0 \quad Q_3 < 0 \quad Q_4 < 0$$

$$Q_1 + Q_2 + Q_3 + Q_4 = 0$$

$$Q_1 = m_{\text{hielo}} c_{\text{hielo}} (T_f - T_{\text{inicial}}^{\text{hielo}}) = 100\,000 \text{ g} \cdot 0.5 \frac{\text{cal}}{\text{g}^\circ\text{C}} (T_f - (-20))^\circ\text{C} = 50\,000 (T_f + 20) \text{ cal}$$

$$T_f > -20^\circ\text{C} \Rightarrow Q_1 = 50\,000 (T_f + 20) > 0 \Rightarrow \text{Hemos comprobado que } Q_1, \text{ un calor absorbido, es mayor que cero.}$$

$$Q_2 = -100\,000 \text{ cal} < 0$$

$$Q_3 = -80\,000 \text{ cal} < 0$$

$$Q_4 = m_{\text{agua}} c_{\text{hielo}} (T_f - T_{\text{inicial}}^{\text{a-h}}) = 1000 \text{ g} \cdot 0.5 \frac{\text{cal}}{\text{g}^\circ\text{C}} (T_f - 0)^\circ\text{C} = 500 T_f \text{ cal}$$

$$T_{\text{inicial}}^{\text{a-h}} = 0^\circ\text{C}$$

$$T_f < 0 \quad (T_f < 0^\circ\text{C}) \Rightarrow Q_4 = 500 T_f < 0 \Rightarrow$$

$\Rightarrow$ Hemos comprobado que Q_4 , un calor cedido, es menor que cero.

$$Q_1 + Q_2 + Q_3 + Q_4 = 0 \Rightarrow 50\,000 (T_f + 20) - 180\,000 + 500 T_f = 0 \Rightarrow$$

$$\Rightarrow 50\,500 T_f + 1000\,000 - 180\,000 = 0 \Rightarrow$$

$$\Rightarrow T_f = -\frac{820\,000}{50\,500} = \boxed{-16.2376^\circ\text{C}} \quad \text{tutor: Iván Cabria Álvaro}$$

$T_f < 0^\circ\text{C}$ y $T_f > -20^\circ\text{C} \Rightarrow$ La hipótesis es correcta.

$$T_f \text{ en } ^\circ\text{F} = 32 + \frac{9}{5} T_f \text{ en } ^\circ\text{C} = \boxed{+2.7723^\circ\text{F}}$$

$$T_f \text{ en } \text{K} = T_f \text{ en } ^\circ\text{C} + 273.15 = \boxed{256.9124 \text{ K}}$$

$$(5.17) \quad V(t_c) = V(0^\circ\text{C})(1 + K t_c) \Rightarrow \frac{V(t_c)}{V(0^\circ\text{C})} = 1 + K t_c \Rightarrow$$

(5.13)

$$\Rightarrow K = \frac{V(t_c)}{V(0^\circ\text{C}) t_c} - \frac{1}{t_c}$$

$$\left. \begin{array}{l} V(100^\circ\text{C}) = 140 \text{ cm}^3 \\ V(0^\circ\text{C}) = 100 \text{ cm}^3 \end{array} \right\} \begin{array}{l} t_c = 100^\circ\text{C} \\ \Rightarrow K = \frac{140}{100 \cdot 100} - \frac{1}{100} (^\circ\text{C})^{-1} \\ \Rightarrow K = 0.004 (^\circ\text{C})^{-1} \end{array}$$

$$V(t_c) \rightarrow 0$$

$$t_c \rightarrow t_{\text{cero absoluto}}$$

$$\Rightarrow 0 = V(0^\circ\text{C})(1 + K t_{\text{cero absoluto}}) \Rightarrow$$

$$\Rightarrow 1 + K t_{\text{cero absoluto}} = 0 \Rightarrow t_{\text{cero absoluto}} = -\frac{1}{K} = -\frac{1}{0.004 (^\circ\text{C})^{-1}} \Rightarrow$$

$$\Rightarrow t_{\text{cero absoluto}} = -250^\circ\text{C}$$

$$(5.18) \quad \left. \begin{array}{l} t_F = 32^\circ\text{F} \leftrightarrow t_c = 0^\circ\text{C} \\ t_F = m t_c + m \end{array} \right\} \Rightarrow 32 = m \cdot 0 + m \Rightarrow m = 32$$

$$\left. \begin{array}{l} t_F = 212^\circ\text{F} \leftrightarrow t_c = 100^\circ\text{C} \\ t_F = m t_c + m \end{array} \right\} \Rightarrow \left. \begin{array}{l} 212 = m \cdot 100 + m \\ m = 32 \end{array} \right\} \Rightarrow 212 = m \cdot 100 + 32 \Rightarrow$$

$$\Rightarrow 180 = m \cdot 100 \Rightarrow m = \frac{180}{100} = \frac{18}{10} = \frac{9 \cdot 2}{5 \cdot 2} = \frac{9}{5}$$

$$t_F = \frac{9}{5} t_c + 32$$

(5.19)

$$T = 273.15 \text{ K} \leftrightarrow t_c = 0^\circ\text{C} \text{ y } T = m t_c + m \Rightarrow 273.15 = m \cdot 0 + m \Rightarrow m = 273.15$$

$$\left. \begin{array}{l} T = 373.15 \text{ K} \leftrightarrow t_c = 100^\circ\text{C} \\ T = m t_c + m \end{array} \right\} \Rightarrow \left. \begin{array}{l} 373.15 = m \cdot 100 + m \\ m = 273.15 \end{array} \right\} \Rightarrow 373.15 = m \cdot 100 + 273.15 \Rightarrow$$

$$\Rightarrow 373.15 - 273.15 = m \cdot 100 \Rightarrow 100 = m \cdot 100 \Rightarrow m = 1$$

$$T = t_c + 273.15$$

Autor: Iván Gabrila Álvarez

5.20 T: temperatura en K. t_F : temperatura en grados Fahrenheit. (5.14)

1ª manera: En función de t_F .

$$T = t_F$$

$$T = m t_F + n = \frac{5}{9} t_F + 255.372 \Rightarrow t_F = \frac{5}{9} t_F + 255.372 \Rightarrow$$

$$\Rightarrow t_F \left(1 - \frac{5}{9}\right) = 255.372 \Rightarrow \frac{4}{9} t_F = 255.372 \Rightarrow t_F = \frac{9}{4} 255.372 \Rightarrow$$

$$\Rightarrow \boxed{t_F = 574.5875^\circ\text{F} = 574.5875\text{ K} = T} \quad 1^\text{a} \text{ manera}$$

2ª manera: En función de T.

Calculamos $t_F = mT + n$:

$$t_F = 32^\circ\text{F} \leftrightarrow T = 273.15\text{ K}$$

$$t_F = mT + n \Rightarrow 32 = m \cdot 273.15 + n \Rightarrow n = 32 - m \cdot 273.15$$

$$t_F = 212^\circ\text{F} \leftrightarrow T = 373.15\text{ K}$$

$$t_F = mT + n \Rightarrow 212 = m \cdot 373.15 + n$$

$$\Rightarrow 212 = m \cdot 373.15 + 32 - m \cdot 273.15 \Rightarrow 212 - 32 = m \cdot 100 \Rightarrow$$

$$\Rightarrow m = \frac{180}{100} = \frac{9}{5} \Rightarrow n = 32 - m \cdot 273.15 = 32 - \frac{9}{5} 273.15$$

$$t_F = mT + n = \frac{9}{5} T + 32 - \frac{9}{5} 273.15 = \frac{9}{5} (T - 273.15) + 32$$

Calculamos $T = t_F$:

$$T = t_F$$

$$t_F = \frac{9}{5} (T - 273.15) + 32 \Rightarrow T = \frac{9}{5} (T - 273.15) + 32 \Rightarrow$$

$$\Rightarrow \frac{5}{9} T = T - 273.15 + \frac{5}{9} 32 \Rightarrow 273.15 - \frac{5}{9} 32 = T - \frac{5}{9} T \Rightarrow$$

$$\Rightarrow \frac{4}{9} T = 273.15 - \frac{5}{9} 32 \Rightarrow T = \frac{9}{4} 273.15 - \frac{5}{4} 32 \Rightarrow T = \frac{9}{4} 273.15 - 40 \Rightarrow$$

$$\Rightarrow \boxed{T = 574.5875\text{ K} = 574.5875^\circ\text{F} = t_F} \quad 2^\text{a} \text{ manera}$$

(5.15)

$$\left. \begin{array}{l} (5.21) \quad t_C = 0^\circ\text{C} \leftrightarrow t_R = 0^\circ\text{R} \\ t_C = m t_R + m \end{array} \right\} \Rightarrow 0 = m \cdot 0 + m \Rightarrow m = 0$$

$$\left. \begin{array}{l} t_C = 100^\circ\text{C} \leftrightarrow t_R = 80^\circ\text{R} \\ t_C = m t_R + m \end{array} \right\} \Rightarrow \begin{array}{l} 100 = m \cdot 80 + m \\ m = 0 \end{array} \Rightarrow 100 = m \cdot 80 \Rightarrow m = \frac{100}{80} \Rightarrow$$

$$\Rightarrow m = \frac{10}{8} = \frac{5 \cdot 2}{4 \cdot 2} = \frac{5}{4}$$

$$\boxed{t_C = \frac{5}{4} t_R}$$

$$\left. \begin{array}{l} (5.22) \quad t_F = 32^\circ\text{F} \leftrightarrow t_R = 0^\circ\text{R} \\ t_F = m t_R + m \end{array} \right\} \Rightarrow 32 = m \cdot 0 + m \Rightarrow m = 32$$

$$\left. \begin{array}{l} t_F = 212^\circ\text{F} \leftrightarrow t_R = 80^\circ\text{R} \\ t_F = m t_R + m \end{array} \right\} \Rightarrow \begin{array}{l} 212 = m \cdot 80 + m \\ m = 32 \end{array} \Rightarrow 212 - 32 = m \cdot 80 \Rightarrow$$

$$\Rightarrow 180 = m \cdot 80 \Rightarrow 18 = m \cdot 8 \Rightarrow m = \frac{18}{8} = \frac{9 \cdot 2}{4 \cdot 2} = \frac{9}{4}$$

$$\boxed{t_F = \frac{9}{4} t_R + 32}$$

$$\left. \begin{array}{l} (5.23) \quad T = 273.15 \text{ K} \leftrightarrow t_R = 0^\circ\text{R} \\ T = m t_R + m \end{array} \right\} \Rightarrow 273.15 = m \cdot 0 + m \Rightarrow m = 273.15$$

$$\left. \begin{array}{l} T = 373.15 \text{ K} \leftrightarrow t_R = 80^\circ\text{R} \\ T = m t_R + m \end{array} \right\} \Rightarrow \begin{array}{l} 373.15 = m \cdot 80 + m \\ m = 273.15 \end{array} \Rightarrow 373.15 - 273.15 = m \cdot 80 \Rightarrow$$

$$\Rightarrow 100 = m \cdot 80 \Rightarrow m = \frac{100}{80} = \frac{5}{4}$$

$$\boxed{T = \frac{5}{4} t_R + 273.15}$$

$$\left. \begin{array}{l} b) \quad T = 0 \text{ K} \\ T = \frac{5}{4} t_R + 273.15 \end{array} \right\} \Rightarrow 0 = \frac{5}{4} t_R + 273.15 \Rightarrow \frac{5}{4} t_R = -273.15 \Rightarrow$$

$$\Rightarrow t_R = -\frac{4}{5} 273.15 = \boxed{-218.52^\circ\text{R}}$$

$$\left. \begin{array}{l} c) \quad \text{13}^\circ\text{manera:} \\ t_R = -220^\circ\text{R} \\ T = \frac{5}{4} t_R + 273.15 \end{array} \right\} \Rightarrow T = \frac{5}{4} (-220) + 273.15 \text{ K} = -275 + 273.15 \text{ K} \Rightarrow$$

$$\Rightarrow T = -1.85 \text{ K} \quad \text{No existen temperaturas absolutas negativas en K.} \Rightarrow \boxed{\text{No es posible alcanzar } -220^\circ\text{R.}}$$

Autor: Iván Gabris Alvares

2ª moneda: $T = 0 \text{ K} \Rightarrow t_R = -218.52^\circ \text{R}$

La temperatura no puede ser $\leq 0 \text{ K} \Rightarrow$ La temperatura no puede ser $\leq -218.52^\circ \text{R}$

$t_R = -220^\circ \text{R}$ es menor que $-218.52^\circ \text{R} \Rightarrow$ No es posible alcanzar -220°R .

(5.24)

Q_1 : calor absorbido por los siete gramos de hielo para pasar de -5°C a 0°C

Q_2 : calor absorbido por los siete gramos de hielo para fundirse

Q_3 : calor absorbido por los siete gramos de agua para pasar de 0°C a $T_f^\circ \text{C}$

Q_4 : calor intercambiado por el gramo de agua para pasar de 10°C a $T_f^\circ \text{C}$

Q_5 : calor cedido por el gramo de vapor de agua para pasar de 110°C a 100°C

Q_6 : calor cedido por el gramo de vapor de agua para condensarse

Q_7 : calor cedido por el gramo de agua para pasar de 100°C a $T_f^\circ \text{C}$

T_f en Q_3, Q_4 y Q_7 : $0^\circ \text{C} \leq T_f \leq 100^\circ \text{C}$

$$Q_1 = 7 \text{ g } 0.5 \frac{\text{cal}}{\text{g}^\circ \text{C}} (0 - (-5))^\circ \text{C} = +17.5 \text{ cal} > 0$$

$$Q_2 = 7 \text{ g } 80 \frac{\text{cal}}{\text{g}} = +560 \text{ cal} > 0$$

$$Q_3 = 7 \text{ g } 1 \frac{\text{cal}}{\text{g}^\circ \text{C}} (T_f - 0)^\circ \text{C} = 7T_f \text{ cal} \geq 0$$

$$Q_4 = 1 \text{ g } 1 \frac{\text{cal}}{\text{g}^\circ \text{C}} (T_f - 10)^\circ \text{C} = (T_f - 10) \text{ cal} \leq 0 \quad Q_4 \text{ puede ser cero, } < 0 \text{ o } > 0$$

$$Q_5 = 1 \text{ g } 0.47 \frac{\text{cal}}{\text{g}^\circ \text{C}} (100 - 110)^\circ \text{C} = -4.7 \text{ cal} < 0$$

$$Q_6 = -1 \text{ g } 540 \frac{\text{cal}}{\text{g}} = -540 \text{ cal} < 0$$

$$Q_7 = 1 \text{ g } 1 \frac{\text{cal}}{\text{g}^\circ \text{C}} (T_f - 100)^\circ \text{C} = (T_f - 100) \text{ cal} \leq 0$$

Suponemos que $T_f = +100^\circ \text{C}$ y que toda la moneda es agua.

$$Q_3 = +700 \text{ cal} > 0 \quad Q_4 = (100 - 10) \text{ cal} = +90 \text{ cal} > 0$$

$$Q_7 = (T_f - 100) = 0 \text{ cal}$$

Autor: Iván Cabrera Alvarado

La suma de todos los calores debe ser cero. Sumamos los calores y comprobamos si la suma es cero.

$$Q_1 + Q_2 + Q_3 + Q_4 + Q_5 + Q_6 + Q_7 = +17.5 + 560 + 700 + 90 - 4.7 - 540 + 0 = +822.8 \text{ cal} > 0 \Rightarrow \text{La hipótesis de } T_f = +100^\circ\text{C no es correcta y } T_f < +100^\circ\text{C}$$

Suponemos que $T_f = 0^\circ\text{C}$ y que toda la mezcla es agua.

$$Q_3 = 7T_f \text{ cal} = 0 \text{ cal} \quad Q_4 = (T_f - 10) \text{ cal} = -10 \text{ cal} < 0$$

$$Q_7 = (T_f - 100) \text{ cal} = -100 \text{ cal} < 0$$

$$Q_1 + Q_2 + Q_3 + Q_4 + Q_5 + Q_6 + Q_7 = +17.5 + 560 + 0 - 10 - 4.7 - 540 - 100 \text{ cal} = -77.2 \text{ cal} < 0 \Rightarrow \text{La hipótesis de } T_f = 0^\circ\text{C no es correcta y } T_f > 0^\circ\text{C}$$

T_f debe estar entre 0°C y 100°C

$$Q_1 + Q_2 + Q_3 + Q_4 + Q_5 + Q_6 + Q_7 = 0 \Rightarrow$$

$$\Rightarrow +17.5 + 560 + 7T_f + (T_f - 10) - 4.7 - 540 + (T_f - 100) = 0 \text{ cal} \Rightarrow$$

$$\Rightarrow -77.2 + 9T_f = 0 \Rightarrow T_f = \frac{+77.2}{9} = \boxed{8.5777^\circ\text{C}}$$

5.25 $m_A = 70\,000\text{ g}$ $m_B = 250\text{ g}$

Suponemos que la temperatura final de la mezcla es 0°C , hacemos unos cálculos de los calores y comparamos esos calores. $t_{\text{final}} = 0^\circ\text{C}$

Q_1 : calor absorbido para calentar $70\,000\text{ g}$ de hielo desde -5°C hasta 0°C .

Q_3 : calor cedido por los 250 g de agua para enfriarse desde 100°C hasta 0°C .

Q_2 : calor cedido por los 250 g de vapor de agua para pasar del estado gaseoso al estado líquido.

$$Q_1 = m_A c_{\text{hielo}} (t_{\text{final}} - t_{\text{hielo}}) = 70\,000\text{ g} \cdot 0.5 \frac{\text{cal}}{\text{g}^\circ\text{C}} (0 - (-5))^\circ\text{C} = 175\,000\text{ cal}$$

$$Q_2 = -m_B L_V = -250\text{ g} \cdot 540 \frac{\text{cal}}{\text{g}} = -135\,000\text{ cal}$$

$$Q_3 = m_B c_{\text{agua}} (t_{\text{final}} - t_{\text{agua}}) = 250\text{ g} \cdot 1 \frac{\text{cal}}{\text{g}^\circ\text{C}} (0 - 100)^\circ\text{C} = -25\,000\text{ cal}$$

$$Q_1 + Q_2 + Q_3 = 15\,000\text{ cal} > 0$$

La suma de los calores es mayor que cero. Debería ser cero. Esto significa que es necesario absorber menos calor, o ceder más calor o los dos efectos.

Para congelar $m_B = 250\text{ g}$ de agua, hacen falta $m_B L_f = 250\text{ g} \cdot 80 \frac{\text{cal}}{\text{g}} = 20\,000\text{ cal}$.

$0 < Q_1 + Q_2 + Q_3 < m_B L_f \Rightarrow$ Cediendo calor para congelar una parte de los 250 g de agua, podemos conseguir que la suma de todos los calores sea cero.

Definimos un nuevo calor

Q_4 : calor cedido por X gramos de agua para pasar del estado líquido al estado sólido. $0 < X < 250\text{ g}$

$$Q_4 = -X L_f = -X \cdot 80 \frac{\text{cal}}{\text{g}} = -X \cdot 80\text{ cal} \quad Q_4 < 0$$

$$Q_1 + Q_2 + Q_3 + Q_4 = 0 \Rightarrow 15\,000\text{ cal} - X \cdot 80\text{ cal} = 0 \Rightarrow$$

$$\Rightarrow X = \frac{15\,000}{80} = \frac{1500}{8} = 187.5\text{ g de agua se congelan y están a } 0^\circ\text{C}.$$

$$250\text{ g} - 187.5\text{ g} = 62.5\text{ g de agua a } 0^\circ\text{C}.$$

Temperatura final: 0°C

Estado final de la mezcla: $70\,187.5\text{ g}$ de hielo y 62.5 g de agua

Autor: Iván Cabria Álvarez

$$\left. \begin{array}{l} \textcircled{5.26} \quad t_F = +14^\circ\text{F} \\ t_C = \frac{5}{9}(t_F - 32) \end{array} \right\} \Rightarrow t_C = \frac{5}{9}(14 - 32)^\circ\text{C} = -\frac{5}{9}18^\circ\text{C} = -10^\circ\text{C} \quad \textcircled{5.19}$$

$$\left. \begin{array}{l} t_F = +212^\circ\text{F} \\ t_C = \frac{5}{9}(t_F - 32) \end{array} \right\} \Rightarrow t_C = \frac{5}{9}(212 - 32)^\circ\text{C} = \frac{5}{9}180^\circ\text{C} = 100^\circ\text{C}$$

Suponemos que la temperatura final de la mezcla, t_{final} es 0°C . Hay varios escenarios en los que $t_{\text{final}} = 0^\circ\text{C}$. En lugar de considerar todos los escenarios de uno en uno, vamos a calcular algunos valores y a comparar esos valores, suponiendo que $t_{\text{final}} = 0^\circ\text{C}$.

Q_1 : calor absorbido para calentar los 100 kg de hielo desde -10°C hasta 0°C .

Q_2 : calor cedido para enfriar el kilo de agua desde 100°C hasta 0°C .

Q_3 : calor cedido para congelar el kilo de agua.

$$Q_1 > 0 \qquad Q_2 < 0 \qquad Q_3 < 0$$

$$Q_1 = m_{\text{hielo}} C_{\text{hielo}} (t_{\text{final}} - t_{\text{hielo}}) = 10^5 \text{ g} \cdot 0.5 \frac{\text{cal}}{\text{g}^\circ\text{C}} (0 - (-10))^\circ\text{C} = 500\,000 \text{ cal} > 0$$

$$Q_2 = m_{\text{agua}} C_{\text{agua}} (t_f - t_{\text{agua}}) = 10^3 \text{ g} \cdot 1 \frac{\text{cal}}{\text{g}^\circ\text{C}} (0 - 100)^\circ\text{C} = -100\,000 \text{ cal} < 0$$

$$Q_3 = -m_{\text{agua}} L_f = -10^3 \text{ g} \cdot 80 \frac{\text{cal}}{\text{g}} = -80\,000 \text{ cal} < 0$$

La suma de Q_1 , Q_2 y Q_3 es mayor que cero: $Q_1 + Q_2 + Q_3 = 320\,000 \text{ cal}$. Esta suma debería ser cero. El hecho de que la suma sea mayor que cero significa que es necesario ceder más calor y absorber menos calor. De esta manera, se conseguirá que la suma sea cero. Por tanto, $t_{\text{final}} < 0^\circ\text{C}$.

Autor: Iván Cabria Álvarez

Suponemos que $t_{\text{final}} < 0^\circ\text{C}$
 Q_1 debe ser redefinido ^{y recalculado}. Q_2 y Q_3 tienen el mismo significado y valor numérico. Queremos que definamos un nuevo calor, Q_4 .

Q_1 : calor absorbido para calentar los 100 kg de hielo desde -10°C hasta $t_{\text{final}}^\circ\text{C}$.

Q_4 : calor cedido para enfriar el kilo de hielo, que antes era agua, desde 0°C hasta $t_{\text{final}}^\circ\text{C}$.

La temperatura final de la mezcla, t_{final} , debe estar entre -10°C y 0°C .

$$Q_1 > 0 \quad Q_2 < 0 \quad Q_3 < 0 \quad Q_4 < 0$$

$$Q_1 + Q_2 + Q_3 + Q_4 = 0$$

$$Q_1 = m_{\text{hielo}} C_{\text{hielo}} (t_{\text{final}} - t_{\text{hielo}}) = 10^5 \text{ g } 0.5 \frac{\text{cal}}{\text{g}^\circ\text{C}} (t_{\text{final}} - (-10))^\circ\text{C}$$

$$= 50\,000 (t_{\text{final}} + 10) \text{ cal}$$

Vamos a comprobar que $Q_1 > 0$:

$$t_{\text{final}}^\circ\text{C} > -10^\circ\text{C} \Rightarrow Q_1 = 50\,000 (t_{\text{final}} + 10) > 0$$

$$Q_2 = -100\,000 \text{ cal} < 0 \quad Q_3 = -80\,000 \text{ cal} < 0$$

$$Q_4 = m_{\text{agua}} C_{\text{hielo}} (t_{\text{final}} - t_{\text{a-h}}) = 10^3 \text{ g } 0.5 \frac{\text{cal}}{\text{g}^\circ\text{C}} (t_{\text{final}} - 0)^\circ\text{C}$$

$t_{\text{a-h}} = 0^\circ\text{C}$: El kilo de agua se ha convertido en un kilo de hielo a 0°C .

Tutor: Iván Cabria Álvarez

$$Q_4 = 500 t_{\text{final}} \text{ cal}$$

Vamos a comprobar que $Q_4 < 0$: $t_{\text{final}}^\circ\text{C} < 0^\circ\text{C} \Rightarrow Q_4 = 500 t_{\text{final}} < 0$

$$Q_1 + Q_2 + Q_3 + Q_4 = 0 \Rightarrow 50\,000 (t_{\text{final}} + 10) - 180\,000 + 500 t_{\text{final}} = 0 \Rightarrow$$

$$\Rightarrow 50\,500 t_{\text{final}} + 500\,000 - 180\,000 = 0 \Rightarrow t_{\text{final}} = -\frac{320\,000}{50\,500} = -6.3366^\circ\text{C}$$

$t_{\text{final}} < 0^\circ\text{C}$ y $t_{\text{final}} > -10^\circ\text{C} \Rightarrow t_{\text{final}}$ es consistente con la hipótesis; la hipótesis es correcta.

$$t_{\text{final}} \text{ en } ^\circ\text{F} = \frac{9}{5} t_{\text{final}} \text{ en } ^\circ\text{C} + 32 = \frac{9}{5} (-6.3366) + 32 = \boxed{+20.59^\circ\text{F}}$$

Vamos a comprobar que $t_{\text{final}} \text{ en } ^\circ\text{F}$ es consistente: $t_{\text{final}} \text{ en } ^\circ\text{F}$ debe estar entre $+14^\circ\text{F}$ (-10°C), y $+32^\circ\text{F}$ (0°C). $t_{\text{final}} \text{ en } ^\circ\text{F} = +20.59^\circ\text{F}$ está entre esos dos valores numéricos. Por tanto, $+20.59^\circ\text{F}$ es consistente.

5.27 a) Convertimos las temperaturas en R a temperaturas en °C: 5.21

$$\left. \begin{aligned} t_R &= t_F + 459.67 \Rightarrow t_F = t_R - 459.67 \\ t_C &= \frac{5}{9}(t_F - 32) \Rightarrow t_F = \frac{9}{5}t_C + 32 \end{aligned} \right\} \Rightarrow t_R - 459.67 = \frac{9}{5}t_C + 32 \Rightarrow$$

$$\Rightarrow t_C = \frac{5}{9}(t_R - 459.67 - 32) \Rightarrow t_C = \frac{5}{9}(t_R - 491.67) = \frac{5}{9}t_R - 273.15$$

$$t_R = +446.67 \text{ R} \Rightarrow t_C = \frac{5}{9}(446.67 - 491.67) = \boxed{-25^\circ\text{C}}$$

$$t_R = +671.67 \text{ R} \Rightarrow t_C = \frac{5}{9}(671.67 - 491.67) = \boxed{+100^\circ\text{C}}$$

b) Calculamos la temperatura final y el estado de la mezcla.

Q_1 : Calor absorbido por los 57.6 kg de hielo para pasar de -25°C a 0°C .

Q_X : Calor absorbido por X gramos de hielo para fundirse.

Q_2 : Calor cedido por el kilo de vapor de agua para condensarse.

Q_3 : Calor cedido por el kilo de agua para pasar de $+100^\circ\text{C}$ a 0°C .

Q_4 : Calor cedido por el kilo de agua para solidificarse.

Q_Y : Calor cedido por Y gramos de agua para solidificarse.

$$Q_1 > 0 \quad Q_X > 0 \quad Q_2 < 0 \quad Q_3 < 0 \quad Q_4 < 0 \quad Q_Y < 0$$

$$Q_1 = m_h c_{e,h} (t_{f,\text{hielo}} - t_{i,\text{hielo}}) = 57600 \text{ g} \cdot 0.5 \frac{\text{cal}}{\text{g}^\circ\text{C}} (0 - (-25))^\circ\text{C} = +720\,000 \text{ cal}$$

$$Q_X = +X \text{ g} L_f = +X \text{ g} \cdot 80 \frac{\text{cal}}{\text{g}} = +80X \text{ cal}$$

$$Q_2 = -m_{\text{agua}} L_v = -1000 \text{ g} \cdot 540 \frac{\text{cal}}{\text{g}} = -540\,000 \text{ cal}$$

$$Q_3 = m_{\text{agua}} c_{e,\text{agua}} (t_{f,\text{agua}} - t_{i,\text{agua}}) = 1000 \text{ g} \cdot 1 \frac{\text{cal}}{\text{g}^\circ\text{C}} (0 - 100)^\circ\text{C} = -100\,000 \text{ cal}$$

$$Q_4 = -m_{\text{agua}} L_f = -1000 \text{ g} \cdot 80 \text{ cal/g} = -80\,000 \text{ cal}$$

$$Q_Y = -Y \text{ g} L_f = -Y \text{ g} \cdot 80 \text{ cal/g} = -80Y \text{ cal}$$

Autor: Iván Gabrín Álvarez

Hay varias formas de resolver este problema.

1ª forma: Suponemos que la t° final es 0°C , que el hielo no se funde y que se congela toda el agua. Si esta hipótesis es cierta, entonces $Q_1 + Q_2 + Q_3 + Q_4 = 0$.

$$Q_1 + Q_2 + Q_3 + Q_4 = 720\,000\text{ cal} - 540\,000\text{ cal} - 100\,000\text{ cal} - 80\,000\text{ cal} = 0\text{ cal}$$

$\Rightarrow$ la hipótesis es cierta $\Rightarrow$ La mezcla consiste en 58.6 Kg de hielo a 0°C .

2ª forma: Suponemos que la t° final es 0°C , que el hielo no se funde y que se congelan Y gramos de agua.

$$Q_1 + Q_2 + Q_3 + Q_Y = 0 \Rightarrow 720\,000 - 540\,000 - 100\,000 - 80Y = 0 \Rightarrow$$

$$\Rightarrow 80\,000 - 80Y = 0 \Rightarrow Y = 1000\text{g} \Rightarrow \text{Se congela toda el agua (el kg de agua)} \Rightarrow$$

$\Rightarrow$ La mezcla consiste en 58.6 Kg de hielo a 0°C .

3ª forma: Suponemos que la t° final es 0°C , que se funden X gramos de hielo y que se congela toda el agua.

$$Q_1 + Q_X + Q_2 + Q_3 + Q_4 = 0 \Rightarrow 720\,000 + 80X - 540\,000 - 100\,000 - 80\,000 = 0 \Rightarrow$$

$$\Rightarrow 0 + 80X = 0 \Rightarrow X = 0 \Rightarrow \text{Ningún gramo de hielo se funde} \Rightarrow$$

$\Rightarrow$ La mezcla consiste en 58.6 Kg de hielo a 0°C .

4ª forma: Suponemos que la t° final es 0°C , que se funden X gramos de hielo y que se congelan Y gramos de agua.

$$Q_1 + Q_X + Q_2 + Q_3 + Q_Y = 0 \Rightarrow 720\,000 + 80X - 540\,000 - 100\,000 - 80Y = 0 \Rightarrow$$

$$\Rightarrow 80\,000 + 80X - 80Y = 0 \Rightarrow 1000 + X - Y = 0 \Rightarrow Y = X + 1000$$

$$\left. \begin{array}{l} Y \leq 1000 \\ Y = X + 1000 \end{array} \right\} \Rightarrow \left. \begin{array}{l} X + 1000 \leq 1000 \\ X \geq 0 \end{array} \right\} \Rightarrow X = 0 \Rightarrow Y = 1000 \Rightarrow$$

$\Rightarrow$ Ningún gramo de hielo se funde ($X = 0$) y se congela toda el agua ($Y = 1000$) $\Rightarrow$ La mezcla consiste en 58.6 Kg de hielo a 0°C .

(5.23)

$$(5.28) \quad t_F = +5^\circ\text{F} \Rightarrow t_C = \frac{5}{9} (t_F - 32) = \frac{5}{9} (-27) = -15^\circ\text{C}$$

$$t_F = +213.8^\circ\text{F} \Rightarrow t_C = \frac{5}{9} (213.8 - 32)^\circ\text{C} = \frac{5}{9} 181.8^\circ\text{C} = +101^\circ\text{C}$$

$$t_{\text{final}} = 0^\circ\text{C} \quad t_{i \text{ hielo}} = -15^\circ\text{C} \quad t_{100} = +100^\circ\text{C} \quad t_{i \text{ vapor}} = +101^\circ\text{C}$$

Q_1 : Calor absorbido por los X gramos de hielo para pasar de -15°C a 0°C .

Q_2 : Calor absorbido por los X gramos de hielo para fundirse.

Q_3 : Calor cedido por los 2000 gramos de vapor de agua para pasar de $+101^\circ\text{C}$ a $+100^\circ\text{C}$.

Q_4 : Calor cedido por los 2000 gramos de vapor de agua para condensarse.

Q_5 : Calor cedido por los 2000 gramos de agua para pasar de $+100^\circ\text{C}$ a 0°C .

$$Q_1 > 0 \quad Q_2 > 0 \quad Q_3 < 0 \quad Q_4 < 0 \quad Q_5 < 0$$

$$Q_1 = X C_{\text{hielo}} (t_{\text{final}} - t_{i \text{ hielo}}) = X \cdot 0.5 \cdot 15 \text{ cal} = X 7.5 \text{ cal} > 0$$

$$Q_2 = X 80 \text{ cal} > 0$$

$$Q_3 = m C_{\text{vapor}} (t_{100} - t_{i \text{ vapor}}) = 2000 \text{ g} \cdot 0.47 \frac{\text{cal}}{\text{g}^\circ\text{C}} (-1)^\circ\text{C} = -940 \text{ cal} < 0$$

$$Q_4 = -m L_v = -2000 \text{ g} \cdot 540 \frac{\text{cal}}{\text{g}} = -1080 \text{ 000 cal} < 0$$

$$Q_5 = m C_{\text{agua}} (t_{\text{final}} - t_{100}) = 2000 \text{ g} \cdot 1 \frac{\text{cal}}{\text{g}^\circ\text{C}} (-100)^\circ\text{C} = -200 \text{ 000 cal} < 0$$

$$Q_1 + Q_2 + Q_3 + Q_4 + Q_5 = 0 \Rightarrow X 7.5 + X 80 - 940 - 1080 \cdot 000 - 200 \cdot 000 = 0 \Rightarrow$$

$$\Rightarrow X 87.5 = 1280940 \Rightarrow \boxed{X = 14639.3140 \text{ g}}$$

Tutor: Iván Abria Álvaro

5.29 a) $\sigma = \frac{1}{S} \frac{dS}{dt_c} \Big|_{t_c=t_0}$
 $S = aL^2 \Rightarrow \frac{dS}{dt_c} = a2L \frac{dL}{dt_c} \Big|_{t_c=t_0} \Rightarrow$

$$\Rightarrow \sigma = \frac{1}{aL^2} a2L \frac{dL}{dt_c} \Big|_{t_c=t_0} = 2 \frac{1}{L} \frac{dL}{dt_c} \Big|_{t_c=t_0} = \boxed{2\alpha}$$

b) $K = \frac{1}{V} \frac{dV}{dt_c} \Big|_{t_c=t_0}$
 $V = bL^3 \Rightarrow \frac{dV}{dt_c} = b3L^2 \frac{dL}{dt_c} \Big|_{t_c=t_0} \Rightarrow$

$$\Rightarrow K = 3 \frac{1}{L} \frac{dL}{dt_c} \Big|_{t_c=t_0} = \boxed{3\alpha}$$

c) $V = V_0(1 + Kt_c) \quad t_0 = 0^\circ\text{C}$

$$r(20^\circ\text{C}) = \frac{3}{2} \text{ cm} \quad r(t_a) = \frac{3.01}{2} \text{ cm}$$

$$V(20^\circ\text{C}) = \frac{4\pi}{3} r(20^\circ\text{C})^3 \quad V(t_a) = \frac{4\pi}{3} r(t_a)^3$$

Primera forma de resolver el problema:

$$\left. \begin{aligned} V = V_0(1 + Kt_c) \Rightarrow V(20^\circ\text{C}) &= V_0(1 + K20) \\ V(t_a) &= V_0(1 + Kt_a) \end{aligned} \right\} \Rightarrow V(t_a) - V(20^\circ\text{C}) = V_0K(t_a - 20)$$

$$V(t_a) = \frac{4\pi}{3} r(t_a)^3$$

$$V(20^\circ\text{C}) = \frac{4\pi}{3} r(20^\circ\text{C})^3$$

$$\Rightarrow \frac{4\pi}{3} (r(t_a)^3 - r(20^\circ\text{C})^3) = V_0K(t_a - 20) \Rightarrow$$

$$\Rightarrow t_a = 20^\circ\text{C} + \frac{4\pi (r(t_a)^3 - r(20^\circ\text{C})^3)}{3V_0K} ^\circ\text{C}$$

Autor: Juan Carlos Alvarado

$$V(20^\circ\text{C}) = V_0(1 + K20) \Rightarrow V_0 = \frac{V(20^\circ\text{C})}{1 + K20} \quad K = 3\alpha \quad \Rightarrow V_0 = \frac{V(20^\circ\text{C})}{1 + 60\alpha}$$

$$t_a = 20^\circ\text{C} + \frac{4\pi (r(t_a)^3 - r(20^\circ\text{C})^3)}{3V_0K}$$

$$K = 3\alpha$$

$$\Rightarrow t_a = 20^\circ\text{C} + \frac{4\pi (r(t_a)^3 - r(20^\circ\text{C})^3) (1 + 60\alpha)}{9\alpha V(20^\circ\text{C})}$$

$$= 20^\circ\text{C} + \frac{4\pi \left(\left(\frac{3.01}{2} \right)^3 - \left(\frac{3}{2} \right)^3 \right) \text{cm}^3 (1 + 60 \cdot 12 \cdot 10^{-6})}{9 \cdot 12 \cdot 10^{-6} (\text{C})^{-1} \frac{4\pi}{3} \left(\frac{3}{2} \right)^3 \text{cm}^3}$$

$$= 20^\circ\text{C} + \frac{8 \left(\left(\frac{3.01}{2} \right)^3 - \left(\frac{3}{2} \right)^3 \right) (1 + 7.2 \cdot 10^{-4})}{36 \cdot 10^{-6} \cdot 27}^\circ\text{C} =$$

$$= 20^\circ\text{C} + 278.9054^\circ\text{C} \Rightarrow \boxed{t_a = 298.9054^\circ\text{C}}$$

Segunda forma de resolver el problema:

$$V(20^\circ\text{C}) = \frac{4\pi}{3} r(20^\circ\text{C})^3 = \frac{4\pi}{3} (1.5 \text{ cm})^3 = 14.13716694 \text{ cm}^3$$

$$V(20^\circ\text{C}) = V_0(1 + K20) \Rightarrow V_0 = \frac{V(20^\circ\text{C})}{1 + K20}$$

$$K = 3\alpha = 36 \cdot 10^{-6} (\text{C})^{-1}$$

$$\Rightarrow V_0 = \frac{14.13716694 \text{ cm}^3}{1 + 36 \cdot 10^{-6} \cdot 20} = 14.126995504 \text{ cm}^3$$

$$V(t_a) = \frac{4\pi}{3} r(t_a)^3 = \frac{4\pi}{3} (1.505 \text{ cm})^3 = 14.27901037 \text{ cm}^3$$

$$\Rightarrow \frac{V(t_a)}{V_0} = \frac{14.27901037}{14.126995504} = 1.01076059$$

$$V(t_a) = V_0(1 + Kt_a) \Rightarrow \frac{V(t_a)}{V_0} = 1 + Kt_a \Rightarrow Kt_a = \frac{V(t_a)}{V_0} - 1 \Rightarrow$$

$$\Rightarrow t_a = \frac{\left(\frac{V(t_a)}{V_0} - 1 \right)}{K} = \frac{1.01076059 - 1}{36 \cdot 10^{-6} (\text{C})^{-1}} = \boxed{298.9054^\circ\text{C}}$$

Autor: Juan Carlos Alvarez

$$(5.30) \quad \frac{dT}{dt} = a(T_{foco} - T(t)) \Rightarrow \frac{dT}{T_{foco} - T(t)} = a dt \quad (5.26)$$

Si $T_{foco} - T(t) > 0 \quad \forall t \geq 0$:

$$\int_{T(0)}^{T(t)} \frac{dT}{T_{foco} - T(t)} = \int_0^t a dt \stackrel{(1)}{\Rightarrow} -\ln\left(\frac{T_{foco} - T(t)}{T_{foco} - T(0)}\right) = at \Rightarrow$$

Nota: $T_{foco} - T(t) > 0$ y $T_{foco} - T(0) > 0$. Por tanto, el paso (1) tiene sentido matemático.

$$\Rightarrow \frac{T_{foco} - T(t)}{T_{foco} - T(0)} = e^{-at} \Rightarrow T_{foco} - T(t) = (T_{foco} - T(0))e^{-at}$$

$$\Rightarrow T(t) = T_{foco} - (T_{foco} - T(0))e^{-at} \Rightarrow$$

$$\Rightarrow \boxed{T(t) = T_{foco} + (T(0) - T_{foco})e^{-at} \quad \text{si } T_{foco} - T(t) > 0 \quad \forall t \geq 0}$$

Nota: Si $a - x > 0$ entonces, la derivada de $f(x) = \ln(a - x)$ es $\frac{df}{dx} = -\frac{1}{(a - x)}$ y la integral indefinida de $\frac{1}{a - x}$ es

$$\int \frac{dx}{a - x} = -\ln(a - x) + C.$$

Si $T_{foco} - T(t) < 0 \quad \forall t \geq 0$:

$$\frac{dT}{T_{foco} - T(t)} = a dt \Rightarrow \frac{dT}{T(t) - T_{foco}} = -a dt \Rightarrow$$

$$\Rightarrow \int_{T(0)}^{T(t)} \frac{dT}{T(t) - T_{foco}} = -a \int_0^t dt \stackrel{(2)}{\Rightarrow} \ln\left(\frac{T(t) - T_{foco}}{T(0) - T_{foco}}\right) = -at \Rightarrow$$

Nota: El paso (2) tiene sentido matemático porque $T(t) - T_{foco} > 0$ y $T(0) - T_{foco} > 0$.

Autor: Iván Cabris Álvarez

$$\Rightarrow \frac{T(t) - T_{foco}}{T(0) - T_{foco}} = e^{-at} \Rightarrow T(t) - T_{foco} = (T(0) - T_{foco}) e^{-at} \Rightarrow \quad (5.27)$$

$$\Rightarrow \boxed{T(t) = T_{foco} + (T(0) - T_{foco}) e^{-at}}$$

si $T_{foco} - T(t) < 0 \quad \forall t \geq 0$

Se obtiene la misma solución para los dos casos.

(5.31) $m_{agua} = 2 \text{ Kg} = 2000 \text{ g}$

$m_{hielo} = 500 \text{ g}$

$m_{metal} = 1 \text{ Kg} = 1000 \text{ g}$

$m_{vapor} = ? \quad t_{vapor} = 400 \text{ K}$

Equilibrio térmico del metal, el agua y el hielo $\Rightarrow t_i = 273.15 \text{ K} = 0^\circ \text{C}$

$t_f = 25^\circ \text{C} = 298.15 \text{ K}$

m_{vapor} está en gramos.

Q_1 : Calor absorbido para calentar el metal desde 0°C hasta 25°C .

Q_2 : Calor absorbido para calentar el agua desde 0°C hasta 25°C .

Q_3 : Calor absorbido para fundir el hielo.

Q_4 : Calor absorbido para calentar 500 g de agua, que antes eran hielo, desde 0°C hasta 25°C .

Q_5 : Calor cedido por m_{vapor} gramos de vapor para pasar de 400 K a $373.15 \text{ K} = 100^\circ \text{C}$.

Autor: Iván Cabrera Álvarez

Q_6 : Calor cedido por m_{vapor} gramos de vapor para condensarse.

Q_7 : Calor cedido por m_{vapor} gramos de agua, que antes eran vapor de agua, para enfriarse desde 100°C hasta 25°C .

$Q_1 > 0 \quad Q_2 > 0 \quad Q_3 > 0 \quad Q_4 > 0 \quad Q_5 < 0 \quad Q_6 < 0 \quad Q_7 < 0$

$Q_1 = m_{metal} C_{metal} (t_f - t_i) = 1000 \text{ g} \cdot 0.2 \frac{\text{cal}}{\text{g}^\circ \text{C}} \cdot 25^\circ \text{C} = 5000 \text{ cal}$

$Q_2 = m_{agua} C_{agua} (t_f - t_i) = 2000 \text{ g} \cdot 1 \frac{\text{cal}}{\text{g}^\circ \text{C}} \cdot 25^\circ \text{C} = 50\,000 \text{ cal}$

$Q_3 = m_{hielo} L_{fusión} = 500 \text{ g} \cdot 80 \frac{\text{cal}}{\text{g}} = 40\,000 \text{ cal}$

$Q_4 = m_{hielo} C_{agua} (t_f - t_i) = 500 \text{ g} \cdot 1 \frac{\text{cal}}{\text{g}^\circ \text{C}} \cdot 25^\circ \text{C} = 12\,500 \text{ cal}$

$$Q_5 = m_{\text{vapor}} C_{\text{vapor}} (375.15 - 400)K = m_{\text{vapor}} 0.47 (-26.85) \text{ cal} \quad (5.28)$$

$$= -m_{\text{vapor}} 12.6195 \text{ cal}$$

$$Q_6 = -m_{\text{vapor}} L_{\text{vaporización}} = -m_{\text{vapor}} 540 \text{ cal}$$

$$Q_7 = m_{\text{vapor}} C_{\text{agua}} (t_f - t_{100}) = m_{\text{vapor}} 1 \frac{\text{cal}}{\text{g}^\circ\text{C}} (25 - 100)^\circ\text{C}$$

$$= -m_{\text{vapor}} 75 \text{ cal}$$

$$\sum_{i=1}^7 Q_i = 0 \Rightarrow 5000 + 50000 + 40000 + 12500 - m_{\text{vapor}} 12.6195$$

$$- m_{\text{vapor}} 540 - m_{\text{vapor}} 75 = 0 \Rightarrow$$

$$\Rightarrow m_{\text{vapor}} = \frac{107500}{627.6195} = \boxed{171.28 \text{ gramos de vapor de agua}}$$

(5.32) $m_h = 2000 \text{ g}$ $T_h = -50^\circ\text{C}$ $L_v = 540 \text{ cal/g}$
 $m_a = 100 \text{ g}$ $T_a = +10^\circ\text{C}$ $L_f = 80 \text{ cal/g}$
 $m_v = 50 \text{ g}$ $T_v = +120^\circ\text{C}$

Calculamos T_{final} en $^\circ\text{C}$

Q_1 : Calor absorbido por los 2000 g de hielo para pasar de -50°C hasta 0°C

$$Q_1 = m_h C_h (T_0 - T_h) = 2000 \text{ g } 0.5 \frac{\text{cal}}{\text{g}^\circ\text{C}} (0 + 50)^\circ\text{C} = 50000 \text{ cal} \quad Q_1 > 0$$

Q_2 : Calor absorbido por los 2000 g de hielo para fundirse a 0°C

$$Q_2 = m_h L_f = 2000 \text{ g } 80 \text{ cal/g} = 160000 \text{ cal} \quad Q_2 > 0$$

Q_3 : Calor cedido por los 50 g de vapor de agua para pasar de $+120^\circ\text{C}$ a $+100^\circ\text{C}$

$$Q_3 = m_v C_{\text{vapor}} (T_{100} - T_{120}) = 50 \text{ g } 0.47 \frac{\text{cal}}{\text{g}^\circ\text{C}} (100 - 120)^\circ\text{C} = -470 \text{ cal} \quad Q_3 < 0$$

Autor: Iván Cabris Álvarez

Q_4 : Calor cedido por los 50 g de vapor de agua para condensarse a $+100^\circ\text{C}$

$$Q_4 = -m_v L_v = -50\text{ g } 540 \frac{\text{cal}}{\text{g}} = -27\,000\text{ cal} \quad Q_4 < 0$$

Q_5 : Calor cedido por los 50 g de agua, que antes eran vapor de agua, para pasar de $+100^\circ\text{C}$ a 0°C $Q_5 < 0$

$$Q_5 = m_v C_{\text{agua}} (T_0 - T_{100}) = 50\text{ g } 1 \frac{\text{cal}}{\text{g}^\circ\text{C}} (0 - 100)^\circ\text{C} = -5\,000\text{ cal}$$

Q_6 : Calor cedido por los 100 g de agua para pasar de $+10^\circ\text{C}$ a 0°C

$$Q_6 = m_a C_{\text{agua}} (T_0 - T_{10}) = 100\text{ g } 1 \frac{\text{cal}}{\text{g}^\circ\text{C}} (0 - 10)^\circ\text{C} = -1\,000\text{ cal}$$

Q_7 : Calor cedido por los 150 g de agua para solidificarse como hielo a 0°C

$$Q_7 = -(m_a + m_v) L_f = -150\text{ g } 80 \frac{\text{cal}}{\text{g}} = -12\,000\text{ cal}$$

Suponemos que $T_{\text{final}} = 0^\circ\text{C}$.

Suponemos que los 2000 g de hielo se funden y que los 150 g de agua se convierten en hielo:

$$Q_1 + Q_2 + Q_3 + Q_4 + Q_5 + Q_6 + Q_7 = 50\,000 + 160\,000 - 470 - 27\,000 - 5\,000 - 1\,000 - 12\,000\text{ cal} = +164\,530\text{ cal} > 0$$

La suma debería ser cero. La suma es mayor que cero. Esto significa que hay demasiado calor absorbido en la suma y/o que falta mucho calor cedido en la suma.

Vamos a descartar el calor absorbido Q_2 y a rehacer la suma:

Descartar Q_2 significa que suponemos que los 2000 g de hielo no se funden.

$$Q_1 + Q_3 + Q_4 + Q_5 + Q_6 + Q_7 = 50\,000 - 470 - 27\,000 - 5\,000 - 1\,000 - 12\,000\text{ cal} = +4\,530\text{ cal} > 0$$

La suma es mayor que cero. Esto significa que sobra calor cedido (Q_1 debería ser menos de 50 000 cal) y que falta calor cedido (los 150 g de hielo deberían bajar su temperatura). Para que Q_1 sea menor que 50 000 cal, T_{final} debería ser menor que cero.

Redefinimos Q_1 : Calor absorbido por los 2000 g de hielo para

pasar de -50°C a $T_{\text{final}}^\circ\text{C}$, $T_{\text{final}} < 0^\circ\text{C}$
 $Q_1 = m_h c_h (T_{\text{final}} - T_h) = 2000\text{ g } 0.5 \frac{\text{cal}}{\text{g}^\circ\text{C}} (T_{\text{final}} + 50)^\circ\text{C} = 1000 (T_{\text{final}} + 50) \text{ cal}$
Definimos un nuevo calor cedido, Q_8 .

Q_8 : Calor cedido por los 150 g de hielo para pasar de 0°C a $T_{\text{final}}^\circ\text{C}$,
 $T_{\text{final}} < 0^\circ\text{C}$

$$Q_8 = (m_a + m_v) c_h (T_{\text{final}} - T_0) = 150\text{ g } 0.5 \frac{\text{cal}}{\text{g}^\circ\text{C}} T_{\text{final}}^\circ\text{C} = 75 T_{\text{final}} \text{ cal}$$

$$T_{\text{final}} < 0^\circ\text{C} \Rightarrow Q_8 < 0$$

Calculamos la suma de todos los calores. Esta suma debe ser cero.

$$Q_1 + Q_3 + Q_4 + Q_5 + Q_6 + Q_7 + Q_8 = 0 \Rightarrow$$

$$\Rightarrow 1000 (T_{\text{final}} + 50) - 470 - 27000 - 5000 - 1000 - 12000 + 75 T_{\text{final}} = 0$$

$$\Rightarrow 1000 (T_{\text{final}} + 50) - 45470 + 75 T_{\text{final}} = 0 \Rightarrow$$

$$\Rightarrow 1075 T_{\text{final}} + 50000 - 45470 = 0 \Rightarrow T_{\text{final}} = -\frac{4530}{1075} = \boxed{-4.21^\circ\text{C}}$$

5.33 a) $m_h = 23000\text{ g}$ $T_h = -5^\circ\text{C}$ $m_v = 80\text{ g}$ $T_v = +120^\circ\text{C}$
 $T_{100} = +100^\circ\text{C}$ $T_0 = 0^\circ\text{C}$

Suponemos que la temperatura final de la mezcla es 0°C : $T_f = 0^\circ\text{C}$

Q_1 : Calor absorbido por los 23000 g de hielo para pasar de -5°C hasta 0°C

$$Q_1 = m_h c_{\text{hielo}} (T_0 - T_h) = 23000\text{ g } 0.5 \frac{\text{cal}}{\text{g}^\circ\text{C}} (0 + 5)^\circ\text{C} = 57500 \text{ cal}$$

$Q_1 > 0$

Q_x : Calor absorbido por x gramos de hielo para fundirse

$$Q_x = x L_f = x \text{ g } 80 \frac{\text{cal}}{\text{g}} = 80x \text{ cal} \quad Q_x \geq 0 \quad 0 \leq x \leq 23000\text{ g}$$

Q_2 : Calor cedido por los 80 g de vapor de agua para pasar de $+120^\circ\text{C}$

hasta $+100^\circ\text{C}$

$$Q_2 = m_v c_{\text{vapor}} (T_{100} - T_v) = 80\text{ g } 0.47 \frac{\text{cal}}{\text{g}^\circ\text{C}} (100 - 120)^\circ\text{C}$$
$$= -752 \text{ cal} \quad Q_2 < 0$$

Q_3 : Calor cedido por los 80 g de vapor de agua para condensarse a $+100^\circ\text{C}$

$$Q_3 = -m_v L_v = -80\text{ g } 540 \frac{\text{cal}}{\text{g}} = -43200 \text{ cal} \quad Q_3 < 0$$

Q_4 : Calor cedido por los 80 g de agua, que inicialmente eran vapor de agua, para pasar de $+100^\circ\text{C}$ hasta 0°C

$$Q_4 = m_v C_{\text{agua}} (T_0 - T_{100}) = 80\text{g} \cdot 1 \frac{\text{cal}}{\text{g}^\circ\text{C}} (0 - 100)^\circ\text{C} = -8000\text{cal}$$

$$Q_4 < 0$$

Q_Y : Calor cedido por Y gramos de agua para solidificarse

$$Q_Y = -Y L_f = -80Y \text{ cal} \quad Q_Y \leq 0 \quad 0 \leq Y \leq 80\text{g}$$

$$Q_1 + Q_x + Q_2 + Q_3 + Q_4 + Q_Y = 0 \Rightarrow$$

$$\Rightarrow 57500 + 80X - 752 - 43200 - 8000 - 80Y = 0 \Rightarrow$$

$$\Rightarrow 5548 + 80(X - Y) = 0 \Rightarrow X - Y = -69.35\text{g} \Rightarrow X - Y < 0$$

$$X \geq 0 \quad Y \geq 0$$

$$X - Y < 0 \Rightarrow X < Y$$

$$X - Y = -69.35\text{g} \Rightarrow Y = X + 69.35\text{g}$$

X gramos de hielo se han convertido en agua

Y gramos de agua se han convertido en hielo

La mezcla final consiste en $23000 - X + Y$ gramos de hielo y

$80 + X - Y$ gramos de agua $\Rightarrow$

$\Rightarrow$ La mezcla final consiste en 23069.35 g de hielo y 10.65 g de agua a 0°C

b) $T_h = ?$ T_h en $^\circ\text{C}$ $T_h < 0^\circ\text{C}$ $Y = 80\text{g}$ $X = 0\text{g}$

$$Q_x = 0 \text{ cal}$$

$$Q_Y = -Y L_f = -80\text{g} \cdot 80 \frac{\text{cal}}{\text{g}} = -6400 \text{ cal} \quad Q_Y < 0$$

$$Q_1 = m_h C_{\text{hielo}} (T_0 - T_h) = 23000\text{g} \cdot 0.5 \frac{\text{cal}}{\text{g}^\circ\text{C}} (0 - T_h)^\circ\text{C} =$$

$$= -11500 T_h \text{ cal} > 0 \quad T_h < 0^\circ\text{C}$$

$$Q_1 + Q_x + Q_2 + Q_3 + Q_4 + Q_Y = 0 \Rightarrow -11500 T_h - 752 - 43200 - 8000 - 6400 = 0$$

$$\Rightarrow -11500 T_h - 58352 = 0 \Rightarrow T_h = \frac{58352}{(-11500)}^\circ\text{C} \Rightarrow \boxed{T_h = -5.07^\circ\text{C}}$$

Autor: Iván Cabria Álvarez

P6.1 Proceso isoterma $\Rightarrow W = nRT \ln\left(\frac{V_f}{V_i}\right) \Rightarrow W = nRT \ln 3$
 $V_f = 3V_i$

$$W = nRT \ln 3 \Rightarrow n = \frac{W}{RT \ln 3}$$

$$\left. \begin{array}{l} W = 1 \text{ kWh} = 1000 \text{ W} \cdot 3600 \text{ s} = 3.6 \text{ MJ} \\ R = 8.3145 \text{ J mol}^{-1} \text{ K}^{-1} \\ T = 300 \text{ K} \end{array} \right\} \Rightarrow n = \frac{3.6 \cdot 10^6 \text{ J}}{8.3145 \text{ J mol}^{-1} \text{ K}^{-1} \cdot 300 \text{ K} \ln 3}$$

$$n = 1314 \text{ moles}$$

6.2 a) Proceso adiabático $\Rightarrow PV^\gamma = \text{cte.} \Rightarrow P_i V_i^\gamma = P_f V_f^\gamma \Rightarrow \left(\frac{V_f}{V_i}\right)^\gamma = \frac{P_i}{P_f}$
 $PV = nRT \Rightarrow \frac{P_i V_i}{P_f V_f} = \frac{n_i T_i}{n_f T_f} \Rightarrow \frac{P_i V_i}{P_f V_f} = 3 \Rightarrow \frac{P_i}{P_f} = 3 \frac{V_f}{V_i}$
 $n_i = n_f \quad T_f = T_i/3$
 $\Rightarrow \left(\frac{V_f}{V_i}\right)^\gamma = 3 \frac{V_f}{V_i} \Rightarrow \left(\frac{V_f}{V_i}\right)^{\gamma-1} = 3 \Rightarrow \frac{V_f}{V_i} = 3^{\frac{1}{\gamma-1}} \Rightarrow \frac{V_f}{V_i} = 15.625$
 $\gamma = 1.4$

b) $\frac{P_f}{P_i} = \frac{V_i}{3V_f} = \frac{1}{3 \cdot 15.625} = 0.0233$
 Otra manera de resolver el apartado a):
 $TV^{\gamma-1} = \text{cte.} \Rightarrow T_i V_i^{\gamma-1} = T_f V_f^{\gamma-1} \Rightarrow \frac{V_f}{V_i} = \left(\frac{T_i}{T_f}\right)^{\frac{1}{\gamma-1}} = 3^{\frac{1}{0.4}} = 3^{5/2} = 15.625$

6.3

$P_1 V_1 T_1$	$P_2 V_2 T_2$
n_1	n_2

 $P_1 = P_2 \quad T_1 = T_2$

a) $PV = nRT$
 $n = \frac{N}{N_A}$
 $\left. \begin{array}{l} \frac{P_1 V_1}{P_2 V_2} = \frac{N_1 T_1}{N_2 T_2} \\ P_1 = P_2 \quad T_1 = T_2 \end{array} \right\} \Rightarrow \frac{V_1}{V_2} = \frac{N_1}{N_2} \Rightarrow$

$\Rightarrow$ Si $V_1 = V_2$, entonces la afirmación será cierta
 Si $V_1 \neq V_2$, entonces la afirmación será falsa

b) $m_1 \neq m_2$ porque se trata de gases diferentes $\Rightarrow$ La afirmación es falsa (6.2)

c) $H_1 = N_1 m_1$ $H_2 = N_2 m_2 \Rightarrow H_1 = H_2$ si $N_1/N_2 = \frac{m_2}{m_1}$

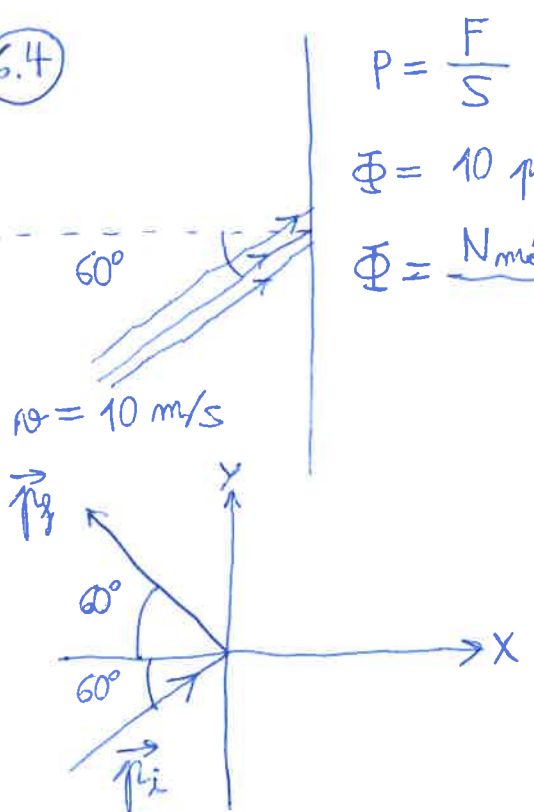
d) $m_i \frac{v_{rms,i}^2}{2} = \frac{3}{2} k_B T_i \quad i=1,2$
 $m_1 \neq m_2 \quad T_1 = T_2 \quad \Rightarrow v_{rms1} \neq v_{rms2} \Rightarrow$ La afirmación es falsa

(6.4)

$P = \frac{F}{S} \quad 1 \text{ Pa} = 1 \frac{\text{N}}{\text{m}^2}$

$\Phi = 10 \text{ partículas/s} \quad p = m v = 0.2 \text{ Kg} \cdot 10 \text{ m/s} = 2 \text{ Kg m/s}$

$\Phi = \frac{N_{\text{moléculas chocan en } \Delta t}}{\Delta t}$



$\vec{p}_i = p \cos 60^\circ \vec{u}_x + p \sin 60^\circ \vec{u}_y$
 $\vec{p}_r = -p \cos 60^\circ \vec{u}_x + p \sin 60^\circ \vec{u}_y$
 $|\vec{p}_i| = |\vec{p}_r| = p \quad p \sin 60^\circ$
 $\cos 60^\circ = \frac{1}{2}$

$\Delta \vec{p}_{\text{molécula}} = \vec{p}_r - \vec{p}_i = -2p \cos 60^\circ \vec{u}_x = -p \vec{u}_x$

En Δt : $\Delta \vec{p}_{\text{total}} = N_{\text{moléculas}} \Delta \vec{p}_{\text{molécula}}$

$F = \frac{|\Delta \vec{p}_{\text{total}}|}{\Delta t} = \frac{N_{\text{moléculas}} |\Delta \vec{p}_{\text{molécula}}|}{\Delta t} = \Phi |\Delta \vec{p}_{\text{molécula}}|$

$F = 10 \text{ moléculas/s} \cdot 2 \text{ Kg m/s molécula} = 20 \text{ N} \quad P = \frac{F}{S} = \frac{20 \text{ N}}{1 \text{ m}^2} = 20 \text{ Pa}$

(6.5) a) $v_{rms} = \sqrt{\frac{3k_B T}{m}} \Rightarrow T = \frac{m v_{rms}^2}{3k_B} = \frac{6.64 \cdot 10^{-27} \text{ Kg} (600 \text{ m/s})^2}{3 \cdot 1.3807 \cdot 10^{-23} \text{ J/K}} = 57.71 \text{ K}$

b) $v_{rms} = \sqrt{\frac{3k_B T}{m}} \Rightarrow \frac{v_{rms2}^2}{v_{rms1}^2} = \frac{T_2}{T_1} \Rightarrow v_{rms2} = v_{rms1} \sqrt{\frac{T_2}{T_1}}$

$v_{rms2} = 600 \frac{\text{m}}{\text{s}} \sqrt{\frac{5800}{57.71}} = 6015 \text{ m/s}$

Otra manera de calcular v_{rms2} :

$v_{rms2} = \sqrt{\frac{3k_B T}{m}} = \sqrt{\frac{3 \cdot 1.3807 \cdot 10^{-23} \text{ J/K}^{-1} \cdot 5800 \text{ K}}{6.64 \cdot 10^{-27} \text{ Kg}}} = 6015 \text{ m/s}$

Autor: Yván Cabrera Álvarez

6.6 a) $N_2 = n N_A = 1 \text{ mol } 6.022 \cdot 10^{23} \text{ moléculas/mol} = 6.022 \cdot 10^{23} \text{ moléculas}$

$$N_0 = 2N_2 = 12.044 \cdot 10^{23} \text{ átomos de oxígeno}$$

b) Hay dos maneras de resolver este problema

Primera manera:

$$\left. \begin{aligned} p &= \frac{m}{V} \\ m &= N_2 m_{O_2} \end{aligned} \right\} \Rightarrow p = \frac{N_2 m_{O_2}}{V}$$

$$\left. \begin{aligned} PV &= nRT \Rightarrow V = \frac{nRT}{P} \\ n &= \frac{N_2}{N_A} \end{aligned} \right\} \Rightarrow V = \frac{N_2 RT}{N_A P}$$

$$\left. \begin{aligned} V &= \frac{N_2 RT}{N_A P} \\ K_B &= \frac{R}{N_A} \end{aligned} \right\} \Rightarrow V = N_2 K_B \frac{T}{P}$$

$$\Rightarrow p = \frac{N_2 m_{O_2}}{N_2 K_B \frac{T}{P}} = \frac{m_{O_2} P}{K_B T}$$

$$P = 770 \text{ mmHg} = \frac{770}{760} \text{ atm} = \frac{770}{760} 1.013 \cdot 10^5 \text{ Pa} = 102633 \text{ Pa}$$

$$p = \frac{5.31 \cdot 10^{-26} \text{ Kg } 102633 \text{ Pa}}{1.3807 \cdot 10^{-23} \text{ J/K } 298.15 \text{ K}} = 1.324 \frac{\text{Kg}}{\text{m}^3} = 1324 \frac{\text{g}}{\text{m}^3} = \frac{1324}{1000} \frac{\text{g}}{\text{dm}^3} = \boxed{1.324 \frac{\text{g}}{\text{L}}}$$

Segunda manera:

$$PV = nRT \Rightarrow V = \frac{nRT}{P} = \frac{1 \text{ mol } 8.31 \text{ J mol}^{-1} \text{ K}^{-1} 298.15 \text{ K}}{102633 \text{ Pa}} = 0.02414 \text{ m}^3$$

$$p = \frac{m}{V} = \frac{N_2 m_{O_2}}{V} = \frac{6.022 \cdot 10^{23} 5.31 \cdot 10^{-26} \text{ Kg}}{0.02414 \text{ m}^3} = 1.324 \frac{\text{Kg}}{\text{m}^3} = \boxed{1.324 \frac{\text{g}}{\text{L}}}$$

$$c) v_{\text{rms}} = \sqrt{\frac{3K_B T}{m_{O_2}}} = \sqrt{\frac{3 \cdot 1.3807 \cdot 10^{-23} \text{ J K}^{-1} 298.15 \text{ K}}{5.31 \cdot 10^{-26} \text{ Kg}}} = \boxed{482 \frac{\text{m}}{\text{s}}}$$

6.7 $V = 20 \text{ cm}^3 = 20 \text{ cm}^3 \cdot 10^{-6} \frac{\text{m}^3}{\text{cm}^3} = 2 \cdot 10^{-5} \text{ m}^3$

$M_{H_2} = 0.001 \text{ g} = 10^{-6} \text{ Kg}$ de gas hidrógeno

a) $n_i = \frac{M_{H_2}}{N_A m_{H_2}} = \frac{10^{-6} \text{ Kg}}{6.022 \cdot 10^{23} \text{ moléculas} \cdot \text{mol}^{-1} 3.32 \cdot 10^{-27} \text{ Kg moléculas}^{-1}} = 0.0005 \text{ moles}$

$P_i = \frac{n_i R T_i}{V_i} = \frac{0.0005 \text{ moles } 8.3145 \text{ J mol}^{-1} \text{ K}^{-1} 290.15 \text{ K}}{2 \cdot 10^{-5} \text{ m}^3} = 60311 \text{ Pa}$

Autor: Iván Cabria Álvarez

$$P_i = \frac{60311 \text{ Pa}}{1.013 \cdot 10^5 \text{ Pa/atm}} = \frac{60311 \text{ Pa}}{1.013 \cdot 10^5} \text{ atm} \cdot 760 \frac{\text{mmHg}}{\text{atm}} = \boxed{452.5 \text{ mmHg}} \quad (6.4)$$

$$P_f = \frac{m_f RT_f}{V_f} = \frac{0.0005 \text{ moles} \cdot 8.3145 \text{ J mol}^{-1} \text{ K}^{-1} \cdot 283.15 \text{ K}}{2 \cdot 10^{-5} \text{ m}^3} = 58856 \text{ Pa}$$

$$P_f = 58856 \text{ Pa} = 0.581 \text{ atm} = \boxed{441.56 \text{ mmHg}}$$

$$b) m_f = \frac{P_f V_f}{RT_f} = \frac{\frac{P_i}{100} V_f}{RT_f} = \frac{588.56 \text{ Pa} \cdot 2 \cdot 10^{-5} \text{ m}^3}{8.3145 \text{ J mol}^{-1} \text{ K}^{-1} \cdot 283.15 \text{ K}} = 0.000005 \text{ moles}$$

$$m_{\text{extraído}} = 0.0005 - 0.000005 \text{ moles} = 4.95 \cdot 10^{-4} \text{ moles de H}_2$$

$$m_{\text{H}_2 \text{ extraído}} = m_{\text{extraído}} N_A m_{\text{H}_2} = 4.95 \cdot 10^{-4} \text{ moles} \cdot 6.022 \cdot 10^{23} \frac{\text{moles}}{\text{mol}} \cdot 3.32 \cdot 10^{-27} \frac{\text{kg}}{\text{molécula}} \\ = 9.8966 \cdot 10^{-7} \text{ kg} \approx 10^{-6} \text{ kg de H}_2 = \boxed{1 \text{ mg de H}_2}$$

6.8



$$m_i = 0.1 \text{ moles}$$

$$S = 50 \text{ cm}^2 = 5 \cdot 10^{-3} \text{ m}^2$$

$$V_i = S h$$

$$T_i = 273 \text{ K}$$

$$\text{Fuerza ejercida por el pistón} = m g = 100 \text{ kg} \cdot 9.8 \frac{\text{m}}{\text{s}^2} = 980 \text{ N}$$

$$P_i = \frac{F}{S} = \frac{980 \text{ N}}{5 \cdot 10^{-3} \text{ m}^2} = 196000 \text{ Pa}$$

Final



$$V_f = S h_f$$

$$P_f = P_i$$

$$m_f = m_i$$

$$a) V_i = \frac{m_i RT_i}{P_i} = \frac{0.1 \text{ moles} \cdot 8.3145 \text{ J mol}^{-1} \text{ K}^{-1} \cdot 273 \text{ K}}{196000 \text{ Pa}} = 1.1581 \cdot 10^{-3} \text{ m}^3$$

$$h = \frac{V_i}{S} = \frac{1.1581 \cdot 10^{-3} \text{ m}^3}{5 \cdot 10^{-3} \text{ m}^2} = \boxed{0.2316 \text{ m}}$$

Autor: Iván Gabia Alvarro

$$b) h_g = h + 10 \text{ cm} = 0.2316 \text{ m} + 0.1 \text{ m} = 0.3316 \text{ m}$$

$$V_g = S h_g = 5 \cdot 10^{-3} \text{ m}^2 \cdot 0.3316 \text{ m} = 1.658 \cdot 10^{-3} \text{ m}^3$$

$$T_g = \frac{P_g V_g}{n_g R} = \frac{196000 \text{ Pa} \cdot 1.658 \cdot 10^{-3} \text{ m}^3}{0.1 \text{ moles} \cdot 8.3145 \text{ J mol}^{-1} \text{ K}^{-1}} = \boxed{391 \text{ K}}$$

$$c) \Delta U = n C_v \Delta T = 0.1 \text{ moles} \cdot 5 \text{ cal mol}^{-1} \text{ K}^{-1} (391 - 273) \text{ K} = 59 \text{ cal} = \boxed{246.86 \text{ J}}$$

$$d) C_p = C_v + R \simeq 5 \frac{\text{cal}}{\text{mol K}} + 2 \frac{\text{cal}}{\text{mol K}} = 7 \frac{\text{cal}}{\text{mol K}}$$

Hay dos maneras de calcular Q

$$\text{Primera manera: } Q = n C_p \Delta T = 0.1 \text{ moles} \cdot 7 \frac{\text{cal}}{\text{mol K}} \cdot 118 \text{ K} = 82.6 \text{ cal} = \boxed{345.27 \text{ J}}$$

$$\text{Segunda manera: } W = P \Delta V = 196000 \text{ Pa} \cdot 0.005 \text{ m}^2 \cdot 0.1 \text{ m} = 98 \text{ J}$$

$$Q = \Delta U + W = 344.86 \text{ J}$$

6.9 Gas ideal monoatómico $\Rightarrow \gamma = 5/3$

$$P_A = 1 \text{ atm} = 1.013 \cdot 10^5 \text{ Pa} \quad V_A = 80 \text{ dm}^3 = 0.08 \text{ m}^3 \quad V_B = 40 \text{ dm}^3 = 0.04 \text{ m}^3$$

$$n = \frac{N_{\text{gas}}}{N_A} = \frac{M_{\text{gas}}}{m_{\text{He}} N_A} = \frac{0.07 \text{ kg}}{6.64 \cdot 10^{-27} \frac{\text{kg}}{\text{átomo}} \cdot 6.022 \cdot 10^{23} \frac{\text{átomos}}{\text{mol}}} = 17.5 \text{ moles}$$

$$a) P_A V_A = n R T_A \Rightarrow T_A = \frac{P_A V_A}{n R} = \frac{1.013 \cdot 10^5 \text{ Pa} \cdot 0.08 \text{ m}^3}{17.5 \text{ moles} \cdot 8.3145 \text{ J mol}^{-1} \text{ K}^{-1}} = 55.7 \text{ K}$$

$$T V^{\gamma-1} = \text{cte} \Rightarrow T_A V_A^{\gamma-1} = T_B V_B^{\gamma-1} \Rightarrow T_B = T_A \left(\frac{V_A}{V_B} \right)^{\gamma-1} = 55.7 \text{ K} \left(\frac{80}{40} \right)^{\frac{5}{3}-1}$$

$$\boxed{T_B = 88.4 \text{ K}}$$

$$b) P_B = \frac{n R T_B}{V_B} = \frac{17.5 \text{ moles} \cdot 8.3145 \text{ J mol}^{-1} \text{ K}^{-1} \cdot 88.4 \text{ K}}{0.04 \text{ m}^3} = 321563.3 \text{ Pa}$$

$$W = \frac{P_B V_B - P_A V_A}{1 - \gamma} = \boxed{-7137.8 \text{ J}} \quad \text{2ª manera: } W = \frac{n R (T_B - T_A)}{1 - \gamma}$$

Autor: Iván Cabria Álvarez

c) $Q=0$ por ser un proceso adiabático $\Rightarrow \Delta U = -W = \boxed{+7137.8 \text{ J}}$

$$\Delta U = Q - W$$

También se puede calcular ΔU de la siguiente manera:

$$\Delta U = n C_V \Delta T = n C_V (T_B - T_A) = n \frac{3}{2} R (T_B - T_A) =$$

$$= 17.5 \text{ moles} \frac{3}{2} 8.3145 \text{ J mol}^{-1} \text{ K}^{-1} (88.4 - 55.7) \text{ K} = 7137.0 \text{ J}$$

6.10) A nivel del mar, la presión de la atmósfera sobre un área S es igual al peso (fuerza) de una columna de mercurio de 760 mm de altura y área S .

$$P = \frac{F}{S} = \frac{m g}{S} = \frac{\rho_{\text{Hg}} V_{\text{Hg}} g}{S} = \frac{\rho_{\text{Hg}} l_{\text{Hg}} S g}{S} = \rho_{\text{Hg}} l_{\text{Hg}} g$$

Hg: Símbolo químico del mercurio

l_{Hg} : altura de la columna del mercurio

ρ_{Hg} : densidad del mercurio

$$P = \rho_{\text{Hg}} l_{\text{Hg}} g = 13590 \frac{\text{kg}}{\text{m}^3} 0.76 \text{ m } 9.81 \frac{\text{m}}{\text{s}^2} = 101321.6 \frac{\text{kg m}^2}{\text{m}^3 \text{ s}^2} =$$

$$= 1.013 \cdot 10^5 \frac{\text{kg m/s}^2}{\text{m}^2} = 1.013 \cdot 10^5 \frac{\text{N}}{\text{m}^2} = 1.013 \cdot 10^5 \text{ Pa}$$

$$\boxed{1 \text{ atm} = 1.013 \cdot 10^5 \text{ Pa}}$$

Autor: Juan Cabria Álvaro

P8.11 a) $N_{O_2} = \frac{M_{O_2}}{m_{O_2}} = \frac{0.001 \text{ Kg}}{5.31 \cdot 10^{-26} \text{ Kg}} = 1.887 \cdot 10^{22} \text{ moléculas}$

$n_i = \frac{N_{O_2}}{N_A} = \frac{1.887 \cdot 10^{22} \text{ moléculas}}{6.022 \cdot 10^{23} \text{ moléculas/mol}} = 0.03133 \text{ moles de } O_2$

$V_i = \frac{n_i R T_i}{P_i} = \frac{0.03133 \text{ moles} \cdot 8.3145 \text{ J mol}^{-1} \text{ K}^{-1} \cdot 320 \text{ K}}{10 \cdot 1.013 \cdot 10^5 \text{ Pa}} = \boxed{8.2 \cdot 10^{-5} \text{ m}^3}$

b) $PV = nRT$
 $V_i = V_f \Rightarrow \frac{P_i}{P_f} = \frac{n_i T_i}{n_f T_f} \Rightarrow n_f = n_i \frac{P_i T_i}{P_i T_f} = n_i \frac{5 T_i}{8 T_f} =$

$= 0.03133 \text{ moles} \cdot \frac{5 \cdot 320}{8 \cdot 300} = 0.02089 \text{ moles de } O_2$

$M_{O_2} \text{ final} = n_f N_A m_{O_2} = 0.02089 \text{ moles} \cdot 6.022 \cdot 10^{23} \text{ moléculas/mol} \cdot 5.3 \cdot 10^{-26} \text{ Kg/molécula}$

$M_{O_2} \text{ final} = \boxed{6.7 \cdot 10^{-4} \text{ Kg}}$

P8.12 a) $P_{10} = 10 \text{ atm}$ $PV = nRT \Rightarrow V_{10} = \frac{nRT}{P_{10}} = \frac{1 \text{ mol} \cdot 8.3145 \text{ J mol}^{-1} \text{ K}^{-1} \cdot 300 \text{ K}}{1.013 \cdot 10^6 \text{ Pa}}$

$V_{10} = 2.46 \cdot 10^{-3} \text{ m}^3$

$V_1 = \frac{nRT}{P_1} = \frac{1 \text{ mol} \cdot 8.3145 \text{ J mol}^{-1} \text{ K}^{-1} \cdot 300 \text{ K}}{1.013 \cdot 10^5 \text{ Pa}} = 0.0246 \text{ m}^3$

$W = P_{ext} (V_1 - V_{10}) = 1.013 \cdot 10^5 \text{ Pa} (0.0246 - 0.00246) \text{ m}^3 = \boxed{2.24 \text{ KJ}}$

b) Etapa a: de 10 a 5 atm $V_5 = \frac{nRT}{P_5} = \frac{8.3145 \cdot 300 \text{ J}}{5 \cdot 1.013 \cdot 10^5 \text{ Pa}} = 0.00492 \text{ m}^3$

$W_a = P_5 (V_5 - V_{10}) = 5 \cdot 1.013 \cdot 10^5 \text{ Pa} (0.00492 - 0.00246) \text{ m}^3 = 1245.99 \text{ J}$

Etapa b: de 5 a 1 atm $W_b = P_1 (V_1 - V_5) = 1.013 \cdot 10^5 \text{ Pa} (0.0246 - 0.00492) \text{ m}^3 = 1994 \text{ J}$

$W = W_a + W_b = \boxed{3.24 \text{ KJ}}$

c) $V_2 = \frac{8.3145 \cdot 300 \text{ J}}{2 \cdot 1.013 \cdot 10^5 \text{ Pa}} = 0.0123 \text{ m}^3$

$W_a = P_5 (V_5 - V_{10}) = 1246 \text{ J}$ $W_b = P_2 (V_2 - V_5) = 2 \cdot 1.013 \cdot 10^5 \text{ Pa} (0.0123 - 0.00492) \text{ m}^3 = 1495 \text{ J}$

$W_c = P_1 (V_1 - V_2) = 1.013 \cdot 10^5 \text{ Pa} (0.0246 - 0.0123) \text{ m}^3 = 1246 \text{ J}$

$W = W_a + W_b + W_c = \boxed{3987 \text{ J}}$

d) $W = nRT \ln \left(\frac{V_1}{V_{10}} \right) = 5743.5 \text{ J}$
 proceso isoterma reversible

P8.13 $m = \frac{M_{H_2}}{m_{H_2} N_A} = \frac{6.5 \cdot 10^{-3} \text{ Kg}}{3.32 \cdot 10^{-27} \text{ Kg/molécula} \cdot 6.022 \cdot 10^{23} \text{ moléculas/mol}} = 3.25 \text{ moles de } H_2$

a) $P = \text{cte.} \Rightarrow W = P (V_f - V_i) = P (2V_i - V_i) = P V_i$

$P = \text{cte.} \Rightarrow P = P_i = P_f \Rightarrow W = P_i V_i \Rightarrow W = n R T_i = 3.25 \text{ moles} \cdot 8.3145 \text{ J mol}^{-1} \text{ K}^{-1} \cdot 300 \text{ K}$

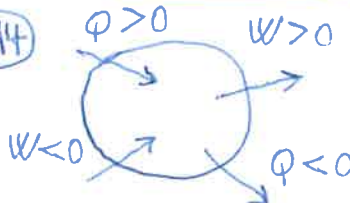
$W = \boxed{8.1 \text{ KJ}}$

Autor: Iván Cabria Álvarez

b) $\left. \begin{array}{l} P = \text{cte.} \\ m = \text{cte.} \\ PV = nRT \end{array} \right\} \Rightarrow \frac{T}{V} = \text{cte.} \Rightarrow \frac{T_i}{V_i} = \frac{T_f}{V_f} \Rightarrow T_f = 2T_i = 600\text{K}$
 $V_f = 2V_i$

$\Delta U = nC_v\Delta T = 3.25 \text{ moles } 4.88 \text{ cal mol}^{-1}\text{K}^{-1} (600 - 300) \text{ K} = 4758 \text{ cal} = \boxed{19.9 \text{ KJ}}$

c) $\Delta U = Q - W \Rightarrow Q = \Delta U + W = 19.9 \text{ KJ} + 8.1 \text{ KJ} = \boxed{28 \text{ KJ}}$

P5/14  $Q_{acb} = +80\text{J}$ $W_{acb} = +30\text{J}$
 U es una función de estado $\Rightarrow \Delta U_{acb} = \Delta U_{adb} = \Delta U_{abc} = -\Delta U_{bca}$

a) $\Delta U_{acb} = Q_{acb} - W_{acb} = (80 - 30)\text{J} = +50\text{J}$
 $\Delta U_{acb} = \Delta U_{adb}$ $\Rightarrow \Delta U_{adb} = +50\text{J}$

$\Delta U_{adb} = Q_{adb} - W_{adb} \Rightarrow Q_{adb} = \Delta U_{adb} + W_{adb}$
 $W_{adb} = +10\text{J}$

$\Rightarrow \boxed{Q_{adb} = +60\text{J}}$

$W_{bca} = -20\text{J}$ (trabajo sobre el sistema)

b) $\Delta U_{bca} = Q_{bca} - W_{bca} \Rightarrow Q_{bca} = \Delta U_{bca} + W_{bca} = -\Delta U_{adb} + W_{bca} = \boxed{-70\text{J}}$

P6/15 Gas ideal monoatómico $\Rightarrow C_v = \frac{3}{2}R$ $C_p = \frac{5}{2}R$ $\gamma = \frac{C_p}{C_v} = \frac{5}{3} = 1.6666$

a) $Q = 0 \Rightarrow \Delta U = -W$

$W = P_{\text{ext}}(V_f - V_i) \Rightarrow W = P_{\text{ext}} nR \left(\frac{T_f}{P_f} - \frac{T_i}{P_i} \right) \Rightarrow \Delta U = -P_{\text{ext}} nR \left(\frac{T_f}{P_f} - \frac{T_i}{P_i} \right)$
 $PV = nRT$

$\Delta U = nC_v(T_f - T_i) = n \frac{3}{2}R(T_f - T_i)$

$\Rightarrow -P_{\text{ext}} \left(\frac{T_f}{P_f} - \frac{T_i}{P_i} \right) = \frac{3}{2}(T_f - T_i)$

$P_{\text{ext}} = 1 \text{ atm}$ $P_f = P_{\text{ext}}$ $P_i = 2 \text{ atm}$ $\Rightarrow -T_f + \frac{T_i}{2} = \frac{3}{2}(T_f - T_i) \Rightarrow$

$\Rightarrow T_f = \frac{4}{5}T_i = \frac{4}{5}300\text{K} = \boxed{240\text{K}}$

b) $V_f = \frac{nRT_f}{P_f} = \frac{1 \text{ mol } 8.3145 \text{ J mol}^{-1}\text{K}^{-1} 240\text{K}}{1.013 \cdot 10^5 \text{ Pa}} = \boxed{0.0197 \text{ m}^3}$

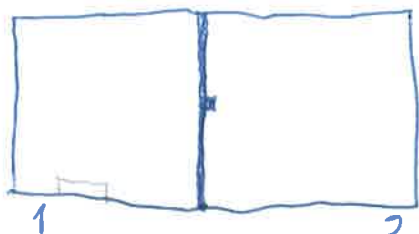
$V_i = \frac{nRT_i}{P_i} = \frac{8.3145 \text{ J K}^{-1} 300\text{K}}{2 \cdot 1.013 \cdot 10^5 \text{ Pa}} = 0.0123 \text{ m}^3$

$W = P_{\text{ext}}(V_f - V_i) = 1.013 \cdot 10^5 \text{ Pa} (0.0197 - 0.0123) \text{ m}^3 = \boxed{+747.59 \text{ J}}$

$\Delta U = Q - W = 0\text{J} - 747.59\text{J} = \boxed{-747.59\text{J}}$

(P616)

(8.9)



$$V_{1i} = V_{2i} = 5L \quad T_{1i} = T_{2i} = 300K$$

$$P_{1i} = P_{2i} = 0.93 \text{ atm} \quad T_{1f} = 500K \quad T_{2f} = 329K$$

$$V_T = V_{1i} + V_{2i} = V_{1f} + V_{2f} = 10L$$

Gas diatómico $\Rightarrow C_V = \frac{5}{2}R$, $C_P = \frac{7}{2}R$ y $\gamma = \frac{7}{5} = 1.4$

a) Proceso adiabático en el recipiente 2 $\Rightarrow T_2 V_2^{\gamma-1} = \text{cte.} \Rightarrow T_{2i} V_{2i}^{\gamma-1} = T_{2f} V_{2f}^{\gamma-1}$

$$\Rightarrow V_{2f} = \left(\frac{T_{2i}}{T_{2f}} \right)^{\frac{1}{\gamma-1}} V_{2i} = \left(\frac{300}{329} \right)^{2.5} 5L = \boxed{3.97L}$$

$$V_{1f} = V_T - V_{2f} = (10 - 3.97)L = \boxed{6.03L}$$

Proceso adiabático en el recipiente 2 $\Rightarrow P_2 V_2^{\gamma} = \text{cte.} \Rightarrow P_{2i} V_{2i}^{\gamma} = P_{2f} V_{2f}^{\gamma} \Rightarrow$

$$\Rightarrow P_{2f} = \left(\frac{V_{2i}}{V_{2f}} \right)^{\gamma} P_{2i} = \left(\frac{5}{3.97} \right)^{1.4} 0.93 \text{ atm} = \boxed{1.285 \text{ atm}}$$

En el equilibrio las presiones a los dos lados del émbolo son iguales $\Rightarrow$

$$\Rightarrow P_{1f} = P_{2f} = \boxed{1.285 \text{ atm}}$$

$$b) n_1 = \frac{P_{1i} V_{1i}}{RT_{1i}} = \frac{0.93 \cdot 1.013 \cdot 10^5 \text{ Pa} \cdot 5 \cdot 10^{-3} \text{ m}^3}{8.3145 \text{ J mol}^{-1} \text{ K}^{-1} \cdot 300 \text{ K}} = 0.1888 \text{ moles}$$

$$\Delta U_1 = n C_V \Delta T_1 = n_1 \frac{5}{2} R (T_{1f} - T_{1i}) = 0.1888 \text{ moles} \frac{5}{2} 8.3145 \text{ J K}^{-1} \text{ mol}^{-1} 200 \text{ K} = 784.89 \text{ J}$$

El sistema no realiza trabajo $\Rightarrow W_1 = -W_2$

$$\left. \begin{aligned} W_2 &= -\Delta U_2 + Q_2 \\ \text{Proceso adiabático en recipiente 2} &\Rightarrow Q_2 = 0 \end{aligned} \right\} \Rightarrow W_2 = -\Delta U_2 = -n_2 C_V (T_{2f} - T_{2i}) \Rightarrow$$

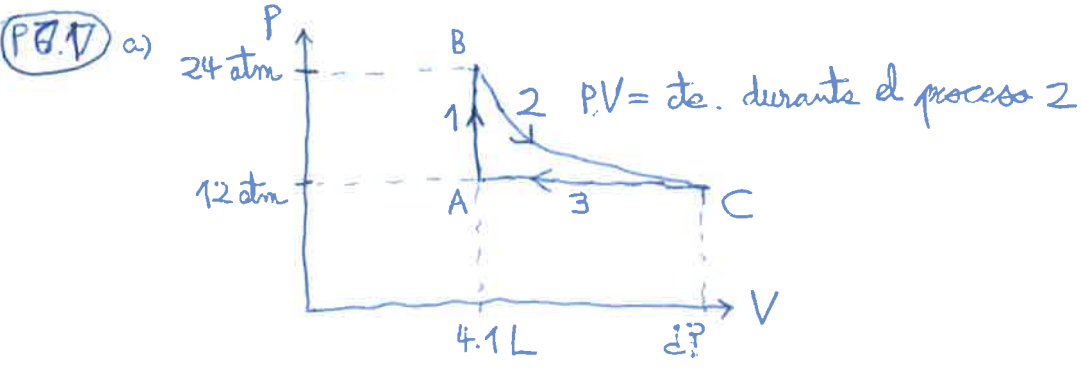
$$n_2 = \frac{P_{2i} V_{2i}}{RT_{2i}} = \frac{P_{1i} V_{1i}}{RT_{1i}} = n_1$$

$$\Rightarrow W_2 = -0.1888 \text{ moles} \frac{5}{2} 8.3145 \text{ J K}^{-1} \text{ mol}^{-1} (329 - 300) \text{ K} = -113.81 \text{ J} \Rightarrow$$

$$\left. \begin{aligned} W_1 &= +113.81 \text{ J} \\ Q_1 &= \Delta U_1 + W_1 \end{aligned} \right\} \Rightarrow \boxed{Q_1 = +898.7 \text{ J}}$$

$$\left. \begin{aligned} \Delta U &= Q - W = (Q_1 + Q_2) - (W_1 + W_2) \\ \Delta U &= Q_1 \quad Q_2 = 0 \end{aligned} \right\} \Rightarrow W_1 = -W_2$$

Autor: Joán Cabria Álvarez



1) Process 1 = Process AB

$V = \text{cte.} \Rightarrow W_{AB} = \int P dV = 0 \Rightarrow \Delta U_{AB} = Q_{AB}$

$P_A V_A = n R T_A \Rightarrow n = \frac{P_A V_A}{R T_A} = \frac{12 \cdot 1.013 \cdot 10^5 \text{ Pa} \cdot 0.0041 \text{ m}^3}{8.3145 \text{ J mol}^{-1} \text{ K}^{-1} \cdot 300 \text{ K}} = 2 \text{ moles}$

$P_B V_B = n R T_B \Rightarrow T_B = \frac{P_B V_B}{n R} = \frac{24 \cdot 1.013 \cdot 10^5 \text{ Pa} \cdot 0.0041 \text{ m}^3}{2 \text{ moles} \cdot 8.3145 \text{ J mol}^{-1} \text{ K}^{-1}} = 600 \text{ K}$

$\Delta U_{AB} = n C_V \Delta T_{AB} = 2 \text{ moles} \cdot 5 \frac{\text{cal}}{\text{mol K}} \cdot 4.184 \frac{\text{J}}{\text{cal}} \cdot (600 - 300) \text{ K} = 12.552 \text{ KJ}$

$Q_{AB} = 12.552 \text{ KJ}$

Process 2 = Process BC $T_B = 600 \text{ K}$ Process isothermo $\Rightarrow T_C = T_B = 600 \text{ K}$

$V_C = \frac{n R T_C}{P_C} = \frac{2 \text{ moles} \cdot 8.3145 \text{ J mol}^{-1} \text{ K}^{-1} \cdot 600 \text{ K}}{12 \cdot 1.013 \cdot 10^5 \text{ Pa}} = 0.0082 \text{ m}^3$

Process isothermo $\Rightarrow W_{BC} = n R T \ln \left(\frac{V_C}{V_B} \right) = 2 \text{ moles} \cdot 8.3145 \text{ J mol}^{-1} \text{ K}^{-1} \cdot 600 \text{ K} \cdot \ln \left(\frac{0.0082}{0.0041} \right) = 6.916 \text{ KJ}$

Process isothermo $\Rightarrow \Delta T_{BC} = 0 \Rightarrow \Delta U_{BC} = 0 \Rightarrow Q_{BC} = W_{BC} = 6.916 \text{ KJ}$

Process 3 = Process CA Process isobaro $\Rightarrow P_C = P_A = 12 \text{ atm}$

$Q_{CA} = n C_P \Delta T_{CA} = n (C_V + R) (T_A - T_C) = 2 (5 \cdot 4.184 + 8.3145) (300 - 600) \text{ J}$

$Q_{CA} = -17.541 \text{ KJ}$

$\Delta U_{CA} = n C_V \Delta T_{CA} = 2 \cdot 5 \cdot 4.184 \cdot (-300) \text{ J} = -12.552 \text{ KJ}$

$\Delta U_{CA} = Q_{CA} - W_{CA} \Rightarrow W_{CA} = -4.988 \text{ KJ}$

Por otra parte, $W_{CA} = P_A (V_A - V_C) = 12 \cdot 1.013 \cdot 10^5 \text{ Pa} (0.0041 - 0.0082) \text{ m}^3 = -4.984 \text{ KJ}$

Tutor: Juan Carlos Alvarado

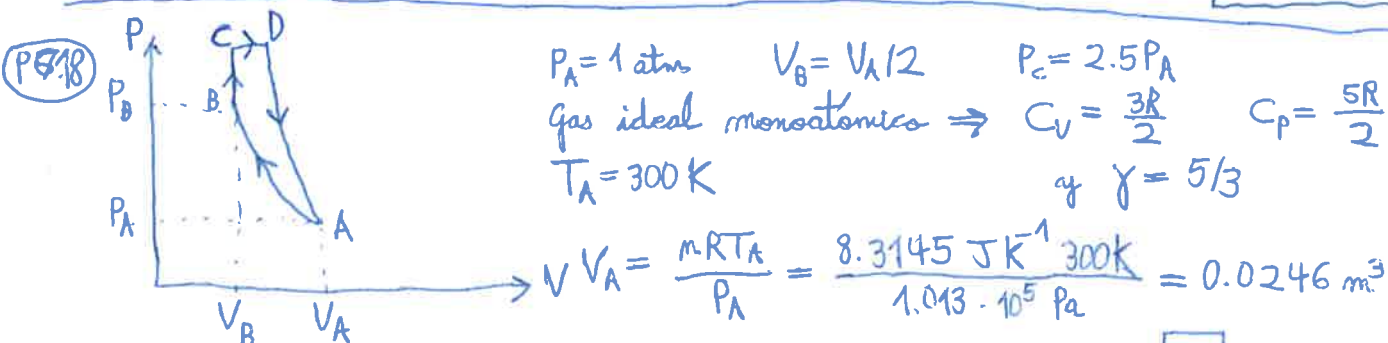
c) Todo el ciclo

Sin hacer cálculos, sabemos que $\Delta U = 0$ por ser un ciclo

$$\Delta U = \Delta U_{AB} + \Delta U_{BC} + \Delta U_{CA} = 12.552 \text{ kJ} + 0 - 12.552 \text{ kJ} = 0$$

$$W = W_{AB} + W_{BC} + W_{CA} = 0 + 6.916 \text{ kJ} - 4.988 \text{ kJ} = +1.927 \text{ kJ}$$

$$Q = Q_{AB} + Q_{BC} + Q_{CA} = 12.552 \text{ kJ} + 6.916 \text{ kJ} - 17.541 \text{ kJ} = +1.927 \text{ kJ}$$



Etapa AB es isoterma $\Rightarrow \Delta T_{AB} = 0 \Rightarrow \Delta U_{AB} = n C_V \Delta T_{AB} = 0$

$$W_{AB} = nRT \ln\left(\frac{V_B}{V_A}\right) = 8.3145 \text{ J K}^{-1} 300 \text{ K} \ln\left(\frac{1}{2}\right) = -1729 \text{ J}$$

$$\Delta U_{AB} = 0 \Rightarrow Q_{AB} = W_{AB} = -1729 \text{ J} \quad \text{isoterma} \Rightarrow P_A V_A = P_B V_B \Rightarrow P_B = 2 P_A$$

$$V_B = V_A/2$$

Etapa BC es isócora $\Rightarrow W_{BC} = 0$

$$\left. \begin{aligned} \Delta U_{BC} &= n C_V \Delta T_{BC} = n C_V (T_C - T_B) \\ P V &= n R T \end{aligned} \right\} \Rightarrow \Delta U_{BC} = \frac{C_V}{R} (P_C V_C - P_B V_B)$$

isócora $\Rightarrow V_C = V_B$
 $C_V = \frac{3}{2} R$

$$\Rightarrow \Delta U_{BC} = \frac{3}{2} V_B (P_C - P_B) = \frac{3}{4} V_A (2.5 - 2) P_A = \frac{3}{8} V_A P_A = \frac{3}{8} 0.0246 \text{ m}^3 1.013 \cdot 10^5 \text{ Pa}$$

$$\Delta U_{BC} = 934.49 \text{ J}$$

$$Q_{BC} = \Delta U_{BC} + W_{BC} = 934.49 \text{ J}$$

Etapa CD es isobara $\Rightarrow P_C = P_D$

tramo DA es adiabático $\Rightarrow P_A V_A^\gamma = P_D V_D^\gamma \Rightarrow V_D = \left(\frac{P_A}{P_D}\right)^{1/\gamma} V_A = \left(\frac{P_A}{P_C}\right)^{1/\gamma} V_A = V_A \left(\frac{1}{2.5}\right)^{1/8}$

$$V_D = 0.0246 \text{ m}^3 \left(\frac{1}{2.5}\right)^{3/5} = 0.0142 \text{ m}^3$$

$$W_{CD} = P_C (V_D - V_C) = 2.5 \cdot 1.013 \cdot 10^5 \text{ Pa} (0.0142 - 0.0123) \text{ m}^3 = +481.18 \text{ J}$$

$$Q_{CD} = n C_P \Delta T_{CD} = n \frac{5}{2} R (T_D - T_C) = \frac{5}{2} (P_D V_D - P_C V_C) = \frac{5}{2} P_C (V_D - V_C) = \frac{5}{2} 2.5 P_A (V_D - V_C) = 633125 \text{ Pa} (0.0142 - 0.0123) \text{ m}^3 = 1202.94 \text{ J}$$

$$\Delta U_{CD} = Q_{CD} - W_{CD} = +721.76 \text{ J}$$

Autor: Iván Gabrín Alvarado

Proceso DA es adiabático $\Rightarrow Q_{DA} = 0$

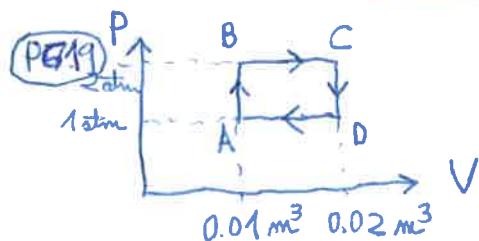
(5.12)

$$\Delta U_{DA} = n C_V \Delta T_{DA} = n \frac{3}{2} R (T_A - T_D)$$

$$T_D = \frac{P_D V_D}{n R} = \frac{P_C V_D}{n R} = \frac{2.5 P_A V_D}{n R} = \frac{2.5 \cdot 1.013 \cdot 10^5 \text{ Pa} \cdot 0.0142 \text{ m}^3}{8.3145 \text{ J K}^{-1}} = 432.52 \text{ K}$$

$$\Delta U_{DA} = \frac{3}{2} 8.3145 \text{ J K}^{-1} (300 - 432.52) \text{ K} = -1653 \text{ J}$$

$$W_{DA} = -\Delta U_{DA} = +1653 \text{ J}$$



a) Ciclo ABCDA Proceso AB es isócoro $\Rightarrow W_{AB} = 0$

Proceso BC es isobárico $\Rightarrow W_{BC} = P_B (V_C - V_B) = 2 \cdot 1.013 \cdot 10^5 \text{ Pa} \cdot 0.01 \text{ m}^3 = 2026 \text{ J}$

Proceso CD es isócoro $\Rightarrow W_{CD} = 0$

Proceso DA es isobárico $\Rightarrow W_{DA} = P_A (V_A - V_D) = -1.013 \cdot 10^5 \text{ Pa} \cdot 0.01 \text{ m}^3$

$$W_{DA} = -1013 \text{ J} \quad W = W_{AB} + W_{BC} + W_{CD} + W_{DA} = 1013 \text{ J}$$

Otra manera de calcular W. W es el área del rectángulo ABCD en este orden

$$W = (2 - 1) \text{ atm} \cdot (0.02 - 0.01) \text{ m}^3 = 1.013 \cdot 10^5 \text{ Pa} \cdot 0.01 \text{ m}^3 = 1013 \text{ J}$$

b) $n = \frac{P_A V_A}{R T_A} = \frac{1.013 \cdot 10^5 \text{ Pa} \cdot 0.01 \text{ m}^3}{8.3145 \text{ J mol}^{-1} \text{ K}^{-1} \cdot 273 \text{ K}} = 0.4463 \text{ moles} \quad nR = 3.7108 \text{ J K}^{-1}$

$$T_B = \frac{P_B V_B}{n R} = 546 \text{ K} \quad T_C = \frac{P_C V_C}{n R} = 1092 \text{ K} \quad T_D = \frac{P_D V_D}{n R} = 546 \text{ K}$$

c) Proceso AB es isócoro $Q_{AB} = n C_V (T_B - T_A) \quad m = \rho V = 1.293 \text{ kg m}^{-3} \cdot 0.01 \text{ m}^3 = 0.01293 \text{ kg}$

$$C_V = \frac{Q}{Y} = \frac{0.237 \text{ cal g}^{-1} \text{ K}^{-1}}{1.42} = 0.1669 \text{ cal g}^{-1} \text{ K}^{-1}$$

$$Q_{AB} = 12.93 \text{ g} \cdot 0.1669 \text{ cal g}^{-1} \text{ K}^{-1} (546 - 273) \text{ K} = 589.1 \text{ cal}$$

Proceso BC es isobárico $Q_{BC} = n C_P (T_C - T_B) = 12.93 \text{ g} \cdot 0.237 \text{ cal g}^{-1} \text{ K}^{-1} (1092 - 546) \text{ K} = 1673.2 \text{ cal}$

Proceso CD es isócoro $Q_{CD} = n C_V (T_D - T_C) = 12.93 \text{ g} \cdot 0.1669 \text{ cal g}^{-1} \text{ K}^{-1} (546 - 1092) \text{ K} = -1178.3 \text{ cal}$

Proceso DA es isobárico $Q_{DA} = n C_P (T_A - T_D) = 12.93 \text{ g} \cdot 0.237 \text{ cal g}^{-1} \text{ K}^{-1} (273 - 546) \text{ K} = -836.6 \text{ cal}$

(6.20) a) Las paredes son fijas. Por tanto, se producirá un proceso a volumen constante o isócoro

b) Según los experimentos, si se calienta un gas a volumen constante, aumenta su temperatura. Autor: Iván Cabris Alvarado

$$\left. \begin{array}{l} V = \text{cte.} \\ PV = nRT \end{array} \right\} \Rightarrow \frac{P}{T} = \frac{nR}{V} = \text{cte.} \Rightarrow \left. \begin{array}{l} T \text{ aumenta} \\ P \text{ aumenta} \end{array} \right\}$$

b) Al ser isócoro, el trabajo será cero $\Rightarrow \Delta U = Q > 0 \Rightarrow T_{\text{final}} > T_{\text{inicial}}$

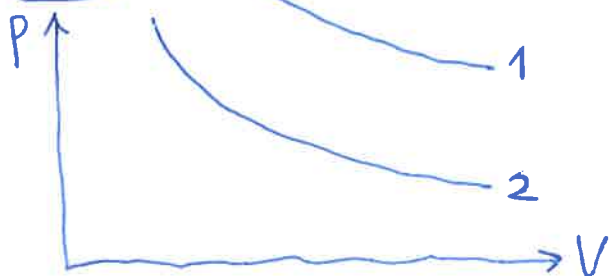
$$\Delta U = n C_V (T_{\text{final}} - T_{\text{inicial}})$$

Q es mayor que cero porque es un calor absorbido por el gas

Si temperatura aumenta

(6.13)

(6.21)



En una isoterma se cumple $PV = \text{constante}$

llamemos A a la constante: $PV = A$

$$PV = nRT \Rightarrow A = nRT$$

Si el número de moles no cambia, como sucede en este problema, entonces A dependerá sólo de T y dependerá linealmente.

A mayor temperatura, mayor valor de A

Las isothermas 1 y 2 cumplen, respectivamente: $PV = A_1 = nRT_1$
y $PV = A_2 = nRT_2$

$P = A_1/V$ es mayor que $P = A_2/V$ para cualquier valor del volumen V.
Por tanto, $A_1 > A_2$

$$\left. \begin{array}{l} A_i = nRT_i \\ A_1 > A_2 \end{array} \right\} \Rightarrow T_1 > T_2 \Rightarrow$$

La isoterma de arriba es la de mayor temperatura

(6.22)

$$W = \int_i^f P dV$$

$$PV^\gamma = a = \text{cte.}$$

$$\Rightarrow W = \int_i^f a V^{-\gamma} dV = a \frac{V^{-\gamma+1}}{(-\gamma+1)} \Big|_i^f =$$

$$= \frac{a V_f^{-\gamma+1} - a V_i^{-\gamma+1}}{(-\gamma+1)}$$

$$PV^\gamma = a \Rightarrow P_i V_i^\gamma = P_f V_f^\gamma = a$$

$$\Rightarrow W = \frac{P_f V_f^\gamma V_f^{-\gamma+1} - P_i V_i^\gamma V_i^{-\gamma+1}}{(-\gamma+1)} =$$

$$= \frac{P_f V_f - P_i V_i}{(-\gamma+1)} = \frac{nRT_f - nRT_i}{(-\gamma+1)} = \frac{nR}{(-\gamma+1)} \Delta T \Rightarrow \boxed{W \propto \Delta T}$$

Autor: Iván Gabris Álvarez

(6.23) a) Demostrar que $TV^{\gamma-1} = \text{cte.}$

$$\begin{aligned} PV^{\gamma} &= a = \text{cte.} \\ PV &= nRT \end{aligned} \Rightarrow P = \frac{nRT}{V} \Rightarrow \frac{nRT}{V} V^{\gamma} = a \Rightarrow$$

$$\Rightarrow TV^{\gamma-1} = \frac{a}{nR} \Rightarrow \boxed{TV^{\gamma-1} = b = \text{cte.}}$$

$n = \text{cte.} \Rightarrow nR = \text{cte.}$
 $R = \text{cte.}$

b) Demostrar que $P^{1-\gamma} T^{\gamma} = \text{cte.}$

$$\begin{aligned} PV^{\gamma} &= a = \text{cte.} \\ PV &= nRT \Rightarrow V = \frac{nRT}{P} \end{aligned} \Rightarrow P \left(\frac{nRT}{P} \right)^{\gamma} = a \Rightarrow P^{1-\gamma} T^{\gamma} = \frac{a}{(nR)^{\gamma}}$$

$$n = \text{cte.} \quad \gamma = \text{cte.} \quad R = \text{cte.} \Rightarrow \boxed{P^{1-\gamma} T^{\gamma} = c = \text{cte.}}$$

(6.24) el objetivo es encontrar la ecuación $d(PV) = nRdT$

$$\circ \quad PdV + VdP = nRdT$$

$$\left. \begin{aligned} PV^{\gamma} &= a \\ a &= \text{constante} \end{aligned} \right\} \Rightarrow d(PV^{\gamma}) = 0 \Rightarrow \gamma V^{\gamma-1} dV + V^{\gamma} dP = 0$$

Multiplicando por $V^{1-\gamma}$ a los dos lados de la ecuación anterior, obtenemos:

$$\gamma dV + VdP = 0$$

Usamos la relación de Mayer para convertir $\gamma dV + VdP = 0$ en una ecuación con tres términos $\Rightarrow$

$$\left. \begin{aligned} C_p - C_v &= R \\ \gamma &= C_p / C_v \end{aligned} \right\} \Rightarrow \gamma = \frac{R}{C_v} + 1$$

a la izquierda

$$\Rightarrow \frac{R}{\gamma} PdV + PdV + VdP = 0 \leftarrow \text{Es una ecuación con tres términos a la izquierda}$$

$$dU = \delta Q - \delta W$$

$$\delta W = PdV$$

$$\text{Proceso adiabático} \Rightarrow \delta Q = 0$$

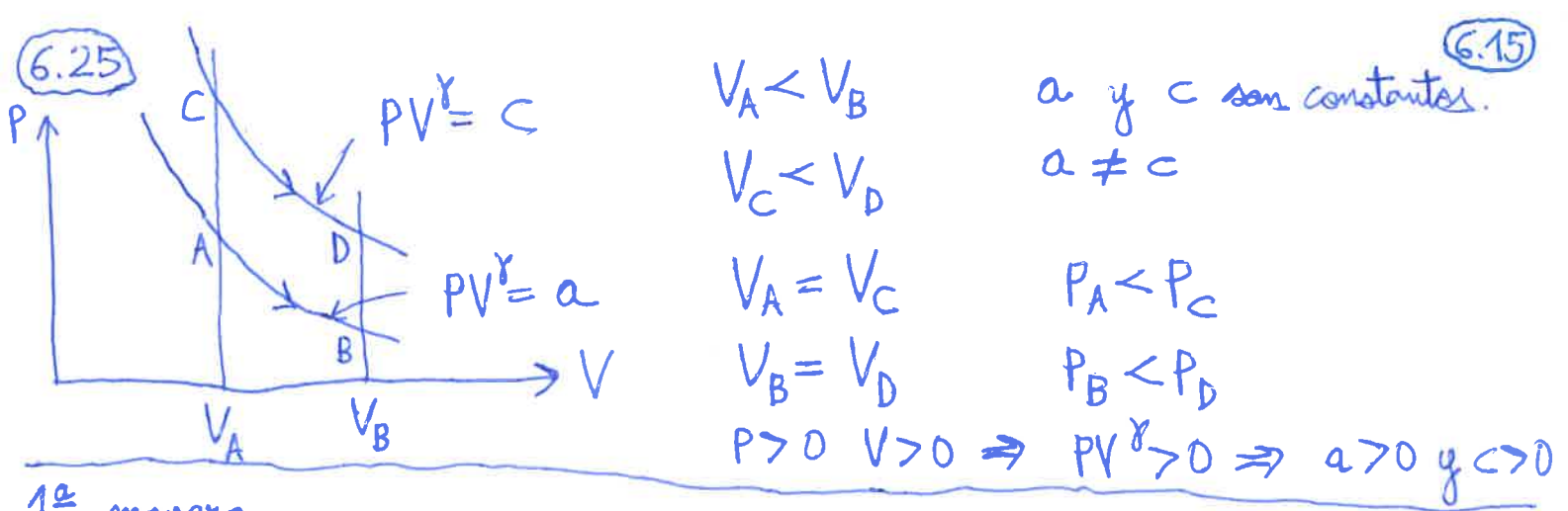
$$dU = nC_v dT$$

$$\left. \begin{aligned} dU &= \delta Q - \delta W \\ \delta Q &= 0 \\ dU &= nC_v dT \end{aligned} \right\} \Rightarrow nC_v dT = -PdV \Rightarrow -nR dT = \frac{R}{\gamma} PdV$$

$$\left. \begin{aligned} \frac{R}{\gamma} PdV + PdV + VdP &= 0 \\ -nR dT &= \frac{R}{\gamma} PdV \end{aligned} \right\} \Rightarrow -nR dT + PdV + VdP = 0 \Rightarrow$$

$$\Rightarrow nR dT = PdV + VdP \Rightarrow nR dT = d(PV) \Rightarrow \boxed{PV = nRT}$$

Autor: Iván Cabria Álvarez



1ª manera

$$W_{AB} = \int_A^B P dV = \frac{a}{1-\gamma} (V_B^{1-\gamma} - V_A^{1-\gamma})$$

$$W_{CD} = \int_C^D P dV = \frac{c}{1-\gamma} (V_D^{1-\gamma} - V_C^{1-\gamma}) \left. \begin{array}{l} V_A = V_C \\ V_B = V_D \end{array} \right\} \Rightarrow W_{CD} = \frac{c}{1-\gamma} (V_B^{1-\gamma} - V_A^{1-\gamma})$$

W_{AB} y W_{CD} se diferencian en ^{las constantes} a y c .

Vamos a calcular cuál de las dos constantes, a y c , es ^{la} mayor:

$$\left. \begin{array}{l} P_A V_A^\gamma = a \\ P_C V_C^\gamma = c \end{array} \right\} \Rightarrow \frac{P_A V_A^\gamma}{P_C V_C^\gamma} = \frac{a}{c} \left. \begin{array}{l} V_A = V_C \\ P_A < P_C \end{array} \right\} \Rightarrow \frac{P_A}{P_C} = \frac{a}{c} \Rightarrow \frac{a}{c} < 1 \Rightarrow$$

$$\Rightarrow a < c$$

$$\left. \begin{array}{l} |W_{AB}| = \frac{|a|}{|1-\gamma|} |(V_B^{1-\gamma} - V_A^{1-\gamma})| \\ |W_{CD}| = \frac{|c|}{|1-\gamma|} |(V_B^{1-\gamma} - V_A^{1-\gamma})| \end{array} \right\} \Rightarrow \frac{|W_{AB}|}{|W_{CD}|} = \frac{|a|}{|c|} \Rightarrow \left. \begin{array}{l} a > 0 \quad c > 0 \Rightarrow |a| = a \quad |c| = c \\ a < c \end{array} \right\} \Rightarrow$$

$$\Rightarrow |W_{CD}| > |W_{AB}|$$

Autor: Yván Gabrín Alvarado

2ª manera

$$W_{AB} = \frac{P_B V_B - P_A V_A}{1-\gamma} = \frac{mR(T_B - T_A)}{1-\gamma}$$

$$T_A V_A^{\gamma-1} = T_B V_B^{\gamma-1} \Rightarrow T_A = T_B \left(\frac{V_B}{V_A} \right)^{\gamma-1} \Rightarrow T_B - T_A = T_B \left(1 - \left(\frac{V_B}{V_A} \right)^{\gamma-1} \right)$$

$$\Rightarrow W_{AB} = \frac{mRT_B}{1-\gamma} \left(1 - \left(\frac{V_B}{V_A} \right)^{\gamma-1} \right)$$

$$W_{CD} = \frac{mRT_D}{1-\gamma} \left(1 - \left(\frac{V_D}{V_C} \right)^{\gamma-1} \right) \Rightarrow W_{CD} = \frac{mRT_D}{1-\gamma} \left(1 - \left(\frac{V_B}{V_A} \right)^{\gamma-1} \right)$$

$$V_A = V_C \quad y \quad V_B = V_D$$

W_{AB} y W_D difieren en T_B y T_D .

¿Cómo a calcular cuál de las dos temperaturas es la mayor:

$$\begin{aligned} PV = mRT &\Rightarrow P_B V_B = m_B R T_B \\ P_D V_D &= m_D R T_D \\ \text{El n.º de moles es el mismo: } m_B &= m_D \end{aligned} \Rightarrow \left. \begin{aligned} \frac{P_B V_B}{P_D V_D} &= \frac{T_B}{T_D} \\ V_B &= V_D \end{aligned} \right\} \Rightarrow$$

$$\Rightarrow \left. \begin{aligned} \frac{P_B}{P_D} &= \frac{T_B}{T_D} \\ P_B &< P_D \end{aligned} \right\} \Rightarrow \frac{T_B}{T_D} < 1 \Rightarrow T_B < T_D$$

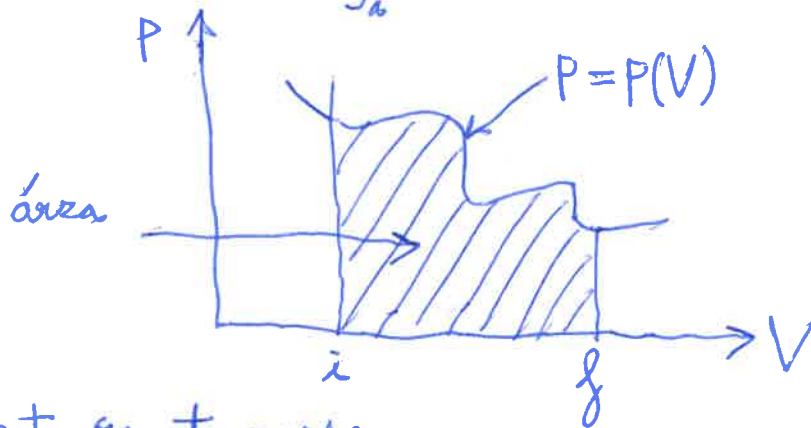
$$\begin{aligned} |W_{AB}| &= \frac{mRT_B}{|1-\gamma|} \left| 1 - \left(\frac{V_B}{V_A} \right)^{\gamma-1} \right| \\ |W_{CD}| &= \frac{mRT_D}{|1-\gamma|} \left| 1 - \left(\frac{V_B}{V_A} \right)^{\gamma-1} \right| \end{aligned} \Rightarrow \left. \begin{aligned} \frac{|W_{AB}|}{|W_{CD}|} &= \frac{T_B}{T_D} \\ T_B &< T_D \end{aligned} \right\} \Rightarrow$$

$$\Rightarrow \boxed{|W_{CD}| > |W_{AB}|}$$

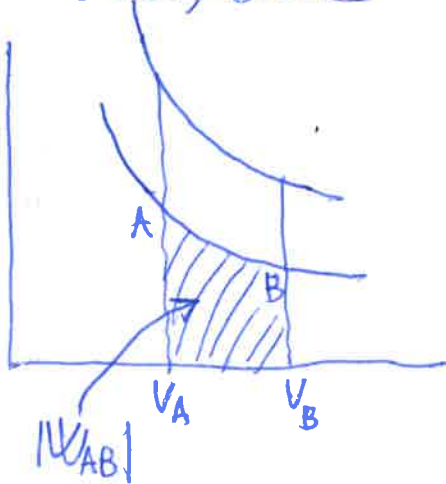
Autores: Iván Cabria Álvarez

$$|W| = \left| \int_i^f P dV \right|$$

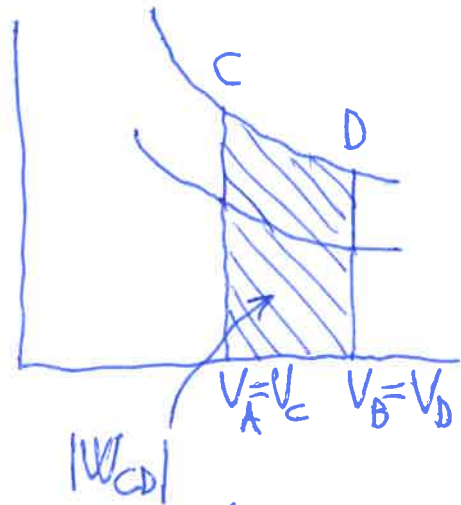
El módulo de la integral $\int_i^f P dV$ es el área indicada en la siguiente figura:



En el presente caso, tenemos:



$$|W_{AB}| = \text{Área } ABV_A V_B$$



$$|W_{CD}| = \text{Área } CDV_A V_B$$

El área $CDV_A V_B$ es mayor que el área $ABV_A V_B$

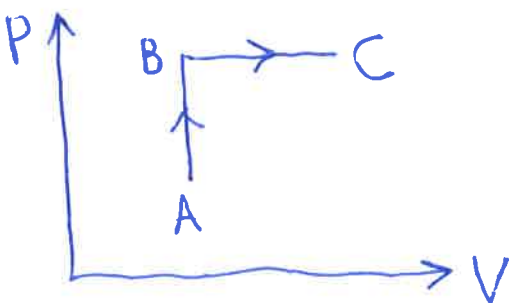
$$|W_{AB}| = \text{Área } ABV_A V_B$$

$$|W_{CD}| = \text{Área } CDV_A V_B$$

$$\Rightarrow |W_{CD}| > |W_{AB}|$$

(6.26)

(6.18)

a) ¿ $T_A \geq T_B$?

$$T = \frac{PV}{mR}$$

$$\Downarrow$$

$$T_A = \frac{P_A V_A}{mR}$$

$$T_B = \frac{P_B V_B}{mR}$$

$$\left. \begin{array}{l} T_A = \frac{P_A V_A}{mR} \\ T_B = \frac{P_B V_B}{mR} \end{array} \right\} \Rightarrow \frac{T_B}{T_A} = \frac{P_B V_B}{P_A V_A} \left. \begin{array}{l} \\ V_B = V_A \end{array} \right\} \Rightarrow$$

$$\Rightarrow \frac{T_B}{T_A} = \frac{P_B}{P_A} \left. \begin{array}{l} \\ P_B > P_A \end{array} \right\} \Rightarrow \boxed{T_B > T_A}$$

$$b) \text{ ¿} T_B \geq T_C? \quad T = \frac{PV}{mR} \Rightarrow \frac{T_C}{T_B} = \frac{P_C V_C}{P_B V_B} \left. \begin{array}{l} \\ P_B = P_C \end{array} \right\} \Rightarrow \frac{T_C}{T_B} = \frac{V_C}{V_B} \left. \begin{array}{l} \\ V_C > V_B \end{array} \right\} \Rightarrow$$

$$\Rightarrow \boxed{T_C > T_B}$$

TEMA 7 SEGUNDO PRINCIPIO DE LA TERMODINÁMICA

$$(7.1) \quad \eta = \frac{T_c - T_f}{T_c} \quad T_c > T_f$$

Calentar el foco caliente: $T_c' = T_c + \Delta T$ $\Delta T > 0$

$$\eta_{\text{calentar}} = \frac{T_c' - T_f}{T_c'} = \frac{T_c + \Delta T - T_f}{T_c + \Delta T}$$

Enfriar el foco frío: $T_f' = T_f - \Delta T$ $\Delta T > 0$

$$\eta_{\text{enfriar}} = \frac{T_c - T_f'}{T_c} = \frac{T_c - T_f + \Delta T}{T_c}$$

$$\frac{\eta_{\text{enfriar}}}{\eta_{\text{calentar}}} = \frac{T_c + \Delta T}{T_c} > 1 \Rightarrow \boxed{\eta_{\text{enfriar}} > \eta_{\text{calentar}}} \Rightarrow$$

$\Rightarrow$ La mejor opción para aumentar el rendimiento es enfriar el foco frío

(7.2a) Expansión adiabática $\Rightarrow Q=0 \Rightarrow \Delta U = Q - W = -W$

$$W = 854 \text{ Kg} \cdot \text{m} = 854 \text{ Kg} \cdot 9.8 \text{ m/s}^2 \cdot \text{m} = 8369.2 \text{ J}$$

$$\Delta U = m C_v \Delta T = m C_v (T_{\text{fr}} - T_{\text{cal}}) = 5 \cdot 4.18 \text{ J K}^{-1} (T_{\text{fr}} - T_{\text{cal}})$$

$$\eta = \frac{T_{\text{cal}} - T_{\text{fr}}}{T_{\text{cal}}} \Rightarrow \frac{T_{\text{fr}}}{T_{\text{cal}}} = \frac{1}{2}$$

$$\eta = 0.5$$

$$T_{\text{fr}} - T_{\text{cal}} = 400 \text{ K}$$

$$T_{\text{fr}} = 400 \text{ K}$$

$$T_{\text{cal}} = 800 \text{ K}$$

$$T_{\text{fr}} - T_{\text{cal}} = 400 \text{ K}$$

$$\Rightarrow T_{\text{fr}} - T_{\text{cal}} = -400 \text{ K}$$



BC: expansión adiabática

$$(b) \quad \left. \begin{array}{l} PV = nRT \\ T_B = 2T_C \end{array} \right\} \Rightarrow \frac{P_B V_B}{P_C V_C} = \frac{T_B}{T_C} = 2 \Rightarrow \frac{P_B V_B}{P_C V_C} = 2$$

$$\Rightarrow \left(\frac{V_C}{V_B} \right)^{\gamma-1} = 2$$

$$\frac{P_B}{P_C} = \left(\frac{V_C}{V_B} \right)^{\gamma}$$

$$C_p - C_v = R$$

$$C_v = 5 \text{ cal mol}^{-1} \text{ K}^{-1}$$

$$R = 2 \text{ cal mol}^{-1} \text{ K}^{-1}$$

$$\Rightarrow C_p = 7 \text{ cal mol}^{-1} \text{ K}^{-1}$$

$$\gamma = C_p / C_v$$

$$\Rightarrow \gamma = \frac{7}{5} = 1.4 \Rightarrow \gamma - 1 = 0.4$$

Autor: Iván Cabria Álvarez

$$\left\{ \begin{aligned} \left(\frac{V_C}{V_B} \right)^{\gamma-1} &= 2 \\ \gamma-1 &= 0.4 \end{aligned} \right\} \Rightarrow \left(\frac{V_C}{V_B} \right)^{0.4} = 2 \Rightarrow \left(\frac{V_C}{V_B} \right)^{2/5} = 2 \Rightarrow \frac{1}{2} = \left(\frac{V_B}{V_C} \right)^{2/5} \Rightarrow$$

$$\Rightarrow \frac{V_B}{V_C} = \frac{1}{2^{5/2}} = \boxed{\frac{1}{4\sqrt{2}}}$$

7.3) proceso isoterma $\Rightarrow \Delta T = 0 \Rightarrow \Delta U = m C_V \Delta T = \boxed{0}$

$$\Delta S = m R \ln \left(\frac{V_f}{V_i} \right) = 2 \frac{\text{cal}}{\text{K}} \ln 4 = \boxed{2.76 \frac{\text{cal}}{\text{K}}}$$

7.4) $\Delta S_1 = m C_{\text{fuerza}} \int_{253.15}^{273.15} \frac{dT}{T} = 1 \text{g} \cdot 0.5 \frac{\text{cal}}{\text{g K}} \ln \left(\frac{273.15}{253.15} \right) = 0.03403 \frac{\text{cal}}{\text{K}}$

$$\Delta S_2 = \frac{m L_f}{T_f} = \frac{80 \text{ cal}}{273.15 \text{ K}} = 0.29288 \text{ cal/K}$$

$$\Delta S_3 = m C_{\text{agua}} \int_{273.15}^{373.15} \frac{dT}{T} = 1 \text{g} \cdot 1 \frac{\text{cal}}{\text{g K}} \ln \left(\frac{373.15}{273.15} \right) = 0.31196 \text{ cal/K}$$

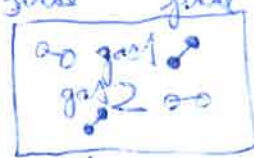
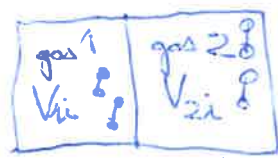
$$\Delta S_4 = \frac{m L_v}{T_v} = \frac{540 \text{ cal}}{373.15 \text{ K}} = 1.44714 \text{ cal/K}$$

$$\Delta S_5 = m C_{\text{vapor}} \int_{373.15}^{423.15} \frac{dT}{T} = 0.47 \frac{\text{cal}}{\text{K}} \ln \left(\frac{423.15}{373.15} \right) = 0.05910 \text{ cal/K}$$

$$\Delta S = \Delta S_1 + \Delta S_2 + \Delta S_3 + \Delta S_4 + \Delta S_5 = \boxed{2.14516 \text{ cal/K}}$$

7.5) $m_1 = 1 \text{ mol}$ $m_2 = 3 \text{ moles}$ $V_T = V_{1i} + V_{2i}$
 $P_1 = P_2 = P_{2i} = P_{1i}$ $V_1 = V_{1i}$ $T_1 = T_2 = T_{2i} = T_{1i}$ $V_2 = V_{2i}$ $V_{2f} = V_{1f} = V_{1i} + V_{2i} = V_T$

Al poner en comunicación los recipientes, los gases se dispersan por el volumen de los dos recipientes: $V_{1i} + V_{2i}$. El volumen ocupado por el gas 1 será el mismo que el volumen ocupado por el gas 2: $V_{1f} = V_{2f}$ y será igual a $V_T = V_{1i} + V_{2i}$



$$V_T = V_{1i} + V_{2i}$$

$$V_T = V_{1f} = V_{2f}$$

Autor: Iván Cabria Álvarez

(7.3)

Variación de la entropía del gas 1: ΔS_1

Proceso isotermo $\Rightarrow \Delta S_1 = mR \ln \left(\frac{V_{1f}}{V_{1i}} \right) = m_1 R \ln \left(\frac{V_1 + V_2}{V_1} \right)$

$$\frac{V_1 + V_2}{V_1} = \frac{\frac{m_1 R T_1}{P_1} + \frac{m_2 R T_2}{P_2}}{\frac{m_1 R T_1}{P_1}} = \frac{m_1 \frac{T_1}{P_1} + m_2 \frac{T_2}{P_2}}{m_1 \frac{T_1}{P_1}} = \frac{\frac{m_1}{P_1} + \frac{m_2}{P_2}}{\frac{m_1}{P_1}} = \frac{m_1 + m_2}{m_1}$$

$\uparrow \quad \uparrow$
 $T_1 = T_2 \quad P_1 = P_2$

$$\Delta S_1 = m_1 R \ln \left(\frac{m_1 + m_2}{m_1} \right) = 1 \text{ mol} \cdot 8.3145 \frac{\text{J}}{\text{mol K}} \ln 4 = 11.53 \text{ J/K}$$

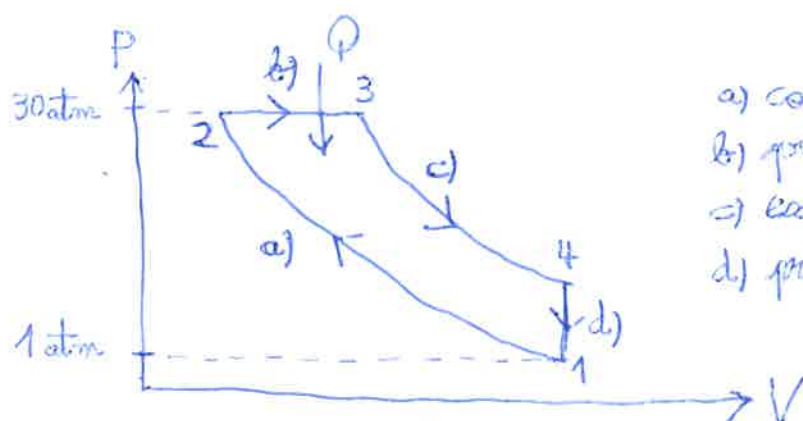
Variación de la entropía del gas 2: ΔS_2

$$\Delta S_2 = m_2 R \ln \left(\frac{V_{2f}}{V_{2i}} \right) = m_2 R \ln \left(\frac{V_1 + V_2}{V_2} \right) = m_2 R \ln \left(\frac{m_1 + m_2}{m_2} \right) =$$

$$= 3 \text{ moles} \cdot 8.3145 \frac{\text{J}}{\text{mol K}} \ln \left(\frac{4}{3} \right) = 7.18 \text{ J/K}$$

$$\Delta S = \Delta S_1 + \Delta S_2 = \boxed{18.71 \text{ J/K}}$$

(7.6) (a)



- a) compresión adiabática hasta 30 atm
- b) proceso a presión constante
- c) expansión adiabática hasta V_1
- d) proceso isócoro

(P_1, V_1, T_1) son la presión, volumen y temperatura inicial, respectivamente

El proceso a presión constante se dibuja como $2 \rightarrow 3$ por lo siguiente:

$$\left. \begin{aligned} Q &= m C_p (T_3 - T_2) \\ Q &> 0 \end{aligned} \right\} \Rightarrow T_3 > T_2 \Rightarrow \frac{T_3}{T_2} > 1$$

$$PV = mRT \Rightarrow \frac{P_3 V_3}{P_2 V_2} = \frac{T_3}{T_2}$$

$$\Rightarrow \frac{P_3 V_3}{P_2 V_2} > 1 \Rightarrow P_3 V_3 > P_2 V_2$$

Autor: Juan Carlos Álvarez

$$P_3 = P_2 \text{ (presión constante)} \Rightarrow P V_3 > P V_2 \Rightarrow V_3 > V_2 \Rightarrow \text{Se dibuja como } 2 \rightarrow 3$$

$V_2 \quad V_3$

(b) $T_1 = (273.15 + 15) \text{ K} = 288.15 \text{ K}$ $P_1 = 1 \text{ atm}$

T_1 y P_1 son datos que están en el enunciado del problema. Falta hallar V_1

El enunciado del problema contiene la densidad del aire en condiciones normales (0°C y 1 atm) y la masa de aire, 1 kg . A partir de estos datos podemos hallar el volumen V a 0°C y 1 atm :

$$\rho_{\text{aire}} = \frac{M}{V_0} \Rightarrow V_0 = \frac{M}{\rho_{\text{aire}}} = \frac{1 \text{ kg}}{1.293 \text{ kg} \cdot \text{m}^{-3}} = 0.7733952 \text{ m}^3 \text{ a } 0^\circ \text{C y } 1 \text{ atm}$$

Para calcular V_1 a partir de V_0 , hacemos lo siguiente: En un proceso a presión constante, $\frac{T_1}{V_1}$ es igual a $\frac{T_0}{V_0}$, según la ley o ecuación del gas ideal, $PV = nRT$.

$$\frac{T_1}{V_1} = \frac{T_0}{V_0} \Rightarrow V_1 = \frac{T_1}{T_0} V_0 = \frac{288.15}{273.15} 0.7733952 \text{ m}^3 = 0.816 \text{ m}^3$$

$$\boxed{P_1 = 1 \text{ atm} \quad V_1 = 0.816 \text{ m}^3 \quad T_1 = 288.15 \text{ K}}$$

Para calcular otras magnitudes necesitamos saber el número de moles. Calculamos a continuación el número de moles:

$$n = \frac{P_1 V_1}{RT_1} = \frac{1.013 \cdot 10^5 \text{ Pa} \cdot 0.816 \text{ m}^3}{8.3145 \text{ J} \cdot \text{mol}^{-1} \text{ K}^{-1} \cdot 288.15 \text{ K}} = 34.502 \text{ moles}$$

Proceso $1 \rightarrow 2$ es una compresión adiabática:

$$P_1 V_1^\gamma = P_2 V_2^\gamma \Rightarrow V_2 = V_1 \left(\frac{P_1}{P_2} \right)^{1/\gamma} = 0.816 \text{ m}^3 \left(\frac{1}{30} \right)^{1/1.4} = 0.0719 \text{ m}^3$$

$$T_2 = \frac{P_2 V_2}{nR} = \frac{30 \cdot 1.013 \cdot 10^5 \text{ Pa} \cdot 0.0719 \text{ m}^3}{34.502 \text{ moles} \cdot 8.3145 \text{ J} \cdot \text{mol}^{-1} \text{ K}^{-1}} = 761.69 \text{ K}$$

$$\boxed{P_2 = 30 \text{ atm} \quad V_2 = 0.0719 \text{ m}^3 \quad T_2 = 761.69 \text{ K}}$$

Autor: Iván Cabria Álvarez

Proceso 2 → 3 es un calentamiento a presión constante:

$$Q = m c_p \Delta T \Rightarrow \Delta T = \frac{Q}{m c_p} = \frac{300 \cdot 10^3 \text{ cal}}{10^3 \text{ g } 0.25 \text{ cal} \cdot \text{g}^{-1} \cdot ^\circ\text{C}^{-1}} = 1200 ^\circ\text{C} = 1200 \text{ K}$$

$$Q = Q_{23}$$

ΔT en $^\circ\text{C}$ es igual que ΔT en grados Kelvin
 T_3 no es igual per $^\circ\text{C}$ que en grados Kelvin
numéricamente

$$\left. \begin{aligned} \Delta T &= T_3 - T_2 \\ T_2 &= 761.69 \text{ K} \end{aligned} \right\} \Rightarrow T_3 = 1961.69 \text{ K} \quad P_3 = P_2$$

$$V_3 = \frac{nRT_3}{P_3} = \frac{34.502 \text{ moles } 8.3145 \text{ J} \cdot \text{mol}^{-1} \cdot \text{K}^{-1} 1961.69 \text{ K}}{30 \cdot 1.013 \cdot 10^5 \text{ Pa}} = 0.185 \text{ m}^3$$

$P_3 = 30 \text{ atm}$ $V_3 = 0.185 \text{ m}^3$ $T_3 = 1961.69 \text{ K}$

Proceso 3 → 4 es una expansión adiabática:

$V_4 = V_1$ según el enunciado del problema

$$P_3 V_3^\gamma = P_4 V_4^\gamma \Rightarrow P_4 = P_3 \left(\frac{V_3}{V_4} \right)^\gamma = 30 \text{ atm} \left(\frac{0.185}{0.816} \right)^{1.4} = 3.7566 \text{ atm}$$

$$T_4 = \frac{P_4 V_4}{nR} = \frac{3.7566 \cdot 1.013 \cdot 10^5 \text{ Pa } 0.816 \text{ m}^3}{34.502 \text{ moles } 8.3145 \text{ J} \cdot \text{mol}^{-1} \cdot \text{K}^{-1}} = 1082.47 \text{ K}$$

$P_4 = 3.7566 \text{ atm}$ $V_4 = 0.816 \text{ m}^3$ $T_4 = 1082.47 \text{ K}$

(c) El rendimiento es $\eta = \frac{W}{Q}$

$$Q_{23} = Q = 300 \text{ kcal} = 1255.2 \text{ kJ} \quad 1 \text{ J} = 1 \text{ Pa} \cdot \text{m}^3$$

$$W = W_{12} + W_{23} + W_{34} + W_{41}$$

$$W_{12} = \frac{P_2 V_2 - P_1 V_1}{1 - \gamma} = \frac{(30 \cdot 0.0719 - 1 \cdot 0.816) 1.013 \cdot 10^5 \text{ Pa} \cdot \text{m}^3}{1 - 1.4} = -339608 \text{ J}$$

$$W_{23} = P_2 (V_3 - V_2) = 30 (0.185 - 0.0719) 1.013 \cdot 10^5 \text{ Pa} \cdot \text{m}^3 = 343711 \text{ J}$$

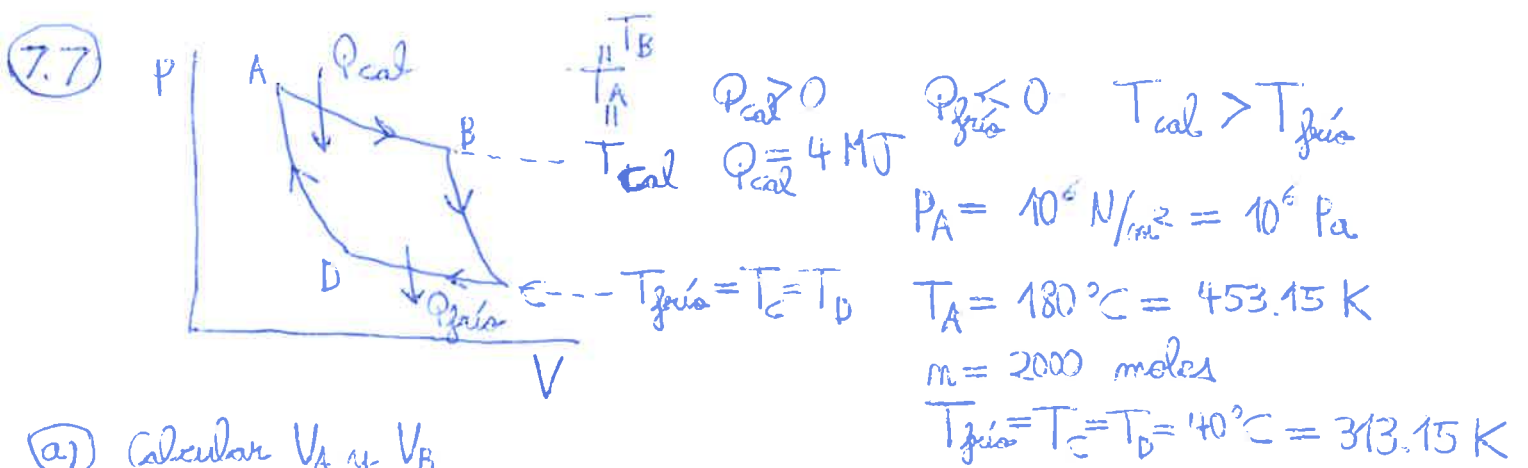
Autor: Juan Cabria Albano

$$W_{34} = \frac{P_4 V_4 - P_3 V_3}{1 - \gamma} = \frac{(3.7566 \cdot 0.816 - 30 \cdot 0.185) 1.013 \cdot 10^5 \text{ Pa} \cdot \text{m}^3}{1 - 1.4} = +629229 \text{ J}$$

$W_{41} = 0$ porque el volumen es constante en el proceso $4 \rightarrow 1$

$$W = W_{12} + W_{23} + W_{34} + W_{41} = 633332 \text{ J}$$

$$\eta = \frac{633332 \text{ J}}{1255200 \text{ J}} = \boxed{0.5}$$



a) Calcular V_A y V_B

$$V_A = \frac{n R T_A}{P_A} = \frac{2000 \text{ moles} \cdot 8.3145 \text{ J} \cdot \text{mol}^{-1} \cdot \text{K}^{-1} \cdot 453.15 \text{ K}}{10^6 \text{ Pa}} = \boxed{7.535 \text{ m}^3}$$

$$Q_{AB} = n R T_A \ln\left(\frac{V_B}{V_A}\right) \Rightarrow \frac{Q_{AB}}{n R T_A} = \ln\left(\frac{V_B}{V_A}\right) \Rightarrow e^{\frac{Q_{AB}}{n R T_A}} = \frac{V_B}{V_A} \Rightarrow$$

$$\Rightarrow V_B = V_A e^{\frac{Q_{AB}}{n R T_A}} \quad \left. \begin{array}{l} Q_{AB} = Q_{cal} \end{array} \right\} \Rightarrow V_B = V_A e^{\frac{Q_c}{n R T_A}}$$

$$V_B = 7.535 \text{ m}^3 e^{\left(\frac{4 \cdot 10^6 \text{ J}}{2000 \text{ moles} \cdot 8.3145 \text{ J} \cdot \text{mol}^{-1} \cdot \text{K}^{-1} \cdot 453.15 \text{ K}}\right)} = \boxed{12.812 \text{ m}^3}$$

b) $W_{\text{expansión}} = W_{AB} + W_{BC}$

Expansión isoterma de A a B $\Rightarrow \left. \begin{array}{l} W_{AB} = Q_{AB} \\ Q_{AB} = Q_{cal} \end{array} \right\} \Rightarrow W_{AB} = 4 \cdot 10^6 \text{ J}$

Autor: Iván Cabria Álvarez

Expansión adiabática de B a C $\Rightarrow Q_{BC}=0 \Rightarrow W_{BC} = -\Delta U_{BC}$

$$\begin{aligned}\Delta U_{BC} &= m C_V \Delta T_{BC} = m C_V (T_C - T_B) = m C_V (T_{fría} - T_{cal}) = \\ &= m (C_p - R) (T_{fría} - T_{cal}) = m \left(\frac{7R}{2} - R \right) (T_{fría} - T_{cal}) = \\ &= m \frac{5}{2} R (T_{fría} - T_{cal}) = 2000 \text{ moles } \frac{5}{2} 8.3145 \text{ J mol}^{-1} \text{ K}^{-1} (313.15 - 453.15) \text{ K} \\ &= -5.82 \cdot 10^6 \text{ J}\end{aligned}$$

$$W_{BC} = +5.82 \cdot 10^6 \text{ J}$$

$$W_{\text{expansión}} = W_{AB} + W_{BC} = \boxed{9.82 \cdot 10^6 \text{ J}}$$

(2) $W_{\text{compresión}} = W_{CD} + W_{DA}$

Compresión isotérmica de C a D $\Rightarrow W_{CD} = Q_{CD} = Q_{fría} < 0$

En un ciclo de Carnot $\frac{|Q_{fría}|}{Q_{cal}} = \frac{T_{fría}}{T_{cal}} \Rightarrow |Q_{fría}| = \frac{T_{fría}}{T_{cal}} Q_{cal}$ } $\Rightarrow$

$$\Rightarrow |W_D| = \frac{T_{fría}}{T_{cal}} Q_{cal} = \frac{313.15}{453.15} 4 \cdot 10^6 \text{ J} = 2.764 \cdot 10^6 \text{ J}$$

$$W_D = -|W_D| = -2.764 \cdot 10^6 \text{ J}$$

En un ciclo de Carnot también se cumple $-\frac{Q_{fría}}{Q_{cal}} = \frac{T_{fría}}{T_{cal}}$. Usando esta ecuación también obtenemos $W_{CD} = -2.764 \cdot 10^6 \text{ J}$

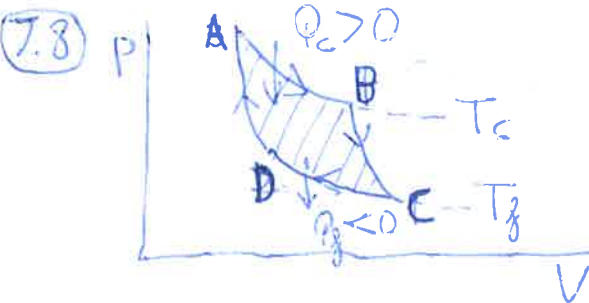
Compresión adiabática de D a A $\Rightarrow W_{DA} = -\Delta U_{DA}$

$$\begin{aligned}\Delta U_{DA} &= m C_V \Delta T_{DA} = m (C_p - R) \Delta T_{DA} = m (C_p - R) (T_A - T_D) = \\ &= m (C_p - R) (T_{cal} - T_{fría}) = 2000 \text{ moles } \frac{5}{2} 8.3145 \text{ J mol}^{-1} \text{ K}^{-1} 140 \text{ K} = \\ &= +5.82 \cdot 10^6 \text{ J}\end{aligned}$$

$$W_{DA} = -5.82 \cdot 10^6 \text{ J}$$

$$W_{\text{compresión}} = W_{CD} + W_{DA} = \boxed{-8.584 \cdot 10^6 \text{ J}}$$

Autor: Juan Cabria Álvarez



$W = \text{área PV} = 30 \text{ Kcal}$
 $T_c > T_f$

a) $\eta_c = \frac{T_c - T_f}{T_c} = \frac{473.15 - 323.15}{473.15} = 0.317$

$\eta_c = \frac{W}{Q_c} \Rightarrow Q_c = \frac{W}{\eta_c} = \frac{30 \text{ Kcal}}{0.317} = \boxed{94.63 \text{ Kcal}}$

$\eta_c = \frac{Q_c - |Q_f|}{Q_c} \Rightarrow |Q_f| = Q_c - \eta Q_c = (1 - \eta) Q_c$

$|Q_f| = (1 - 0.317) 94.63 \text{ Kcal} = \boxed{64.63 \text{ Kcal}}$

b) $\Delta S_{AB} = \frac{Q_{AB}}{T_A} = \frac{Q_{cal}}{T_{cal}} = \boxed{200 \frac{\text{cal}}{\text{K}}}$ $\Delta S_{CD} = \frac{Q_{CD}}{T_D} = \frac{Q_{frio}}{T_{frio}} = \boxed{-200 \frac{\text{cal}}{\text{K}}}$
 por ser procesos adiabáticos $\Delta S_{BC} = \Delta S_{DA} = 0$
 $\Delta S_{frio} = 0$

7.9 $\Delta S_1 = \frac{Q_{fusión}}{T_{fusión}} = \frac{m \cdot L_f}{T_{fusión}} = \frac{0.1 \text{ kg} \cdot 3 \text{ Kcal kg}^{-1}}{234 \text{ K}} = 1.282 \cdot 10^{-3} \frac{\text{Kcal}}{\text{K}} =$
 $= 5.36 \text{ J/K}$

$\Delta S_2 = \int_{T_{fusión}}^{T_2} \frac{m \cdot c_p dT}{T} = m \cdot c_p \int_{T_{fusión}}^{T_2} \frac{dT}{T} = m \cdot c_p \ln \left(\frac{T_2}{T_{fusión}} \right) =$
 $= 100 \text{ g} \cdot 0.0335 \frac{\text{cal}}{\text{g K}} \ln \left(\frac{373}{234} \right) = 6.54 \text{ J/K}$

$\Delta S_{total} = \Delta S_1 + \Delta S_2 = \boxed{11.9 \text{ J/K}}$

7.10 Primera hay que asegurarse si se funde todo el hielo, b) si se funde una parte y c) si se funde todo el hielo y después el agua aumenta su temperatura por encima de 0°C

Para fundir todo el hielo, 2 kg, hace falta la siguiente cantidad de calor: $Q_T = m_{hi} L_f = 2000 \text{ g} \cdot 80 \frac{\text{cal}}{\text{g}} = 160 \text{ 000 cal}$

El calor cedido por el kg de agua al enfriarse desde 298.15 K hasta 273.15 K es: $Q_{ced} = m_{agua} c_{agua} (298.15 - 273.15) \text{ K} = 4000 \text{ g} \cdot 1 \frac{\text{cal}}{\text{g K}} \cdot 25 \text{ K} \Rightarrow$
 $\Rightarrow Q_{ced} = 25000 \text{ cal}$ Autor: Juan Cabria Albarr

El calor cedido por el agua es inferior al calor necesario para fundir todo el hielo. (7.9)

Suponemos que se funden x gramos de hielo:

$$Q_{\text{abs}} = x L_f$$

$$Q_{\text{abs}} = Q_{\text{ced}} \Rightarrow x L_f = m_{\text{agua}} C_{\text{agua}} (298.15 - 273.15) \text{ K} \Rightarrow$$

$$\Rightarrow x = \frac{m_{\text{agua}} C_{\text{agua}} 25 \text{ K}}{L_f} = \frac{25000 \text{ cal}}{80 \text{ cal/g}} = 312.5 \text{ g}$$

Variación de la entropía del hielo:

$$\Delta S_{\text{hielo}} = \frac{x L_f}{T_f} = \frac{25000 \text{ cal}}{273.15 \text{ K}} = 91.525 \frac{\text{cal}}{\text{K}} = 382.94 \text{ J/K}$$

Variación de la entropía del agua

$$\Delta S_{\text{agua}} = m_{\text{agua}} C_{\text{agua}} \int_{298.15 \text{ K}}^{273.15 \text{ K}} \frac{dT}{T} = 1000 \text{ g} \cdot 1 \frac{\text{cal}}{\text{g K}} \ln\left(\frac{273.15}{298.15}\right)$$

$$\Delta S_{\text{agua}} = -87.576 \frac{\text{cal}}{\text{K}} = -366.416 \text{ J/K}$$

$$\Delta S_{\text{total}} = \Delta S_{\text{hielo}} + \Delta S_{\text{agua}} = \boxed{+16.52 \text{ J/K}}$$

$$7.11(a) \quad m_{\text{agua}} = \rho_{\text{agua}} V_{\text{agua}} = 1 \frac{\text{Kg}}{\text{L}} \cdot 0.001 \text{ m}^3 = 10^3 \frac{\text{Kg}}{\text{m}^3} \cdot 0.001 \text{ m}^3 = 1 \text{ Kg}$$

$$\Delta S_{\text{agua}} = m_{\text{agua}} C_{\text{agua}} \int_{293}^{373} \frac{dT}{T} = m_{\text{agua}} C_{\text{agua}} \ln\left(\frac{373}{293}\right) =$$

$$= 1000 \text{ g} \cdot 1 \frac{\text{cal}}{\text{g K}} \ln\left(\frac{373}{293}\right) = 241.405 \frac{\text{cal}}{\text{K}} = \boxed{1010.042 \text{ J/K}}$$

$$\Delta S_{\text{foco}} = \frac{Q_{\text{foco}}}{T_{\text{foco}}} = \frac{Q_{\text{cedido al agua por el foco}}}{T_{\text{foco}}} = - \frac{m_{\text{agua}} C_{\text{agua}} \Delta T}{T_{\text{foco}}} =$$

$$= - \frac{1000 \text{ g} \cdot 1 \text{ cal g}^{-1} \text{ K}^{-1} \cdot 80 \text{ K}}{373 \text{ K}} = -214.477 \text{ cal/K} = \boxed{-897.373 \text{ J/K}}$$

$$\Delta S_{\text{universo}} = \Delta S_{\text{foco}} + \Delta S_{\text{agua}} = \boxed{112.669 \text{ J/K}}$$

Autor: Iván Cabrera Alvarado

$$\begin{aligned}
 \textcircled{b)} \Delta S_{\text{agua}} &= m_{\text{agua}} C_{\text{e agua}} \int_{293}^{323} \frac{dT}{T} + m_{\text{agua}} C_{\text{e agua}} \int_{323}^{373} \frac{dT}{T} = \\
 &= m_{\text{agua}} C_{\text{e agua}} \int_{293}^{373} \frac{dT}{T} = m_{\text{agua}} C_{\text{e agua}} \ln \left(\frac{373}{293} \right) = 1000 \text{ g} \cdot 1 \frac{\text{cal}}{\text{g K}} \ln \left(\frac{373}{293} \right) \\
 &= 241.406 \frac{\text{cal}}{\text{K}} = \boxed{1010.042 \text{ J/K}}
 \end{aligned}$$

$$T_i = 293 \text{ K} \quad T_{\text{foco1}} = 323 \text{ K} \quad T_{\text{foco2}} = 373 \text{ K}$$

$$\Delta S_{\text{foco1}} = - \frac{m_{\text{agua}} C_{\text{e agua}} (T_{\text{foco1}} - T_i)}{T_{\text{foco1}}} = -92.879 \text{ cal/K} = \boxed{-388.607 \text{ J/K}}$$

$$\Delta S_{\text{foco2}} = - \frac{m_{\text{agua}} C_{\text{e agua}} (T_{\text{foco2}} - T_{\text{foco1}})}{T_{\text{foco2}}} = -134.048 \text{ cal/K} = \boxed{-560.858 \text{ J/K}}$$

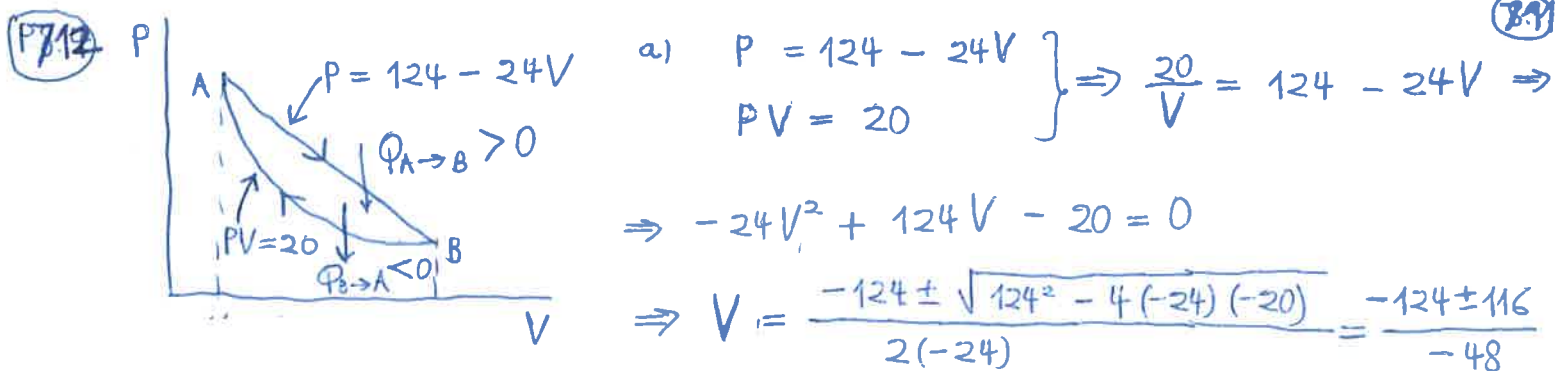
$$\Delta S_{\text{universo}} = \Delta S_{\text{agua}} + \Delta S_{\text{foco1}} + \Delta S_{\text{foco2}} = \boxed{+60.577 \text{ J/K}}$$

$$\textcircled{c)} \Delta S_{\text{agua}} = m_{\text{agua}} C_{\text{e agua}} \int_{293}^{373} \frac{dT}{T} = m_{\text{agua}} C_{\text{e agua}} \ln \left(\frac{373}{293} \right) = \boxed{1010.042 \text{ J/K}}$$

Al ser un proceso reversible (infinitas focas), la entropía del Universo permanece constante $\Rightarrow \boxed{\Delta S_{\text{universo}} = 0}$

$$\Delta S_{\text{universo}} = \Delta S_{\text{agua}} + \Delta S_{\text{focas}} \Rightarrow \Delta S_{\text{focas}} = \Delta S_{\text{universo}} - \Delta S_{\text{agua}} \quad \Rightarrow$$

$$\Rightarrow \Delta S_{\text{focas}} = \boxed{-1010.042 \text{ J/K}}$$



$$V = \begin{cases} 5 \text{ m}^3 \\ 1/6 \text{ m}^3 \end{cases} \Rightarrow \begin{cases} V_A = 1/6 \text{ m}^3 \\ V_B = 5 \text{ m}^3 \end{cases}$$

$$W_{AB} = \int_{V_A}^{V_B} P dV = \int_{V_A}^{V_B} (124 - 24V) dV = 124V - 12V^2 \Big|_{V_A}^{V_B} = 124(V_B - V_A) - 12(V_B^2 - V_A^2)$$

$$= 124(5 - 1/6) \text{ J} - 12(5^2 - (1/6)^2) \text{ J} = 599.333 \text{ J} - 299.667 \text{ J} = 299.667 \text{ J}$$

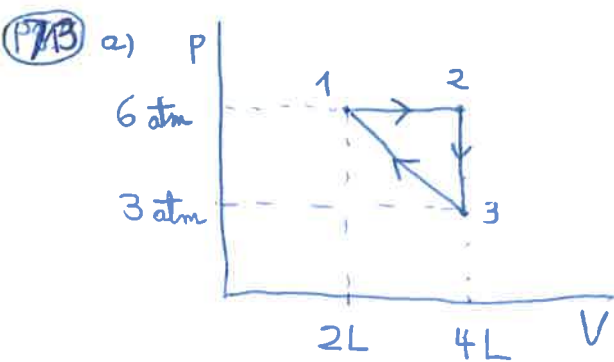
$$W_{BA} = \int_B^A P dV = 20 \int_B^A \frac{dV}{V} \text{ J} = 20 \ln\left(\frac{V_A}{V_B}\right) \text{ J} = 20 \ln\left(\frac{1/6}{5}\right) \text{ J} = 20 \ln\left(\frac{1}{30}\right) \text{ J} = -68.024 \text{ J}$$

$$W = W_{AB} + W_{BA} = \boxed{231.643 \text{ J}}$$

b) $\Delta U_{BA} = m C_V \Delta T_{BA}$
 proceso B $\rightarrow$ A es isotermo $\Rightarrow T_B = T_A \Rightarrow \Delta T_{BA} = 0 \Rightarrow \Delta U_{BA} = 0$
 $\Delta U_{BA} = Q_{BA} - W_{BA} \Rightarrow Q_{BA} = W_{BA}$
 $\Delta S_{BA} = \int_B^A \frac{dQ}{T} = \frac{1}{T_A} \int_B^A dQ = \frac{Q_{BA}}{T_A}$
 $T_A = \frac{P_A V_A}{m R} = \frac{20}{1.83145} \text{ K} = 2.405 \text{ K}$

$$\Rightarrow \Delta S_{BA} = \frac{W_{BA}}{T_A} = \frac{-68.024 \text{ J}}{2.405 \text{ K}} = \boxed{-28.284 \text{ J K}^{-1}}$$

c) $\eta = \frac{W_{\text{útil}}}{Q_{\text{absorbido}}} = \frac{W}{Q_{A \rightarrow B}}$
 $\Delta U_{A \rightarrow B \rightarrow A} = \Delta U_{A \rightarrow B} + \Delta U_{B \rightarrow A}$
 $\Delta U_{A \rightarrow B \rightarrow A} = 0 \Rightarrow \Delta U_{A \rightarrow B} = 0$
 $\Delta U_{B \rightarrow A} = 0 \Rightarrow \Delta U_{A \rightarrow B} = Q_{A \rightarrow B} - W_{A \rightarrow B}$
 $\Rightarrow Q_{A \rightarrow B} = W_{A \rightarrow B}$
 $\Rightarrow \eta = \frac{W}{W_{A \rightarrow B}} = \frac{231.643}{299.667} = \boxed{0.773}$
 Profesor: Iván Cabria Álvarez



b) Estado 2

$$P_2 = P_1 = \boxed{6 \text{ atm}}$$

$$V_2 = 2V_1 = \boxed{4 \text{ L}}$$

De 1 a 2 proceso isobaro $\Rightarrow P_1 = P_2$
 $PV = nRT \Rightarrow$

$$\Rightarrow \frac{T_2}{V_2} = \frac{T_1}{V_1} \Rightarrow T_2 = T_1 \frac{V_2}{V_1} = 2T_1$$

$$T_1 = \frac{P_1 V_1}{nR} = \frac{6 \cdot 1.013 \cdot 10^5 \text{ Pa} \cdot 2 \cdot 10^{-3} \text{ m}^3}{0.5 \text{ moles} \cdot 8.3145 \text{ J mol}^{-1} \text{ K}^{-1}} = 292.405 \text{ K}$$

$$\boxed{T_2 = 584.81 \text{ K}}$$

Estado 3 $P_3 = 3 \text{ atm}$

De 2 a 3 proceso isocoro $\Rightarrow V = \text{cte.} \Rightarrow V_3 = V_2 \Rightarrow \boxed{V_3 = 4 \text{ L}}$

$PV = nRT \Rightarrow \frac{T}{P} = \text{cte.} \Rightarrow \frac{T_2}{P_2} = \frac{T_3}{P_3} \Rightarrow$

$$\Rightarrow T_3 = P_3 \frac{T_2}{P_2} = 3 \text{ atm} \frac{584.81 \text{ K}}{6 \text{ atm}} = \boxed{292.405 \text{ K}}$$

c) De 1 a 2: $Q_{1 \rightarrow 2} = n C_p \Delta T_{1 \rightarrow 2} = n \frac{7}{2} R (T_2 - T_1) = 0.5 \text{ moles} \cdot 3.5 \cdot 8.3145 \text{ J mol}^{-1} \text{ K}^{-1} \cdot$

$$(584.81 - 292.405) \text{ K} = \boxed{4254.6 \text{ J}}$$

$$W_{1 \rightarrow 2} = \int_1^2 P dV = P_2 \int_1^2 dV = P_2 (V_2 - V_1) = 6 \cdot 1.013 \cdot 10^5 \text{ Pa} (4 - 2) \cdot 10^{-3} \text{ m}^3 = \boxed{1215.6 \text{ J}}$$

De 2 a 3: $Q_{2 \rightarrow 3} = n C_v (T_3 - T_2) = n \frac{5}{2} R (T_3 - T_2) = 0.5 \cdot 2.5 \cdot 8.3145 \text{ J K}^{-1} (292.405 - 584.81) \text{ K} = \boxed{-3039 \text{ J}}$

$$W_{2 \rightarrow 3} = \int_2^3 P dV = \boxed{0}$$

De 3 a 1: $W = W_{1 \rightarrow 2} + W_{2 \rightarrow 3} + W_{3 \rightarrow 1}$

$$W = \frac{1}{2} (V_3 - V_1) (P_1 - P_3) = \frac{1}{2} (4 - 2) \cdot 10^{-3} (6 - 3) \cdot 1.013 \cdot 10^5 \text{ Pa} = \boxed{303.9 \text{ J}}$$

$$W_{3 \rightarrow 1} = W - W_{1 \rightarrow 2} - W_{2 \rightarrow 3} = 303.9 \text{ J} - 1215.6 \text{ J} - 0 = \boxed{-911.7 \text{ J}}$$

$\Delta U_{3 \rightarrow 1} = n C_v (T_1 - T_3) \Rightarrow \Delta U_{3 \rightarrow 1} = 0$
 $T_1 = T_3$

$\Delta U_{3 \rightarrow 1} = Q_{3 \rightarrow 1} - W_{3 \rightarrow 1} \Rightarrow Q_{3 \rightarrow 1} = W_{3 \rightarrow 1} = \boxed{-911.7 \text{ J}}$

Autor: Iván Cabria Álvarez

$$d) \eta = \frac{W_{\text{util}}}{Q_{\text{absorbido}}} = \frac{303.9 \text{ J}}{Q_{1 \rightarrow 2}} = \frac{303.9 \text{ J}}{4254.6 \text{ J}} = \boxed{0.071}$$

7.8

e) De 1 a 2:

$$\Delta S_{1 \rightarrow 2} = \int_1^2 \frac{dQ}{T} = n C_p \int_1^2 \frac{dT}{T} = n C_p \ln\left(\frac{T_2}{T_1}\right) = n \frac{7}{2} R \ln\left(\frac{2T_1}{T_1}\right) =$$

$$= 0.5 \text{ moles} \cdot 3.5 \cdot 8.3145 \text{ J mol}^{-1} \text{ K}^{-1} \ln 2 = \boxed{10.086 \text{ J K}^{-1}}$$

De 2 a 3:

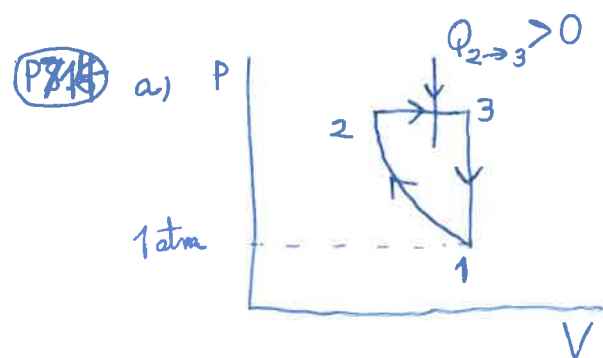
$$\Delta S_{2 \rightarrow 3} = \int_2^3 \frac{dQ}{T} = n C_v \int_2^3 \frac{dT}{T} = n \frac{5}{2} R \ln\left(\frac{T_3}{T_2}\right) = n \frac{5}{2} R \ln\left(\frac{1}{2}\right)$$

$$= 0.5 \text{ moles} \cdot 2.5 \cdot 8.3145 \text{ J mol}^{-1} \text{ K}^{-1} \ln 2 = \boxed{-7.204 \text{ J K}^{-1}}$$

De 3 a 1:

$$\Delta S_{1 \rightarrow 2 \rightarrow 3 \rightarrow 1} = \Delta S_{1 \rightarrow 2} + \Delta S_{2 \rightarrow 3} + \Delta S_{3 \rightarrow 1} \Rightarrow \Delta S_{3 \rightarrow 1} = -\Delta S_{1 \rightarrow 2} - \Delta S_{2 \rightarrow 3} = \boxed{-2.882 \frac{\text{J}}{\text{K}}}$$

$$\Delta S_{1 \rightarrow 2 \rightarrow 3 \rightarrow 1} = 0$$



$$b) \eta = \frac{W}{Q_{\text{absorbido}}}$$

$$W = W_{1 \rightarrow 2} + W_{2 \rightarrow 3} + W_{3 \rightarrow 1}$$

$$Q_{\text{absorbido}} = Q_{2 \rightarrow 3}$$

$$T_2 = 2T_1$$

$$W_{1 \rightarrow 2} = \frac{P_2 V_2 - P_1 V_1}{1 - \gamma} = \frac{n R T_2 - n R T_1}{1 - \gamma} = \frac{n R}{1 - \gamma} T_1 = \frac{P_1 V_1}{1 - \gamma}$$

$$W_{2 \rightarrow 3} = \int_2^3 P dV = P_2 \int_2^3 dV = P_2 (V_3 - V_2) = P_3 V_3 - P_2 V_2$$

$P_2 = P_3$

$$W_{3 \rightarrow 1} = 0$$

$$W = \frac{P_1 V_1}{1 - \gamma} + P_3 V_3 - P_2 V_2$$

Autor: Juan Cabria Alvarro

$$Q_{2 \rightarrow 3} = n C_p (T_3 - T_2) = \frac{n C_p}{n R} (P_3 V_3 - P_2 V_2) = \frac{C_p}{R} (P_3 V_3 - P_2 V_2) = \frac{C_p \gamma}{R} (P_3 V_3 - P_2 V_2)$$

$$\gamma = \frac{C_p}{C_v} = \frac{C_v + R}{C_v} \approx \frac{5}{3}$$

$$\frac{C_v}{R} \approx \frac{3}{2} \Rightarrow \frac{C_p \gamma}{R} \approx \frac{3}{2} \cdot \frac{5}{3} = \frac{5}{2}$$

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1} \Rightarrow \frac{V_1}{V_2} = \left(\frac{T_2}{T_1} \right)^{\frac{1}{\gamma-1}} = (2)^{\frac{1}{\gamma-1}} = 2^{\frac{1}{\frac{5}{3}-1}} = 2^{\frac{3}{2}} = 2\sqrt{2} \quad (3.14)$$

$$\Rightarrow V_2 = \frac{V_1}{2\sqrt{2}}$$

$$\left. \begin{array}{l} P_2 V_2 = n R T_2 \\ P_1 V_1 = n R T_1 \end{array} \right\} \Rightarrow \frac{P_2}{P_1} = \frac{T_2}{T_1} \frac{V_1}{V_2} = 2 \frac{V_1}{V_2} = 2^2 \sqrt{2} = 4\sqrt{2}$$

$$\Rightarrow P_2 = P_1 4\sqrt{2}$$

$$W_{2 \rightarrow 3} = P_3 V_3 - P_2 V_2 = \underset{\substack{P_3 = P_2 \\ V_3 = V_1}}{P_2 (V_1 - V_2)} = P_1 4\sqrt{2} \left(V_1 - \frac{V_1}{2\sqrt{2}} \right) = P_1 V_1 4\sqrt{2} \left(1 - \frac{1}{2\sqrt{2}} \right)$$

$$= P_1 V_1 (4\sqrt{2} - 2) = 2 P_1 V_1 (2\sqrt{2} - 1)$$

$$Q_{2 \rightarrow 3} = \frac{5}{2} (P_3 V_3 - P_2 V_2) = \frac{5}{2} 2 P_1 V_1 (2\sqrt{2} - 1) = 5 P_1 V_1 (2\sqrt{2} - 1)$$

$$W = \frac{P_1 V_1}{1-\gamma} + 2 P_1 V_1 (2\sqrt{2} - 1) = -\frac{3 P_1 V_1}{2} + 2 P_1 V_1 (2\sqrt{2} - 1) =$$

$$= P_1 V_1 \left(-\frac{3}{2} + 4\sqrt{2} - 2 \right) = P_1 V_1 \left(-\frac{7}{2} + 4\sqrt{2} \right)$$

$$\eta = \frac{W}{Q_{2 \rightarrow 3}} = \frac{-\frac{7}{2} + 4\sqrt{2}}{5(2\sqrt{2} - 1)} = \boxed{0.236}$$

$$\eta = \frac{Q_{2 \rightarrow 3} - |Q_{3 \rightarrow 1}|}{Q_{2 \rightarrow 3}} = 1 - \frac{|C_V (1 - 4\sqrt{2})|}{C_P (4\sqrt{2} - 2)}$$

$Q_{3 \rightarrow 1} < 0$

c) Proceso isobárico, de 2 a 3:

$$\Delta S_{2 \rightarrow 3} = \int_2^3 \frac{dQ}{T} = n C_P \int_2^3 \frac{dT}{T} = n C_P \ln \left(\frac{T_3}{T_2} \right) = 1 \text{ mol } 5 \text{ cal mol}^{-1} \text{ K}^{-1} \ln \left(\frac{T_3}{T_2} \right)$$

$$P_2 = P_3 \Rightarrow \frac{T_2}{V_2} = \frac{T_3}{V_3} \Rightarrow \frac{T_3}{T_2} = \frac{V_3}{V_2} = \frac{V_1}{V_2} = 2\sqrt{2}$$

$$\Delta S_{2 \rightarrow 3} = 5 \text{ cal K}^{-1} \ln(2\sqrt{2}) = 5 \text{ cal K}^{-1} \ln(2^{3/2}) = \boxed{7.5 \text{ cal K}^{-1} \ln 2}$$

Proceso isócoro, de 3 a 1:

$$\Delta S_{3 \rightarrow 1} = \int_3^1 \frac{dQ}{T} = n C_V \int_3^1 \frac{dT}{T} = n C_V \ln \left(\frac{T_1}{T_3} \right) = n C_V \ln \left(\frac{1}{4\sqrt{2}} \right) =$$

$$\frac{T_1}{T_3} = \frac{T_2}{2T_3} = \frac{1}{2^2 \sqrt{2}}$$

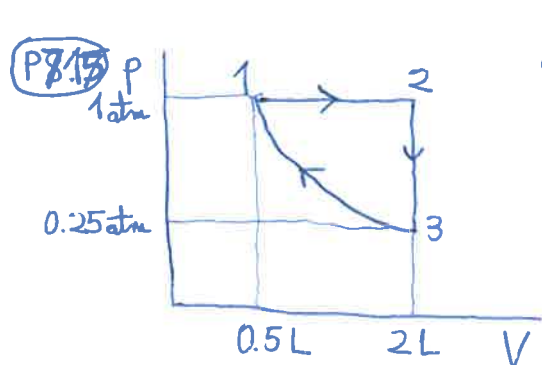
$$= -3 \text{ cal K}^{-1} \ln(2^{5/2}) = \boxed{-7.5 \text{ cal K}^{-1} \ln 2}$$

Autor: Iván Cabrera Álvarez

$$\Delta S_{1 \rightarrow 2} = 0 \text{ por ser adiabática}$$

(7.15)

$$\Delta S_{1 \rightarrow 2 \rightarrow 3 \rightarrow 1} = \Delta S_{1 \rightarrow 2} + \Delta S_{2 \rightarrow 3} + \Delta S_{3 \rightarrow 1} = 0 + 7.5 \ln 2 \frac{\text{cal}}{\text{K}} - 7.5 \ln 2 \frac{\text{cal}}{\text{K}} = 0 \text{ como esperábamos}$$



$$a) T_1 = \frac{P_1 V_1}{n R} = \frac{1.013 \cdot 10^5 \text{ Pa} \cdot 0.5 \cdot 10^{-3} \text{ m}^3}{1 \text{ mol} \cdot 8.3145 \text{ J mol}^{-1} \text{ K}^{-1}} = \boxed{6.1 \text{ K}}$$

$$\text{Proceso } 1 \rightarrow 2 \text{ es isóbarico} \Rightarrow \frac{T_1}{V_1} = \frac{T_2}{V_2} \Rightarrow$$

$$\Rightarrow T_2 = \frac{V_2}{V_1} T_1 = \frac{2}{0.5} 6.1 \text{ K} = \boxed{24.4 \text{ K}}$$

$$\text{Proceso } 2 \rightarrow 3 \text{ es isócorico} \Rightarrow \frac{T_2}{P_2} = \frac{T_3}{P_3} \Rightarrow T_3 = \frac{P_3}{P_2} T_2 = \frac{0.25}{1} 24.4 \text{ K} = \boxed{6.1 \text{ K}}$$

b) Proceso $1 \rightarrow 2$, isóbarico

$$W_{1 \rightarrow 2} = P_1 (V_2 - V_1) = 1.013 \cdot 10^5 \text{ Pa} (2 - 0.5) 10^{-3} \text{ m}^3 = \boxed{151.95 \text{ J}}$$

$$Q_{1 \rightarrow 2} = n C_p (T_2 - T_1) = 1 \text{ mol} \cdot 5 \text{ cal mol}^{-1} \text{ K}^{-1} (24.4 - 6.1) \text{ K} = 91.5 \text{ cal} = \boxed{382.836 \text{ J}}$$

$$\Delta U_{1 \rightarrow 2} = Q_{1 \rightarrow 2} - W_{1 \rightarrow 2} = \boxed{230.886 \text{ J}}$$

$$\Delta S_{1 \rightarrow 2} = \int_1^2 \frac{dQ}{T} = n C_p \int_1^2 \frac{dT}{T} = n C_p \ln \left(\frac{T_2}{T_1} \right) = 1 \text{ mol} \cdot \frac{5 \text{ cal}}{\text{mol K}} \ln \left(\frac{24.4}{6.1} \right) = 6.931 \frac{\text{cal}}{\text{K}} = \boxed{29 \text{ J/K}}$$

Proceso $2 \rightarrow 3$, isócorico

$$W_{2 \rightarrow 3} = \boxed{0} \quad Q_{2 \rightarrow 3} = n C_v (T_3 - T_2) = 1 \text{ mol} \cdot 3 \frac{\text{cal}}{\text{mol K}} (6.1 - 24.4) \text{ K} = -54.9 \text{ cal} = \boxed{-229.7 \text{ J}}$$

$$\Delta U_{2 \rightarrow 3} = Q_{2 \rightarrow 3} - W_{2 \rightarrow 3} = \boxed{-229.7 \text{ J}}$$

$$\Delta S_{2 \rightarrow 3} = \int_2^3 \frac{dQ}{T} = n C_v \int_2^3 \frac{dT}{T} = n C_v \ln \left(\frac{T_3}{T_2} \right) = n C_v \ln \left(\frac{1}{4} \right) = 1 \text{ mol} \cdot 3 \frac{\text{cal}}{\text{mol K}} \ln 4 = -4.159 \frac{\text{cal}}{\text{K}} = \boxed{-17.4 \text{ J/K}}$$

Autor: Iván Cabria Álvarez

Proceso $3 \rightarrow 1$, isoterma

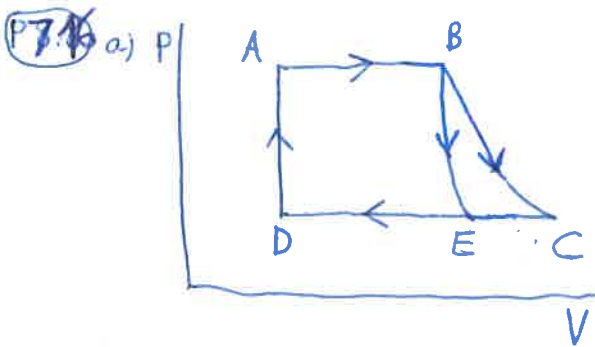
$$W_{3 \rightarrow 1} = n R T_1 \ln \left(\frac{V_1}{V_3} \right) = 1 \text{ mol } 8.3145 \frac{\text{J}}{\text{mol K}} 6.1 \text{ K} \ln \left(\frac{0.5}{2} \right) = \boxed{-70.31 \text{ J}}$$

$$\Delta U_{3 \rightarrow 1} = 0$$

$$Q_{3 \rightarrow 1} = \Delta U_{3 \rightarrow 1} + W_{3 \rightarrow 1} = \boxed{-70.31 \text{ J}}$$

$$\Delta S_{3 \rightarrow 1} = \int_3^1 \frac{dQ}{T} = \frac{1}{T_1} \int_3^1 dQ = \frac{Q_{3 \rightarrow 1}}{T_1} = \frac{-70.31 \text{ J}}{6.1 \text{ K}} = \boxed{-11.526 \frac{\text{J}}{\text{K}}}$$

$$c) \eta = \frac{W}{Q_{\text{absorbida}}} = \frac{W}{Q_{1 \rightarrow 2}} = \frac{151.95 \text{ J} + 0 - 70.31 \text{ J}}{382.836 \text{ J}} = \boxed{0.213}$$



b) En una isoterma $PV = \text{cte.}$

" " adiabática $PV^\gamma = \text{cte.}$ $\gamma > 1$

$\gamma > 1 \Rightarrow P(V)$ tiene una pendiente más pronunciada $\Rightarrow$ $B \rightarrow C$ es adiabática y $B \rightarrow A$ es isoterma

$$c) W_{A \rightarrow B} = P_A (V_B - V_A) \quad Q_{A \rightarrow B} = n C_p (T_B - T_A)$$

$$W_{A \rightarrow B} = 4 \cdot 1.013 \cdot 10^5 \text{ Pa } (16.4 - 8.2) \cdot 10^{-3} \text{ m}^3 = 3322.64 \text{ J}$$

$$Q_{A \rightarrow B} = 1 \text{ mol } 5 \frac{\text{cal}}{\text{mol K}} (800 - 400) \text{ K} = 2000 \text{ cal} = 8368 \text{ J} > 0$$

$$W_{B \rightarrow C} = n R T_B \ln \left(\frac{V_C}{V_B} \right) = 8.3145 \frac{\text{J}}{\text{K}} 800 \text{ K} \ln \left(\frac{65.6}{16.4} \right) = 9221.08 \text{ J}$$

$$Q_{B \rightarrow C} = W_{B \rightarrow C} = 9221.08 \text{ J} > 0$$

$$W_{C \rightarrow D} = P_C (V_D - V_C) = 1.013 \cdot 10^5 \text{ Pa } (8.2 - 65.6) \cdot 10^{-3} \text{ m}^3 = -5814.62 \text{ J}$$

$$Q_{C \rightarrow D} = n C_p (T_D - T_C) = 5 \text{ cal K}^{-1} (50 - 800) \text{ K} = -3750 \text{ cal} = -15690 \text{ J}$$

$$W_{D \rightarrow A} = 0 \quad Q_{D \rightarrow A} = n C_v (T_A - T_D) = 3 \text{ cal K}^{-1} (400 - 50) \text{ K} = 1050 \text{ cal} = 4393.2 \text{ J} > 0$$

$$\eta = \frac{W}{Q_{\text{absorbida}}} = \frac{3322.64 \text{ J} + 9221.08 \text{ J} - 5814.62 \text{ J} + 0 \text{ J}}{8368 \text{ J} + 9221.08 \text{ J} + 4393.2 \text{ J}} = \frac{6729.1 \text{ J}}{21982.28 \text{ J}} = \boxed{0.306}$$

Autor: Iván Cabrera Álvarez

$$W_{B \rightarrow E} = \frac{P_E V_E - P_B V_B}{1 - \gamma} = \frac{(43.6 \cdot 1.013 \cdot 10^5 \cdot 10^{-3} - 4 \cdot 164 \cdot 1.013 \cdot 10^5 \cdot 10^{-3}) \text{ J}}{1 - \frac{5}{3}} =$$

$$= + 3342.9 \text{ J}$$

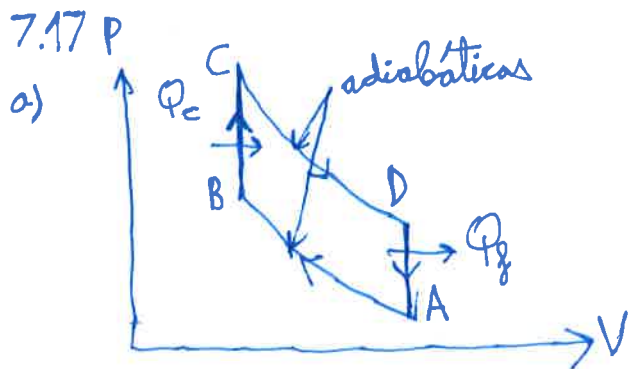
$$Q_{B \rightarrow E} = 0$$

$$W_{E \rightarrow D} = P_D (V_D - V_E) = 1.013 \cdot 10^5 \text{ Pa} (8.2 - 43.6) \cdot 10^{-3} \text{ m}^3 = - 3586.02 \text{ J}$$

$$Q_{E \rightarrow D} = m C_p (T_D - T_E) = 5 \text{ cal K}^{-1} (50 - 530) \text{ K} = - 2400 \text{ cal} = - 10041.6 \text{ J}$$

$$\eta_{ABEDA} = \frac{W}{Q_{\text{absorbido}}} = \frac{3342.9 \text{ J} + 3342.9 \text{ J} - 3586.02 \text{ J} + 0 \text{ J}}{8368 \text{ J} + 0 \text{ J} + 4393.2 \text{ J}} = \boxed{0.241}$$

El ciclo ABCDA tiene mayor rendimiento que el ciclo ABEDA



$$b) \eta = \frac{Q_c + Q_f}{Q_c} = 1 + \frac{Q_f}{Q_c}$$

$$Q_f < 0 \quad Q_c > 0$$

$$Q_c = Q_{BC} = m C_v (T_C - T_B)$$

$$Q_f = Q_{DA} = m C_v (T_A - T_D)$$

$$\eta = 1 + \frac{Q_f}{Q_c} = \boxed{1 + \frac{(T_A - T_D)}{(T_C - T_B)}}$$

$$c) \text{ Proceso BC es a } V = \text{cte.} \Rightarrow \frac{T}{P} = \text{cte.} \Rightarrow \frac{T_B}{P_B} = \frac{T_C}{P_C} \Rightarrow T_C = \frac{P_C}{P_B} T_B$$

$$\text{Proceso DA es a } V = \text{cte.} \Rightarrow \frac{T}{P} = \text{cte.} \Rightarrow \frac{T_A}{P_A} = \frac{T_D}{P_D} \Rightarrow T_D = \frac{P_D}{P_A} T_A$$

$$\left. \begin{array}{l} \text{Proceso CD es adiabático} \Rightarrow P_D V_D^\gamma = P_C V_C^\gamma \\ \text{Proceso AB es adiabático} \Rightarrow P_A V_A^\gamma = P_B V_B^\gamma \end{array} \right\} \Rightarrow \frac{P_D}{P_A} \left(\frac{V_D}{V_A} \right)^\gamma = \frac{P_C}{P_B} \left(\frac{V_C}{V_B} \right)^\gamma \Rightarrow$$

$$\text{Proceso BC es a } V = \text{cte.} \Rightarrow V_B = V_C$$

$$\text{Proceso DA es a } V = \text{cte.} \Rightarrow V_D = V_A$$

$$\Rightarrow \frac{P_D}{P_A} = \frac{P_C}{P_B}$$

Autor: Iván Cabria Álvarez

$$\left. \begin{aligned} \frac{P_D}{P_A} &= \frac{P_C}{P_B} \\ T_C &= \frac{P_C}{P_B} T_B \end{aligned} \right\} \Rightarrow T_C = \frac{P_D}{P_A} T_B$$

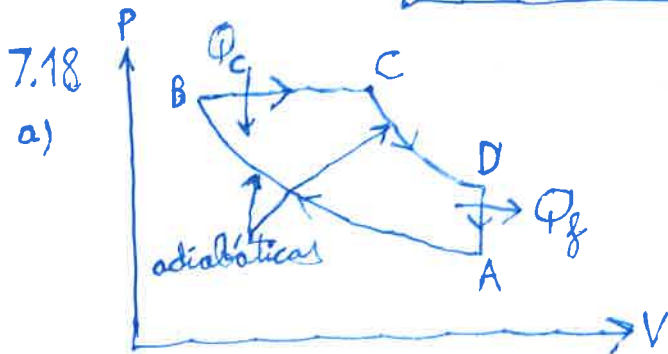
$$\eta = 1 + \frac{T_A - T_D}{T_C - T_B} = 1 + \frac{\left(T_A - \frac{P_D}{P_A} T_A\right)}{\left(\frac{P_D}{P_A} T_B - T_B\right)} = 1 + \frac{T_A}{T_B} \frac{\left(1 - \frac{P_D}{P_A}\right)}{\left(\frac{P_D}{P_A} - 1\right)}$$

$$= \boxed{1 - \frac{T_A}{T_B}}$$

d) Proceso AB es adiabático $\Rightarrow TV^{\gamma-1} = \text{cte.} \Rightarrow T_A V_A^{\gamma-1} = T_B V_B^{\gamma-1} \Rightarrow$

$$\Rightarrow \frac{T_A}{T_B} = \left(\frac{V_B}{V_A}\right)^{\gamma-1} = \frac{1}{2^{\gamma-1}}$$

$$\eta = 1 - \frac{T_A}{T_B} = \boxed{1 - \frac{1}{2^{\gamma-1}}}$$



$$b) \eta = 1 + \frac{Q_f}{Q_c}$$

$$Q_c > 0 \\ Q_f < 0$$

$$Q_c = Q_{BC} = m C_p (T_C - T_B)$$

$$Q_f = Q_{DA} = m C_v (T_A - T_D)$$

$$\eta = 1 + \frac{Q_f}{Q_c} = 1 + \frac{C_v}{C_p} \frac{(T_A - T_D)}{(T_C - T_B)} = \boxed{1 + \frac{1}{\gamma} \frac{(T_A - T_D)}{(T_C - T_B)}}$$

c) Proceso BC es a $P = \text{cte.} \Rightarrow \frac{T}{V} = \text{cte.} \Rightarrow \frac{T_B}{V_B} = \frac{T_C}{V_C} \Rightarrow T_C = \frac{V_C}{V_B} T_B$

Proceso DA es a $V = \text{cte.} \Rightarrow \frac{T}{P} = \text{cte.} \Rightarrow \frac{T_D}{P_D} = \frac{T_A}{P_A} \Rightarrow T_D = \frac{P_D}{P_A} T_A$

$$\eta = 1 + \frac{1}{\gamma} \frac{(T_A - T_D)}{(T_C - T_B)} = 1 + \frac{1}{\gamma} \frac{\left(T_A - \frac{P_D}{P_A} T_A\right)}{\left(\frac{V_C}{V_B} T_B - T_B\right)} = \boxed{1 + \frac{1}{\gamma} \frac{T_A}{T_B} \frac{\left(1 - \frac{P_D}{P_A}\right)}{\left(\frac{V_C}{V_B} - 1\right)}}$$

Autor: Iván Abria Álvarez

(7.19)

d) Proceso AB es adiabático $\Rightarrow TV^{\gamma-1} = \text{cte.} \Rightarrow T_A V_A^{\gamma-1} = T_B V_B^{\gamma-1} \Rightarrow$

$$\Rightarrow \frac{T_A}{T_B} = \left(\frac{V_B}{V_A} \right)^{\gamma-1} = \frac{1}{2^{\gamma-1}}$$

Buscamos la relación entre P_D/P_A y $\alpha = V_C/V_B$:

$$\left. \begin{array}{l} \text{Proceso CD es adiabático} \Rightarrow P_D V_D^{\gamma} = P_C V_C^{\gamma} \\ \text{Proceso AB es adiabático} \Rightarrow P_A V_A^{\gamma} = P_B V_B^{\gamma} \end{array} \right\} \Rightarrow \frac{P_D}{P_A} \left(\frac{V_D}{V_A} \right)^{\gamma} = \frac{P_C}{P_B} \left(\frac{V_C}{V_B} \right)^{\gamma}$$

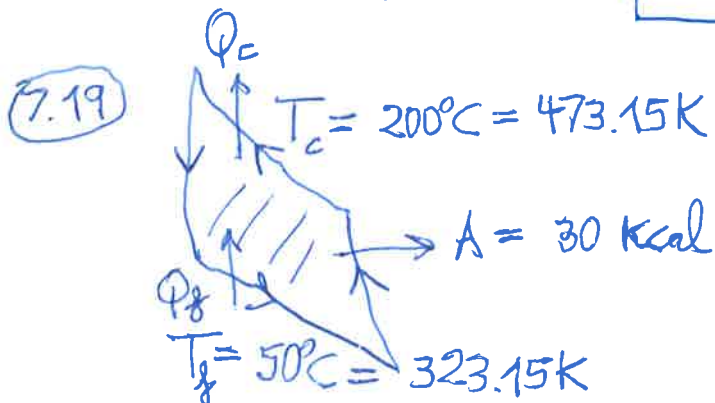
Proceso DA es a $V = \text{cte} \Rightarrow V_D = V_A$

$$\Rightarrow \frac{P_D}{P_A} = \frac{P_C}{P_B} \left(\frac{V_C}{V_B} \right)^{\gamma} \left. \begin{array}{l} \text{Proceso BC es a } P = \text{cte.} \Rightarrow P_B = P_C \end{array} \right\} \Rightarrow \frac{P_D}{P_A} = \left(\frac{V_C}{V_B} \right)^{\gamma} = \alpha^{\gamma}$$

Proceso BC es a $P = \text{cte.} \Rightarrow P_B = P_C$

$$\eta = 1 + \frac{1}{\gamma} \frac{T_A}{T_B} \frac{\left(1 - \frac{P_D}{P_A} \right)}{\left(\frac{V_C}{V_B} - 1 \right)} = 1 + \frac{1}{\gamma} \frac{T_A}{T_B} \frac{(1 - \alpha^{\gamma})}{(\alpha - 1)}$$

$$= 1 - \frac{T_A}{T_B} \frac{(\alpha^{\gamma} - 1)}{\gamma(\alpha - 1)} = \boxed{1 - \frac{(\alpha^{\gamma} - 1)}{2^{\gamma-1} \gamma (\alpha - 1)}}$$



a) $W < 0$
 $|W| = A \Rightarrow W = -A = \boxed{-30\text{ Kcal}}$

b) $E_f = \frac{T_f}{T_c - T_f} = \frac{323.15\text{K}}{150\text{K}} = \boxed{2.154333}$

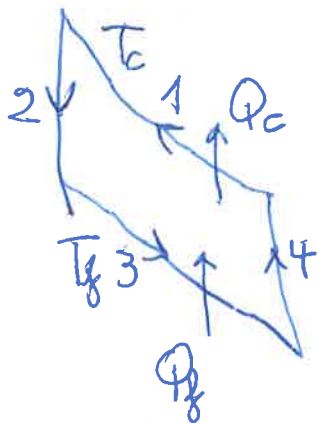
c) $E_f = \frac{Q_f}{|W|} \Rightarrow Q_f = E_f |W| = \boxed{+64.63\text{ Kcal}}$

$|W| = |Q_c| - Q_f \Rightarrow |Q_c| = |W| + Q_f = 30\text{ Kcal} + 64.63\text{ Kcal} = 94.63\text{ Kcal}$

$Q_c < 0 \Rightarrow \boxed{Q_c = -94.63\text{ Kcal}}$

Autor: Iván Abria Álvarez

d)



$\Delta S_2 = \Delta S_4 = 0$ porque las etapas 2 y 4 son adiabáticas

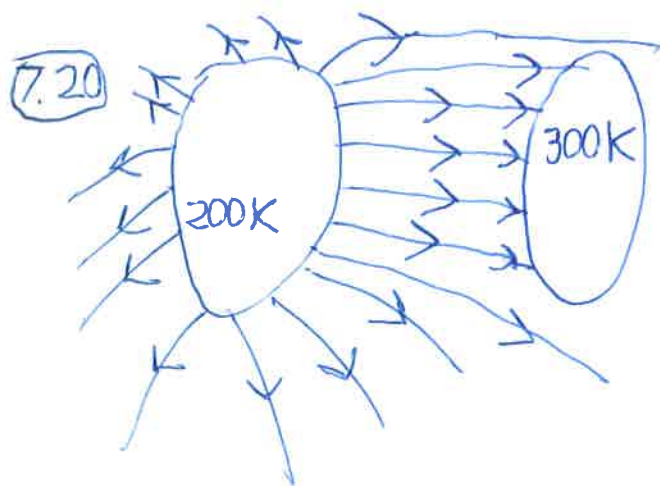
(7.20)

$$\Delta S_1 = \frac{Q_c}{T_c} = \frac{-94.63 \text{ Kcal}}{473.15 \text{ K}} = \boxed{-0.2 \frac{\text{Kcal}}{\text{K}}}$$

$$\Delta S_3 = \frac{Q_f}{T_f} = \frac{+64.63 \text{ Kcal}}{323.15 \text{ K}} = \boxed{+0.2 \frac{\text{Kcal}}{\text{K}}}$$

ΔS de todo el ciclo ^{reversible} debe ser cero. Lo comprobamos:

$$\Delta S = \Delta S_1 + \Delta S_2 + \Delta S_3 + \Delta S_4 = (-0.2 + 0 + 0.2 + 0) \frac{\text{Kcal}}{\text{K}} = 0$$



El objeto a 300K también emite calor por radiación térmica y también calienta al objeto que está a 200K.

El objeto a 300K emite más calor que el objeto a 200K, porque el calor emitido por radiación es una función creciente de la temperatura del objeto: A mayor

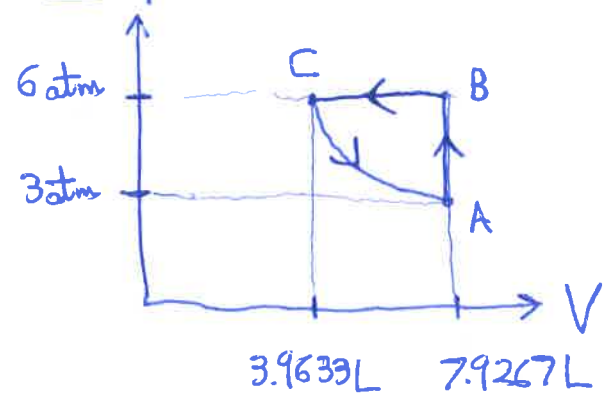
temperatura, mayor calor emitido. Los dos objetos emiten calor por radiación y calientan al otro objeto. El objeto de 300K calienta más al objeto de 200K que este objeto al objeto de 300K.

El balance neto de las emisiones de calor por radiación ^{y recepciones} es que el objeto de 300K cede calor al objeto de 200K y por tanto, si pasa calor espontáneamente del objeto caliente al frío y no al revés.

Autor: Juan Carlos Álvarez

(7.21)

(7.21)



Calculamos y/o escribimos la presión, el volumen y la temperatura en A, B y C.

Estado A:
 $P_A = 3 \text{ atm}$

$T_A = 290 \text{ K}$

$$V_A = \frac{nRT_A}{P_A} = \frac{1 \text{ mol} \cdot 0.082 \text{ atm} \cdot \text{L} \cdot \text{mol}^{-1} \cdot \text{K}^{-1} \cdot 290 \text{ K}}{3 \text{ atm}} = 7.9267 \text{ L}$$

Estado B:

$P_B = 6 \text{ atm}$

$P_B = 2P_A$

$$V_B = V_A = 7.9267 \text{ L} \quad T_B = \frac{P_B V_B}{nR} = \frac{2P_A V_B}{nR} = \frac{2P_A V_A}{nR} = 2T_A = 580 \text{ K}$$

Estado C:

$P_C = P_B = 6 \text{ atm} \quad V_C = V_A/2 = 3.9633 \text{ L}$

Entre los estados C y A se produce una expansión isoterma. Por tanto,
 $T_C = T_A = 290 \text{ K}$.

También podemos comprobar que $T_C = T_A$:

$$T_C = \frac{P_C V_C}{nR} = \frac{P_B V_C}{nR} = \frac{2P_A V_C}{nR} = \frac{2P_A V_A/2}{nR} = \frac{P_A V_A}{nR} = T_A$$

Más adelante observaremos que no es necesario calcular los valores numéricos de P, V y T y que sí es necesario calcular los cocientes de algunas temperaturas y volúmenes. Calculamos los cambios de entropía de los tres procesos.

Proceso AB: $V = \text{cte.}$

$$\Delta S_{AB} = \int_A^B \frac{\delta Q}{T} = \int_A^B \frac{n C_V dT}{T} = n C_V \ln \left(\frac{T_B}{T_A} \right) = n C_V \ln \left(\frac{2T_A}{T_A} \right) = n C_V \ln 2 \approx 1 \text{ mol} \cdot 3 \frac{\text{cal}}{\text{mol K}} \ln 2 = \boxed{+ 2.0794 \frac{\text{cal}}{\text{K}}}$$

Autor: Juan Cabria Álvarez

Processo BC: $P = \text{cte.}$

$$\Delta S_{BC} = \int_B^C \frac{\delta Q}{T} = \int_B^C m C_p \frac{dT}{T} = m C_p \ln\left(\frac{T_C}{T_B}\right) = m C_p \ln\left(\frac{T_A}{2T_A}\right) =$$

$$= -m C_p \ln 2 \approx -1 \text{ mol } 5 \frac{\text{cal}}{\text{mol K}} \ln 2 = \boxed{-3.4657 \frac{\text{cal}}{\text{K}}}$$

Processo CA: $T = \text{cte.}$

$$\Delta S_{CA} = \int_C^A \frac{\delta Q}{T} = \frac{1}{T_A} \int_C^A \delta Q = \frac{Q_{CA}}{T_A}$$

Em um processo isotermo $\Delta U = 0 \Rightarrow \Delta U_{CA} = 0$

$$\Delta U_{CA} = Q_{CA} - W_{CA} \Rightarrow Q_{CA} = W_{CA}$$

$$W_{CA} = \int_C^A P dV = \int_C^A m R T \frac{dV}{V} = m R T_A \int_C^A \frac{dV}{V} = m R T_A \ln\left(\frac{V_A}{V_C}\right)$$

$$\Rightarrow \Delta S_{CA} = m R \ln\left(\frac{V_A}{V_C}\right) = m R \ln\left(\frac{2V}{V}\right) = m R \ln 2 =$$

$$\approx 1 \text{ mol } 2 \frac{\text{cal}}{\text{mol K}} \ln 2 = \boxed{+1.3863 \frac{\text{cal}}{\text{K}}}$$

Outra forma de calcular ΔS_{CA} :

Se trata de um ciclo de processos reversíveis $\Rightarrow \Delta S_{\text{ciclo}} = 0 \Rightarrow$

$$\Delta S_{\text{ciclo}} = \Delta S_{AB} + \Delta S_{BC} + \Delta S_{CA}$$

$$\Rightarrow 0 = \Delta S_{AB} + \Delta S_{BC} + \Delta S_{CA} \Rightarrow \Delta S_{CA} = -\Delta S_{AB} - \Delta S_{BC} =$$

$$= -2.0794 \frac{\text{cal}}{\text{K}} + 3.4657 \frac{\text{cal}}{\text{K}} = \boxed{+1.3863 \frac{\text{cal}}{\text{K}}}$$

Autor: Yván Cabrera Alvarado

7.22 $m = 1000 \text{ g de agua}$

$$T_{\text{inicial agua}} = 363.15 \text{ K}$$

$$T_{\text{final agua}} = 298.15 \text{ K}$$

Agua:

$$\Delta S_{\text{agua}} = \int_{T_{\text{inicial agua}}}^{T_{\text{final agua}}} \frac{\delta Q}{T} = \int_{T_{\text{inicial agua}}}^{T_{\text{final agua}}} \frac{m C_{\text{agua}} dT}{T} = m C_{\text{agua}} \ln \left(\frac{T_{\text{final agua}}}{T_{\text{inicial agua}}} \right) =$$

$$= 1000 \text{ g} \cdot 1 \frac{\text{cal}}{\text{g K}} \ln \left(\frac{298.15}{363.15} \right) = -197.22 \frac{\text{cal}}{\text{K}} = \boxed{-825.17 \frac{\text{J}}{\text{K}}}$$

Habitación:

$$T = T_i = T_f = 298.15 \text{ K} \quad T_f = T_{\text{final habitación}} = T_{\text{final agua}}$$

$$\Delta S_{\text{habitación}} = \int_i^f \frac{\delta Q}{T} = \frac{1}{T} \int_i^f \delta Q = \frac{Q_{\text{habitación}}}{T}$$

$$Q_{\text{habitación}} + Q_{\text{agua}} = 0 \quad Q_{\text{agua}} < 0 \quad Q_{\text{habitación}} > 0$$

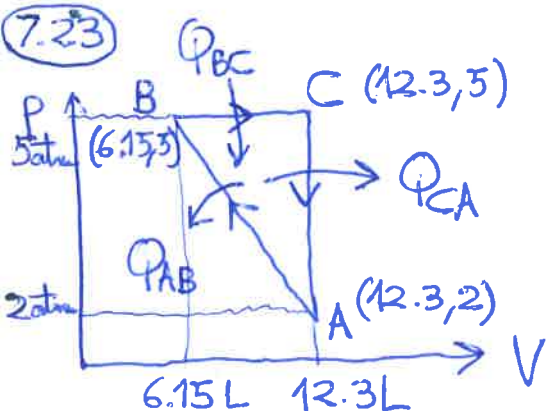
$$Q_{\text{agua}} = m C_{\text{agua}} (T_{\text{final agua}} - T_{\text{inicial agua}}) = 1000 \text{ g} \cdot 1 \frac{\text{cal}}{\text{g K}} (298.15 - 363.15) \text{ K}$$

$$= -65000 \text{ cal} = -271\,960 \text{ J}$$

$$Q_{\text{habitación}} = -Q_{\text{agua}} = +271\,960 \text{ J}$$

$$\Delta S_{\text{habitación}} = \frac{271\,960 \text{ J}}{298.15 \text{ K}} = \boxed{+912.16 \frac{\text{J}}{\text{K}}}$$

Autor: Yoán Abria Álvarez



Proceso AB: Polinomial con respecto a V.

$$P = aV + b$$

$$\left. \begin{aligned} 2 &= a(12.3) + b \\ 5 &= a(6.15) + b \end{aligned} \right\} \Rightarrow 2 - 5 = a(6.15) \Rightarrow$$

$$\Rightarrow a = \frac{-3}{6.15} \frac{\text{atm}}{\text{L}}$$

$$V_A = 12.3 \text{ L} \quad V_B = 6.15 \text{ L}$$

$$b = 2 - a(12.3) = 2 + \frac{3}{6.15} 12.3 = 2 + 6 \text{ atm} = 8 \text{ atm}$$

$$W_{AB} = \int_A^B P dV = \int_{V_A}^{V_B} (aV + b) dV = \int_{V_A}^{V_B} \left(-\frac{3}{6.15} V + 8 \right) dV =$$

$$= -\frac{3}{6.15} \frac{V^2}{2} \Big|_{V_A}^{V_B} + 8V \Big|_{V_A}^{V_B} = -\frac{3}{2 \cdot 6.15} \left[(6.15)^2 - (12.3)^2 \right] +$$

$$+ 8(6.15 - 12.3) \text{ atm L} = \boxed{-21.5250 \text{ atm L} = -523.3158 \text{ cal} =}$$

$$\boxed{= -2180.4825 \text{ J}}$$

$$\Delta U_{AB} = n C_V (T_B - T_A)$$

$$T_A = 300 \text{ K}$$

$$C_V = 3 \frac{\text{cal}}{\text{mol K}}$$

$$n = \frac{P_A V_A}{RT_A} = \frac{2 \cdot 12.3 \text{ L}}{0.082 \text{ atm L mol}^{-1} \text{ K}^{-1} 300 \text{ K}} = 1 \text{ mol}$$

$$T_B = \frac{P_B V_B}{nR} = \frac{5 \text{ atm} 6.15 \text{ L}}{1 \text{ mol} 0.082 \text{ atm L mol}^{-1} \text{ K}^{-1}} = 375 \text{ K}$$

$$\Delta U_{AB} = 1 \text{ mol} 3 \frac{\text{cal}}{\text{mol K}} (375 - 300) \text{ K} = \boxed{225 \text{ cal} = 937.5 \text{ J}}$$

$$Q_{AB} = \Delta U_{AB} + W_{AB} = 225 \text{ cal} - 523.3158 \text{ cal} = \boxed{-298.3158 \text{ cal}} \\ = \boxed{-1242.9825 \text{ J}}$$

Autor: Yeán Cabris Álvarez

7.24

Proceso BC: $P = \text{cte.}$

(7.25)

$$Q_{BC} = n C_p (T_C - T_B)$$

$$C_p = C_v + R \approx 3 \text{ cal/mol/K} + R \approx 5 \text{ cal/mol/K}$$

$$T_C = \frac{P_C V_C}{n R} = \frac{5 \text{ atm } 12.3 \text{ L}}{1 \text{ mol } 0.082 \text{ atm L/mol}^{-1} \text{ K}^{-1}} = 750 \text{ K}$$

$$Q_{BC} \approx 1 \text{ mol } 5 \frac{\text{cal}}{\text{mol K}} (750 - 375) \text{ K} = \boxed{1875 \text{ cal} = 7812.5 \text{ J}}$$

$$W_{BC} = P_B (V_C - V_B) = 5 \text{ atm } (12.3 - 6.15) \text{ L} = \boxed{30.75 \text{ atm L}} = \boxed{3114.9750 \text{ J} = 747.594 \text{ cal}}$$

$$\Delta U_{BC} = Q_{BC} - W_{BC} = 1875 \text{ cal} - 747.594 \text{ cal} = \boxed{1127.4 \text{ cal} = 4697.5 \text{ J}}$$

Otra forma de calcular ΔU_{BC} :

$$\Delta U_{BC} = n C_v (T_C - T_B) = 1 \text{ mol } 3 \frac{\text{cal}}{\text{mol K}} (750 - 375) \text{ K} = \boxed{1125 \text{ cal}} = \boxed{4687.5 \text{ J}}$$

La diferencia numérica entre las dos formas de calcular ΔU_{BC} proviene de la aproximación $R \approx 2 \text{ cal/mol K}$.

Proceso CA: $V = \text{cte.}$

$W_{CA} = 0$ por ser un proceso a volumen constante.

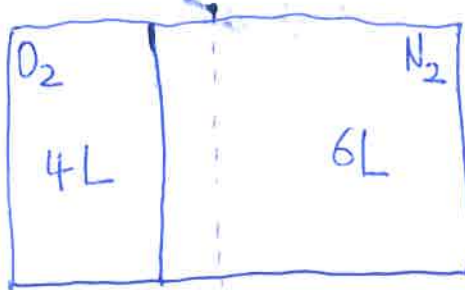
$$Q_{CA} = n C_v (T_A - T_C) = 1 \text{ mol } 3 \frac{\text{cal}}{\text{mol K}} (300 - 750) \text{ K} = \boxed{-1350 \text{ cal}} = \boxed{-5625 \text{ J}}$$

$$\Delta U_{CA} = n C_v (T_A - T_C) = \boxed{-1350 \text{ cal} = -5625 \text{ J}}$$

$$\eta = \frac{W}{Q_{\text{absorbido}}} = \frac{W_{AB} + W_{BC} + W_{CA}}{Q_{BC}} = \frac{-523.3148 \text{ cal} + 747.594 \text{ cal} + 0 \text{ cal}}{1875 \text{ cal}} = \boxed{0.1196}$$

Autor: Iván Gabrís Álvarez

(7.24)



$$T_{iO_2} = T_{iN_2} = 300 \text{ K} = T_{fO_2} = T_{fN_2}$$

$$P_{iO_2} = P_{iN_2} = 0.1 \text{ MPa} = 10^5 \text{ Pa}$$

$$V_{iO_2} = 4 \text{ L} = 0.004 \text{ m}^3$$

$$V_{fO_2} = 10 \text{ L} = 0.01 \text{ m}^3$$

$$V_{iN_2} = 6 \text{ L} = 0.006 \text{ m}^3 \quad V_{fN_2} = 10 \text{ L} = 0.01 \text{ m}^3$$

1ª forma de resolver este problema:

$$\left. \begin{aligned} dU &= \delta Q - PdV \\ dU &= mC_v dT \\ dS &= \frac{\delta Q}{T} \end{aligned} \right\} \Rightarrow dS = mC_v \frac{dT}{T} + \frac{P}{T} dV \quad \left. \begin{aligned} PV &= mRT \Rightarrow \frac{P}{T} = \frac{mR}{V} \end{aligned} \right\} \Rightarrow dS = mC_v \frac{dT}{T} + mR \frac{dV}{V} \Rightarrow$$

$$\Rightarrow \Delta S = mC_v \ln\left(\frac{T_f}{T_i}\right) + mR \ln\left(\frac{V_f}{V_i}\right) \quad \Delta S = \int_i^f dS$$

$$\Delta S_{\text{sistema}} = \Delta S_{O_2} + \Delta S_{N_2}$$

$$\Delta S_{O_2} = m_{O_2} C_{vO_2} \ln\left(\frac{T_{fO_2}}{T_{iO_2}}\right) + m_{O_2} R \ln\left(\frac{V_{fO_2}}{V_{iO_2}}\right) = m_{O_2} C_{vO_2} \ln 1 + m_{O_2} R \ln\left(\frac{10}{4}\right) = 0 + m_{O_2} R \ln\left(\frac{5}{2}\right) = m_{O_2} R \ln\left(\frac{5}{2}\right)$$

$$P_{iO_2} V_{iO_2} = m_{O_2} R T_{iO_2} \Rightarrow m_{O_2} R = \frac{P_{iO_2} V_{iO_2}}{T_{iO_2}} = \frac{10^5 \text{ Pa} \cdot 4 \cdot 10^{-3} \text{ m}^3}{300 \text{ K}} \Rightarrow$$

$$\Rightarrow m_{O_2} R = 1.3 \text{ J/K} \Rightarrow \Delta S_{O_2} = 1.3 \ln\left(\frac{5}{2}\right) \text{ J/K} = +1.221721 \text{ J/K}$$

Calculamos ΔS_{N_2} :

$$\begin{aligned} \Delta S_{N_2} &= m_{N_2} C_{vN_2} \ln\left(\frac{T_{fN_2}}{T_{iN_2}}\right) + m_{N_2} R \ln\left(\frac{V_{fN_2}}{V_{iN_2}}\right) = m_{N_2} C_{vN_2} \ln 1 + m_{N_2} R \ln\left(\frac{10}{6}\right) \\ &= m_{N_2} R \ln\left(\frac{5}{3}\right) = \frac{P_{iN_2} V_{iN_2}}{T_{iN_2}} \ln\left(\frac{5}{3}\right) = \\ &= \frac{10^5 \text{ Pa} \cdot 0.006 \text{ m}^3}{300 \text{ K}} \ln\left(\frac{5}{3}\right) = +1.0216512 \text{ J/K} \end{aligned}$$

$$\Delta S_{\text{sistema}} = +2.2433722 \text{ J/K} \simeq \boxed{+2.24 \text{ J/K}}$$

Autor: Iván Cabria Álvarez

(7.26)

2ª forma de resolver este problema:

$$\left. \begin{aligned} dU &= \delta Q - PdV \Rightarrow \delta Q = dU + PdV \\ dU &= mC_v dT \end{aligned} \right\} \Rightarrow \delta Q = mC_v dT + PdV \Rightarrow dS = \frac{\delta Q}{T}$$

$$\Rightarrow dS = mC_v \frac{dT}{T} + \frac{P}{T} dV$$

$$PV = mRT \Rightarrow PdV + VdP = mRdT \Rightarrow PdV = mRdT - VdP \Rightarrow$$

$$\Rightarrow dS = mC_v \frac{dT}{T} + \frac{mRdT}{T} - \frac{VdP}{T} = m(C_v + R) \frac{dT}{T} - \frac{VdP}{T} \Rightarrow$$

$$C_p = C_v + R$$

$$\Rightarrow dS = mC_p \frac{dT}{T} - \frac{VdP}{T} \Rightarrow dS = mC_p \frac{dT}{T} - mR \frac{dP}{P} \Rightarrow$$

$$PV = mRT \Rightarrow \frac{V}{T} = \frac{mR}{P}$$

$$\Rightarrow \Delta S = \int_i^f dS = mC_p \ln\left(\frac{T_f}{T_i}\right) - mR \ln\left(\frac{P_f}{P_i}\right)$$

$$\Delta S_{\text{sistema}} = \Delta S_{O_2} + \Delta S_{N_2}$$

Calculamos ΔS_{O_2} :

$$\Delta S_{O_2} = m_{O_2} C_{pO_2} \ln\left(\frac{T_{fO_2}}{T_{iO_2}}\right) - m_{O_2} R \ln\left(\frac{P_{fO_2}}{P_{iO_2}}\right) = m_{O_2} C_{pO_2} \ln 1 - m_{O_2} R \ln\left(\frac{P_{fO_2}}{P_{iO_2}}\right) =$$

$$= -m_{O_2} R \ln\left(\frac{P_{fO_2}}{P_{iO_2}}\right) = -\frac{P_{iO_2} V_{iO_2}}{T_{iO_2}} \ln\left(\frac{P_{fO_2}}{P_{iO_2}}\right)$$

Calculamos P_{fO_2}/P_{iO_2} :

$$\frac{P_{fO_2}}{P_{iO_2}} = \frac{m_{O_2} R T_{fO_2}}{V_{fO_2}} \frac{V_{iO_2}}{m_{O_2} R T_{iO_2}} = \frac{V_{iO_2}}{V_{fO_2}} = \frac{4L}{10L} = \frac{2}{5}$$

$$\text{Portanto: } \Delta S_{O_2} = -\frac{10^5 \text{ Pa} \cdot 0.004 \text{ m}^3}{300 \text{ K}} \ln\left(\frac{2}{5}\right) = +1.221721 \text{ J/K}$$

Autor: João Cabria Álvares

Calculamos ΔS_{N_2} :

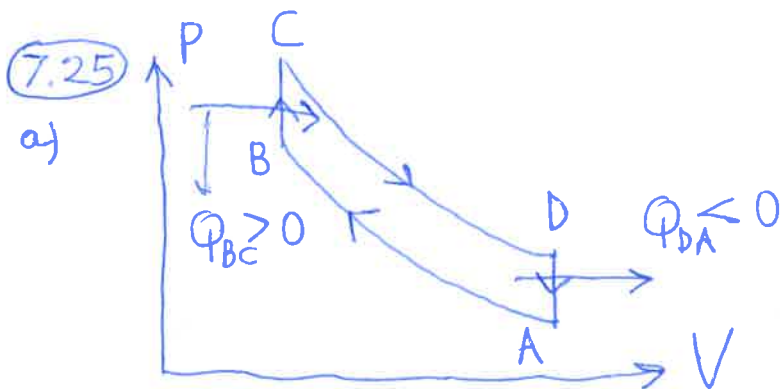
$$\begin{aligned}\Delta S_{N_2} &= m_{N_2} C_{PN_2} \ln \left(\frac{T_{fN_2}}{T_{iN_2}} \right) - m_{N_2} R \ln \left(\frac{P_{fN_2}}{P_{iN_2}} \right) = m_{N_2} C_{PN_2} \ln 1 - m_{N_2} R \ln \left(\frac{P_{fN_2}}{P_{iN_2}} \right) \\ &= -m_{N_2} R \ln \left(\frac{P_{fN_2}}{P_{iN_2}} \right) = -\frac{P_{iN_2} V_{iN_2}}{T_{iN_2}} \ln \left(\frac{P_{fN_2}}{P_{iN_2}} \right)\end{aligned}$$

Calculamos P_{fN_2}/P_{iN_2} :

$$\frac{P_{fN_2}}{P_{iN_2}} = \frac{m_{N_2} R T_{fN_2}}{V_{fN_2}} \frac{V_{iN_2}}{m_{N_2} R T_{iN_2}} = \frac{V_{iN_2}}{V_{fN_2}} = \frac{6L}{10L} = \frac{3}{5}$$

Por tanto: $\Delta S_{N_2} = -\frac{10^5 \text{ Pa} \cdot 0.006 \text{ m}^3}{300 \text{ K}} \ln \left(\frac{3}{5} \right) = +1.0216512 \text{ J/K}$

$\Delta S_{\text{sistema}} = +2.2433722 \text{ J/K} \approx \boxed{+2.24 \text{ J/K}}$



$T_A = 300 \text{ K} \quad V_A = 0.004 \text{ m}^3$

$P_A = 100 \text{ kPa} = 10^5 \text{ Pa}$

$\gamma = 7/5 = 1.4$

$Q_{BC} = +3700 \text{ J} > 0$

Calculamos P_B, T_B, V_B :

$V_B = V_A/8 = \boxed{5 \cdot 10^{-4} \text{ m}^3}$

El proceso AB es adiabático $\Rightarrow PV^\gamma = \text{cte.}$
 $TV^{\gamma-1} = \text{cte.}$

AB adiabático $\Rightarrow P_B V_B^\gamma = P_A V_A^\gamma \Rightarrow P_B = P_A \left(\frac{V_A}{V_B} \right)^\gamma = 10^5 \text{ Pa} \left(\frac{0.004}{0.0005} \right)^{1.4} =$

$= \boxed{1837917.4 \text{ Pa}}$

$T_B V_B^{\gamma-1} = T_A V_A^{\gamma-1} \Rightarrow T_B = T_A \left(\frac{V_A}{V_B} \right)^{\gamma-1} = 300 \text{ K} \cdot 8^{0.4} = \boxed{689.22 \text{ K}}$

Autor: Iván Cabrera Álvarez

Calculamos P_C , T_C y V_C :

(7.29)

El proceso BC es a volumen constante $\Rightarrow Q_{BC} = m C_V (T_C - T_B) \Rightarrow$
 $\Rightarrow T_C = T_B + \frac{Q_{BC}}{m C_V}$

No conocemos los valores numéricos de m y C_V en el S.I.

Calculamos C_V :

$$\left. \begin{aligned} \gamma &= \frac{C_p}{C_V} \\ C_p &= C_V + R \end{aligned} \right\} \Rightarrow \gamma = \frac{C_V + R}{C_V} = 1 + \frac{R}{C_V} \Rightarrow \gamma - 1 = \frac{R}{C_V} \Rightarrow$$

$$\Rightarrow C_V = \frac{R}{\gamma - 1} = \frac{R}{\frac{7}{5} - 1} = \frac{R}{(2/5)} = \frac{5}{2} R$$

Calculamos m , el número de moles:

$$m = \frac{P_A V_A}{R T_A} = \frac{10^5 \text{ Pa } 0.004 \text{ m}^3}{8.3145 \text{ J mol}^{-1} \text{ K}^{-1} 300 \text{ K}} = 0.1603624 \text{ moles}$$

Calculamos T_C :

$$T_C = T_B + \frac{Q_{BC}}{m C_V} = 689.22 \text{ K} + \frac{3700 \text{ J}}{0.1603624 \text{ moles } \frac{5}{2} 8.3145 \text{ J mol}^{-1} \text{ K}^{-1}} =$$
$$= \boxed{1799.22 \text{ K}}$$

El proceso BC es a volumen constante $\Rightarrow \frac{P_C}{T_C} = \frac{P_B}{T_B} \Rightarrow$

$$\Rightarrow P_C = T_C \frac{P_B}{T_B} = 1799.22 \text{ K } \frac{1837917.4 \text{ Pa}}{689.22 \text{ K}} = \boxed{4797913.2 \text{ Pa}}$$

$$V_C = V_B = \boxed{5 \cdot 10^{-4} \text{ m}^3}$$

Calculamos P_D , T_D y V_D :

El proceso DA es a volumen constante $\Rightarrow V_D = V_A = \boxed{0.004 \text{ m}^3}$

El proceso CD es adiabático $\Rightarrow P_D V_D^\gamma = P_C V_C^\gamma \Rightarrow$

$$\Rightarrow P_D = P_C \left(\frac{V_C}{V_D} \right)^\gamma = 4797913.2 \text{ Pa} \left(\frac{0.0005}{0.004} \right)^{1.4} = \boxed{261051.63 \text{ Pa}}$$

El proceso CD es adiabático $\Rightarrow T_D V_D^{\gamma-1} = T_C V_C^{\gamma-1} \Rightarrow T_D = T_C \left(\frac{V_C}{V_D} \right)^{\gamma-1} =$

$$= 1799.22 \text{ K} \left(\frac{0.0005}{0.004} \right)^{0.4} = \boxed{783.15599 \text{ K}}$$

Autor: Juan Cabria Álvarez

$$b) Q_{\text{absorbido}} = Q_{BC} = \boxed{3700 \text{ J} > 0}$$

$$Q_{\text{cedido}} = Q_{DA} = n C_V (T_A - T_D) = 0.1603624 \text{ moles} \frac{5}{2} 8.3145 \frac{\text{J}}{\text{mol K}} \times (300 - 783.15599) \text{ K} = \boxed{-1610.52 \text{ J}}$$

$$c) W = W_{AB} + W_{BC} + W_{CD} + W_{DA}$$

Los procesos BC y DA son a volumen constante $\Rightarrow W_{BC} = 0$ y $W_{DA} = 0$

$$W_{AB} = \frac{P_B V_B - P_A V_A}{1 - \gamma} = \frac{1837917.4 \text{ Pa} \cdot 0.0005 \text{ m}^3 - 10^5 \text{ Pa} \cdot 0.004 \text{ m}^3}{(1 - 1.4)} =$$

$$= -1297.3968 \text{ J}$$

$$W_{CD} = \frac{P_D V_D - P_C V_C}{1 - \gamma} = \frac{261051.63 \text{ Pa} \cdot 0.004 \text{ m}^3 - 4797913.2 \text{ Pa} \cdot 0.0005 \text{ m}^3}{(1 - 1.4)} =$$

$$= +3386.8752 \text{ J}$$

$$W = -1297.3968 \text{ J} + 0 \text{ J} + 3386.8752 \text{ J} + 0 \text{ J} = \boxed{+2089.4784 \text{ J}}$$

d) Primera forma de calcular η :

$$\eta = 1 + \frac{Q_{\text{frio}}}{Q_{\text{caliente}}} = 1 + \frac{Q_{\text{cedido}}}{Q_{\text{absorbido}}} = 1 - \frac{1610.52}{3700} = \boxed{0.5647}$$

Segunda forma de calcular η :

$$\eta = \frac{W}{Q_{\text{absorbido}}} = \frac{2089.4784}{3700} = \boxed{0.5647}$$

e) Los procesos AB y CD son adiabáticos $\Rightarrow \boxed{\Delta S_{AB} = 0}$ y $\boxed{\Delta S_{CD} = 0}$

$$\Delta S_{BC} = \int_{BC} \frac{\delta Q}{T} = \int_{BC} \frac{n C_V dT}{T} = \int_{T_B}^{T_C} \frac{n C_V dT}{T} = n C_V \ln \left(\frac{T_C}{T_B} \right) =$$

$$= 0.1603624 \text{ moles} \frac{5}{2} 8.3145 \frac{\text{J}}{\text{mol K}} \ln \left(\frac{1799.22}{689.22} \right) = \boxed{+3.1985 \frac{\text{J}}{\text{K}}}$$

Autor: Iván Cabrera Álvarez

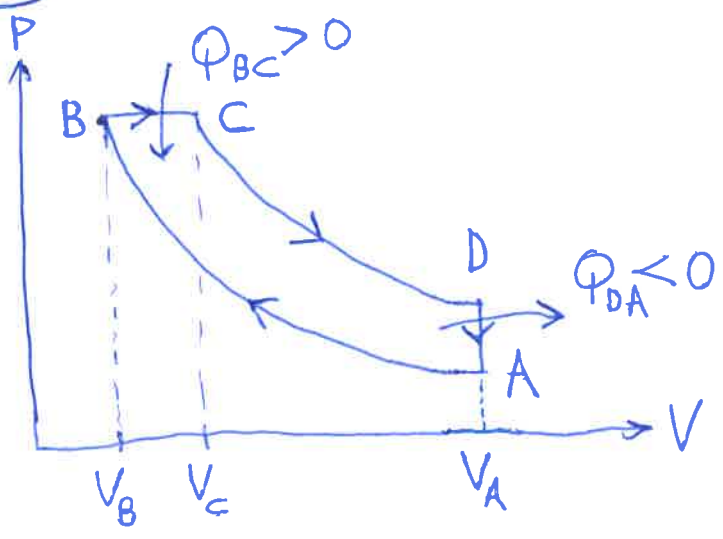
$$\Delta S_{DA} = \int_{DA} \frac{\delta Q}{T} = \int_{T_D}^{T_A} \frac{n C_v dT}{T} = n C_v \ln \left(\frac{T_A}{T_D} \right) = n \frac{5}{2} R \ln \left(\frac{T_A}{T_D} \right)$$

$$= 0.1603624 \text{ moles} \cdot \frac{5}{2} \cdot 8.3145 \frac{\text{J}}{\text{mol} \cdot \text{K}} \ln \left(\frac{300}{783.15599} \right) = \boxed{-3.1985 \frac{\text{J}}{\text{K}}}$$

Comprobamos que el cambio de la entropía del ciclo reversible ABCDA es cero:

$$\Delta S_{\text{ciclo}} = \Delta S_{AB} + \Delta S_{BC} + \Delta S_{CD} + \Delta S_{DA} = (0 + 3.1985 + 0 - 3.1985) \frac{\text{J}}{\text{K}} = 0 \frac{\text{J}}{\text{K}}$$

7.26



$$T_A = 300 \text{ K}$$

$$V_A = 2 \text{ L} = 0.002 \text{ m}^3$$

$$P_A = 0.1 \text{ MPa} = 10^5 \text{ Pa}$$

$$\gamma = \frac{C_p}{C_v} = 1.4$$

$$V_B = V_A / 20 = 0.0001 \text{ m}^3$$

$$V_C = 2 V_B = 0.0002 \text{ m}^3$$

$$V_D = V_A = 0.002 \text{ m}^3$$

a)
$$n = \frac{P_A V_A}{R T_A} = \frac{10^5 \text{ Pa} \cdot 0.002 \text{ m}^3}{8.3145 \text{ J mol}^{-1} \text{ K}^{-1} \cdot 300 \text{ K}} = 0.0801812 \text{ moles}$$

$\approx \boxed{0.0802 \text{ moles}}$

Calculamos P_B y T_B :

AB es un proceso adiabático $\Rightarrow P_A V_A^\gamma = P_B V_B^\gamma \Rightarrow P_B = P_A \left(\frac{V_A}{V_B} \right)^\gamma \Rightarrow$

$$\Rightarrow P_B = 10^5 \left(\frac{0.002}{0.0001} \right)^{1.4} = 10^5 \text{ Pa} \cdot (20)^{1.4} = \boxed{6628908 \text{ Pa}}$$

tutor: Iván Cabris Álvarez

$$T_A V_A^{\gamma-1} = T_B V_B^{\gamma-1} \Rightarrow T_B = T_A \left(\frac{V_A}{V_B} \right)^{\gamma-1} = 300 \text{ K} (20)^{0.4} = \boxed{994.3362 \text{ K}} \quad (7.32)$$

Calculamos P_C y T_C :

$$BC \text{ es un proceso isóbaro} \Rightarrow P_C = P_B \Rightarrow \boxed{P_C = 6\,628\,908 \text{ Pa}}$$

$$T_C = \frac{P_C V_C}{nR} = \frac{6\,628\,908 \text{ Pa} \cdot 0.0002 \text{ m}^3}{0.0801812 \text{ moles} \cdot 8.3145 \text{ J mol}^{-1} \text{ K}^{-1}} = \boxed{1988.6726 \text{ K}}$$

Otra forma de calcular T_C :

$$BC \text{ es un proceso isóbaro} \Rightarrow \frac{T_C}{V_C} = \frac{T_B}{V_B} \Rightarrow T_C = T_B \frac{V_C}{V_B} \Rightarrow$$

$$\Rightarrow T_C = 994.3362 \text{ K} \cdot 2 = \boxed{1988.6724 \text{ K}}$$

Calculamos P_D y T_D :

$$CD \text{ es un proceso adiabático} \Rightarrow P_D V_D^{\gamma} = P_C V_C^{\gamma} \Rightarrow P_D = P_C \left(\frac{V_C}{V_D} \right)^{\gamma} \Rightarrow$$

$$\Rightarrow P_D = 6\,628\,908 \text{ Pa} \left(\frac{0.0002}{0.002} \right)^{1.4} = 6\,628\,908 \text{ Pa} (0.1)^{1.4} = \boxed{2.6390 \cdot 10^5 \text{ Pa}}$$

$$T_D V_D^{\gamma-1} = T_C V_C^{\gamma-1} \Rightarrow T_D = T_C \left(\frac{V_C}{V_D} \right)^{\gamma-1} = 1988.67 \text{ K} \left(\frac{0.0002}{0.002} \right)^{0.4} = \boxed{791.7 \text{ K}}$$

Otra forma de calcular P_D :

$$P_D = \frac{n R T_D}{V_D} = \frac{0.0801812 \text{ moles} \cdot 8.3145 \text{ J mol}^{-1} \text{ K}^{-1} \cdot 791.7 \text{ K}}{0.002 \text{ m}^3} = \boxed{263900 \text{ Pa}}$$

Autor: Juan Cabria Álvarez

$$b) W_{AB} = \frac{P_B V_B - P_A V_A}{1 - \gamma} = \frac{6628908 \text{ Pa } 0.0001 \text{ m}^3 - 10^5 \text{ Pa } 0.002 \text{ m}^3}{(-0.4)} =$$

$$= \boxed{-1157.2270 \text{ J}}$$

$$W_{BC} = P_B (V_C - V_B) = 6628908 \text{ Pa } (0.0002 - 0.0001) \text{ m}^3 = \boxed{662.8908 \text{ J}}$$

$$W_{CD} = \frac{P_D V_D - P_C V_C}{1 - \gamma} = \frac{263900 \text{ Pa } 0.002 \text{ m}^3 - 6628908 \text{ Pa } 0.0002 \text{ m}^3}{(-0.4)} =$$

$$= \boxed{+1994.9540 \text{ J}}$$

$$W_{DA} = 0 \text{ por ser DA un proceso isócoro.}$$

$$Q_{AB} = 0 \text{ por ser AB un proceso adiabático.}$$

$$Q_{CD} = 0 \text{ por ser CD un proceso adiabático.}$$

Para calcular Q_{BC} y Q_{DA} , necesitamos C_V y C_P . Calculamos C_V y C_P :

$$\left. \begin{array}{l} \gamma = C_P / C_V \\ C_P = C_V + R \end{array} \right\} \Rightarrow \gamma = 1 + \frac{R}{C_V} \Rightarrow \frac{R}{C_V} = \gamma - 1 \Rightarrow C_V = \frac{R}{\gamma - 1}$$

$$\left. \begin{array}{l} \gamma = 1.4 = 7/5 \\ C_V = \frac{R}{\gamma - 1} \end{array} \right\} \Rightarrow C_V = \frac{R}{\left(\frac{7}{5} - 1\right)} = \frac{5}{2} R$$

$$C_P = C_V + R = \frac{5}{2} R + R = \frac{7}{2} R$$

Calculamos Q_{BC} y Q_{DA} :

$$Q_{BC} = n C_P (T_C - T_B) = 0.0801812 \text{ moles } \frac{7}{2} 8.3145 \text{ J mol}^{-1} \text{ K}^{-1} \cdot (1988.6724 - 994.3362) \text{ K} = \boxed{+2320.1175 \text{ J}}$$

$$Q_{DA} = n C_V (T_A - T_D) = 0.0801812 \text{ moles } \frac{5}{2} 8.3145 \text{ J mol}^{-1} \text{ K}^{-1} \cdot (300 - 791.7) \text{ K} = \boxed{-819.5 \text{ J}}$$

Autor: Iván Cabria Álvarez

$$c) \eta = 1 + \frac{Q_{\text{fuera}}}{Q_{\text{caliente}}} = 1 + \frac{Q_{DA}}{Q_{BC}} =$$

$$= 1 - \frac{819.5 \text{ J}}{2320.1175 \text{ J}} = 0.646785 \approx \boxed{0.647}$$

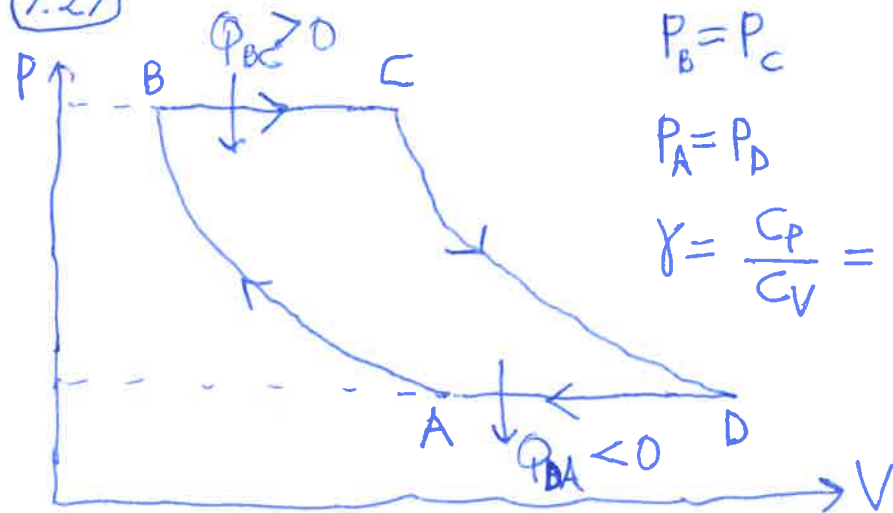
Otra forma de calcular η :

$$\eta = \frac{W}{Q_{\text{abs}}} = \frac{W_{AB} + W_{BC} + W_{CD} + W_{DA}}{Q_{BC}} =$$

$$= \frac{(-1157.2270 + 662.8908 + 1994.9540 + 0) \text{ J}}{2320.1175 \text{ J}} =$$

$$= 0.646785 \approx \boxed{0.647}$$

(7.27)



$$P_B = P_C$$

$$P_B > P_A$$

$$P_A = P_D$$

$$P_B = z P_A \quad z > 1$$

$$\gamma = \frac{C_p}{C_v} = 1.4$$

$$\eta = 1 + \frac{Q_{\text{cedido}}}{Q_{\text{absorbido}}} = 1 + \frac{Q_{DA}}{Q_{BC}}$$

$$Q_{DA} = m C_p (T_A - T_D) < 0$$

$$Q_{BC} = m C_p (T_C - T_B) > 0$$

Primera forma:

$$\eta = 1 + \frac{(T_A - T_D)}{(T_C - T_B)} = 1 + \frac{T_A \left(1 - \frac{T_D}{T_A}\right)}{T_B \left(\frac{T_C}{T_B} - 1\right)} = 1 - \frac{T_A (1 - T_D/T_A)}{T_B (1 - T_C/T_B)}$$

Autor: Yoán Cabrera Álvarez

AB proceso adiabático $\Rightarrow T^\gamma P^{1-\gamma} = \text{cte.} \Rightarrow T_A^\gamma P_A^{1-\gamma} = T_B^\gamma P_B^{1-\gamma} \Rightarrow$
 $\Rightarrow T_A/T_B = (P_B/P_A)^{(1-\gamma)/\gamma} = z^{(1-\gamma)/\gamma} = f \quad f \equiv z^{(1-\gamma)/\gamma}$

CD proceso adiabático $\Rightarrow T_C^\gamma P_C^{1-\gamma} = T_D^\gamma P_D^{1-\gamma} \Rightarrow$
 $\Rightarrow T_C/T_D = (P_D/P_C)^{(1-\gamma)/\gamma} = (P_A/P_B)^{(1-\gamma)/\gamma} = \frac{1}{z^{(1-\gamma)/\gamma}} = \frac{1}{f} \Rightarrow$

$$\Rightarrow f = \frac{T_D}{T_C}$$

temos obtenido dos expresiones de f :

$$\left. \begin{aligned} f &= \frac{T_A}{T_B} \\ f &= \frac{T_D}{T_C} \end{aligned} \right\} \Rightarrow \frac{T_A}{T_B} = \frac{T_D}{T_C} \Rightarrow \frac{T_C}{T_B} = \frac{T_D}{T_A} \Rightarrow$$

$$\left. \begin{aligned} \eta &= 1 - \frac{T_A}{T_B} \frac{(1 - T_D/T_A)}{(1 - T_C/T_B)} \end{aligned} \right\} \Rightarrow$$

$$\Rightarrow \eta = 1 - \frac{T_A}{T_B}$$

$$f = \frac{T_A}{T_B}$$

$$f = z^{(1-\gamma)/\gamma}$$

$$\left. \begin{aligned} \eta &= 1 - \frac{T_A}{T_B} \\ f &= \frac{T_A}{T_B} \\ f &= z^{(1-\gamma)/\gamma} \end{aligned} \right\} \Rightarrow \boxed{\eta = 1 - z^{(1-\gamma)/\gamma} = 1 - \frac{1}{z^{(\gamma-1)/\gamma}}}$$

Segunda forma:

$$\eta = 1 + \frac{(T_A - T_D)}{(T_C - T_B)} = 1 + \frac{(P_A V_A - P_D V_D)}{(P_C V_C - P_B V_B)} = 1 + \frac{P_A (V_A - V_D)}{P_B (V_C - V_B)} =$$

$$= 1 + \frac{1}{z} \frac{(V_A - V_D)}{(V_C - V_B)}$$

Autor: Iván Cabria Álvarez

$$\begin{aligned}
 P_A V_A^\gamma &= P_B V_B^\gamma \Rightarrow V_A = \left(\frac{P_B}{P_A} \right)^{1/\gamma} V_B = 2^{1/\gamma} V_B \\
 P_C V_C^\gamma &= P_D V_D^\gamma \Rightarrow V_D = \left(\frac{P_C}{P_D} \right)^{1/\gamma} V_C = \left(\frac{P_B}{P_A} \right)^{1/\gamma} V_C = 2^{1/\gamma} V_C \\
 \Rightarrow V_A - V_D &= 2^{1/\gamma} (V_B - V_C) = -2^{1/\gamma} (V_C - V_B) \\
 \eta &= 1 + \frac{1}{\gamma} \frac{(V_A - V_D)}{(V_C - V_B)} \\
 \Rightarrow \eta &= 1 - \frac{2^{1/\gamma}}{\gamma} = 1 - 2^{\left(\frac{1}{\gamma} - 1\right)} = 1 - 2^{(1-\gamma)/\gamma} = \\
 &= 1 - \frac{1}{2^{(\gamma-1)/\gamma}}
 \end{aligned}
 \tag{7.36}$$

$$\begin{aligned}
 &\text{7.28} \\
 &\text{Diagram: A P-V diagram showing a thermodynamic cycle with states A, B, C, and D. The process goes from A to B (isothermal expansion), B to C (isobaric expansion), C to D (isobaric compression), and D to A (isothermal compression). Arrows indicate the direction of the cycle: A → B → C → D → A. The volume axis is labeled V and the pressure axis is labeled P.} \\
 &V_B = V_A / 10 \quad V_C = 2V_B = V_A / 5 \quad V_D = V_A \\
 &P_B = P_A \left(\frac{V_A}{V_B} \right)^\gamma = P_A 10^\gamma = 10^5 \text{ Pa } 10^{1.4} = 2511886.4 \text{ Pa} \\
 &P_B = P_C \quad \gamma = \frac{C_p}{C_v} = \frac{7}{5} = 1.4 \\
 &C_p = \frac{7}{2} R \quad C_v = \frac{5}{2} R \\
 &P_D = P_C \left(\frac{V_C}{V_D} \right)^\gamma = P_B \left(\frac{V_A}{5V_A} \right)^\gamma = P_B \left(\frac{1}{5} \right)^\gamma = P_A \frac{10^\gamma}{5^\gamma} = P_A 2^\gamma = \\
 &= 10^5 \text{ Pa } 2^{1.4} = 263901.6 \text{ Pa} \\
 &n = \frac{P_A V_A}{R T_A} = \frac{10^5 \text{ Pa } 0.0249435 \text{ m}^3}{8.3145 \frac{\text{J}}{\text{mol K}} 300 \text{ K}} = 1 \text{ mol}
 \end{aligned}$$

Autor: Iván Gloria Alvarado

$$a) \Delta U = m C_V (T_f - T_i) = m \frac{5}{2} R (T_f - T_i)$$

$$\Delta U_{AB} = m \frac{5}{2} R (T_B - T_A) \quad \text{Calculamos } T_B:$$

$$AB \text{ processo adiabático} \Rightarrow T_B V_B^{\gamma-1} = T_A V_A^{\gamma-1} \Rightarrow T_B = T_A \left(\frac{V_A}{V_B} \right)^{\gamma-1} = T_A 10^{\gamma-1} \Rightarrow$$

$$\Rightarrow T_B = 300 \text{ K } (10)^{0.4} = 753.57 \text{ K}$$

$$\Delta U_{AB} = m \frac{5}{2} R (T_B - T_A) = 1 \text{ mol } 2.5 \cdot 8.3145 \frac{\text{J}}{\text{mol K}} (753.57 - 300) \text{ K} =$$

$$= \boxed{+9428.02 \text{ J}}$$

$$\Delta U_{BC} = m \frac{5}{2} R (T_C - T_B)$$

$$BC \text{ processo isóbarico} \Rightarrow T_C = T_B \frac{V_C}{V_B} \Rightarrow T_C = 2 T_B = 1507.14 \text{ K}$$

$$\Delta U_{BC} = 1 \text{ mol } 2.5 \cdot 8.3145 \frac{\text{J}}{\text{mol K}} (1507.14 - 753.57) \text{ K} = \boxed{+15663.89 \text{ J}}$$

$$\Delta U_{CD} = m \frac{5}{2} R (T_D - T_C)$$

$$CD \text{ processo adiabático} \Rightarrow T_D V_D^{\gamma-1} = T_C V_C^{\gamma-1} \Rightarrow T_D = T_C \left(\frac{V_C}{V_D} \right)^{\gamma-1}$$

$$\Rightarrow T_D = T_C \left(\frac{V_A}{5V_A} \right)^{\gamma-1} = \frac{T_C}{5^{\gamma-1}} = \frac{1507.14 \text{ K}}{5^{0.4}} = 791.71 \text{ K}$$

$$\Delta U_{CD} = 1 \text{ mol } 2.5 \cdot 8.3145 \frac{\text{J}}{\text{mol K}} (791.71 - 1507.14) \text{ K} = \boxed{-14871.11 \text{ J}}$$

$$\Delta U_{DA} = m \frac{5}{2} R (T_A - T_D) = 1 \text{ mol } 2.5 \cdot 8.3145 \frac{\text{J}}{\text{mol K}} (300 - 791.71) \text{ K} =$$

$$= \boxed{-10220.8 \text{ J}}$$

$$\Delta U_{\text{ciclo}} = \Delta U_{AB} + \Delta U_{BC} + \Delta U_{CD} + \Delta U_{DA} = 0, \text{ como esperávamos.}$$

$$b) \boxed{\Delta S_{AB} = 0} \text{ porque AB es um processo adiabático}$$

$$\boxed{\Delta S_{CD} = 0} \text{ porque CD es um processo adiabático}$$

$$\Delta S_{BC} = \int_B^C \frac{\delta Q}{T} = \int_{T_B}^{T_C} \frac{m C_p dT}{T} = m C_p \ln \left(\frac{T_C}{T_B} \right) = m \frac{7}{2} R \ln \left(\frac{T_C}{T_B} \right)$$

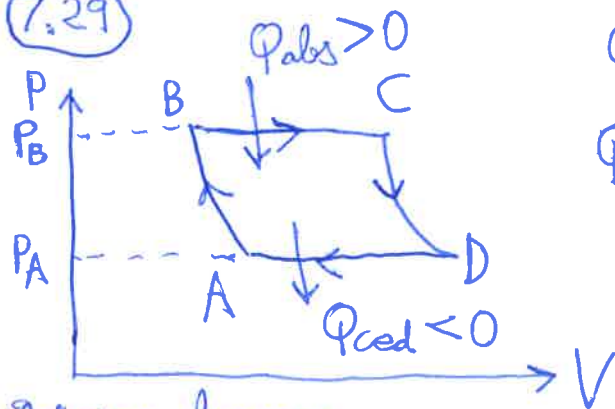
Professor: Iván Cabria Álvarez

$$\Delta S_{BC} = 1 \text{ mol } 3.5 \cdot 8.3145 \frac{\text{J}}{\text{mol K}} \ln \left(\frac{1507.14}{753.57} \right) = \boxed{+20.17 \frac{\text{J}}{\text{K}}} \quad (7.38)$$

$$\begin{aligned} \Delta S_{DA} &= \int_D^A \frac{\delta Q}{T} = \int_{T_D}^{T_A} \frac{m C_V dT}{T} = m C_V \ln \left(\frac{T_A}{T_D} \right) = \\ &= m \frac{5}{2} R \ln \left(\frac{T_A}{T_D} \right) = 1 \text{ mol } 2.5 \cdot 8.3145 \frac{\text{J}}{\text{mol K}} \ln \left(\frac{300}{791.71} \right) = \\ &= \boxed{-20.17 \frac{\text{J}}{\text{K}}} \end{aligned}$$

$$\Delta S_{\text{ciclo}} = \Delta S_{AB} + \Delta S_{BC} + \Delta S_{CD} + \Delta S_{DA} = 0, \text{ como esperábamos.}$$

(7.29)



$$Q_{\text{abs}} = Q_{BC} = m C_P (T_C - T_B) > 0$$

$$Q_{\text{ced}} = Q_{DA} = m C_P (T_A - T_D) < 0$$

$$\eta = 1 + \frac{Q_{\text{ced}}}{Q_{\text{abs}}} = 1 + \frac{T_A - T_D}{T_C - T_B}$$

Primera forma:

$$DA \text{ isóbara} \Rightarrow \frac{T_A}{V_A} = \frac{T_D}{V_D} \Rightarrow T_D = \frac{V_D}{V_A} T_A$$

$$BC \text{ isóbara} \Rightarrow \frac{T_B}{V_B} = \frac{T_C}{V_C} \Rightarrow T_C = \frac{V_C}{V_B} T_B$$

$$\eta = 1 + \frac{T_A (1 - V_D/V_A)}{T_B (V_C/V_B - 1)} = 1 - \frac{T_A}{T_B} \frac{(1 - \frac{V_D}{V_A})}{(1 - \frac{V_C}{V_B})}$$

$$AB \text{ adiabática} \Rightarrow P_A V_A^\gamma = P_B V_B^\gamma \Rightarrow P_B/P_A = V_A^\gamma/V_B^\gamma$$

$$CD \text{ adiabática} \Rightarrow P_C V_C^\gamma = P_D V_D^\gamma \Rightarrow P_C/P_D = V_D^\gamma/V_C^\gamma$$

$$\left. \begin{array}{l} P_C/P_D = V_D^\gamma/V_C^\gamma \\ P_C = P_B \text{ y } P_D = P_A \end{array} \right\} \Rightarrow P_B/P_A = V_D^\gamma/V_C^\gamma$$

Autor: Juan Carlos Álvarez

$$\left. \begin{aligned} P_B/P_A &= V_A^\gamma/V_B^\gamma \\ P_B/P_A &= V_D^\gamma/V_C^\gamma \end{aligned} \right\} \Rightarrow \frac{V_A^\gamma}{V_B^\gamma} = \frac{V_D^\gamma}{V_C^\gamma} \Rightarrow \frac{V_A}{V_B} = \frac{V_D}{V_C} \Rightarrow \frac{V_D}{V_A} = \frac{V_C}{V_B}$$

$$\left. \begin{aligned} \eta &= 1 - \frac{T_A}{T_B} \frac{(1 - V_D/V_A)}{(1 - V_C/V_B)} \\ \frac{V_D}{V_A} &= \frac{V_C}{V_B} \end{aligned} \right\} \Rightarrow \eta = 1 - \frac{T_A}{T_B}$$

$$\eta = 1 - \frac{T_A}{T_B} = 1 - \frac{300 \text{ K}}{365.7 \text{ K}} = \boxed{0.1797}$$

Segunda forma:

$$\begin{aligned} P^{1-\gamma} T^\gamma &= \text{cte.} \Rightarrow P_A^{1-\gamma} T_A^\gamma = P_B^{1-\gamma} T_B^\gamma \Rightarrow \left(\frac{T_A}{T_B} \right)^\gamma = \left(\frac{P_B}{P_A} \right)^{1-\gamma} \\ \Rightarrow \frac{T_A}{T_B} &= \left(\frac{P_B}{P_A} \right)^{\frac{1-\gamma}{\gamma}} \Rightarrow T_A = T_B \left(\frac{P_B}{P_A} \right)^{\frac{1-\gamma}{\gamma}} \Rightarrow T_B = T_A \left(\frac{P_A}{P_B} \right)^{\frac{1-\gamma}{\gamma}} \end{aligned}$$

$$P^{1-\gamma} T^\gamma = \text{cte.} \Rightarrow P_C^{1-\gamma} T_C^\gamma = P_D^{1-\gamma} T_D^\gamma \Rightarrow T_C = T_D \left(\frac{P_D}{P_C} \right)^{\frac{1-\gamma}{\gamma}}$$

$$\left. \begin{aligned} P_C &= P_B \quad \text{y} \quad P_D = P_A \\ T_C &= T_D \left(\frac{P_D}{P_C} \right)^{\frac{1-\gamma}{\gamma}} \end{aligned} \right\} \Rightarrow T_C = T_D \left(\frac{P_A}{P_B} \right)^{\frac{1-\gamma}{\gamma}}$$

$$\left. \begin{aligned} T_B &= T_A \left(\frac{P_A}{P_B} \right)^{\frac{1-\gamma}{\gamma}} \Rightarrow \frac{T_B}{T_A} = \left(\frac{P_A}{P_B} \right)^{\frac{1-\gamma}{\gamma}} \\ T_C &= T_D \left(\frac{P_A}{P_B} \right)^{\frac{1-\gamma}{\gamma}} \Rightarrow \frac{T_C}{T_D} = \left(\frac{P_A}{P_B} \right)^{\frac{1-\gamma}{\gamma}} \end{aligned} \right\} \Rightarrow \frac{T_B}{T_A} = \frac{T_C}{T_D} \Rightarrow$$

$$\begin{aligned} \Rightarrow T_C &= T_D \frac{T_B}{T_A} \Rightarrow T_C - T_B = T_D \frac{T_B}{T_A} - T_B = T_B \left(\frac{T_D}{T_A} - 1 \right) = \\ &= \frac{T_B}{T_A} (T_D - T_A) = - (T_A - T_D) \frac{T_B}{T_A} \end{aligned}$$

Autor: Irén Gabriela Álvarez

7.40

$$\left. \begin{aligned} \eta &= 1 + \frac{T_A - T_D}{T_C - T_B} \\ T_C - T_B &= -(T_A - T_D) \frac{T_B}{T_A} \end{aligned} \right\} \Rightarrow \eta = 1 - \frac{(T_A - T_D)}{(T_A - T_D) \frac{T_B}{T_A}} =$$

$$= 1 - \frac{T_A}{T_B} = 1 - \frac{300 \text{ K}}{365.7 \text{ K}} = \boxed{0.1797}$$

tercera forma:

El rendimiento del ciclo de Brayton es $\eta = 1 - r^{\frac{1-\gamma}{\gamma}}$ con $r = \frac{P_B}{P_A}$

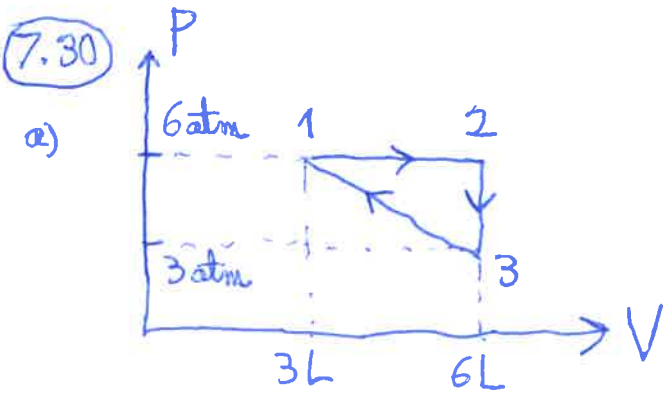
Calculamos r en función de las temperaturas:

$$P^{1-\gamma} T^{\gamma} = \text{cte.} \Rightarrow P_A^{1-\gamma} T_A^{\gamma} = P_B^{1-\gamma} T_B^{\gamma} \Rightarrow \left(\frac{T_A}{T_B} \right)^{\gamma} = \left(\frac{P_B}{P_A} \right)^{1-\gamma}$$

$$\left. \begin{aligned} r &= \frac{P_B}{P_A} \\ \left(\frac{T_A}{T_B} \right)^{\gamma} &= \left(\frac{P_B}{P_A} \right)^{1-\gamma} \end{aligned} \right\} \Rightarrow \left(\frac{T_A}{T_B} \right)^{\gamma} = r^{1-\gamma} \Rightarrow r^{\frac{1-\gamma}{\gamma}} = \frac{T_A}{T_B} \Rightarrow$$

$$\Rightarrow \eta = 1 - r^{\frac{1-\gamma}{\gamma}} = 1 - \frac{T_A}{T_B} = 1 - \frac{300 \text{ K}}{365.7 \text{ K}} = \boxed{0.1797}$$

7.30



$$\begin{aligned} V_2 &= 2V_1 \\ P_3 &= P_1/2 \\ P_1 &= 6 \text{ atm} \\ V_1 &= 3 \text{ L} \end{aligned}$$

b) Calculamos T_1 : $T_1 = \frac{P_1 V_1}{nR} = \frac{6 \text{ atm } 3 \text{ L}}{1 \text{ mol } 0.082 \frac{\text{atm L}}{\text{mol K}}} = 219.5122 \text{ K}$

Calculamos P_2, V_2 y T_2 :

$$V_2 = 2V_1 = 6 \text{ L} = \boxed{0.006 \text{ m}^3}$$

$$P_2 = P_1 = 6 \text{ atm} = \boxed{6.078 \cdot 10^5 \text{ Pa}}$$

Autor: Juan Cabria Alvaro

$$T_2 = \frac{P_2 V_2}{nR} = \frac{P_1 2V_1}{nR} = 2T_1 = \boxed{439.0244 \text{ K}}$$

(7.41)

Calculamos P_3, V_3 y T_3 :

$$V_3 = V_2 = \boxed{0.006 \text{ m}^3} \quad P_3 = P_1/2 = 3 \text{ atm} = \boxed{303900 \text{ Pa}}$$

$$T_3 = \frac{P_3 V_3}{nR} = \frac{(P_1/2) V_2}{nR} = \frac{\frac{P_1}{2} 2V_1}{nR} = \frac{P_1 V_1}{nR} = T_1 = \boxed{219.5122 \text{ K}}$$

c) Proceso $1 \rightarrow 2$: $P = \text{cte.}$ $P_1 = P_2$

$$Q_{12} > 0 \quad Q_{12} = n C_p (T_2 - T_1) \quad \text{Gas diatómico} \Rightarrow C_v = \frac{5}{2} R \text{ y } C_p = \frac{7}{2} R$$

$$Q_{12} = 1 \text{ mol } \frac{7}{2} 8.3145 \frac{\text{J}}{\text{mol K}} 219.5122 \text{ K} = \boxed{+6387.97 \text{ J}}$$

$Q_{12} > 0$, porque es un calor absorbido por el gas.

$$W_{12} = P_1 (V_2 - V_1) = P_1 (2V_1 - V_1) = P_1 V_1 = 6 \text{ atm} \cdot 3 \text{ L} = 18 \text{ atm L} = \\ = 18 \cdot 101300 \text{ Pa} \cdot 0.001 \text{ m}^3 = \boxed{+1823.4 \text{ J}}$$

$W_{12} > 0$, porque es un trabajo realizado por el gas.

Proceso $2 \rightarrow 3$: $V = \text{cte.}$ $V_2 = V_3$

$$Q_{23} = n C_v (T_3 - T_2) = n \frac{5}{2} R (T_3 - T_2) = n \frac{5}{2} R (T_1 - 2T_1) = -n \frac{5}{2} R T_1 = \\ = -1 \text{ mol } \frac{5}{2} 8.3145 \frac{\text{J}}{\text{mol K}} 219.5122 \text{ K} = \boxed{-4562.84 \text{ J}}$$

$Q_{23} < 0$, porque es un calor cedido por el gas.

$W_{23} = 0$, porque es un proceso a volumen constante.

Proceso $3 \rightarrow 1$: $P = aV + b$

Autor: Xosé Cabris Álvarez

Calculamos a y b :

V	P	
6L	3atm	$3 = 6a + b$
3L	6atm	$6 = 3a + b$

$$\left. \begin{array}{l} 3 = 6a + b \\ 6 = 3a + b \end{array} \right\} \Rightarrow \left. \begin{array}{l} -3 = -6a - b \\ 6 = 3a + b \end{array} \right\} \Rightarrow$$

$$\Rightarrow -3 + 6 = -3a \Rightarrow 3 = -3a \Rightarrow a = -1 \frac{\text{atm}}{\text{L}}$$

$$\left. \begin{aligned} 3 &= 6a + b \Rightarrow b = 3 - 6a \\ a &= -1 \frac{\text{atm}}{\text{L}} \end{aligned} \right\} \Rightarrow b = 3 + 6 \text{ atm} = 9 \text{ atm}$$

$$a = -1 \frac{\text{atm}}{\text{L}} \quad b = 9 \text{ atm} \quad P = 9 - V \text{ atm, L}$$

$$\left. \begin{aligned} \Delta U_{31} &= m C_V (T_1 - T_3) = m \frac{5}{2} R (T_1 - T_1) = 0 \\ \Delta U_{31} &= Q_{31} - W_{31} \end{aligned} \right\} \Rightarrow Q_{31} = W_{31}$$

Calculamos W_{31} :

$$\begin{aligned} W_{31} &= \int_3^1 P dV = \int_3^1 (aV + b) dV = a \frac{V^2}{2} + bV \Big|_3^1 = \frac{a}{2} (V_1^2 - V_3^2) \\ &+ b(V_1 - V_3) = -\frac{1}{2} \frac{\text{atm}}{\text{L}} ((3\text{L})^2 - (6\text{L})^2) + 9 \text{ atm} (3\text{L} - 6\text{L}) = \\ &= -\frac{1}{2} (9 - 36) \text{ atm L} - 27 \text{ atm L} = \left(\frac{27}{2} - 27 \right) \text{ atm L} = -\frac{27}{2} \text{ atm L} = \\ &= -\frac{27}{2} 101300 \text{ Pa } 10^{-3} \text{ m}^3 = \boxed{-1367.55 \text{ J}} \end{aligned}$$

$$Q_{31} = W_{31} = \boxed{-1367.55 \text{ J}}$$

$Q_{31} < 0$, porque es un calor cedido por el gas.

$W_{31} < 0$, porque es un trabajo realizado sobre el gas.

$$\begin{aligned} d) \Delta U_{12} &= m C_V (T_2 - T_1) = m \frac{5}{2} R (T_2 - T_1) = m \frac{5}{2} R (2T_1 - T_1) = \\ &= m \frac{5}{2} R T_1 = 1 \text{ mol } \frac{5}{2} 8.3145 \frac{\text{J}}{\text{mol K}} 219.5122 \text{ K} = \boxed{+4562.84 \text{ J}} \end{aligned}$$

$$\begin{aligned} \Delta U_{23} &= m C_V (T_3 - T_2) = m \frac{5}{2} R (T_1 - 2T_1) = -m \frac{5}{2} R T_1 = -\Delta U_{12} = \\ &= \boxed{-4562.84 \text{ J}} \end{aligned}$$

tutor: Yohan Cabria Alvarado

$$\Delta U_{31} = m C_V (T_1 - T_3) = m \frac{5}{2} R (T_1 - T_1) = \boxed{0}$$

$$\Delta U_{\text{ciclo}} = \Delta U_{12} + \Delta U_{23} + \Delta U_{31} = \boxed{0}$$

La energía interna es una función de estado $\Rightarrow \Delta U_{\text{ciclo reversible}} = 0$, tal y como hemos obtenido.

Calculamos el cambio de la entropía:

(7.43)

$$\Delta S_{12} = \int_1^2 n C_p \frac{dT}{T} = n C_p \ln \left(\frac{T_2}{T_1} \right) = n \frac{7}{2} R \ln \left(\frac{2T_1}{T_1} \right) = n \frac{7}{2} R \ln 2 =$$

$$= 1 \text{ mol} \cdot \frac{7}{2} \cdot 8.3145 \frac{\text{J}}{\text{mol K}} \ln 2 = \boxed{+ 20.17 \frac{\text{J}}{\text{K}}}$$

$$\Delta S_{23} = \int_2^3 n C_v \frac{dT}{T} = n C_v \ln \left(\frac{T_3}{T_2} \right) = n C_v \ln \left(\frac{T_1}{2T_1} \right) =$$

$$= n \frac{5}{2} R \ln \left(\frac{1}{2} \right) = - n \frac{5}{2} R \ln 2 = - 1 \text{ mol} \cdot \frac{5}{2} \cdot 8.3145 \frac{\text{J}}{\text{mol K}} \ln 2 =$$

$$= \boxed{- 14.41 \frac{\text{J}}{\text{K}}}$$

$$\Delta S_{31} = \int_3^1 n C_v \frac{dT}{T} + \int_3^1 n R \frac{dV}{V} = n C_v \ln \left(\frac{T_1}{T_3} \right) + n R \ln \left(\frac{V_1}{V_3} \right) =$$

$$= n \frac{5}{2} R \ln \left(\frac{T_1}{T_1} \right) + n R \ln \left(\frac{V_1}{2V_1} \right) = 0 - n R \ln 2 =$$

$$= - 1 \text{ mol} \cdot 8.3145 \frac{\text{J}}{\text{mol K}} \ln 2 = \boxed{- 5.76 \frac{\text{J}}{\text{K}}}$$

$$\Delta S_{\text{ciclo}} = \Delta S_{12} + \Delta S_{23} + \Delta S_{31} = (+20.17 - 14.41 - 5.76) \frac{\text{J}}{\text{K}} = \boxed{0}$$

La entropía es una función de estado $\Rightarrow \Delta S_{\text{ciclo}} = 0$, tal y como hemos obtenido.

$$e) Q_{\text{absorbido}} = Q_{12} > 0 \quad Q_{\text{cedido}} = Q_{23} + Q_{31} < 0$$

$$\eta = 1 + \frac{Q_{\text{cedido}}}{Q_{\text{absorbido}}} = 1 + \frac{(-4562.84 - 1367.55) \text{ J}}{6387.97 \text{ J}} = \boxed{0.0716}$$

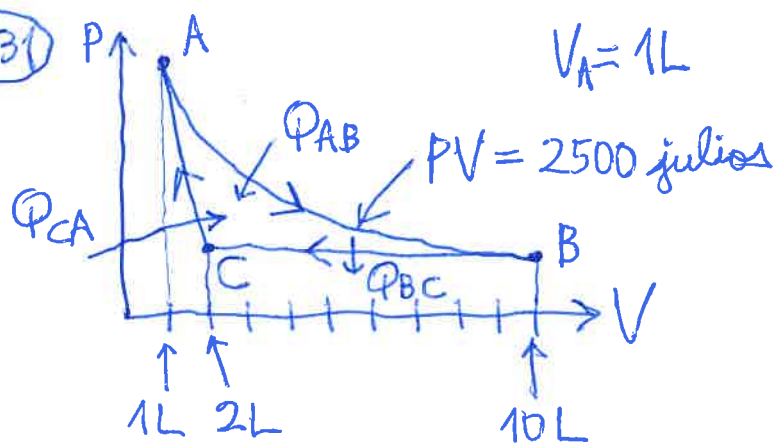
Otra manera de calcular el rendimiento:

$$\eta = \frac{W_{\text{ciclo}}}{Q_{\text{absorbido}}} = \frac{W_{12} + W_{23} + W_{31}}{Q_{\text{absorbido}}} = \frac{(1823.4 + 0 - 1367.55) \text{ J}}{6387.97 \text{ J}} =$$

$$= \boxed{0.07136}$$

Autor: Iván Cabria Álvarez

(7.3) $V_A = 1L$ $V_B = 10L$ $V_C = 2L$ $P_B = P_C$ (7.44)



$$\eta = \frac{W_{\text{ciclo}}}{Q_{\text{abs ciclo}}}$$

$$W_{\text{ciclo}} = W_{AB} + W_{BC} + W_{CA}$$

$$Q_{AB} > 0 \quad Q_{BC} < 0 \quad Q_{CA} > 0$$

$$Q_{\text{abs ciclo}} = Q_{AB} + Q_{CA}$$

AB es un proceso que cumple $PV = \text{cte.} \Rightarrow$ AB es una expansión isoterma.

$$PV = 2500 \text{ julios} \Rightarrow P_A V_A = P_B V_B = 2500 \text{ julios}$$

$Q_{AB} > 0$ porque se trata de una expansión isoterma

$Q_{BC} < 0$ porque el proceso BC es una compresión isobara
vamos a comprobar que Q_{BC} es negativo:

$$\left. \begin{aligned} Q_{BC} &= m C_p (T_C - T_B) \\ PV &= nRT \Rightarrow nT = \frac{PV}{R} \end{aligned} \right\} \Rightarrow Q_{BC} = \frac{C_p}{R} (P_C V_C - P_B V_B)$$

BC es un proceso isobaro $\Rightarrow P_C = P_B$

$$\Rightarrow Q_{BC} = \frac{C_p}{R} (P_B V_C - P_B V_B) = \frac{C_p P_B}{R} (V_C - V_B) \Rightarrow Q_{BC} < 0$$

$V_C < V_B$

Calculamos W_{ciclo} :

$$W_{AB} = n R T_A \ln\left(\frac{V_B}{V_A}\right) = P_A V_A \ln\left(\frac{V_B}{V_A}\right) = 2500 \text{ julios} \ln\left(\frac{10}{1}\right) =$$

$$= 5756.46 \text{ julios}$$

$$W_{BC} = P_B (V_C - V_B)$$

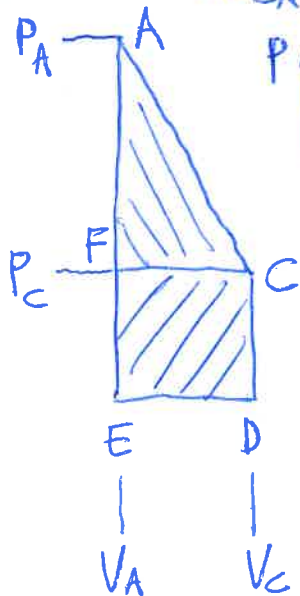
Calculamos P_B : $P_A V_A = P_B V_B = 2500 \text{ julios} \Rightarrow P_B = \frac{2500 \text{ julios}}{V_B} =$

$$= \frac{2500 \text{ julios}}{10^{-2} \text{ m}^3} = 2.5 \cdot 10^5 \text{ Pa}$$

Autor: Iván Cabria Álvaro

$$W_{BC} = P_B (V_C - V_B) = 2.5 \cdot 10^5 \text{ Pa} (2 - 1) \cdot 10^{-3} \text{ m}^3 = -2000 \text{ julios} \quad (7.45)$$

Calculamos W_{CA} : $W_{CA} = - \text{Área ACDE}$



$$\text{Área ACDE} = \text{Área triángulo ACF} +$$

$$\text{Área rectángulo CDEF} =$$

$$= (V_C - V_A) \frac{(P_A - P_C)}{2} + (V_C - V_A) P_C$$

$$P_C = P_B \Rightarrow P_C = 2.5 \cdot 10^5 \text{ Pa}$$

Calculamos P_A : $P_A V_A = 2500 \text{ julios} \Rightarrow$

$$\Rightarrow P_A = \frac{2500 \text{ julios}}{V_A} = \frac{2500 \text{ julios}}{10^{-3} \text{ m}^3} = 25 \cdot 10^5 \text{ Pa}$$

$$\text{Área ACDE} = (2 - 1) \cdot 10^{-3} \text{ m}^3 \frac{(25 + 2.5) \cdot 10^5 \text{ Pa}}{2} + (2 - 1) \cdot 10^{-3} \text{ m}^3 2.5 \cdot 10^5 \text{ Pa} =$$

$$= 1125 \text{ julios} + 250 \text{ julios} = 1375 \text{ julios} \Rightarrow W_{CA} = -1375 \text{ julios}$$

¶ Hemos calculado los tres trabajos que componen W_{ciclo} . Calculamos W_{ciclo} :

$$W_{\text{ciclo}} = W_{AB} + W_{BC} + W_{CA} = 5756.46 - 2000 - 1375 \text{ J} =$$

$$= +2381.46 \text{ J}$$

Calculamos $Q_{\text{abs ciclo}}$: $Q_{\text{abs ciclo}} = Q_{AB} + Q_{CA}$

AB es un proceso isoterma $\Rightarrow Q_{AB} = W_{AB} \Rightarrow Q_{AB} = 5756.46 \text{ J}$

Calculamos Q_{CA} : $Q_{CA} = \Delta U_{CA} + W_{CA}$

Calculamos ΔU_{CA} :

$$\Delta U_{\text{ciclo}} = \Delta U_{AB} + \Delta U_{BC} + \Delta U_{CA}$$

$$\Delta U_{\text{ciclo}} = 0$$

AB es un proceso isoterma $\Rightarrow \Delta U_{AB} = 0$

$$\left. \begin{aligned} \Rightarrow \Delta U_{CA} &= -\Delta U_{BC} \\ \Delta U_{BC} &= m C_V (T_C - T_B) \end{aligned} \right\} \Rightarrow$$

$$\Rightarrow \Delta U_{CA} = m C_V (T_B - T_C)$$

Calculamos T_B en función de otras magnitudes:

$$\left. \begin{aligned} P_B V_B &= 2500 \text{ julios} \\ P_B V_B &= n R T_B \end{aligned} \right\} \Rightarrow T_B = \frac{2500 \text{ julios}}{n R}$$

Autor: Iván Cabrera Álvarez

Calculamos T_C en función de otras magnitudes:

$$P_C V_C = n R T_C$$

$$P_C = P_B$$

$$V_C = 2L$$

$$V_B = 10L \Rightarrow V_C = \frac{V_B}{5} \Rightarrow P_C V_C = P_C \frac{V_B}{5}$$

$$\left. \begin{array}{l} P_C V_C = P_C \frac{V_B}{5} \\ P_C V_C = \frac{P_B V_B}{5} \end{array} \right\} \Rightarrow P_C V_C = \frac{P_B V_B}{5} \Rightarrow$$

$$\Rightarrow \frac{P_B V_B}{5} = n R T_C$$

$$P_B V_B = 2500 \text{ julios}$$

$$\frac{P_B V_B}{5} = n R T_C$$

$$\left. \begin{array}{l} P_B V_B = 2500 \text{ julios} \\ \frac{P_B V_B}{5} = n R T_C \end{array} \right\} \Rightarrow 500 \text{ julios} = n R T_C \Rightarrow T_C = \frac{500 \text{ julios}}{n R}$$

Calculamos ΔU_{CA} :

$$\Delta U_{CA} = n C_V (T_B - T_C) = n C_V \left(\frac{2500}{n R} - \frac{500}{n R} \right) \text{ julios} = \frac{C_V}{R} 2000 \text{ julios} = \frac{3}{2} 2000 \text{ julios} = 3000 \text{ julios}$$

Calculamos Q_{CA} : $Q_{CA} = \Delta U_{CA} + W_{CA} = 3000 - 1375 \text{ julios} = +1625 \text{ julios}$

$Q_{\text{absorido}} = Q_{AB} + Q_{CA} = 5756.46 + 1625 \text{ julios} = 7381.46 \text{ julios}$

$$\eta = \frac{W_{\text{neto}}}{Q_{\text{absorido}}} = \frac{2381.46 \text{ J}}{7381.46 \text{ J}} = \boxed{0.323}$$

Otra manera de calcular el rendimiento η :

$$\eta = 1 + \frac{Q_{\text{cedido cdo}}}{Q_{\text{absorido}}}$$

$$Q_{\text{cedido cdo}} = Q_{BC}$$

$$Q_{BC} = n C_P (T_C - T_B) = \frac{C_P}{R} (P_C V_C - P_B V_B) = \frac{C_P P_B}{R} (V_C - V_B) = \frac{5}{2} P_B (V_C - V_B) = \frac{5}{2} 2.5 \cdot 10^5 \text{ Pa} (2 - 10) 10^{-3} \text{ m}^3 = -5000 \text{ J}$$

$$\eta = 1 + \frac{(-5000 \text{ J})}{7381.46 \text{ J}} = \boxed{0.323}$$

Otra forma de calcular W_{CA} :

(7.47)

$$W_{CA} = \int_C^A P dV = \int_C^A P(V) dV = \int_{V_C}^{V_A} (aV + b) dV = a \frac{V^2}{2} \Big|_{V_C}^{V_A} + bV \Big|_{V_C}^{V_A} =$$

$$= \frac{a}{2} (V_A^2 - V_C^2) + b(V_A - V_C)$$

Calculamos a y b en unidades del SI:

$$\left. \begin{array}{l} P_A = aV_A + b \\ P_C = aV_C + b \end{array} \right\} \Rightarrow P_A - P_C = a(V_A - V_C) \Rightarrow a = \frac{P_A - P_C}{V_A - V_C} =$$

$$= \frac{(25 \cdot 10^5 - 2.5 \cdot 10^5) \text{ Pa}}{(10^{-3} - 2 \cdot 10^{-3}) \text{ m}^3} = -2.25 \cdot 10^9 \frac{\text{Pa}}{\text{m}^3}$$

$$P_A = aV_A + b \Rightarrow b = P_A - aV_A = (25 \cdot 10^5 + 2.25 \cdot 10^9 \cdot 10^{-3}) \text{ Pa} =$$

$$= (2.5 \cdot 10^5 + 2.25 \cdot 10^6) \text{ Pa} = 4.75 \cdot 10^6 \text{ Pa}$$

$$W_{CA} = -\frac{2.25 \cdot 10^9}{2} \frac{\text{Pa}}{\text{m}^3} (10^{-6} - 4 \cdot 10^{-6}) \text{ m}^6 +$$

$$+ 4.75 \cdot 10^6 \text{ Pa} (10^{-3} - 2 \cdot 10^{-3}) \text{ m}^3 = \frac{6750}{2} \text{ julios} - 4750 \text{ julios} =$$

$$= -1375 \text{ julios} \quad 1 \text{ julio} = 1 \text{ Pa m}^3$$

(7.32) $\Delta S = \sum_{i=1}^7 \Delta S_i$ $m_h = 2000 \text{ g}$ $m_a = 100 \text{ g}$ $m_v = 50 \text{ g}$

$$\Delta S_1 = m_h C_h \ln \left(\frac{T_{\text{final en K}}}{T_h \text{ en K}} \right) = 2000 \text{ g} \cdot 0.5 \frac{\text{cal}}{\text{g K}} \ln \left(\frac{273.15 - 4.21}{273.15 - 50} \right) =$$

$$= +186.64 \text{ cal/K}$$

$$\Delta S_2 = m_v C_{\text{vapor}} \ln \left(\frac{T_{100 \text{ en K}}}{T_{120 \text{ en K}}} \right) = 50 \text{ g} \cdot 0.47 \frac{\text{cal}}{\text{g K}} \ln \left(\frac{373.15}{393.15} \right) = -1.23 \frac{\text{cal}}{\text{K}}$$

$$\Delta S_3 = -\frac{m_v L_v}{T_{100 \text{ en K}}} = -\frac{50 \text{ g} \cdot 540 \text{ cal/g}}{373.15 \text{ K}} = -72.36 \text{ cal/K}$$

Autor: Iván Cabris Alvarro

$$\Delta S_4 = m_v C_{\text{agua}} \ln \left(\frac{T_0 \text{ en K}}{T_{100} \text{ en K}} \right) = 50 \text{ g } 1 \frac{\text{cal}}{\text{g K}} \ln \left(\frac{273.15}{373.15} \right) = \textcircled{-7.48}$$

$$= -15.60 \text{ cal/K}$$

$$\Delta S_5 = m_a C_{\text{agua}} \ln \left(\frac{T_0 \text{ en K}}{T_{10} \text{ en K}} \right) = 100 \text{ g } 1 \frac{\text{cal}}{\text{g K}} \ln \left(\frac{273.15}{283.15} \right) =$$

$$= -3.60 \text{ cal/K}$$

$$\Delta S_6 = - \frac{(m_a + m_v) L_f}{T_0 \text{ en K}} = - \frac{12000 \text{ cal}}{273.15 \text{ K}} = -43.93 \text{ cal/K}$$

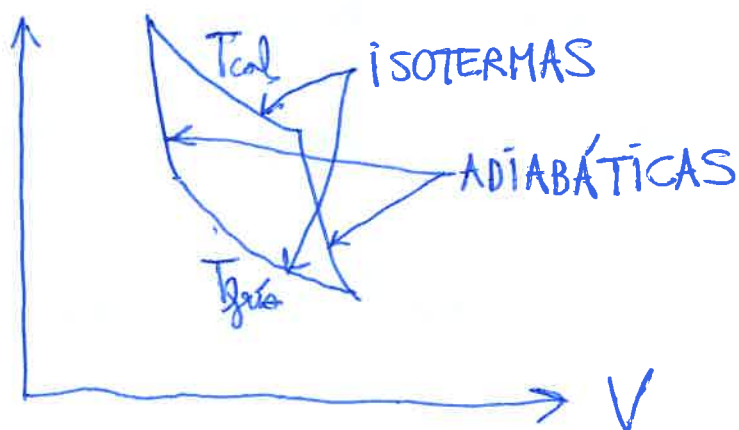
$$\Delta S_7 = (m_a + m_v) C_h \ln \left(\frac{T_{\text{final}} \text{ en K}}{T_0 \text{ en K}} \right) = 150 \text{ g } 0.5 \frac{\text{cal}}{\text{g K}} \ln \left(\frac{273.15 - 4.2}{273.15} \right)$$

$$= -1.16 \text{ cal/K}$$

$$\Delta S = +186.64 - 1.23 - 72.36 - 15.60 - 3.60 - 43.93 - 1.16 \text{ cal/K}$$

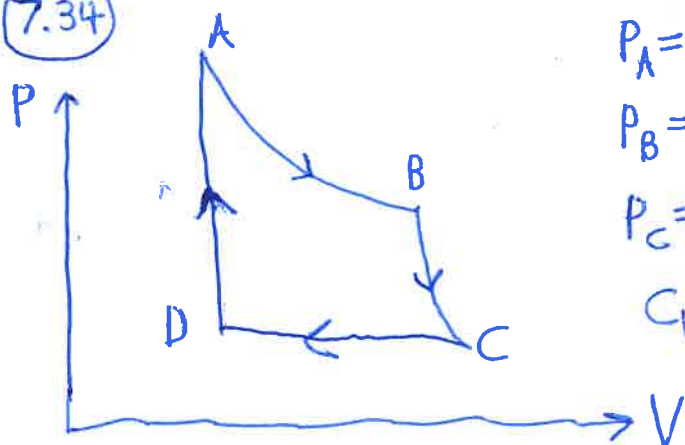
$$= \boxed{+48.76 \text{ cal/K}}$$

$$\textcircled{7.33} \quad \eta_c = 1 - \frac{T_{\text{gas}}}{T_{\text{cal}}} = 1 - \frac{6.1}{24.4} = 0.75$$



7.34

7.49



$$P_A = 200\,000 \text{ Pa}$$

$$P_B = 100\,000 \text{ Pa}$$

$$P_C = 50\,000 \text{ Pa} = P_D$$

$$C_V = \frac{3}{2}R \Rightarrow C_P = \frac{5}{2}R \Rightarrow \gamma = \frac{C_P}{C_V} = \frac{5}{3}$$

$$P_A V_A = P_B V_B = 3000 \text{ J}$$

$$V_A = V_D$$

$$P_B V_B^\gamma = P_C V_C^\gamma$$

$$\eta = \frac{W_{\text{ciclo}}}{Q_{\text{absorbido}}}$$

$$\text{Calculamos } W_{\text{ciclo}}: W_{\text{ciclo}} = W_{AB} + W_{BC} + W_{CD} + W_{DA}$$

En el proceso AB se cumple $PV = \text{cte} \Rightarrow AB$ es isoterma $\Rightarrow$
 $\Rightarrow W_{AB} = m R T_A \ln\left(\frac{V_B}{V_A}\right) = P_A V_A \ln\left(\frac{V_B}{V_A}\right) = 3000 \text{ J} \ln\left(\frac{V_B}{V_A}\right)$

$$P_A V_A = P_B V_B \Rightarrow \frac{V_B}{V_A} = \frac{P_A}{P_B} \Rightarrow \frac{V_B}{V_A} = \frac{200\,000}{100\,000} = 2$$

$$W_{AB} = 3000 \text{ J} \ln\left(\frac{V_B}{V_A}\right) = 3000 \text{ J} \ln 2 = 2079.4415 \text{ J}$$

El proceso BC es adiabático $\Rightarrow W_{BC} = \frac{P_C V_C - P_B V_B}{1 - \gamma}$

Calculamos V_C : $P_B V_B^\gamma = P_C V_C^\gamma \Rightarrow \left(\frac{V_C}{V_B}\right)^\gamma = \frac{P_B}{P_C} \Rightarrow \frac{V_C}{V_B} = \left(\frac{P_B}{P_C}\right)^{1/\gamma} \Rightarrow$

$$\Rightarrow V_C = V_B \left(\frac{P_B}{P_C}\right)^{1/\gamma} = V_B 2^{3/5}$$

Calculamos V_B : $P_B V_B = 3000 \text{ J} \Rightarrow V_B = \frac{3000 \text{ J}}{P_B} = \frac{3000 \text{ J}}{100\,000 \text{ Pa}} = 0.03 \text{ m}^3$

$$V_C = V_B 2^{3/5} = 0.03 \cdot 2^{3/5} \text{ m}^3 = 0.0454715 \text{ m}^3$$

$$W_{BC} = \frac{P_C V_C - P_B V_B}{1 - \gamma} = \frac{50\,000 \text{ Pa} \cdot 0.03 \cdot 2^{3/5} \text{ m}^3 - 3000 \text{ J}}{1 - \frac{5}{3}}$$

Autor: Iván Cabris Álvarez

$$W_{BC} = -\frac{3}{2} (5 \cdot 10^4 \cdot 0.03 \cdot 2^{3/5} - 3000) \text{ J} = +1089.6377 \text{ J} \quad (7.50)$$

$$W_{CD} = P_D (V_D - V_C) = P_D (V_A - V_C)$$

$$\text{Calculamos } V_A: P_A V_A = 3000 \text{ J} \Rightarrow V_A = \frac{3000 \text{ J}}{P_A} = \frac{3000 \text{ J}}{2 \cdot 10^5 \text{ Pa}} = 0.015 \text{ m}^3$$

$$W_{CD} = P_D (V_A - V_C) = 5 \cdot 10^4 \text{ Pa} (0.015 - 0.03 \cdot 2^{3/5}) \text{ m}^3 = -1523.5748 \text{ J}$$

$$W_{DA} = 0 \text{ porque el proceso DA es isócora}$$

$$W_{\text{ciclo}} = W_{AB} + W_{BC} + W_{CD} + W_{DA} = 2079.4415 + 1089.6377 - 1523.5748 + 0 = +1645.5044 \text{ J}$$

Calculamos el calor absorbido en el ciclo, $Q_{\text{absorbido}}$. Solo los valores mayores que cero son parte de $Q_{\text{absorbido}}$.

$$\text{El proceso AB es isoterma} \Rightarrow Q_{AB} = W_{AB} = 2079.4415 \text{ J} > 0$$

$$\text{El proceso BC es adiabático} \Rightarrow Q_{BC} = 0$$

$$\text{El proceso CD es una compresión isobara} \Rightarrow Q_{CD} < 0$$

Demostremos que $Q_{CD} < 0$:

$$Q_{CD} = m C_p (T_D - T_C) = \frac{C_p}{R} (P_D V_D - P_C V_C) = \frac{C_p}{R} P_D (V_D - V_C) \Bigg\} \Rightarrow$$

$$V_D < V_C \Rightarrow V_D - V_C < 0$$

$$\Rightarrow Q_{CD} < 0 \quad Q_{CD} \text{ no es parte de } Q_{\text{absorbido}}$$

$$\begin{aligned} \text{El proceso DA es isócora} \Rightarrow Q_{DA} &= m C_v (T_A - T_D) = \frac{C_v}{R} (P_A V_A - P_D V_D) = \\ &= \frac{C_v}{R} V_A (P_A - P_D) = \frac{3}{2} V_A (P_A - P_D) = \frac{3}{2} V_A (P_A - \frac{P_A}{4}) = \frac{3}{2} P_A V_A (1 - \frac{1}{4}) = \\ &= \frac{9}{8} P_A V_A = \frac{9}{8} 3000 \text{ J} = 3375 \text{ J} > 0 \end{aligned}$$

$$Q_{\text{absorbido}} = Q_{AB} + Q_{DA} = 2079.4415 + 3375 \text{ J} = 5454.4415 \text{ J}$$

$$\eta = \frac{W_{\text{ciclo}}}{Q_{\text{absorbido}}} = \frac{1645.5044}{5454.4415} = \boxed{0.3017}$$

Autor: Iván Cabria Alvares

(7.35) $m_h = 23000 \text{ g}$ $m_v = 80 \text{ g}$ $T_0 = 273.15 \text{ K}$ $T_h = 268.15 \text{ K}$ (7.51)
 $m_l = 69.35 \text{ g}$ $T_{100} = 373.15 \text{ K}$ $T_v = 393.15 \text{ K}$

Proceso 1: Los 23000 g de hielo se calientan desde -5°C hasta 0°C

Proceso 2: Los 80 g de vapor de agua se enfrían desde $+120^\circ\text{C}$ hasta $+100^\circ\text{C}$

Proceso 3: Los 80 g de vapor de agua se condensan a $+100^\circ\text{C}$

Proceso 4: Los 80 g de agua, que inicialmente eran vapor de agua, se enfrían desde $+100^\circ\text{C}$ hasta 0°C

Proceso 5: 69.35 g de agua se convierten en hielo

$$\Delta S_1 = m_h C_{\text{hielo}} \ln \left(\frac{T_0 \text{ en K}}{T_h \text{ en K}} \right) = 23000 \text{ g} \cdot 0.5 \frac{\text{cal}}{\text{g K}} \ln \left(\frac{273.15}{268.15} \right) =$$

$$= 212.46 \frac{\text{cal}}{\text{K}}$$

$$\Delta S_2 = m_v C_{\text{agua}} \ln \left(\frac{T_{100} \text{ en K}}{T_v \text{ en K}} \right) = 80 \text{ g} \cdot 0.47 \frac{\text{cal}}{\text{g K}} \ln \left(\frac{373.15}{393.15} \right) =$$

$$= -1.96 \frac{\text{cal}}{\text{K}}$$

$$\Delta S_3 = - \frac{m_v L_v}{T_{100} \text{ en K}} = - \frac{80 \text{ g} \cdot 540 \text{ cal/g}}{373.15 \text{ K}} = -115.77 \frac{\text{cal}}{\text{K}}$$

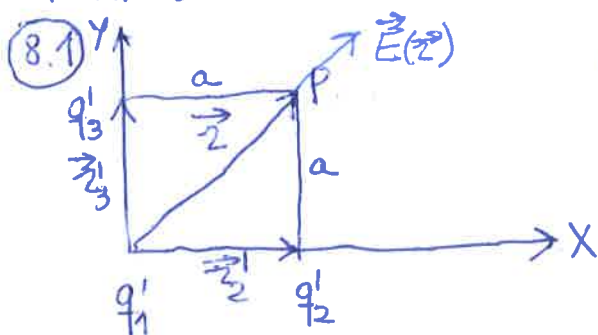
$$\Delta S_4 = m_v C_{\text{agua}} \ln \left(\frac{T_0 \text{ en K}}{T_{100} \text{ en K}} \right) = 80 \text{ g} \cdot 1 \frac{\text{cal}}{\text{g K}} \ln \left(\frac{273.15}{373.15} \right) =$$

$$= -24.96 \text{ cal/K}$$

$$\Delta S_5 = - \frac{m L_f}{T_0 \text{ en K}} = - \frac{69.35 \text{ g} \cdot 80 \text{ cal/g}}{273.15 \text{ K}} = -20.31 \text{ cal/K}$$

$$\Delta S = \sum_{i=1}^5 \Delta S_i = 212.46 - 1.96 - 115.77 - 24.96 - 20.31 \text{ cal/K} =$$

$$= \boxed{+49.46 \frac{\text{cal}}{\text{K}}}$$



$$a = 5 \text{ cm} = 0.05 \text{ m}$$

$$q = 3 \text{ nC} = 3 \cdot 10^{-9} \text{ C}$$

$$\vec{r}_1 = \vec{0}$$

$$q'_1 = -q$$

$$\vec{r}_2 = a \vec{u}_x$$

$$q'_2 = +q$$

$$\vec{r}_3 = a \vec{u}_y$$

$$q'_3 = +q$$

$$\vec{r} = a \vec{u}_x + a \vec{u}_y$$

$$\vec{r} - \vec{r}_1 = a \vec{u}_x + a \vec{u}_y$$

$$\vec{r} - \vec{r}_2 = a \vec{u}_y$$

$$\vec{r} - \vec{r}_3 = a \vec{u}_x$$

$$|\vec{r} - \vec{r}_1| = \sqrt{2} a$$

$$|\vec{r} - \vec{r}_2| = a$$

$$|\vec{r} - \vec{r}_3| = a$$

$$\begin{aligned} \text{a) } \vec{E}(\vec{r}) &= \sum_{i=1}^3 \frac{q'_i (\vec{r} - \vec{r}_i)}{4\pi\epsilon_0 |\vec{r} - \vec{r}_i|^3} = -\frac{q}{4\pi\epsilon_0} \frac{(a\vec{u}_x + a\vec{u}_y)}{2\sqrt{2} a^3} + \frac{q}{4\pi\epsilon_0} \frac{a\vec{u}_y}{a^3} \\ &+ \frac{q}{4\pi\epsilon_0} \frac{a\vec{u}_x}{a^3} = \frac{q}{4\pi\epsilon_0 a^2} \left[-\frac{(\vec{u}_x + \vec{u}_y)}{2\sqrt{2}} + \vec{u}_y + \vec{u}_x \right] = \\ &= \frac{q}{4\pi\epsilon_0 a^2} \left(1 - \frac{1}{2\sqrt{2}} \right) (\vec{u}_x + \vec{u}_y) = \frac{3 \cdot 10^{-9} \text{ C} (1 - 2^{-3/2}) (\vec{u}_x + \vec{u}_y)}{4\pi \cdot 8.854 \cdot 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2} (0.05 \text{ m})^2} \\ &= \boxed{6.97 \cdot 10^3 (\vec{u}_x + \vec{u}_y) \text{ N/C}} \end{aligned}$$

$$\begin{aligned} \text{b) } \phi(\vec{r}) &= \sum_{i=1}^3 \frac{q'_i}{4\pi\epsilon_0 |\vec{r} - \vec{r}_i|} = -\frac{q}{4\pi\epsilon_0 \sqrt{2} a} + \frac{q}{4\pi\epsilon_0 a} + \frac{q}{4\pi\epsilon_0 a} \\ &= \frac{q}{4\pi\epsilon_0 a} \left(-\frac{1}{\sqrt{2}} + 1 + 1 \right) = \frac{q}{4\pi\epsilon_0 a} \left(2 - \frac{1}{\sqrt{2}} \right) = \frac{3 \cdot 10^{-9} \text{ C} (2 - 1/\sqrt{2})}{4\pi \cdot 8.854 \cdot 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2} \cdot 0.05 \text{ m}} \\ &= 697.2 \frac{\text{N}}{\text{C}} \text{ m} = \boxed{697.2 \text{ V}} \end{aligned}$$

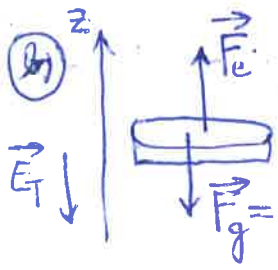
$$\begin{aligned} \text{c) } \vec{F}(\vec{r}) &= q_p \vec{E}(\vec{r}) = q \vec{E}(\vec{r}) = 3 \cdot 10^{-9} \text{ C} \cdot 6.97 \cdot 10^3 (\vec{u}_x + \vec{u}_y) \text{ N/C} \\ &= \boxed{2.09 \cdot 10^{-5} (\vec{u}_x + \vec{u}_y) \text{ N}} \end{aligned}$$

8.2(a) $\vec{F}_g = m\vec{g}$ $\vec{F}_g$ $\vec{F}_e$ $\vec{E}_T$ $E_T = 150 \text{ N/C}$

$$F_e = |q_e \vec{E}_T| = e E_T = 1.66 \cdot 10^{-19} \text{ C} \cdot 150 \text{ N/C} = 2.49 \cdot 10^{-17} \text{ N}$$

$$F_g = mg = 9.1 \cdot 10^{-31} \text{ kg} \cdot 9.8 \text{ m/s}^2 = 8.92 \cdot 10^{-30} \text{ N}$$

$$\frac{F_e}{F_g} \simeq 2.79 \cdot 10^{12}$$



$$\vec{F}_g = -mg \vec{u}_z \quad \vec{F}_e = q \vec{E}_T = \begin{cases} -|q|E_T \vec{u}_z & q > 0 \\ +|q|E_T \vec{u}_z & q < 0 \end{cases} = -q E_T \vec{u}_z \quad q \neq 0$$

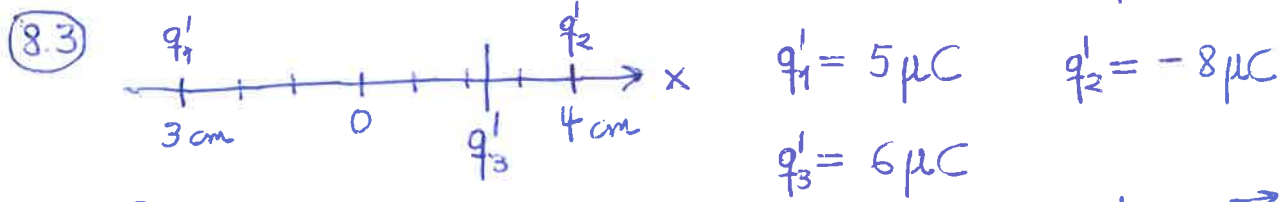
$$\vec{E}_T = -E_T \vec{u}_z$$

$$\vec{F}_e + \vec{F}_g = \vec{0} \Rightarrow \vec{F}_e \text{ debe apuntar en el sentido } z > 0 \Rightarrow \vec{F}_e = +|q|E_T \vec{u}_z \quad q < 0$$

$$\Rightarrow +|q|E_T - mg = 0 \Rightarrow |q| = mg/E_T$$

$$q = -\frac{mg}{E_T} = -\frac{0.003 \text{ kg} \cdot 9.8 \text{ m/s}^2}{150 \text{ N/C}} = -1.96 \cdot 10^{-4} \text{ C}$$

$$\vec{F}_e + \vec{F}_g = \vec{0} \Rightarrow -q E_T \vec{u}_z - mg \vec{u}_z = \vec{0} \Rightarrow q = -\frac{mg}{E_T} = -1.96 \cdot 10^{-4} \text{ C}$$



$$\vec{r} = \vec{0} \quad \vec{r}_1 = -0.03 \vec{u}_x \text{ m} \quad \vec{r}_2 = +0.04 \vec{u}_x \text{ m} \quad \vec{r}_3 = a \vec{u}_x \text{ m}$$

$$\vec{r} - \vec{r}_1 = 0.03 \vec{u}_x \text{ m} \quad \vec{r} - \vec{r}_2 = -0.04 \vec{u}_x \text{ m} \quad \vec{r} - \vec{r}_3 = -a \vec{u}_x \text{ m}$$

$$|\vec{r} - \vec{r}_1| = 0.03 \text{ m} \quad |\vec{r} - \vec{r}_2| = 0.04 \text{ m} \quad |\vec{r} - \vec{r}_3| = a \text{ m}$$

$$\vec{E}_1 + \vec{E}_2 + \vec{E}_3 = \vec{0}$$

$$\sum_{i=1}^3 \frac{q_i (\vec{r} - \vec{r}_i)}{4\pi\epsilon_0 |\vec{r} - \vec{r}_i|^3} = 0$$

$$\frac{q_1 \cdot 0.03 \vec{u}_x \text{ m}}{4\pi\epsilon_0 (0.03 \text{ m})^3} - \frac{q_2 \cdot 0.04 \vec{u}_x \text{ m}}{4\pi\epsilon_0 (0.04 \text{ m})^3} - \frac{q_3 \cdot a \vec{u}_x \text{ m}}{4\pi\epsilon_0 |a|^3 \text{ m}^3} = 0$$

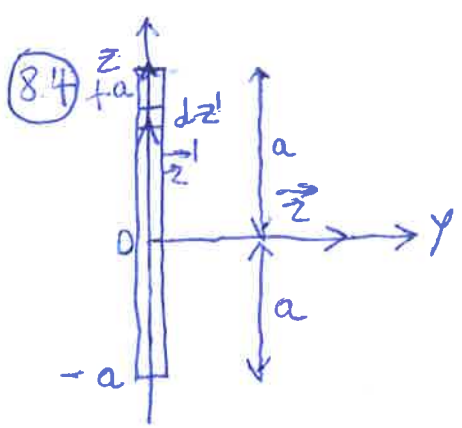
Autor: Iván Cabris Álvarez

$$\frac{q_1' 0.03}{(0.03)^3} - \frac{q_2' 0.04}{(0.04)^3} - \frac{q_3' a}{|a|^3} = 0 \quad q_1', q_2' \text{ y } q_3' \text{ en C}$$

$a \text{ en m}$

$$\frac{5 \cdot 10^{-6}}{(0.03)^2} + \frac{8 \cdot 10^{-6}}{(0.04)^2} - \frac{6 \cdot 10^{-6} a}{|a|^3} = 0 \Rightarrow \frac{5}{(0.03)^2} + \frac{8}{(0.04)^2} - \frac{6a}{|a|^3} = 0$$

$$\Rightarrow \frac{6}{a} = \frac{5}{(0.03)^2} + \frac{8}{(0.04)^2} \Rightarrow a = + \sqrt{\frac{6}{\frac{5}{(0.03)^2} + \frac{8}{(0.04)^2}}} = 0.0238 \text{ m} = \boxed{2.38 \text{ cm}}$$



(a) $\varphi = \int_{-a}^{+a} \lambda dz = \lambda_0 \int_{-a}^{+a} dz = \boxed{\lambda_0 2a}$

$dl = dz$

(b) $\vec{r}' = z' \vec{u}_z \quad \vec{r} = y \vec{u}_y$

$\vec{r} - \vec{r}' = y \vec{u}_y - z' \vec{u}_z \quad |\vec{r} - \vec{r}'| = \sqrt{y^2 + z'^2}$

$dl' = dz'$

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\lambda dl' (\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3} = \frac{1}{4\pi\epsilon_0} \int_{-a}^{+a} \frac{\lambda_0 dz' (y \vec{u}_y - z' \vec{u}_z)}{(y^2 + z'^2)^{3/2}}$$

$$= \frac{\lambda_0}{4\pi\epsilon_0} \left[y \vec{u}_y \int_{-a}^{+a} \frac{dz'}{(y^2 + z'^2)^{3/2}} - \vec{u}_z \int_{-a}^{+a} \frac{z' dz'}{(y^2 + z'^2)^{3/2}} \right]$$

$$\int_{-a}^{+a} \frac{dz'}{(y^2 + z'^2)^{3/2}} = \frac{z'}{y^2 \sqrt{y^2 + z'^2}} \Big|_{z'=-a}^{z'=+a} = \frac{2a}{y^2 \sqrt{y^2 + a^2}}$$

$$\int_{-a}^{+a} \frac{z' dz'}{(y^2 + z'^2)^{3/2}} = - \frac{1}{\sqrt{y^2 + z'^2}} \Big|_{z'=-a}^{z'=+a} = 0$$

$$\vec{E}(\vec{r}) = \frac{\lambda_0}{4\pi\epsilon_0} y \vec{u}_y \frac{2a}{y^2 \sqrt{y^2 + a^2}} = \frac{\lambda_0 2a \vec{u}_y}{4\pi\epsilon_0 y \sqrt{y^2 + a^2}} = \boxed{\frac{\lambda_0 a \vec{u}_y}{2\pi\epsilon_0 y \sqrt{y^2 + a^2}}}$$

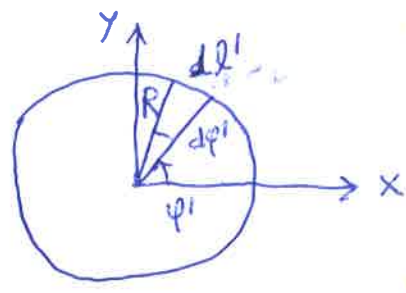
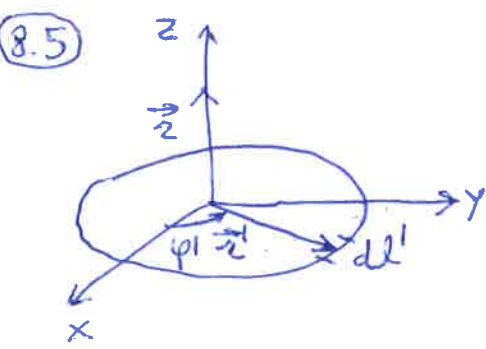
Profesor: Iván Cabrera Álvarez

$$\textcircled{c)} \phi(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\lambda d\ell'}{|\vec{r} - \vec{r}'|} = \frac{1}{4\pi\epsilon_0} \int_{-a}^a \frac{\lambda_0 dz'}{\sqrt{y^2 + z'^2}} = \frac{\lambda_0}{4\pi\epsilon_0} \ln(z' + \sqrt{y^2 + z'^2}) \Big|_{z'=-a}^{z'=+a}$$

$$= \frac{\lambda_0}{4\pi\epsilon_0} \left[\ln(a + \sqrt{y^2 + a^2}) - \ln(-a + \sqrt{y^2 + a^2}) \right] = \boxed{\frac{\lambda_0}{4\pi\epsilon_0} \ln \left(\frac{a + \sqrt{y^2 + a^2}}{-a + \sqrt{y^2 + a^2}} \right)}$$

$$\textcircled{d)} \lim_{a \rightarrow \infty} \vec{E}(\vec{r}) = \lim_{a \rightarrow \infty} \frac{\lambda_0}{2\pi\epsilon_0} \frac{a \vec{u}_y}{y \sqrt{y^2 + a^2}} = \lim_{a \rightarrow \infty} \frac{\lambda_0}{2\pi\epsilon_0 y} \frac{\vec{u}_y}{\sqrt{(\frac{y}{a})^2 + 1}}$$

$$= \boxed{\frac{\lambda_0 \vec{u}_y}{2\pi\epsilon_0 y}}$$



$$\begin{aligned} \rho' &= R \\ \vec{r}' &= R \cos \phi' \vec{u}_x + R \sin \phi' \vec{u}_y \\ \vec{r} &= z \vec{u}_z \\ \vec{r} - \vec{r}' &= -R \cos \phi' \vec{u}_x - R \sin \phi' \vec{u}_y + z \vec{u}_z \\ |\vec{r} - \vec{r}'| &= \sqrt{R^2 + z^2} \\ dl' &= R d\phi' \end{aligned}$$

$$\textcircled{a)} \phi = \int \lambda d\ell' = \int_0^{2\pi} \lambda_0 R d\phi' = \boxed{\lambda_0 R 2\pi}$$

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\lambda d\ell'}{|\vec{r} - \vec{r}'|^3} (\vec{r} - \vec{r}') = \frac{1}{4\pi\epsilon_0} \int_0^{2\pi} \frac{\lambda_0 R d\phi' (-R \cos \phi' \vec{u}_x - R \sin \phi' \vec{u}_y + z \vec{u}_z)}{(R^2 + z^2)^{3/2}}$$

$$= \frac{\lambda_0 R}{4\pi\epsilon_0} \left[-R \vec{u}_x \int_0^{2\pi} \cos \phi' d\phi' - R \vec{u}_y \int_0^{2\pi} \sin \phi' d\phi' + z \vec{u}_z \int_0^{2\pi} d\phi' \right] \frac{1}{(R^2 + z^2)^{3/2}}$$

$$= \frac{\lambda_0 R}{4\pi\epsilon_0 (R^2 + z^2)^{3/2}} [-0 - 0 + z \vec{u}_z 2\pi] = \boxed{\frac{\lambda_0 R z \vec{u}_z}{2\epsilon_0 (R^2 + z^2)^{3/2}}}$$

$$\phi(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\lambda d\ell'}{|\vec{r} - \vec{r}'|} = \frac{1}{4\pi\epsilon_0} \int_0^{2\pi} \frac{\lambda_0 R d\phi'}{\sqrt{R^2 + z^2}} = \frac{\lambda_0 R 2\pi}{4\pi\epsilon_0 \sqrt{R^2 + z^2}} = \boxed{\frac{\lambda_0 R}{2\epsilon_0 \sqrt{R^2 + z^2}}}$$

autor: Iván Cabris Álvarez

$$(b) \quad \varphi = \int \lambda dl' = \int_0^{2\pi} \lambda_0 \sin \varphi' R d\varphi' = \lambda_0 R \int_0^{2\pi} \sin \varphi' d\varphi' = \boxed{0}$$

$$\vec{E}(\vec{z}) = \frac{1}{4\pi\epsilon_0} \int_0^{2\pi} \frac{\lambda_0 \sin \varphi' R d\varphi'}{(R^2 + z^2)^{3/2}} (-R \cos \varphi' \vec{u}_x - R \sin \varphi' \vec{u}_y + z \vec{u}_z)$$

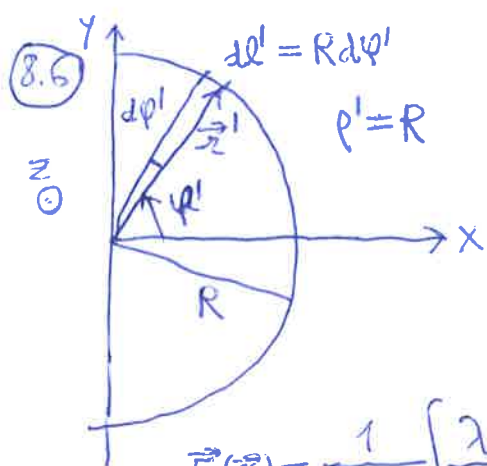
$$= \frac{\lambda_0 R}{4\pi\epsilon_0 (R^2 + z^2)^{3/2}} \left[-R \int_0^{2\pi} \sin \varphi' \cos \varphi' d\varphi' \vec{u}_x - R \int_0^{2\pi} (\sin \varphi')^2 d\varphi' \vec{u}_y + z \vec{u}_z \int_0^{2\pi} \sin \varphi' d\varphi' \right]$$

$$\int_0^{2\pi} \sin \varphi' \cos \varphi' d\varphi' = -\frac{\cos 2\varphi'}{4} \Big|_0^{2\pi} = 0$$

$$\int_0^{2\pi} (\sin \varphi')^2 d\varphi' = \frac{\varphi'}{2} \Big|_0^{2\pi} - \frac{\sin 2\varphi'}{4} \Big|_0^{2\pi} = \pi \quad \int_0^{2\pi} \sin \varphi' d\varphi' = 0$$

$$\vec{E}(\vec{z}) = \frac{\lambda_0 R}{4\pi\epsilon_0 (R^2 + z^2)^{3/2}} \left[-0 - R\pi \vec{u}_y \right] = \boxed{-\frac{\lambda_0 R^2 \vec{u}_y}{4\epsilon_0 (R^2 + z^2)^{3/2}}}$$

$$\phi(\vec{z}) = \frac{1}{4\pi\epsilon_0} \int_0^{2\pi} \frac{\lambda_0 \sin \varphi' R d\varphi'}{\sqrt{R^2 + z^2}} = \frac{\lambda_0 R}{4\pi\epsilon_0 \sqrt{R^2 + z^2}} \int_0^{2\pi} \sin \varphi' d\varphi' = \boxed{0}$$



$$(a) \quad \varphi = \int \lambda dl = \int_{-\pi/2}^{\pi/2} \lambda_0 \cos \varphi R d\varphi = \lambda_0 R \int_{-\pi/2}^{\pi/2} \cos \varphi d\varphi = \boxed{2\lambda_0 R}$$

$$(b) \quad \vec{z} = \vec{0} \quad \vec{z}' = R \cos \varphi' \vec{u}_x + R \sin \varphi' \vec{u}_y$$

$$\vec{z} - \vec{z}' = -R \cos \varphi' \vec{u}_x - R \sin \varphi' \vec{u}_y \quad |\vec{z} - \vec{z}'| = R$$

$$\vec{E}(\vec{z}) = \frac{1}{4\pi\epsilon_0} \int \frac{\lambda dl'}{|\vec{z} - \vec{z}'|^3} (\vec{z} - \vec{z}') = \frac{1}{4\pi\epsilon_0} \int_{-\pi/2}^{\pi/2} \frac{\lambda_0 \cos \varphi' R d\varphi'}{R^3} (-R \cos \varphi' \vec{u}_x - R \sin \varphi' \vec{u}_y)$$

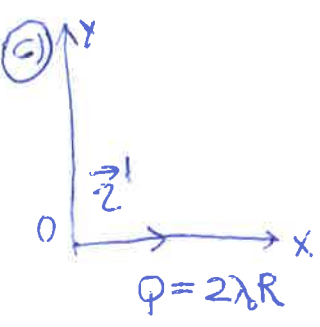
$$= \frac{\lambda_0}{4\pi\epsilon_0 R^2} \left[-R \vec{u}_x \int_{-\pi/2}^{\pi/2} (\cos \varphi')^2 d\varphi' - R \vec{u}_y \int_{-\pi/2}^{\pi/2} \cos \varphi' \sin \varphi' d\varphi' \right]$$

$$\int_{-\pi/2}^{\pi/2} (\cos \varphi')^2 d\varphi' = \frac{\varphi'}{2} \Big|_{-\pi/2}^{\pi/2} + \frac{\sin 2\varphi'}{4} \Big|_{-\pi/2}^{\pi/2} = \frac{\pi}{4} + \frac{\sin \pi}{4} - \left(-\frac{\pi}{4} + \frac{\sin(-\pi)}{4} \right) = \frac{\pi}{2}$$

$$\int_{-\pi/2}^{\pi/2} \cos \varphi' \sin \varphi' d\varphi' = -\frac{\cos 2\varphi'}{4} \Big|_{-\pi/2}^{\pi/2} = -\frac{\cos \pi}{4} + \frac{\cos(-\pi)}{4} = -\frac{\cos \pi}{4} + \frac{\cos \pi}{4} = 0$$

Autor: Iván Cabris Álvarez

$$\vec{E}(\vec{r}) = \frac{\lambda_0}{4\pi\epsilon_0 R^2} \left[-R\vec{u}_x \frac{\pi}{2} - 0 \right] = \boxed{-\frac{\lambda_0}{8\epsilon_0 R} \vec{u}_x}$$



$$\vec{z}' = x' \vec{u}_x \quad \vec{z} = \vec{0} \quad \vec{z} - \vec{z}' = -x' \vec{u}_x$$

$$\vec{E}_{\phi}(\vec{z}) = \frac{-\phi x' \vec{u}_x}{4\pi\epsilon_0 |x'|^3}$$

$$\vec{E}(\vec{z}) = -\frac{\lambda_0}{8\epsilon_0 R} \vec{u}_x$$

$$\vec{E}_{\phi}(\vec{z}) = \vec{E}(\vec{z})$$

$$\Rightarrow \frac{-\phi x' \vec{u}_x}{4\pi\epsilon_0 |x'|^3} = -\frac{\lambda_0 \vec{u}_x}{8\epsilon_0 R} \Rightarrow$$

$$\Rightarrow \frac{\phi x'}{4\pi\epsilon_0 |x'|^3} = \frac{\lambda_0}{8\epsilon_0 R} \Rightarrow \frac{x'}{|x'|^3} = \frac{\lambda_0 \pi}{2\phi R} \Rightarrow \frac{x'}{|x'|^3} = \frac{\pi}{4R^2} \quad \phi = 2\lambda_0 R$$

Para que $\vec{E}_{\phi}(\vec{z})$ y $\vec{E}(\vec{z})$ tengan el mismo sentido $x' > 0$

$$x' > 0 \Rightarrow |x'|^3 = x'^3 \Rightarrow \frac{x'}{|x'|^3} = \frac{1}{x'^2} \Rightarrow \frac{1}{x'^2} = \frac{\pi}{4R^2} \Rightarrow \boxed{x' = \frac{2R}{\sqrt{\pi}}}$$

$$\textcircled{d)} \phi(\vec{z}) = \frac{1}{4\pi\epsilon_0} \int_{-\pi/2}^{\pi/2} \frac{\lambda_0 \cos\psi' R d\psi'}{R} = \frac{\lambda_0}{4\pi\epsilon_0} \int_{-\pi/2}^{\pi/2} \cos\psi' d\psi' = \frac{\lambda_0}{4\pi\epsilon_0} \sin\psi' \Big|_{-\pi/2}^{\pi/2} =$$

$$= \boxed{\frac{\lambda_0}{2\pi\epsilon_0}}$$

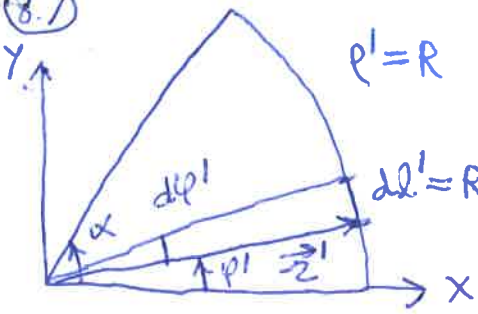
$$\textcircled{e)} \phi_{\phi}(\vec{z}) = \frac{\phi}{4\pi\epsilon_0 |x'|} \quad \vec{z} = \vec{0} \quad \vec{z}' = x' \vec{u}_x$$

$$\phi(\vec{z}) = \frac{\lambda_0}{2\pi\epsilon_0} \Rightarrow \frac{\phi}{4\pi\epsilon_0 |x'|} = \frac{\lambda_0}{2\pi\epsilon_0} \Rightarrow |x'| = \frac{\phi}{2\lambda_0} \Rightarrow \phi = 2\lambda_0 R$$

$$\Rightarrow \boxed{|x'| = R}$$

8.7

8.7



$$\vec{r} = \vec{0} \quad \vec{r}' = R \cos \varphi' \vec{u}_x + R \sin \varphi' \vec{u}_y$$

$$\vec{r} - \vec{r}' = -R \cos \varphi' \vec{u}_x - R \sin \varphi' \vec{u}_y \quad |\vec{r} - \vec{r}'| = R$$

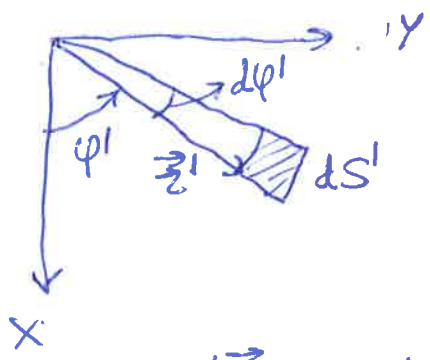
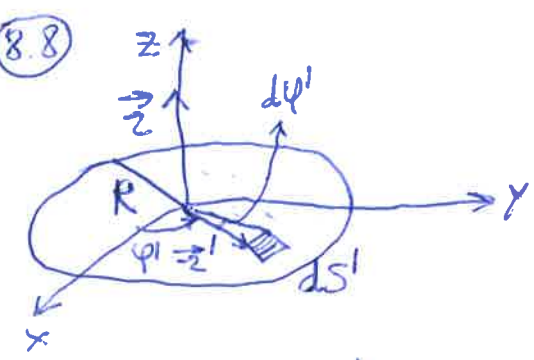
$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\lambda d\vec{l}'}{|\vec{r} - \vec{r}'|^3} (\vec{r} - \vec{r}') =$$

$$= \frac{1}{4\pi\epsilon_0} \int_0^\alpha \frac{\lambda_0 R d\varphi'}{R^3} (-R \cos \varphi' \vec{u}_x - R \sin \varphi' \vec{u}_y) =$$

$$= -\frac{\lambda_0}{4\pi\epsilon_0 R} \int_0^\alpha (\cos \varphi' \vec{u}_x + \sin \varphi' \vec{u}_y) d\varphi' = -\frac{\lambda_0}{4\pi\epsilon_0 R} \left[\sin \varphi' \Big|_0^\alpha \vec{u}_x - \cos \varphi' \Big|_0^\alpha \vec{u}_y \right]$$

$$= -\frac{\lambda_0}{4\pi\epsilon_0 R} \left[\sin \alpha \vec{u}_x - (\cos \alpha - 1) \vec{u}_y \right]$$

8.8



$$\vec{r} = z \vec{u}_z \quad \vec{r}' = \rho' \cos \varphi' \vec{u}_x + \rho' \sin \varphi' \vec{u}_y$$

$$\vec{r} - \vec{r}' = -\rho' \cos \varphi' \vec{u}_x - \rho' \sin \varphi' \vec{u}_y + z \vec{u}_z$$

$$|\vec{r} - \vec{r}'| = \sqrt{\rho'^2 + z^2}$$

$$dS' = \rho' d\varphi' d\rho'$$

$$a) \varphi = \int \sigma dS = \int_0^{2\pi} \int_0^R \sigma_0 \rho' d\varphi' d\rho' = \sigma_0 \int_0^{2\pi} d\varphi' \int_0^R \rho' d\rho' = \sigma_0 2\pi \frac{R^2}{2} = \boxed{\sigma_0 \pi R^2}$$

$$b) \vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\sigma dS'}{|\vec{r} - \vec{r}'|^3} (\vec{r} - \vec{r}') = \frac{1}{4\pi\epsilon_0} \int_0^{2\pi} d\varphi' \int_0^R \rho' d\rho' \frac{\sigma_0 (-\rho' \cos \varphi' \vec{u}_x - \rho' \sin \varphi' \vec{u}_y + z \vec{u}_z)}{(\rho'^2 + z^2)^{3/2}}$$

$$= \frac{\sigma_0}{4\pi\epsilon_0} \left[-\int_0^{2\pi} \cos \varphi' d\varphi' \int_0^R \frac{\rho'^2}{(\rho'^2 + z^2)^{3/2}} d\rho' \vec{u}_x - \int_0^{2\pi} \sin \varphi' d\varphi' \int_0^R \frac{\rho'^2 d\rho'}{(\rho'^2 + z^2)^{3/2}} \vec{u}_y + z \vec{u}_z \int_0^{2\pi} d\varphi' \int_0^R \frac{\rho' d\rho'}{(\rho'^2 + z^2)^{3/2}} \right]$$

$$\int_0^R \frac{\rho' d\rho'}{(\rho'^2 + z^2)^{3/2}} = -\frac{1}{\sqrt{\rho'^2 + z^2}} \Big|_{\rho'=0}^{\rho'=R} = -\frac{1}{\sqrt{z^2 + R^2}} + \frac{1}{|z|}$$

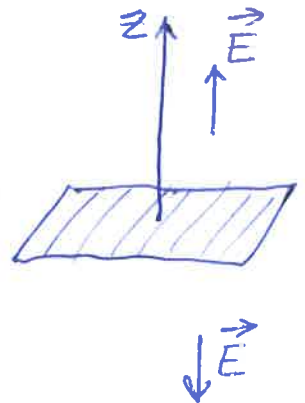
$$\vec{E}(\vec{r}) = \frac{\sigma_0}{4\pi\epsilon_0} \left[-0 - 0 + z \vec{u}_z 2\pi \left(-\frac{1}{\sqrt{z^2 + R^2}} + \frac{1}{|z|} \right) \right] = \boxed{\frac{\sigma_0}{2\epsilon_0} \left(\frac{z}{|z|} - \frac{z}{\sqrt{z^2 + R^2}} \right) \vec{u}_z}$$

Autor: Juan Carlos Álvarez

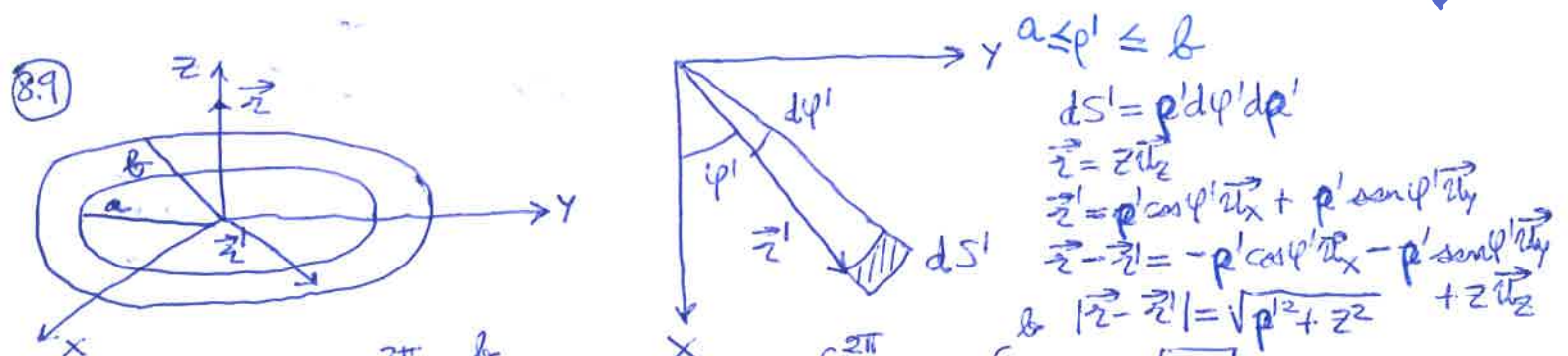
$$\textcircled{c)} \phi(\vec{z}) = \frac{1}{4\pi\epsilon_0} \int \frac{\sigma dS'}{|\vec{z} - \vec{z}'|} = \frac{\sigma_0}{4\pi\epsilon_0} \int_0^{2\pi} d\varphi' \int_0^R \frac{\rho' d\rho'}{\sqrt{\rho'^2 + z^2}} = \frac{\sigma_0}{4\pi\epsilon_0} 2\pi (\sqrt{R^2 + z^2} - |z|) \quad (8.8)$$

$$\int_0^R \frac{\rho' d\rho'}{\sqrt{\rho'^2 + z^2}} = \sqrt{\rho'^2 + z^2} \Big|_{\rho'=0}^{\rho'=R} = \sqrt{R^2 + z^2} - |z|$$

$$\boxed{\phi(\vec{z}) = \frac{\sigma_0}{2\epsilon_0} (\sqrt{R^2 + z^2} - |z|)}$$



$$\textcircled{d)} \vec{E} = \lim_{R \rightarrow \infty} \frac{\sigma_0}{2\epsilon_0} \left(\frac{z}{|z|} - \frac{z}{\sqrt{z^2 + R^2}} \right) \vec{u}_z = \frac{\sigma_0}{2\epsilon_0} \frac{z}{|z|} \vec{u}_z$$



$$\textcircled{a)} Q = \int \sigma dS' = \int_0^{2\pi} d\varphi' \int_a^b \rho' d\rho' \sigma_0 \cos\varphi' = \sigma_0 \int_0^{2\pi} \cos\varphi' d\varphi' \int_a^b \rho' d\rho' = \boxed{0}$$

$$\textcircled{b)} \vec{E}(\vec{z}) = \frac{1}{4\pi\epsilon_0} \int \frac{\sigma dS'}{|\vec{z} - \vec{z}'|^3} (\vec{z} - \vec{z}') = \frac{\sigma_0}{4\pi\epsilon_0} \int_0^{2\pi} d\varphi' \int_a^b \rho' d\rho' \frac{\cos\varphi' (-\rho' \cos\varphi' \vec{u}_x - \rho' \sin\varphi' \vec{u}_y + z \vec{u}_z)}{(\rho'^2 + z^2)^{3/2}}$$

$$= \frac{\sigma_0}{4\pi\epsilon_0} \left[- \int_0^{2\pi} (\cos\varphi')^2 d\varphi' \int_a^b \frac{\rho'^2 d\rho'}{(\rho'^2 + z^2)^{3/2}} - \int_0^{2\pi} \sin\varphi' \cos\varphi' d\varphi' \int_a^b \frac{\rho'^2 d\rho'}{(\rho'^2 + z^2)^{3/2}} + \right.$$

$$\left. + z \vec{u}_z \int_0^{2\pi} \cos\varphi' d\varphi' \int_a^b \frac{\rho' d\rho'}{(\rho'^2 + z^2)^{3/2}} \right]$$

$$\int_0^{2\pi} (\cos\varphi')^2 d\varphi' = \frac{\varphi'}{2} \Big|_0^{2\pi} + \frac{\sin 2\varphi'}{4} \Big|_0^{2\pi} = \pi$$

$$\int_0^{2\pi} \sin\varphi' \cos\varphi' d\varphi' = -\frac{\cos 2\varphi'}{4} \Big|_0^{2\pi} = 0$$

$$\int_a^b \frac{\rho'^2 d\rho'}{(\rho'^2 + z^2)^{3/2}} = -\frac{\rho'}{\sqrt{\rho'^2 + z^2}} \Big|_a^b + \ln(\rho' + \sqrt{\rho'^2 + z^2}) \Big|_a^b = -\frac{b}{\sqrt{b^2 + z^2}} + \frac{a}{\sqrt{a^2 + z^2}}$$

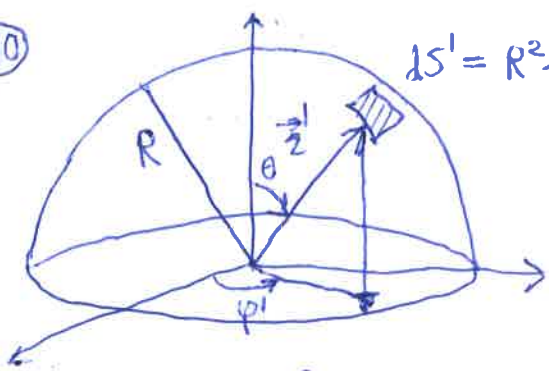
$$+ \ln \left(\frac{b + \sqrt{b^2 + z^2}}{a + \sqrt{a^2 + z^2}} \right)$$

Autor: Iván Cabris Álvarez

$$\vec{E}(\vec{z}) = \frac{\sigma_0}{4\pi\epsilon_0} \left[-\pi \int_a^b \frac{\rho'^{1/2} d\rho'}{(\rho'^2 + z^2)^{3/2}} \vec{u}_x \right] = \boxed{-\frac{\sigma_0}{4\epsilon_0} \left[\frac{a}{\sqrt{a^2 + z^2}} - \frac{b}{\sqrt{b^2 + z^2}} + \ln \left(\frac{b + \sqrt{b^2 + z^2}}{a + \sqrt{a^2 + z^2}} \right) \right] \vec{u}_x}$$

(2) $\phi(\vec{z}) = \frac{1}{4\pi\epsilon_0} \int \frac{\sigma dS'}{|\vec{z} - \vec{z}'|} = \frac{\sigma_0}{4\pi\epsilon_0} \int_0^{2\pi} d\varphi' \cos\varphi' \int_a^b \frac{\rho' d\rho'}{a\sqrt{\rho'^2 + z^2}} = \boxed{0}$

8.10



$dS' = R^2 \sin\theta' d\theta' d\varphi'$ $|\vec{z}'| = R$
 $\vec{z} = 0$

$\vec{z}' = R \sin\theta' \cos\varphi' \vec{u}_x + R \sin\theta' \sin\varphi' \vec{u}_y + R \cos\theta' \vec{u}_z$
 $|\vec{z} - \vec{z}'| = R$

$$\vec{E}(\vec{z}) = \frac{1}{4\pi\epsilon_0} \int \frac{\sigma dS'}{|\vec{z} - \vec{z}'|^3} (\vec{z} - \vec{z}') = \frac{1}{4\pi\epsilon_0} \int_0^{\pi/2} d\theta' \int_0^{2\pi} d\varphi' \frac{\sigma_0 R^2 \sin\theta'}{R^3} (-R) (\sin\theta' \cos\varphi' \vec{u}_x + \sin\theta' \sin\varphi' \vec{u}_y + \cos\theta' \vec{u}_z)$$

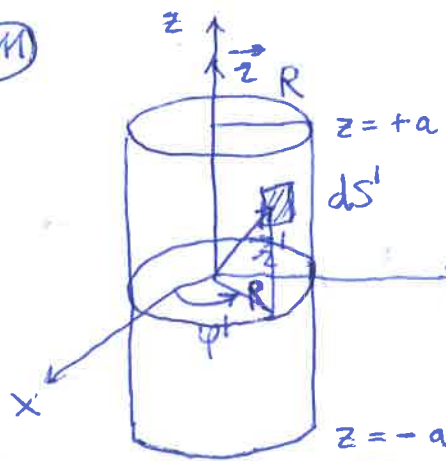
$$= -\frac{\sigma_0}{4\pi\epsilon_0} \left[\int_0^{\pi/2} d\theta' \int_0^{2\pi} d\varphi' (\sin\theta')^2 \cos\varphi' \vec{u}_x + \int_0^{\pi/2} d\theta' \int_0^{2\pi} d\varphi' (\sin\theta')^2 \sin\varphi' \vec{u}_y + \int_0^{\pi/2} d\theta' \sin\theta' \cos\theta' \int_0^{2\pi} d\varphi' \vec{u}_z \right]$$

$\int_0^{\pi/2} d\theta' \sin\theta' \cos\theta' = -\frac{\cos 2\theta'}{4} \Big|_0^{\pi/2} = -\frac{\cos\pi}{4} + \frac{1}{4} = \frac{1}{2}$

$$\vec{E}(\vec{z}) = -\frac{\sigma_0}{4\pi\epsilon_0} \left[0 + 0 + \frac{1}{2} 2\pi \vec{u}_z \right] = \boxed{-\frac{\sigma_0}{4\epsilon_0} \vec{u}_z}$$

Autor: Iván Gabris Alvaro

8.11



$$dS' = R d\phi' dz' \quad \rho' = R$$

$$\vec{z} = z \vec{u}_z$$

$$\vec{z}' = R \cos \phi' \vec{u}_x + R \sin \phi' \vec{u}_y + z' \vec{u}_z$$

$$\vec{z} - \vec{z}' = -R \cos \phi' \vec{u}_x - R \sin \phi' \vec{u}_y + (z - z') \vec{u}_z$$

$$|\vec{z} - \vec{z}'| = \sqrt{R^2 + (z - z')^2}$$

a) $Q = \int \sigma dS' = \int_0^{2\pi} d\phi' \int_{-a}^a R dz' \sigma_0 = \sigma_0 R 2\pi 2a = \boxed{4\pi a \sigma_0 R}$

b)
$$\vec{E}(\vec{z}) = \frac{1}{4\pi\epsilon_0} \int_0^{2\pi} d\phi' \int_{-a}^a R dz' \sigma_0 \frac{(-R \cos \phi' \vec{u}_x - R \sin \phi' \vec{u}_y + (z - z') \vec{u}_z)}{(R^2 + (z - z')^2)^{3/2}}$$

$$= \frac{\sigma_0 R}{4\pi\epsilon_0} \left[-R \int_0^{2\pi} d\phi' \int_{-a}^a \frac{\cos \phi' \vec{u}_x}{(R^2 + (z - z')^2)^{3/2}} - R \int_0^{2\pi} \sin \phi' d\phi' \int_{-a}^a \frac{dz' \vec{u}_y}{(R^2 + (z - z')^2)^{3/2}} + \int_0^{2\pi} d\phi' \int_{-a}^a \frac{(z - z') dz' \vec{u}_z}{(R^2 + (z - z')^2)^{3/2}} \right]$$

$$\int_{-a}^a \frac{(z - z') dz'}{(R^2 + (z - z')^2)^{3/2}} = \left\{ \begin{array}{l} u = z - z' \\ du = -dz' \end{array} \right\} = \int_{z+a}^{z-a} \frac{-u du}{(R^2 + u^2)^{3/2}} = \left. \frac{1}{\sqrt{R^2 + u^2}} \right|_{u=z+a}^{u=z-a}$$

$$= \frac{1}{\sqrt{R^2 + (z-a)^2}} - \frac{1}{\sqrt{R^2 + (z+a)^2}}$$

$$\vec{E}(\vec{z}) = \frac{\sigma_0 R}{4\pi\epsilon_0} \left[-0 - 0 + 2\pi \left(\frac{1}{\sqrt{R^2 + (z-a)^2}} - \frac{1}{\sqrt{R^2 + (z+a)^2}} \right) \vec{u}_z \right]$$

$$\boxed{\vec{E}(\vec{z}) = \frac{\sigma_0 R}{2\epsilon_0} \left(\frac{1}{\sqrt{R^2 + (z-a)^2}} - \frac{1}{\sqrt{R^2 + (z+a)^2}} \right) \vec{u}_z}$$

c)
$$\phi(\vec{z}) = \frac{1}{4\pi\epsilon_0} \int_0^{2\pi} d\phi' \int_{-a}^a \frac{R dz' \sigma_0}{\sqrt{R^2 + (z - z')^2}} = \frac{\sigma_0 R}{4\pi\epsilon_0} 2\pi \int_{-a}^a \frac{dz'}{\sqrt{R^2 + (z - z')^2}}$$

$$\int_{-a}^a \frac{dz'}{\sqrt{R^2 + (z - z')^2}} = \left\{ \begin{array}{l} u = z - z' \\ du = -dz' \end{array} \right\} = - \int_{z+a}^{z-a} \frac{du}{\sqrt{R^2 + u^2}} = - \ln(u + \sqrt{R^2 + u^2}) \Big|_{u=z+a}^{u=z-a}$$

$$= - \ln \left(\frac{z-a + \sqrt{R^2 + (z-a)^2}}{z+a + \sqrt{R^2 + (z+a)^2}} \right)$$

$$\boxed{\phi(\vec{z}) = -\frac{\sigma_0 R}{2\epsilon_0} \ln \left(\frac{z-a + \sqrt{R^2 + (z-a)^2}}{z+a + \sqrt{R^2 + (z+a)^2}} \right)}$$

tutor: Iván Gabrila Alvarado

8.12



$$dV' = r'^2 dr' \sin \theta' d\theta' d\varphi'$$

$$\vec{r} = 0$$

$$|\vec{r}'| = r'$$

$$|\vec{r} - \vec{r}'| = r'$$

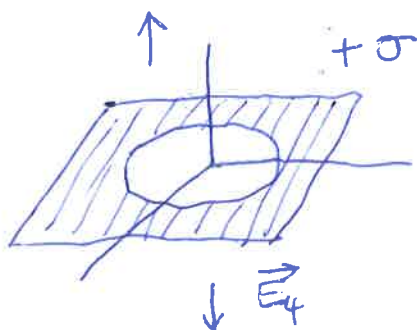
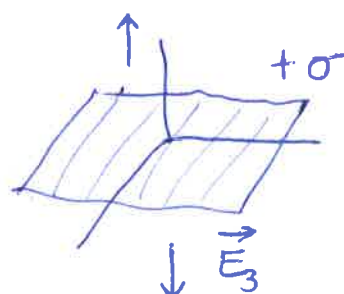
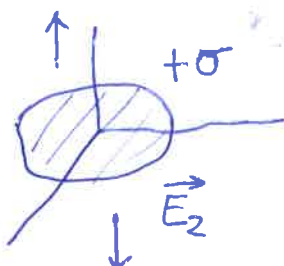
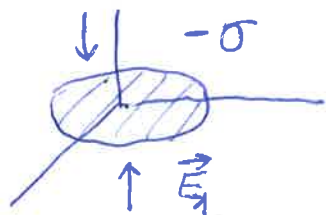
$$\phi(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho dV'}{|\vec{r} - \vec{r}'|} = \frac{1}{4\pi\epsilon_0} \int_0^{2\pi} d\varphi' \int_0^\pi \sin \theta' d\theta' \int_0^R \frac{r'^2}{r'} \rho(r') dr'$$

$$= \frac{1}{\epsilon_0} \int_0^R r' \rho(r') dr'$$

$$a) \phi(\vec{r}) = \frac{1}{\epsilon_0} \int_0^R r' \rho_0 dr' = \boxed{\frac{\rho_0 R^2}{\epsilon_0 2}}$$

$$b) \phi(\vec{r}) = \frac{1}{\epsilon_0} \int_0^R \frac{r'^2}{a} \rho_0 dr' = \boxed{\frac{\rho_0 R^3}{\epsilon_0 3a}}$$

8.13



$$\vec{E} = \vec{E}_1 + \vec{E}_4$$

$$\vec{E}_3 = \vec{E}_2 + \vec{E}_4 \Rightarrow \vec{E}_4 = \vec{E}_3 - \vec{E}_2$$

$$\Rightarrow \vec{E} = \vec{E}_1 + \vec{E}_3 - \vec{E}_2$$

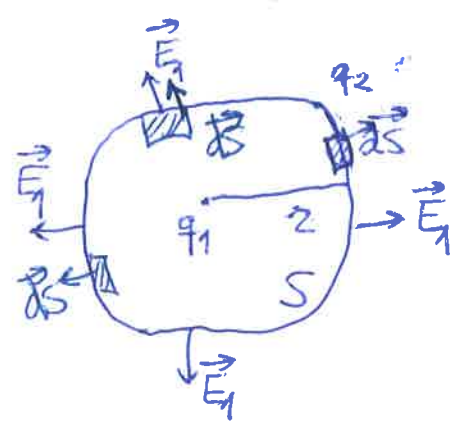
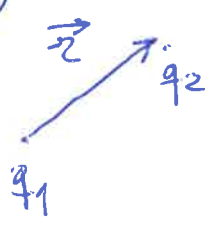
$$\vec{E} = -\frac{\sigma}{2\epsilon_0} \left(\frac{z}{|z|} - \frac{z}{\sqrt{z^2 + R^2}} \right) \vec{u}_z + \frac{\sigma}{2\epsilon_0} \frac{z}{|z|} \vec{u}_z - \frac{\sigma}{2\epsilon_0} \left(\frac{z}{|z|} - \frac{z}{\sqrt{z^2 + R^2}} \right) \vec{u}_z$$

$$= -\frac{\sigma}{\epsilon_0} \left(\frac{z}{|z|} - \frac{z}{\sqrt{z^2 + R^2}} \right) \vec{u}_z + \frac{\sigma}{2\epsilon_0} \frac{z}{|z|} \vec{u}_z = \frac{\sigma}{\epsilon_0} \left(-\frac{z}{|z|} + \frac{z}{2|z|} + \frac{z}{\sqrt{z^2 + R^2}} \right) \vec{u}_z$$

$$= \boxed{\frac{\sigma}{\epsilon_0} \left(-\frac{z}{2|z|} + \frac{z}{\sqrt{z^2 + R^2}} \right) \vec{u}_z}$$

tutor: Iván Cabria Álvarez

8.14



$\phi_{en.5} = 9.1$

$$d\vec{S} = r^2 \sin\theta d\theta d\varphi \vec{u}_r$$

$$r = |\vec{r}|$$

$$\vec{u}_2 = \frac{\vec{z}}{2} \quad \vec{E}_1 = E_1 \vec{u}_2$$

$$\vec{E}_1 \parallel \vec{dS} \text{ en } S$$

$$\oint_S \vec{E} \cdot d\vec{S} = \frac{Q_{encl}}{\epsilon_0}$$

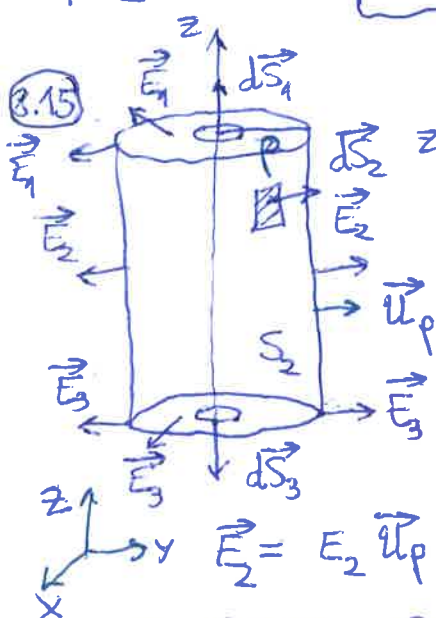
$$\oint_S \vec{E} \cdot d\vec{S} = \int_0^\pi \sin\theta d\theta \int_0^{2\pi} d\phi r^2 E_1 = E_1 4\pi r^2$$

$$\Rightarrow E_1 4\pi z^2 = \frac{q_1}{\epsilon_0} \Rightarrow$$

$$P_{\text{ens}} = 9.1$$

$$\Rightarrow E_1 = \frac{q_1}{4\pi\epsilon_0 r^2} \Rightarrow \vec{E} = \frac{q_1}{4\pi\epsilon_0 r^2} \frac{\vec{r}}{r}$$

$$\vec{F}_{1 \rightarrow 2} = q_2 \vec{E}_1 = \frac{q_1 q_2}{4\pi\epsilon_0 r^2} \hat{r}$$



Cilindro de radio p y altura $2a$ $S = S_1 + S_2 + S_3$

$$\vec{E}_1 \cdot d\vec{S}_1 = E_1 dS_1 \cos 90^\circ = 0 \quad \text{en } S_1$$

$$\vec{E}_2 \cdot d\vec{S}_2 = E_2 dS_2 = E_2 \rho d\varphi dz \quad \text{em } S_2$$

$$\vec{E}_3 \cdot d\vec{S}_3 = E_3 dS_3 \cos 90^\circ = 0 \text{ en } S_3$$

$\vec{E}_1$ es el campo en la superficie S_1 del círculo superior

$\vec{E}_3 \parallel \parallel \parallel \parallel \parallel \dots \parallel S_3 \parallel \parallel$ inferior

$\vec{E}_2 \parallel \parallel \parallel \parallel \parallel \parallel$ lateral δ_2 del cilindro

$$\oint_S \vec{E} \cdot d\vec{S} = \int_{S_1} \vec{E}_1 \cdot d\vec{S}_1 + \int_{S_2} \vec{E}_2 \cdot d\vec{S}_2 + \int_{S_3} \vec{E}_3 \cdot d\vec{S}_3 = 0 + \int_{-a}^{+a} dz \int_0^{2\pi} d\phi E_z + 0 = E_z 4\pi a$$

$$Q_{\text{ens}} = 2\lambda_0 a \quad (\text{for problem 8.4})$$

$$\oint_S \vec{E} \cdot d\vec{S} = \frac{\rho_{en} S}{\epsilon_0} \Rightarrow E_2 \rho 4\pi a = \frac{2\lambda a}{\epsilon_0} \Rightarrow E_2 = \frac{\lambda}{2\pi\epsilon_0 \rho} \Rightarrow \vec{E}_2 = \frac{\lambda}{2\pi\epsilon_0} \frac{\vec{u}_\rho}{\rho}$$

fio finito de longitud $2a$

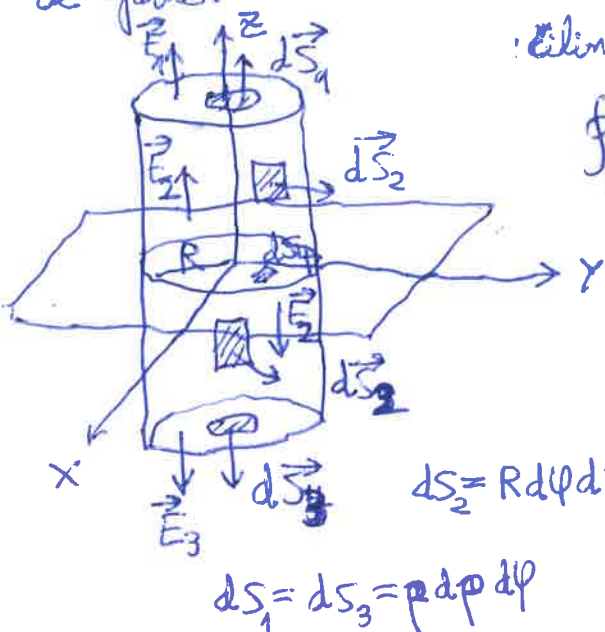
$$\vec{E} = \lim_{a \rightarrow \infty} \vec{E}_2 = \frac{\lambda}{2\pi\epsilon_0} \vec{r}_p$$

$\vec{r}_p$ está en el plano xy



Autor: Yván Cobris Álvarez

8.16 En el problema 8.8 d), calculamos que $\vec{E} = \frac{\sigma_0}{2\epsilon_0} \frac{z}{|z|} \vec{u}_z$.
 Vamos a calcular $\vec{E}$ y a obtener el mismo resultado, pero usando el teorema de Gauss.



Cilindro de radio R

$$\oint_S \vec{E} \cdot d\vec{S} = \int_{S_1} \vec{E}_1 \cdot d\vec{S}_1 + \int_{S_2} \vec{E}_2 \cdot d\vec{S}_2 + \int_{S_3} \vec{E}_3 \cdot d\vec{S}_3$$

$$\vec{E}_1 \cdot d\vec{S}_1 = E_1 dS_1 \cos 0^\circ = E_1 dS_1$$

$$\vec{E}_2 \cdot d\vec{S}_2 = E_2 dS_2 \cos 90^\circ = 0 \quad \text{en la parte superior, } z > 0, \text{ y también en la inferior, } z < 0$$

$$\vec{E}_3 \cdot d\vec{S}_3 = E_3 dS_3 \cos 0^\circ = E_3 dS_3$$

$$E_1 = E_3 = E = |\vec{E}| \quad \vec{E} = \begin{cases} \vec{E}_1 & z > 0 \\ \vec{E}_3 & z < 0 \end{cases}$$

$$\vec{E}_1 = E_1 \vec{u}_z \quad z > 0 \quad \vec{E}_3 = -E_3 \vec{u}_z \quad z < 0$$

$$\int_{S_1} \vec{E}_1 \cdot d\vec{S}_1 = \int_0^R \int_0^{2\pi} E \, d\varphi \, dp = E \pi R^2$$

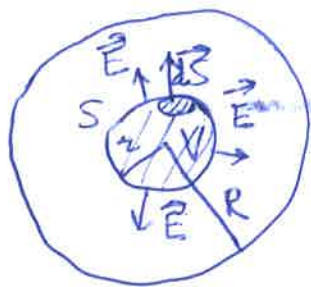
$$\int_{S_3} \vec{E}_3 \cdot d\vec{S}_3 = E \pi R^2$$

$$\oint_S \vec{E} \cdot d\vec{S} = E \pi R^2 + 0 + E \pi R^2 = E 2\pi R^2$$

$$Q_{\text{em } S} = \int_{S_4} \sigma \, dS_4 = \sigma_0 \int_{S_4} dS_4 = \sigma_0 \pi R^2$$

$$\text{Gauss} \Rightarrow E 2\pi R^2 = \frac{\sigma_0 \pi R^2}{\epsilon_0} \Rightarrow E = \frac{\sigma_0}{2\epsilon_0} \Rightarrow \left. \begin{aligned} \vec{E}_1 &= \frac{\sigma_0}{2\epsilon_0} \vec{u}_z \quad z > 0 \\ \vec{E}_3 &= -\frac{\sigma_0}{2\epsilon_0} \vec{u}_z \quad z < 0 \end{aligned} \right\} \Rightarrow$$

$$\Rightarrow \boxed{\vec{E} = \frac{\sigma_0}{2\epsilon_0} \frac{z}{|z|} \vec{u}_z}$$

8.17 $z \leq R$ 

$$\vec{E} = E \vec{u}_r$$

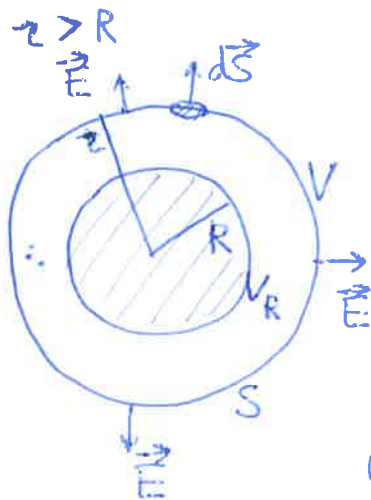
$$d\vec{S} = dS \vec{u}_r$$

$$dS = z^2 \sin \theta d\theta d\varphi$$

$$\oint_S \vec{E} \cdot d\vec{S} = \oint_S E dS = \int_0^{2\pi} d\varphi \int_0^\pi \sin \theta d\theta z^2 E = 4\pi z^2 E$$

$$Q_{\text{enc}S} = \int_V \rho dV = \rho_0 \int_V dV = \rho_0 \frac{4\pi}{3} z^3$$

$$\text{Gauss} \Rightarrow 4\pi z^2 E = \frac{\rho_0 4\pi z^3}{3\epsilon_0} \Rightarrow E = \frac{\rho_0 z}{3\epsilon_0} \Rightarrow \boxed{\vec{E} = \frac{\rho_0 z}{3\epsilon_0} \vec{u}_r \quad z \leq R}$$



$$\oint_S \vec{E} \cdot d\vec{S} = \oint_S E dS = \int_0^{2\pi} d\varphi \int_0^\pi \sin \theta d\theta z^2 E = 4\pi z^2 E$$

$$\vec{E} = E \vec{u}_r \quad d\vec{S} = dS \vec{u}_r$$

$$Q_{\text{enc}S} = \int_V \rho dV = \int_{V_R} \rho dV = \rho_0 \int_{V_R} dV = \rho_0 \frac{4\pi}{3} R^3$$

$$\text{Gauss} \Rightarrow 4\pi z^2 E = \frac{\rho_0 4\pi R^3}{3\epsilon_0} \Rightarrow E = \frac{\rho_0 R^3}{3\epsilon_0 z^2} \Rightarrow$$

$$\Rightarrow \boxed{\vec{E} = \frac{\rho_0 R^3}{3\epsilon_0 z^2} \vec{u}_r \quad z \geq R}$$

$$\left. \begin{aligned} \vec{E} &= -\vec{\nabla} \phi \\ \vec{\nabla} \phi &= \frac{\partial \phi}{\partial z} \vec{u}_z + \end{aligned} \right\} \Rightarrow$$

+ otros términos en $\vec{u}_\theta$ y $\vec{u}_\phi$

$$\frac{\partial \phi}{\partial z} = \begin{cases} -\frac{\rho_0 z}{3\epsilon_0} & z \leq R \\ -\frac{\rho_0 R^3}{3\epsilon_0 z^2} & z \geq R \end{cases}$$

$$\Rightarrow \phi(z) = \begin{cases} -\frac{\rho_0 z^2}{6\epsilon_0} + C & z \leq R \\ +\frac{\rho_0 R^3}{3\epsilon_0 z} & z \geq R \end{cases}$$

$$\phi \text{ debe ser continuo en } z=R \Rightarrow -\frac{\rho_0 R^2}{6\epsilon_0} + C = \frac{\rho_0 R^3}{3\epsilon_0 R} \Rightarrow C = \frac{\rho_0 R^2}{\epsilon_0} \left(\frac{1}{3} + \frac{1}{6} \right)$$

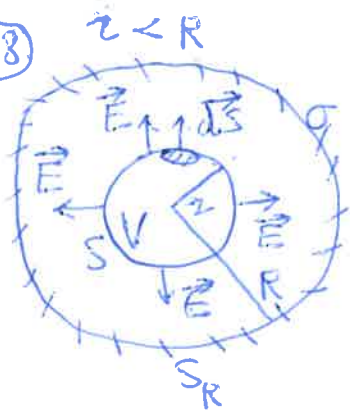
$$\Rightarrow C = \frac{\rho_0 R^2}{2\epsilon_0}$$

$$\boxed{\phi(z) = \begin{cases} -\frac{\rho_0 z^2}{6\epsilon_0} + \frac{\rho_0 R^2}{2\epsilon_0} & z \leq R \\ +\frac{\rho_0 R^3}{3\epsilon_0 z} & z \geq R \end{cases}}$$

Autor: Iván Cabris Alvarro

8.18

8.15



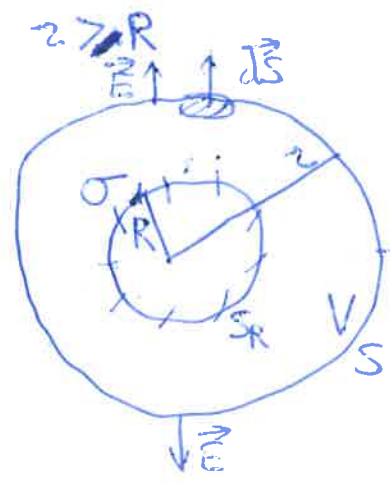
$$\vec{E} = E \vec{u}_r \quad d\vec{S} = dS \vec{u}_r$$

$$\oint_S \vec{E} \cdot d\vec{S} = \oint_S E dS = E \oint_S dS = E 4\pi r^2$$

La superficie de una esfera de radio r es $4\pi r^2$

$Q_{\text{en } S} = \int_S \sigma dS = 0$ Toda la carga está en la superficie de la esfera de radio $R > r$. Por tanto, dentro de la esfera de radio r , es decir, dentro del volumen V , la carga neta es cero.

Gauss $\Rightarrow E 4\pi r^2 = 0 \Rightarrow E = 0$



$$\oint_S \vec{E} \cdot d\vec{S} = E 4\pi r^2 \quad \vec{E} = E \vec{u}_r \quad d\vec{S} = dS \vec{u}_r$$

$$Q_{\text{en } S} = \int_V \rho dV = \int_{S_R} \sigma dS = \sigma_0 \int_{S_R} dS = \sigma_0 4\pi R^2$$

Gauss $\Rightarrow E 4\pi r^2 = \frac{\sigma_0 4\pi R^2}{\epsilon_0} \Rightarrow E = \frac{\sigma_0 R^2}{\epsilon_0 r^2}$

$$\vec{E} = \begin{cases} 0 & r < R \\ \frac{\sigma_0 R^2}{\epsilon_0 r^2} \vec{u}_r & r \geq R \end{cases}$$

$$\vec{E} = -\vec{\nabla} \phi \Rightarrow \vec{\nabla} \phi = \frac{\partial \phi}{\partial r} \vec{u}_r$$

$$\frac{\partial \phi}{\partial r} = \begin{cases} 0 & r < R \\ -\frac{\sigma_0 R^2}{\epsilon_0 r^2} & r \geq R \end{cases} \Rightarrow \phi(r) = \begin{cases} C & r \leq R \\ +\frac{\sigma_0 R^2}{\epsilon_0 r} & r \geq R \end{cases}$$

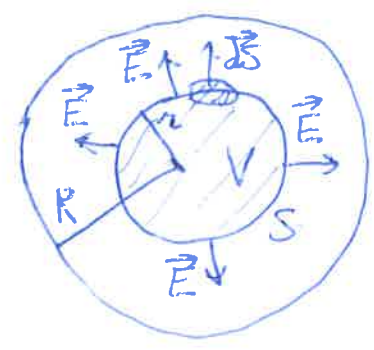
ϕ debe ser continua en $r=R \Rightarrow C = \frac{\sigma_0 R^2}{\epsilon_0 R} = \frac{\sigma_0 R}{\epsilon_0}$

$$\phi(r) = \begin{cases} +\frac{\sigma_0 R}{\epsilon_0} & r \leq R \\ +\frac{\sigma_0 R^2}{\epsilon_0 r} & r \geq R \end{cases}$$

Autor: Iván Cabria Álvarez

8.19) Same Igual que el problema 8.17, pero con $\rho = \rho_0 z^2 / R^2$

$z < R$



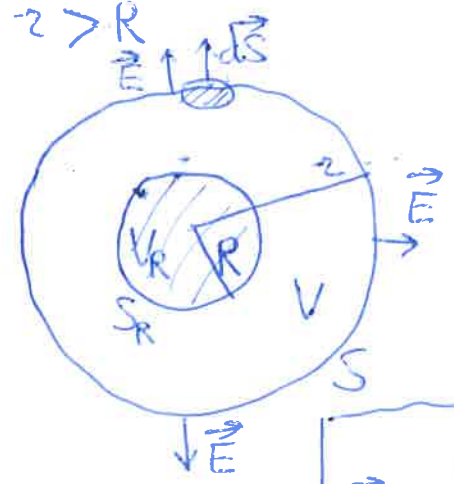
$$\oint_S \vec{E} \cdot d\vec{S} = E 4\pi z^2 \quad dV = z^2 \sin\theta d\theta d\phi dz$$

$$Q_{enc S} = \int_V \rho dV = \int_0^{2\pi} \int_0^\pi \int_0^z \rho_0 \frac{z^2}{R^2} z^2 dz d\theta d\phi$$

$$= 4\pi \int_0^z \rho_0 \frac{z^4}{R^2} dz = 4\pi \rho_0 \frac{z^5}{5R^2}$$

Gauss $\Rightarrow E 4\pi z^2 = \frac{4\pi \rho_0 z^5}{5R^2 \epsilon_0} \Rightarrow E = \frac{4\pi \rho_0 z^5}{5 \cdot 4\pi z^2 \epsilon_0 R^2} = \frac{\rho_0 z^3}{5R^2 \epsilon_0}$

$z > R$



$$\oint_S \vec{E} \cdot d\vec{S} = E 4\pi z^2$$

$$Q_{enc S} = \int_V \rho dV = \int_{V_R} \rho dV = 4\pi \int_0^R \rho_0 \frac{z^4}{R^2} dz = \frac{4\pi \rho_0 R^3}{5}$$

Gauss $\Rightarrow E 4\pi z^2 = \frac{4\pi \rho_0 R^3}{5 \epsilon_0} \Rightarrow E = \frac{\rho_0 R^3}{5 \epsilon_0 z^2}$

$$\vec{E} = \begin{cases} \frac{\rho_0 z^3}{5R^2 \epsilon_0} \vec{u}_r & z < R \\ \frac{\rho_0 R^3}{5 \epsilon_0 z^2} \vec{u}_r & z > R \end{cases}$$

$$\left. \begin{aligned} \vec{E} &= -\vec{\nabla} \phi \\ \vec{\nabla} \phi &= \frac{\partial \phi}{\partial z} \vec{u}_z \end{aligned} \right\} \Rightarrow \frac{\partial \phi}{\partial z} = \begin{cases} -\frac{\rho_0 z^3}{5R^2 \epsilon_0} & z < R \\ -\frac{\rho_0 R^3}{5 \epsilon_0 z^2} & z > R \end{cases} \Rightarrow \phi(z) = \begin{cases} -\frac{\rho_0 z^4}{20 \epsilon_0 R^2} + C_1 & z \leq R \\ +\frac{\rho_0 R^3}{5 \epsilon_0 z} + C_2 & z \geq R \end{cases}$$

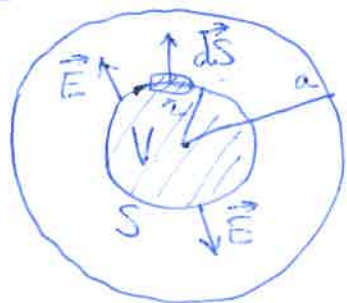
$\phi \rightarrow 0 \Rightarrow C_2 = 0$

ϕ debe ser continua en $z=R \Rightarrow -\frac{\rho_0 R^4}{20 \epsilon_0 R^2} + C_1 = \frac{\rho_0 R^3}{5 \epsilon_0 R} \Rightarrow C_1 = \frac{\rho_0 R^2}{\epsilon_0} \left(\frac{1}{5} + \frac{1}{20} \right)$

$\Rightarrow C_1 = \frac{\rho_0 R^2}{4 \epsilon_0}$

$$\phi(z) = \begin{cases} -\frac{\rho_0 z^4}{20 \epsilon_0 R^2} + \frac{\rho_0 R^2}{4 \epsilon_0} & z \leq R \\ +\frac{\rho_0 R^3}{5 \epsilon_0 z} & z \geq R \end{cases}$$

8.20 $z < a$



$$\oint_S \vec{E} \cdot d\vec{S} = E 4\pi z^2$$

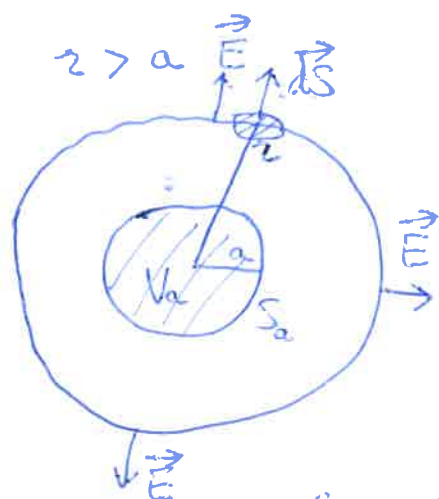
$$Q_{\text{en } S} = \int_V \rho dV = \int_0^{2\pi} \int_0^{\pi} \int_0^z z^2 dz \rho(z) \sin\theta d\theta d\varphi =$$

$$= 4\pi \int_0^z z^2 \rho(z) dz = 4\pi \int_0^z z^2 \rho_0 \left(\frac{3}{4} - \frac{z}{a} \right) dz =$$

$$= 4\pi \rho_0 \int_0^z \left(\frac{3z^2}{4} - \frac{z^3}{a} \right) dz = 4\pi \rho_0 \left[\frac{z^3}{4} - \frac{z^4}{4a} \right]_0^z = 4\pi \rho_0 \left(\frac{z^3}{4} - \frac{z^4}{4a} \right)$$

$$= \pi \rho_0 \left(z^3 - \frac{z^4}{a} \right) \neq \pi \rho_0 z^2 \left(z - \frac{z^2}{a} \right)$$

$$\text{Gauss} \Rightarrow E 4\pi z^2 = \frac{\pi \rho_0 z^2 (z - \frac{z^2}{a})}{\epsilon_0} \Rightarrow E = \frac{\rho_0}{4\epsilon_0} \left(z - \frac{z^2}{a} \right)$$



$$\oint_S \vec{E} \cdot d\vec{S} = E 4\pi z^2$$

$$Q_{\text{en } S} = \int_V \rho dV = \int_{V_a} \rho dV = 4\pi \int_0^a z^2 \rho_0 \left(\frac{3}{4} - \frac{z}{a} \right) dz$$

$$= \pi \rho_0 a^2 \left(a - \frac{a^2}{a} \right) = 0$$

$$\text{Gauss} \Rightarrow E 4\pi z^2 = 0 \Rightarrow E = 0$$

$$\vec{E} = \begin{cases} \frac{\rho_0}{4\epsilon_0} \left(z - \frac{z^2}{a} \right) \vec{u}_z & z < a \\ 0 & z > a \end{cases}$$

$$\vec{E} = -\vec{\nabla} \phi \Rightarrow \frac{\partial \phi}{\partial z} = \begin{cases} -\frac{\rho_0}{4\epsilon_0} \left(z - \frac{z^2}{a} \right) & z < a \\ 0 & z > a \end{cases} \Rightarrow$$

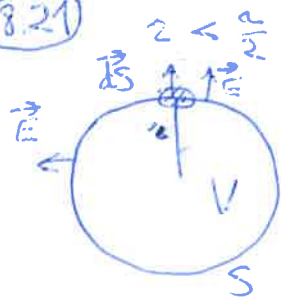
$$\Rightarrow \phi(z) = \begin{cases} -\frac{\rho_0 z^2}{8\epsilon_0} + \frac{\rho_0 z^3}{12\epsilon_0 a} & z < a \\ C & z > a \end{cases} \quad \begin{matrix} \phi \text{ debe ser continua} \\ \text{en } z=a \Rightarrow C = -\frac{\rho_0 a^2}{24\epsilon_0} \end{matrix}$$

$$\phi(z) = \begin{cases} -\frac{\rho_0 z^2}{8\epsilon_0} + \frac{\rho_0 z^3}{12\epsilon_0 a} & z < a \\ -\frac{\rho_0 a^2}{24\epsilon_0} & z \geq a \end{cases}$$

Autor: Iván Gabriel Álvarez

8.17

8.21



$$\oint_S \vec{E} \cdot d\vec{S} = E 4\pi r^2$$

$$Q_{enS} = \int_V \rho dV = 0$$

$$\text{Gauss} \Rightarrow E 4\pi r^2 = 0 \Rightarrow E = 0$$

$$\frac{a}{2} < r < a$$

$$\oint_S \vec{E} \cdot d\vec{S} = E 4\pi r^2$$

$$Q_{enS} = \int_V \rho dV = \int_0^{2\pi} d\varphi \int_0^\pi \sin\theta d\theta \int_0^r r^2 dr \rho(r) = 4\pi \int_0^r r^2 \rho(r) dr =$$

$$= 4\pi \left[\int_0^{a/2} r^2 \rho(r) dr + \int_{a/2}^r r^2 \rho(r) dr \right] = 4\pi \left[0 + \int_{a/2}^r r^2 \rho(r) dr \right] =$$

$$= 4\pi \int_{a/2}^r \rho_0 \sqrt{\frac{r}{a}} r^2 dr = \frac{4\pi \rho_0}{\sqrt{a}} \int_{a/2}^r r^{5/2} dr = \frac{4\pi \rho_0}{\sqrt{a}} \left. \frac{r^{7/2}}{7/2} \right|_{a/2}^r =$$

$$= \frac{8\pi \rho_0}{7\sqrt{a}} \left(r^{7/2} - \left(\frac{a}{2} \right)^{7/2} \right)$$

$$\text{Gauss} \Rightarrow E 4\pi r^2 = \frac{8\pi \rho_0}{7\sqrt{a} \epsilon_0} \left(r^{7/2} - \left(\frac{a}{2} \right)^{7/2} \right) \Rightarrow E = \frac{2\rho_0}{7\epsilon_0 \sqrt{a} r^2} \left(r^{7/2} - \left(\frac{a}{2} \right)^{7/2} \right)$$

$$r > a$$

$$\oint_S \vec{E} \cdot d\vec{S} = E 4\pi r^2$$

$$Q_{enS} =$$

$$\int_V \rho dV = \int_{V_a} \rho dV = 4\pi \int_{a/2}^a \rho_0 \sqrt{\frac{r}{a}} r^2 dr =$$

$$= \frac{8\pi \rho_0}{7\sqrt{a}} \left(a^{7/2} - \left(\frac{a}{2} \right)^{7/2} \right) = \frac{8\pi \rho_0 a^3 \sqrt{a}}{7\sqrt{a}} \left(1 - \frac{1}{8\sqrt{2}} \right) = \frac{8\pi \rho_0 a^3}{7} \left(1 - \frac{1}{8\sqrt{2}} \right)$$

$$\text{Gauss} \Rightarrow E 4\pi r^2 = \frac{8\pi \rho_0 a^3}{7\epsilon_0} \left(1 - \frac{1}{8\sqrt{2}} \right) \Rightarrow E = \frac{2\rho_0 a^3}{7\epsilon_0 r^2} \left(1 - \frac{1}{8\sqrt{2}} \right)$$

Autor: Juan Cabrera Álvarez

$$\vec{E} = \begin{cases} 0 & r < \frac{a}{2} \\ \frac{2\rho_0}{7\epsilon_0\sqrt{a}} r^2 \left(r^{7/2} - \left(\frac{a}{2}\right)^{7/2} \right) \vec{u}_r & \frac{a}{2} < r < a \\ \frac{2\rho_0 a^3}{7\epsilon_0 r^2} \left(1 - \frac{1}{8\sqrt{2}} \right) \vec{u}_r & r > a \end{cases}$$

$$\vec{E} = -\vec{\nabla}\phi \Rightarrow \frac{\partial\phi}{\partial r} = \begin{cases} 0 & r < \frac{a}{2} \\ \frac{-2\rho_0}{7\epsilon_0\sqrt{a}} r^2 \left(r^{7/2} - \left(\frac{a}{2}\right)^{7/2} \right) & \frac{a}{2} < r < a \\ \frac{-2\rho_0 a^3}{7\epsilon_0 r^2} \left(1 - \frac{1}{8\sqrt{2}} \right) & r > a \end{cases} \Rightarrow$$

$$\Rightarrow \phi(r) = \begin{cases} C & r < \frac{a}{2} \\ -\frac{2\rho_0}{7\epsilon_0\sqrt{a}} \left(\frac{2}{5} r^{5/2} + \frac{1}{2} \left(\frac{a}{2}\right)^{7/2} \right) + D & \frac{a}{2} < r < a \\ + \frac{2\rho_0 a^3}{7\epsilon_0 r} \left(1 - \frac{1}{8\sqrt{2}} \right) & r > a \end{cases}$$

$$\phi \text{ debe ser continua en } r=a \Rightarrow -\frac{2\rho_0}{7\epsilon_0\sqrt{a}} \left(\frac{2}{5} a^{5/2} + \frac{1}{2} \left(\frac{a}{2}\right)^{7/2} \right) + D = \frac{2\rho_0 a^3}{7\epsilon_0 a} \left(1 - \frac{1}{8\sqrt{2}} \right)$$

$$\Rightarrow -\frac{2\rho_0 a^{5/2}}{7\epsilon_0\sqrt{a}} \left(\frac{2}{5} + \frac{1}{8\sqrt{2}} \right) + D = \frac{2\rho_0 a^2}{7\epsilon_0} \left(1 - \frac{1}{8\sqrt{2}} \right)$$

$$\Rightarrow D = \frac{2\rho_0 a^2}{7\epsilon_0} \left(1 - \frac{1}{8\sqrt{2}} + \frac{2}{5} + \frac{1}{8\sqrt{2}} \right) = \frac{2\rho_0 a^2}{7\epsilon_0} \left(1 + \frac{2}{5} \right) = \frac{2\rho_0 a^2}{5\epsilon_0}$$

$$\phi \text{ debe ser continua en } r=\frac{a}{2} \Rightarrow C = -\frac{2\rho_0}{7\epsilon_0\sqrt{a}} \left(\frac{2}{5} \left(\frac{a}{2}\right)^{5/2} + \frac{2}{a} \left(\frac{a}{2}\right)^{7/2} \right) + \frac{2\rho_0 a^2}{5\epsilon_0}$$

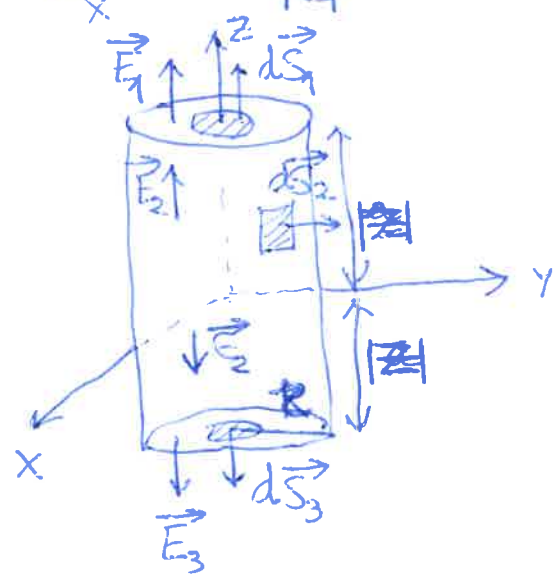
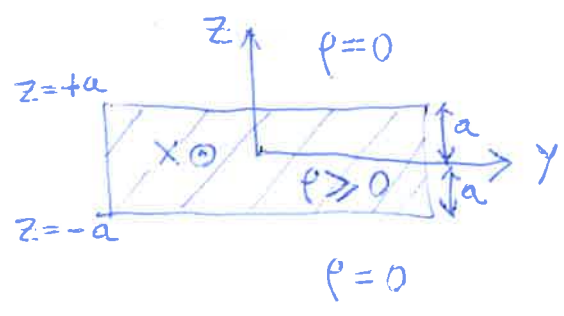
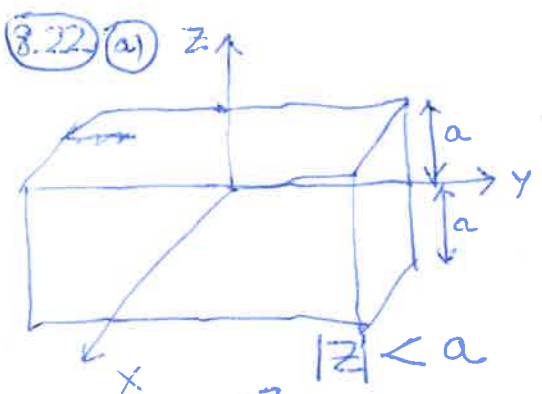
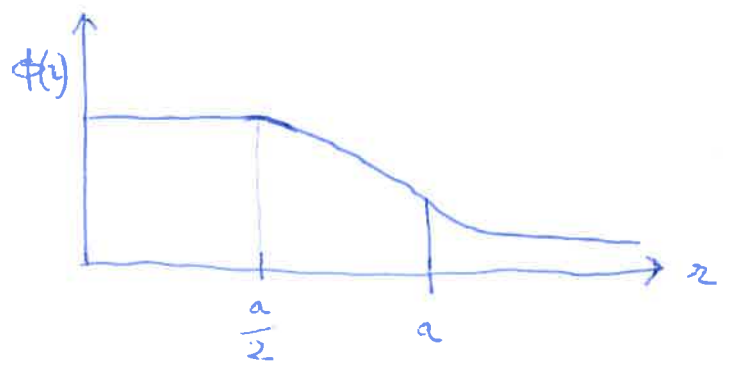
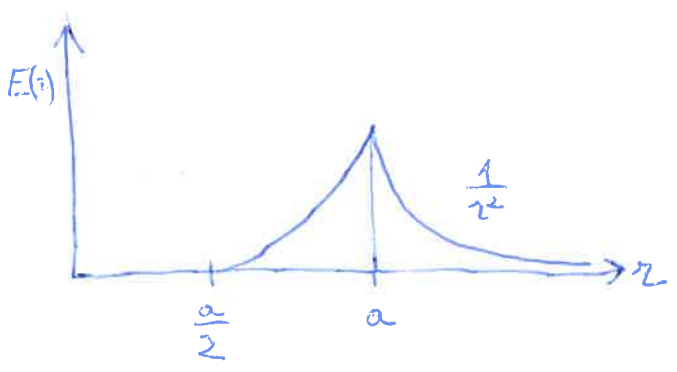
$$\Rightarrow C = -\frac{2\rho_0 a^{5/2}}{7\epsilon_0\sqrt{a}} \left(\frac{2}{5} \frac{1}{4\sqrt{2}} + \frac{1}{4\sqrt{2}} \right) + \frac{2\rho_0 a^2}{5\epsilon_0}$$

$$= -\frac{2\rho_0 a^2}{7\epsilon_0 4\sqrt{2}} \left(\frac{2}{5} + 1 \right) + \frac{2\rho_0 a^2}{5\epsilon_0} = -\frac{2\rho_0 a^2}{7\epsilon_0 4\sqrt{2}} \frac{7}{5} + \frac{2\rho_0 a^2}{5\epsilon_0}$$

Autor: Iván Cabris Flores

$$C = \frac{-2\rho_0 a^2}{5\epsilon_0 4\sqrt{2}} + \frac{2\rho_0 a^2}{5\epsilon_0} = \frac{2\rho_0 a^2}{5\epsilon_0} \left(1 - \frac{1}{4\sqrt{2}}\right)$$

$$\phi(z) = \begin{cases} \frac{2\rho_0 a^2}{5\epsilon_0} \left(1 - \frac{1}{4\sqrt{2}}\right) & z \leq \frac{a}{2} \\ -\frac{2\rho_0}{7\epsilon_0 \sqrt{a}} \left(\frac{2}{5} z^{5/2} + \frac{1}{2} \left(\frac{a}{2}\right)^{7/2} \right) + \frac{2\rho_0 a^2}{5\epsilon_0} & \frac{a}{2} \leq z \leq a \\ \frac{2\rho_0 a^3}{7\epsilon_0 z} \left(1 - \frac{1}{8\sqrt{2}}\right) & z \geq a \end{cases}$$



$$\begin{aligned} \vec{E}_1 &= E_1 \vec{u}_z \\ \vec{E}_2 &= \begin{cases} E_2 \vec{u}_z & z > 0 \\ -E_2 \vec{u}_z & z < 0 \end{cases} \\ \vec{E}_3 &= -E_3 \vec{u}_z \end{aligned}$$

$E_1 = E_3 = E > 0$
 $dS_1 = dS_3$

$$\begin{aligned} \vec{E}_1 \cdot d\vec{S}_1 &= E_1 dS_1 \cos 0^\circ = E_1 dS_1 \\ \vec{E}_2 \cdot d\vec{S}_2 &= E_2 dS_2 \cos 90^\circ = 0 \\ \vec{E}_3 \cdot d\vec{S}_3 &= E_3 dS_3 \cos 0^\circ = E_3 dS_3 \end{aligned}$$

$$\oint_S \vec{E} \cdot d\vec{S} = \int_{S_1} \vec{E}_1 \cdot d\vec{S}_1 + \int_{S_2} \vec{E}_2 \cdot d\vec{S}_2 + \int_{S_3} \vec{E}_3 \cdot d\vec{S}_3 = \int_{S_1} E_1 dS_1 + \int_{S_3} E_3 dS_3 = E_1 \int_{S_1} dS_1 + E_3 \int_{S_3} dS_3$$

$$\oint_S \vec{E} \cdot d\vec{S} = 2E \int_{S_1} dS_1 = 2E \pi r^2$$

$$Q_{enS} = \int_V \rho dV = \int_0^{2\pi} d\varphi \int_0^r dr r \int_{-z}^z dz \rho(z) = 2\pi \frac{r^2}{2} \int_{-z}^z dz \rho_0 \left(1 - \frac{|z|}{a}\right) =$$

$$= \pi r^2 \rho_0 \int_{-z}^z \left(1 - \frac{|z|}{a}\right) dz = \pi r^2 \rho_0 \left[\int_{-z}^0 \left(1 + \frac{z}{a}\right) dz + \int_0^z \left(1 - \frac{z}{a}\right) dz \right]$$

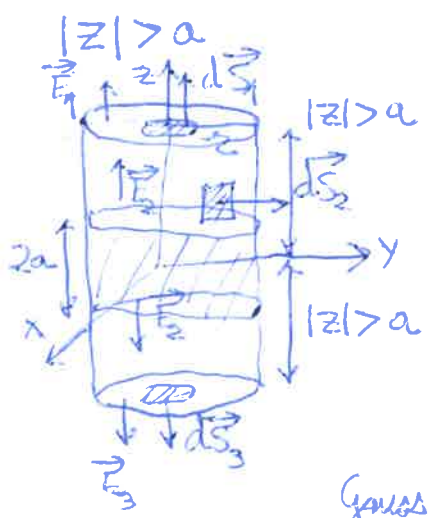
$$= \pi r^2 \rho_0 \left[\left(z + \frac{z^2}{2a}\right) \Big|_{-z}^0 + \left(z - \frac{z^2}{2a}\right) \Big|_0^z \right] = \pi r^2 \rho_0 \left[z - \frac{(-z)^2}{2a} + z - \frac{z^2}{2a} \right]$$

$$= 2\pi r^2 \rho_0 \left(z - \frac{z^2}{2a}\right)$$

$$Gauss \Rightarrow 2E \pi r^2 = \frac{2\pi r^2 \rho_0}{\epsilon_0} \left(z - \frac{z^2}{2a}\right) \Rightarrow E = \frac{\rho_0}{\epsilon_0} \left(z - \frac{z^2}{2a}\right)$$

$$\vec{E} = \begin{cases} \frac{\rho_0}{\epsilon_0} \left(z - \frac{z^2}{2a}\right) \vec{u}_z & 0 < z < a \\ -\frac{\rho_0}{\epsilon_0} \left(z - \frac{z^2}{2a}\right) \vec{u}_z & -a < z < 0 \end{cases}$$

$$= \frac{\rho_0}{\epsilon_0} \left(z - \frac{z^2}{2a}\right) \frac{z}{|z|} \vec{u}_z \quad |z| < a$$



$$\oint_S \vec{E} \cdot d\vec{S} = 2E \pi r^2$$

$$Q_{enS} = \int_V \rho dV = \int_{V_a} \rho dV = \pi r^2 \rho_0 \int_{-a}^a \left(1 - \frac{|z|}{a}\right) dz$$

$$= 2\pi r^2 \rho_0 \left(a - \frac{a^2}{2a}\right) = \pi r^2 \rho_0 a$$

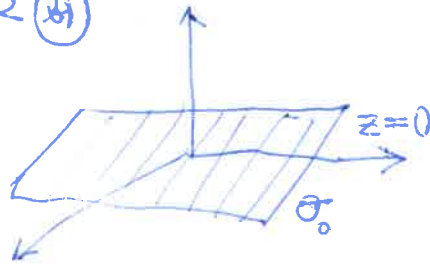
$$Gauss \Rightarrow 2E \pi r^2 = \pi r^2 \rho_0 a / \epsilon_0 \Rightarrow E = \frac{\rho_0 a}{2\epsilon_0}$$

$$\vec{E} = \begin{cases} \frac{\rho_0 a}{2\epsilon_0} \vec{u}_z & z > a \\ -\frac{\rho_0 a}{2\epsilon_0} \vec{u}_z & z < -a \end{cases}$$

$$= \frac{\rho_0 a}{2\epsilon_0} \frac{z}{|z|} \vec{u}_z \quad |z| > a$$

r

8.22 (b)



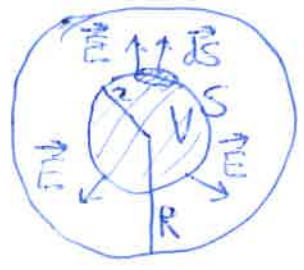
$$\left. \begin{aligned} \vec{E}_{\sigma_0} &= \frac{\sigma_0}{2\epsilon_0} \frac{z}{|z|} \vec{u}_z \\ \vec{E}_{|z|>a} &= \frac{\rho_0 a}{2\epsilon_0} \frac{z}{|z|} \vec{u}_z \\ \vec{E}_{\sigma_0} &= \vec{E}_{|z|>a} \end{aligned} \right\} \Rightarrow \frac{\sigma_0}{2\epsilon_0} = \frac{\rho_0 a}{2\epsilon_0} \Rightarrow$$

$$\Rightarrow \boxed{\sigma_0 = \rho_0 a}$$

8.23 (a)

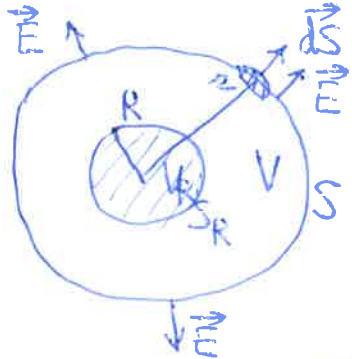
$$\begin{aligned} \Phi_T &= \int_V \rho dV = \int_0^{2\pi} d\varphi \int_0^\pi \sin\theta d\theta \int_0^R r^2 dr \rho_0 \left(\frac{r}{a}\right) = 4\pi \frac{\rho_0}{a} \int_0^R r^3 dr = \frac{4\pi\rho_0}{a} \frac{R^4}{4} \\ &= \boxed{\frac{\pi\rho_0 R^4}{a}} \end{aligned}$$

(b) $z < R$



$$\begin{aligned} \oint_S \vec{E} \cdot d\vec{S} &= E 4\pi z^2 \\ \Phi_{en.S} &= \int_V \rho dV = \int_0^{2\pi} d\varphi \int_0^\pi \sin\theta d\theta \int_0^z r^2 dr \rho_0 \left(\frac{r}{a}\right) = \frac{\pi\rho_0 z^4}{a} \\ \text{Gauss} \Rightarrow E 4\pi z^2 &= \frac{\pi\rho_0 z^4}{\epsilon_0 a} \Rightarrow E = \frac{\pi\rho_0 z^4}{4\pi\epsilon_0 a z^2} = \frac{\rho_0 z^2}{4\epsilon_0 a} \end{aligned}$$

$z > R$



$$\begin{aligned} \oint_S \vec{E} \cdot d\vec{S} &= E 4\pi z^2 \\ \Phi_{en.S} &= \int_V \rho dV = \int_{V_R} \rho dV = \Phi_T = \frac{\pi\rho_0 R^4}{a} \\ \text{Gauss} \Rightarrow E 4\pi z^2 &= \frac{\pi\rho_0 R^4}{a\epsilon_0} \Rightarrow E = \frac{\pi\rho_0 R^4}{4\pi\epsilon_0 a z^2} = \frac{\rho_0 R^4}{4\epsilon_0 a z^2} \end{aligned}$$

$$\boxed{\vec{E} = \begin{cases} \frac{\rho_0 z^2}{4a\epsilon_0} \vec{u}_z & z < R \\ \frac{\rho_0 R^4}{4a\epsilon_0 z^2} \vec{u}_z & z > R \end{cases}}$$

8.23 (c)

$$\vec{E} = -\vec{\nabla}\phi \Rightarrow \frac{\partial \phi}{\partial z} = \begin{cases} -\frac{\rho_0 z^2}{4a\epsilon_0} & z < R \\ -\frac{\rho_0 R^4}{4a\epsilon_0 z^2} & z > R \end{cases} \Rightarrow \phi(z) = \begin{cases} -\frac{\rho_0 z^3}{12a\epsilon_0} + C & z \leq R \\ +\frac{\rho_0 R^4}{4a\epsilon_0 z} & z \geq R \end{cases}$$

ϕ debe ser continua en $z=R \Rightarrow -\frac{\rho_0 R^3}{12a\epsilon_0} + C = \frac{\rho_0 R^4}{4a\epsilon_0 R} \Rightarrow$

$$\Rightarrow C = \frac{\rho_0 R^3}{a\epsilon_0} \left(\frac{1}{4} + \frac{1}{12} \right) = \frac{\rho_0 R^3}{a\epsilon_0} \frac{4}{12} = \frac{\rho_0 R^3}{3a\epsilon_0}$$

$$\phi(z) = \begin{cases} -\frac{\rho_0 z^3}{12a\epsilon_0} + \frac{\rho_0 R^3}{3a\epsilon_0} & z \leq R \\ +\frac{\rho_0 R^4}{4a\epsilon_0 z} & z \geq R \end{cases}$$

1.) $\vec{F}(R) = q_0 \vec{E}(R) = \boxed{\frac{q_0 \rho_0 R^2}{4a\epsilon_0} \vec{e}_z}$

$$V(R) = q_0 \phi(R) = \frac{q_0 \rho_0}{4a\epsilon_0} \frac{R^4}{R} = \boxed{\frac{q_0 \rho_0 R^3}{4a\epsilon_0}}$$

2.) $E_z = T_z + V_z = 0 + V(R)$

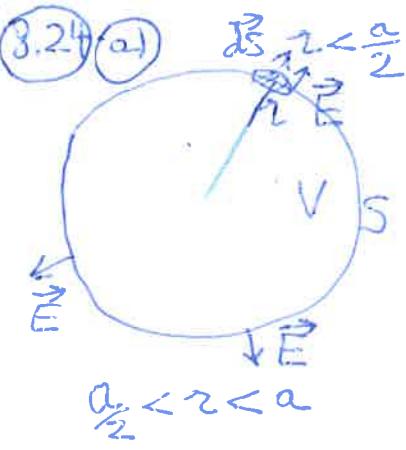
$$\left. \begin{aligned} E_z &= T_z + V_z = \frac{m}{2} \dot{\theta}_z^2 + V(2R) \\ E_z &= E_z \end{aligned} \right\} \Rightarrow V(R) = \frac{m}{2} \dot{\theta}_z^2 + V(2R) \Rightarrow$$

$$\Rightarrow (V(R) - V(2R)) \frac{2}{m} = \dot{\theta}_z^2 \Rightarrow \dot{\theta}_z = \sqrt{\frac{2(V(R) - V(2R))}{m}}$$

$$V(2R) = q_0 \phi(2R) = q_0 \frac{\rho_0 R^4}{4a\epsilon_0 2R} = \frac{q_0 \rho_0 R^3}{8a\epsilon_0} \Rightarrow V(R) - V(2R) = \frac{q_0 \rho_0 R^3}{4a\epsilon_0} - \frac{q_0 \rho_0 R^3}{8a\epsilon_0} = \frac{q_0 \rho_0 R^3}{8a\epsilon_0}$$

$$\dot{\theta}_z = \sqrt{\frac{2 q_0 \rho_0 R^3}{m 8a\epsilon_0}} = \boxed{\sqrt{\frac{q_0 \rho_0 R^3}{m 4a\epsilon_0}}}$$

Autor: Iván Cabria Álvarez

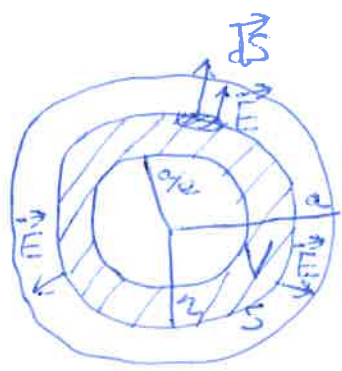


$$\oint_S \vec{E} \cdot d\vec{S} = E 4\pi r^2$$

$$Q_{enS} = \int_V \rho dV = 0 \Rightarrow E 4\pi r^2 = 0 \Rightarrow E = 0$$

Gauss

$$\vec{E} = E \vec{u}_r \quad d\vec{S} = dS \vec{u}_r$$



$$\oint_S \vec{E} \cdot d\vec{S} = E 4\pi r^2$$

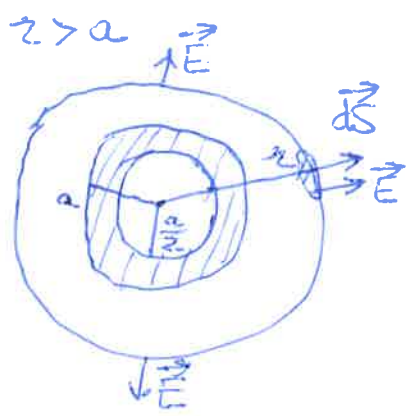
$$Q_{enS} = \int_V \rho dV = \int_0^{2\pi} \int_0^{\pi} \int_0^r r^2 dr \sin\theta d\theta d\phi \rho(r) =$$

$$= 4\pi \int_0^r r^2 \rho(r) dr = 4\pi \int_{a/2}^r r \rho_0 a dr = 4\pi \rho_0 a \frac{r^2}{2} \Big|_{a/2}^r =$$

$$= 2\pi \rho_0 a \left(r^2 - \frac{a^2}{4} \right)$$

Gauss $\Rightarrow E 4\pi r^2 = \frac{2\pi \rho_0 a}{\epsilon_0} \left(r^2 - \frac{a^2}{4} \right) \Rightarrow E = \frac{2\pi \rho_0 a}{4\pi \epsilon_0 r^2} \left(r^2 - \frac{a^2}{4} \right) = \frac{\rho_0 a}{2\epsilon_0 r^2} \left(r^2 - \frac{a^2}{4} \right)$

$$= \frac{\rho_0 a}{2\epsilon_0} \left(1 - \frac{a^2}{4r^2} \right)$$



$$\oint_S \vec{E} \cdot d\vec{S} = E 4\pi r^2$$

$$Q_{enS} = 4\pi \int_0^r r^2 \rho(r) dr = 4\pi \int_{a/2}^a r \rho_0 a dr =$$

$$= 4\pi \rho_0 a \int_{a/2}^a r dr = 4\pi \rho_0 a \frac{r^2}{2} \Big|_{a/2}^a = 2\pi \rho_0 a \left(a^2 - \frac{a^2}{4} \right)$$

$$= 2\pi \rho_0 a^3 \left(1 - \frac{1}{4} \right) = 2\pi \rho_0 a^3 \frac{3}{4} = \frac{3}{2} \pi \rho_0 a^3$$

Gauss $\Rightarrow E 4\pi r^2 = \frac{3\pi \rho_0 a^3}{2\epsilon_0} \Rightarrow E = \frac{3 \rho_0 a^3}{8\epsilon_0 r^2}$

$$\vec{E} = \begin{cases} 0 & r < \frac{a}{2} \\ \frac{\rho_0 a}{2\epsilon_0} \left(1 - \frac{a^2}{4r^2} \right) \vec{u}_r & \frac{a}{2} < r < a \\ \frac{3\rho_0 a^3}{8\epsilon_0 r^2} \vec{u}_r & r > a \end{cases}$$

(8.25)

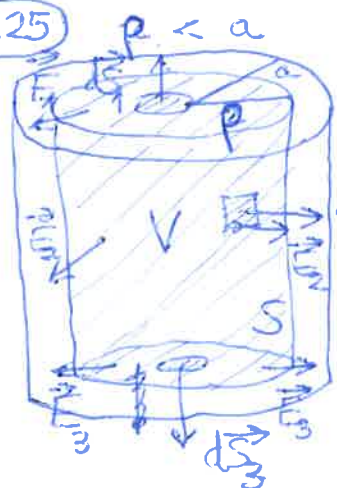
$$8.24 \text{ (b)} \quad \frac{a}{2} < \frac{2a}{3} < a \Rightarrow \vec{E}_e = -e\vec{E}\left(\frac{2a}{3}\right) = -e \frac{\rho_0 a}{2\epsilon_0} \left(1 - \frac{a^2}{4\left(\frac{2a}{3}\right)^2}\right) \vec{u}_z$$

$$= -\frac{e\rho_0 a}{2\epsilon_0} \left(1 - \frac{9}{16}\right) \vec{u}_z = \boxed{-\frac{e\rho_0 a}{2\epsilon_0} \frac{7}{16} \vec{u}_z}$$

$$\vec{E}_e = -e\vec{E}\left(\frac{a}{3}\right) = \boxed{0}$$

$\uparrow$
 $\frac{a}{3} < \frac{a}{2}$

8.25



$$\begin{aligned} \vec{E}_1 \cdot d\vec{S}_1 &= 0 \\ \vec{E}_3 \cdot d\vec{S}_3 &= 0 \\ \vec{E}_2 \cdot d\vec{S}_2 &= E_2 dS_2 \quad E = E_2 \end{aligned}$$

$$\oint_S \vec{E} \cdot d\vec{S} = \sum_{i=1}^3 \vec{E}_i \cdot d\vec{S}_i = \int_{S_2} E_2 dS_2$$

$$\oint_S \vec{E} \cdot d\vec{S} = E \int_{S_2} dS_2 = E 2\pi\rho 2L = E 4\pi\rho L$$

$$Q_{enS} = \int_V \rho dV = \int_0^{2\pi} d\varphi \int_{-L}^L dz \int_0^\rho \rho(\rho) d\rho = 4\pi L \int_0^\rho \rho(\rho) d\rho =$$

$$= 4\pi L \int_0^\rho \rho_0 \rho \sin\left(\frac{\pi\rho}{a}\right) d\rho = 4\pi L \rho_0 \int_0^\rho \rho \sin\left(\frac{\pi\rho}{a}\right) d\rho$$

$$\int_0^\rho \rho \sin\left(\frac{\pi\rho}{a}\right) d\rho = \frac{\sin\left(\frac{\pi\rho}{a}\right)}{\left(\frac{\pi}{a}\right)^2} - \frac{\rho \cos\left(\frac{\pi\rho}{a}\right)}{\left(\frac{\pi}{a}\right)} = \frac{a^2}{\pi^2} \sin\left(\frac{\pi\rho}{a}\right) - \frac{a}{\pi} \rho \cos\left(\frac{\pi\rho}{a}\right)$$

$$Q_{enS} = 4\pi L \rho_0 \left[\frac{a^2}{\pi^2} \sin\left(\frac{\pi\rho}{a}\right) - \frac{a}{\pi} \rho \cos\left(\frac{\pi\rho}{a}\right) \right] = 4aL\rho_0 \left(\frac{a}{\pi} \sin\left(\frac{\pi\rho}{a}\right) - \rho \cos\left(\frac{\pi\rho}{a}\right) \right)$$

$$\text{Gauss} \Rightarrow E 4\pi\rho L = \frac{4aL\rho_0}{\epsilon_0} \left(\frac{a}{\pi} \sin\left(\frac{\pi\rho}{a}\right) - \rho \cos\left(\frac{\pi\rho}{a}\right) \right) \Rightarrow$$

$$\Rightarrow E = \frac{4aL\rho_0}{4\pi\rho L\epsilon_0} \left(\frac{a}{\pi} \sin\left(\frac{\pi\rho}{a}\right) - \rho \cos\left(\frac{\pi\rho}{a}\right) \right) \Rightarrow \boxed{\vec{E} = \frac{a\rho_0}{\pi\rho\epsilon_0} \left(\frac{a}{\pi} \sin\left(\frac{\pi\rho}{a}\right) - \rho \cos\left(\frac{\pi\rho}{a}\right) \right) \vec{u}_z}$$

$\rho < a$

 $\rho > a$

$$\oint_S \vec{E} \cdot d\vec{S} = E 4\pi\rho L$$

$$Q_{enS} = 4\pi L \int_0^\rho \rho \rho(\rho) d\rho = 4\pi L \int_0^a \rho \rho_0 \sin\left(\frac{\pi\rho}{a}\right) d\rho = 4aL\rho_0 \left(\frac{a}{\pi} \sin\pi - a \cos\pi \right)$$

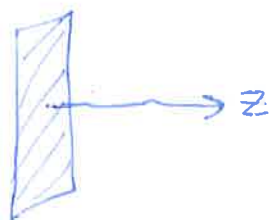
$$= 4a^2 L \rho_0$$

Autor: Iván Abria Alvarado

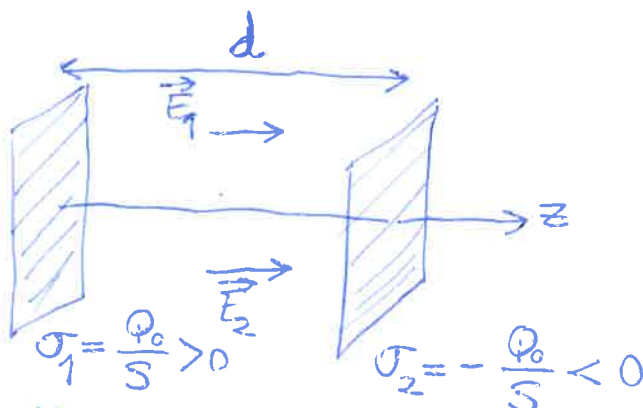
Gauss $\Rightarrow E 4\pi \rho L = \frac{4L \rho_0 a^2}{\epsilon_0} \Rightarrow E = \frac{4L \rho_0 a^2}{4\pi \rho L \epsilon_0} = \frac{\rho_0 a^2}{\pi \rho \epsilon_0}$

$$\vec{E} = \frac{\rho_0 a^2}{\pi \rho \epsilon_0} \vec{u}_z \quad \rho > a$$

8.26. Sin usar el teorema de Gauss:



$$\vec{E} = \frac{\sigma_0}{2\epsilon_0} \frac{z}{|z|} \vec{u}_z$$



$$\vec{E}_1 = \frac{\sigma_1}{2\epsilon_0} \frac{z}{|z|} \vec{u}_z$$

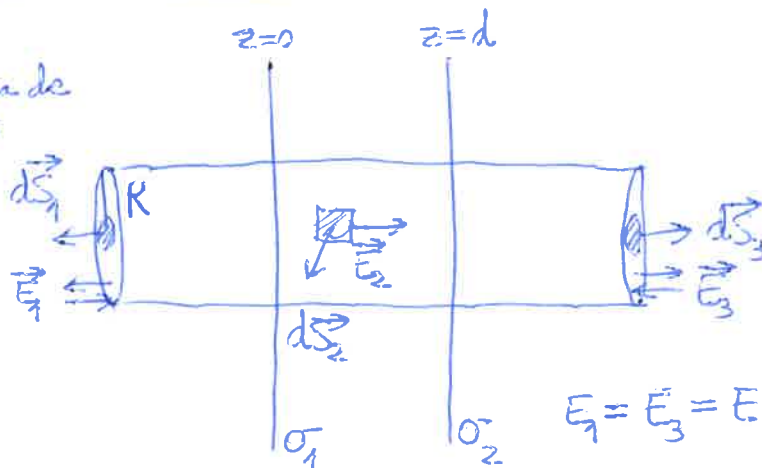
$$\vec{E}_2 = \frac{\sigma_2}{2\epsilon_0} \frac{(z-d)}{|z-d|} \vec{u}_z$$

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \frac{\sigma_1}{2\epsilon_0} \frac{z}{|z|} \vec{u}_z + \frac{\sigma_2}{2\epsilon_0} \frac{(z-d)}{|z-d|} \vec{u}_z = \frac{\rho_0}{2\epsilon_0 S} \frac{z}{|z|} \vec{u}_z - \frac{\rho_0}{2\epsilon_0 S} \frac{(z-d)}{|z-d|} \vec{u}_z$$

$$\vec{E} = \frac{\rho_0}{2\epsilon_0 S} \vec{u}_z + \frac{\rho_0}{2\epsilon_0 S} \vec{u}_z = \frac{\rho_0}{\epsilon_0 S} \vec{u}_z \quad 0 < z < d$$

Usando el teorema de Gauss:

Campo fuera de las placas:



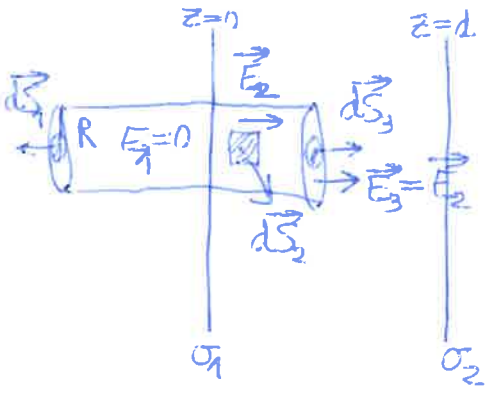
$$\vec{E}_1 \cdot d\vec{S}_1 = \pm E_1 dS_1 \quad \vec{E}_2 \cdot d\vec{S}_2 = 0 \quad \vec{E}_3 \cdot d\vec{S}_3 = \pm E_3 dS_3 \quad S_1 = S_3$$

$$\oint_S \vec{E} \cdot d\vec{S} = \sum_{i=1}^3 \int_{S_i} \vec{E}_i \cdot d\vec{S}_i = \int_{S_1} E_1 dS_1 + \int_{S_3} E_3 dS_3 = 2 \int_{S_1} E_1 dS_1 = 2 E_1 \int_{S_1} dS_1 = 2 E_1 \pi R^2 \quad \parallel \quad 2 E \pi R^2$$

$$Q_{\text{en}} S = \sigma_1 S_R + \sigma_2 S_R = (\sigma_1 + \sigma_2) \pi R^2 = \left(\frac{\rho_0}{S} - \frac{\rho_0}{S} \right) \pi R^2 = 0$$

$$\text{Gauss} \Rightarrow 2 E \pi R^2 = 0 \Rightarrow E = 0 \Rightarrow \vec{E}_1 = 0 \text{ y } \vec{E}_3 = 0$$

Autor: Iván Cabria Álvarez



$$\vec{E}_1 \cdot d\vec{S}_1 = 0$$

$$\vec{E}_1 = 0$$

$$\vec{E}_2 \cdot d\vec{S}_2 = E_2 dS_2 \cos 90^\circ = 0$$

$$\vec{E}_3 \cdot d\vec{S}_3 = E_3 dS_3$$

$$E_2 = E_3 = E$$

$$\vec{E}_2 = E \vec{u}_z$$

$$\vec{E} = \vec{E}_2$$

$$\oint_S \vec{E} \cdot d\vec{S} = \sum_{i=1}^3 \int_{S_i} \vec{E}_i \cdot d\vec{S}_i = \int_{S_3} E_3 dS_3 = E_3 \int_{S_3} dS_3 = E_3 \pi R^2$$

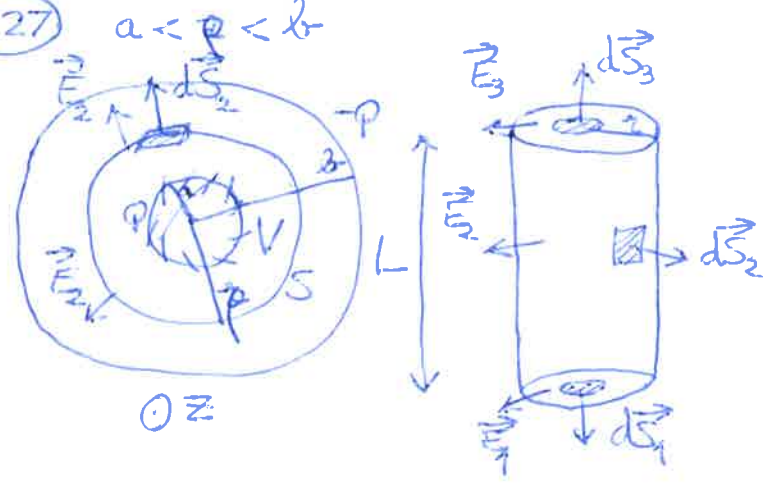
$$= E \pi R^2$$

$$Q_{\text{en } S} = \sigma_1 S_R = \frac{\rho_0}{S} \pi R^2$$

$$\text{Gauss} \Rightarrow E \pi R^2 = \frac{\rho_0}{S} \frac{\pi R^2}{\epsilon_0} \Rightarrow E = \frac{\rho_0}{S \epsilon_0} \Rightarrow \boxed{\vec{E} = \frac{\rho_0}{S \epsilon_0} \vec{u}_z \quad 0 < z < d}$$

$$\vec{E} = -\vec{\nabla} \phi \Rightarrow \frac{\partial \phi}{\partial z} = -\frac{\rho_0}{S \epsilon_0} \Rightarrow \boxed{\phi(z) = -\frac{\rho_0 z}{S \epsilon_0} \quad 0 < z < d}$$

8.27 $a < \rho < b$



$$\vec{E}_1 \cdot d\vec{S}_1 = 0$$

$$\vec{E}_3 \cdot d\vec{S}_3 = 0$$

$$\vec{E}_2 \cdot d\vec{S}_2 = E_2 dS_2$$

$$E_2 = E$$

$$\vec{E}_2 = \vec{E} = E \vec{u}_z$$

$$\oint_S \vec{E} \cdot d\vec{S} = \sum_{i=1}^3 \int_{S_i} \vec{E}_i \cdot d\vec{S}_i = \int_{S_2} E_2 dS_2$$

$$= E \int_{S_2} dS_2 = E 2\pi \rho L$$

$$Q_{\text{en } S} = +Q$$

$$\text{Gauss} \Rightarrow E 2\pi \rho L = \frac{Q}{\epsilon_0} \Rightarrow \vec{E} = \frac{Q}{2\pi \epsilon_0 \rho L} \vec{u}_z$$

$\rho < a$ Nota: no es necesario calcular $\vec{E}$ en $\rho < a$

$$\oint_S \vec{E} \cdot d\vec{S} = E 2\pi \rho L$$

$$\text{Gauss} \Rightarrow E = 0$$

$$Q_{\text{en } S} = 0$$

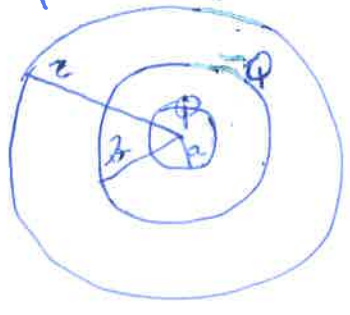
Autor: Juan Carlos Alvarado

$\rho > b$ Nota: no es necesario calcular $\vec{E}$ en $\rho > b$

$$\oint \vec{E} \cdot d\vec{s} = E 2\pi \rho L$$

Gauss $\Rightarrow E=0$

$$Q_{\text{enc}} = Q + (-Q) = 0$$



$$\vec{E} = -\vec{\nabla}\phi \Rightarrow \frac{\partial \phi}{\partial \rho} = \begin{cases} 0 & \rho < a \\ -\frac{Q}{2\pi\epsilon_0 \rho L} & a < \rho < b \\ 0 & \rho > b \end{cases} \Rightarrow \phi(\rho) = \begin{cases} C & \rho \leq a \\ -\frac{Q}{2\pi\epsilon_0 L} \ln(\rho/\rho_0) & a < \rho < b \\ D & \rho \geq b \end{cases}$$

$\phi(z)$ debe ser continua en $\rho = a \Rightarrow C = -\frac{Q}{2\pi\epsilon_0 L} \ln a$

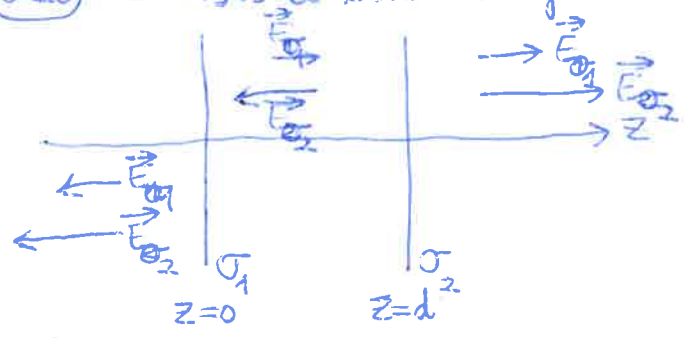
ρ y ρ_0 están en unidades u.
 $\rho_0 = 1u.$

$\phi(z)$ debe ser continua en $\rho = b \Rightarrow D = -\frac{Q}{2\pi\epsilon_0 L} \ln b$

$$\Delta\phi = \phi(a) - \phi(b) = -\frac{Q}{2\pi\epsilon_0 L} \ln a + \frac{Q}{2\pi\epsilon_0 L} \ln b = \frac{Q}{2\pi\epsilon_0 L} \ln\left(\frac{b}{a}\right) \Rightarrow$$

$$\Rightarrow Q = \frac{2\pi L \epsilon_0 \Delta\phi}{\ln\left(\frac{b}{a}\right)}$$

8.28 Sin usar el teorema de Gauss:



$$\sigma_1 > \sigma_2 \quad \sigma_2 > \sigma_1 \quad \sigma_1 < \sigma_2$$

$$\vec{E}_1 = \frac{\sigma_1}{2\epsilon_0} \frac{z}{|z|} \vec{u}_z$$

$$\vec{E}_2 = \frac{\sigma_2}{2\epsilon_0} \frac{(z-d)}{|z-d|} \vec{u}_z$$

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

$$z < 0 \quad \vec{E} = -\frac{\sigma_1}{2\epsilon_0} \vec{u}_z - \frac{\sigma_2}{2\epsilon_0} \vec{u}_z = -\frac{(\sigma_1 + \sigma_2)}{2\epsilon_0} \vec{u}_z = \boxed{-\frac{3}{\epsilon_0} \vec{u}_z \frac{C}{m^2}} \quad z < 0$$

$$0 < z < d \quad \vec{E} = \frac{\sigma_1}{2\epsilon_0} \vec{u}_z - \frac{\sigma_2}{2\epsilon_0} \vec{u}_z = \frac{(\sigma_1 - \sigma_2)}{2\epsilon_0} \vec{u}_z = \boxed{-\frac{\vec{u}_z}{\epsilon_0} \frac{C}{m^2}} \quad 0 < z < d$$

$$z > d \quad \vec{E} = \frac{\sigma_1}{2\epsilon_0} \vec{u}_z + \frac{\sigma_2}{2\epsilon_0} \vec{u}_z = \frac{(\sigma_1 + \sigma_2)}{2\epsilon_0} \vec{u}_z = \boxed{+\frac{3}{\epsilon_0} \vec{u}_z \frac{C}{m^2}} \quad z > d$$

Autor: Iván Cabris Alvarado

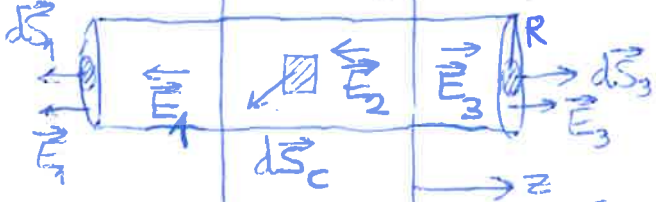
Usando el teorema de Gauss: $\vec{E}_1 = E_1 \vec{u}_z$ $\vec{E}_2 = -E_2 \vec{u}_z$ $\vec{E}_3 = E_3 \vec{u}_z$

$$\sigma_1 < \sigma_2 \Rightarrow \vec{E} = -E_2 \vec{u}_z$$

$$\vec{E}_1 \cdot d\vec{S}_1 = E_1 dS_1 \cos 0^\circ = E_1 dS_1$$

$$\vec{E}_2 \cdot d\vec{S}_2 = E_2 dS_2 \cos 90^\circ = 0$$

$$\vec{E}_3 \cdot d\vec{S}_3 = E_3 dS_3 \cos 0^\circ = E_3 dS_3$$



$$\oint_S \vec{E} \cdot d\vec{S} =$$

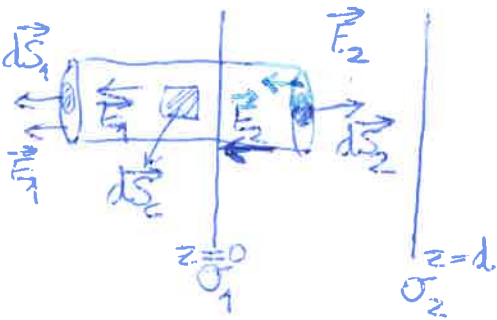
$$\int_{S_1} E_1 dS_1 + \int_{S_3} E_3 dS_3 + \int_{S_2} \vec{E}_2 \cdot d\vec{S}_2 =$$

$$\vec{E} = \begin{cases} \vec{E}_1 & z < 0 \\ \vec{E}_2 & 0 < z < d \\ \vec{E}_3 & z > d \end{cases}$$

$$= E_1 \int_{S_1} dS_1 + E_3 \int_{S_3} dS_3 = (E_1 + E_3) \pi R^2$$

$$Q_{en.S} = \sigma_1 \pi R^2 + \sigma_2 \pi R^2 = (\sigma_1 + \sigma_2) \pi R^2$$

$$\text{Gauss} \Rightarrow (E_1 + E_3) \pi R^2 = \frac{(\sigma_1 + \sigma_2)}{\epsilon_0} \pi R^2 \Rightarrow E_1 + E_3 = \frac{(\sigma_1 + \sigma_2)}{\epsilon_0}$$



$$\oint_S \vec{E} \cdot d\vec{S} = \int_{S_1} \vec{E}_1 \cdot d\vec{S}_1 + \int_{S_2} \vec{E}_2 \cdot d\vec{S}_2 + \int_{S_3} \vec{E}_3 \cdot d\vec{S}_3$$

$$\vec{E}_1 \cdot d\vec{S}_1 = E_1 dS_1 \quad \vec{E}_2 \cdot d\vec{S}_2 = -E_2 dS_2$$

$$\vec{E} = \begin{cases} \vec{E}_1 & z < 0 \\ \vec{E}_2 & 0 < z < d \end{cases} \quad \vec{E}_3 \cdot d\vec{S}_3 = 0$$

$$\oint_S \vec{E} \cdot d\vec{S} = E_1 \int_{S_1} dS_1 - E_2 \int_{S_2} dS_2 = (E_1 - E_2) \pi R^2$$

$$Q_{en.S} = \sigma_1 \pi R^2$$

$$\text{Gauss} \Rightarrow (E_1 - E_2) \pi R^2 = \frac{\sigma_1}{\epsilon_0} \pi R^2 \Rightarrow E_1 - E_2 = \frac{\sigma_1}{\epsilon_0}$$

$$E_1 + E_3 = \frac{\sigma_1 + \sigma_2}{\epsilon_0}$$

Por simetría $E_1 = E_3 \Rightarrow$

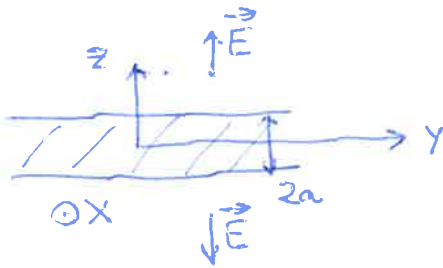
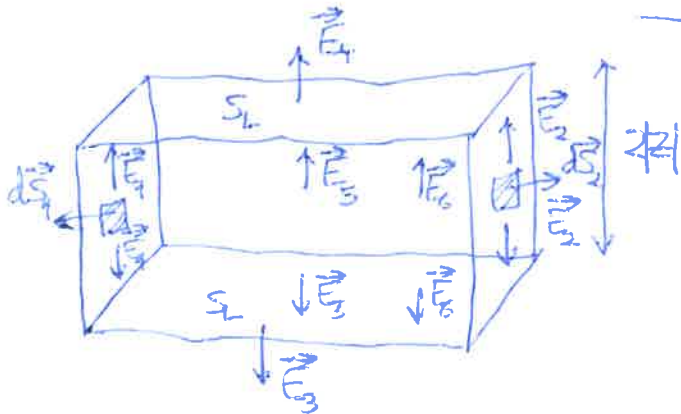
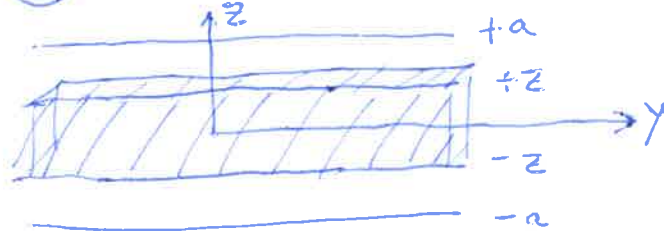
$$E_1 - E_2 = \frac{\sigma_1}{\epsilon_0}$$

$$\left. \begin{aligned} \Rightarrow 2E_1 &= \frac{\sigma_1 + \sigma_2}{\epsilon_0} \Rightarrow E_1 = \frac{\sigma_1 + \sigma_2}{2\epsilon_0} \\ E_1 - E_2 &= \frac{\sigma_1}{\epsilon_0} \Rightarrow E_2 = E_1 - \frac{\sigma_1}{\epsilon_0} \end{aligned} \right\} \Rightarrow E_2 = \frac{\sigma_1 + \sigma_2 - 2\sigma_1}{2\epsilon_0} = \frac{\sigma_2 - \sigma_1}{2\epsilon_0}$$

$$E_3 = E_1 \Rightarrow E_3 = \frac{\sigma_1 + \sigma_2}{2\epsilon_0}$$

$$\vec{E} = \begin{cases} \vec{E}_1 & z < 0 \\ \vec{E}_2 & 0 < z < d \\ \vec{E}_3 & z > d \end{cases} = \begin{cases} -\frac{(\sigma_1 + \sigma_2)}{2\epsilon_0} \vec{u}_z \\ -\frac{(\sigma_2 - \sigma_1)}{2\epsilon_0} \vec{u}_z \\ +\frac{(\sigma_1 + \sigma_2)}{2\epsilon_0} \vec{u}_z \end{cases} = \begin{cases} -\frac{3}{\epsilon_0} \vec{u}_z \frac{C}{m^2} & z < 0 \\ -\frac{\vec{u}_z}{\epsilon_0} \frac{C}{m^2} & 0 < z < d \\ +\frac{3}{\epsilon_0} \vec{u}_z \frac{C}{m^2} & z > d \end{cases}$$

8.29

a) $|z| < a$ 

$$\vec{E}_3 = -E \vec{u}_z \quad \vec{E}_4 = E \vec{u}_z \quad E_3 = E_4 = E$$

$$\vec{E}_2 \cdot d\vec{S}_2 = 0 \quad \vec{E}_1 \cdot d\vec{S}_1 = 0$$

$$\vec{E}_3 \cdot d\vec{S}_3 = E_3 dS_3 \quad \vec{E}_4 \cdot d\vec{S}_4 = E_4 dS_4$$

$$\vec{E}_5 \cdot d\vec{S}_5 = 0 \quad \vec{E}_6 \cdot d\vec{S}_6 = 0$$

$$\oint_S \vec{E} \cdot d\vec{S} = \sum_{i=1}^6 \int_{S_i} \vec{E}_i \cdot d\vec{S}_i = \int_{S_3} \vec{E}_3 \cdot d\vec{S}_3 + \int_{S_4} \vec{E}_4 \cdot d\vec{S}_4 = E_3 \int_{S_3} dS_3 + E_4 \int_{S_4} dS_4 =$$

$$= E_3 S_L + E_4 S_L = 2ES_L$$

$$Q_{\text{en } S} = \int_V \rho dV = \int_0^{L_x} dx \int_0^{L_y} dy \int_{-z}^z dz \rho(z) = S_L \int_{-z}^z dz \rho(z) = S_L \int_{-z}^z dz \rho_0 \left(1 - \frac{z^2}{a^2}\right)$$

Autor: Iván Cabrera Alvarado

$$Q_{enS} = S_L \rho_0 \left[z - \frac{z^3}{3a^2} \right]_z^{-z} = S_L \rho_0 \left(z - \frac{z^3}{3a^2} - \left(-z - \frac{(-z)^3}{3a^2} \right) \right)$$

$$= S_L \rho_0 \left(z - \frac{z^3}{3a^2} + z - \frac{z^3}{3a^2} \right) = 2S_L \rho_0 \left(z - \frac{z^3}{3a^2} \right)$$

$$Q_{enS} \Rightarrow 2ES_L = \frac{2S_L \rho_0}{\epsilon_0} \left(z - \frac{z^3}{3a^2} \right) \Rightarrow E = \frac{\rho_0}{\epsilon_0} \left(z - \frac{z^3}{3a^2} \right) = \frac{\rho_0 z}{\epsilon_0} \left(1 - \frac{z^2}{3a^2} \right)$$

$$\boxed{\vec{E} = \frac{z \rho_0 z}{\epsilon_0} \left(1 - \frac{z^2}{3a^2} \right) \vec{u}_z \quad |z| < a}$$

$$|z| > a$$

$$\oint_S \vec{E} \cdot d\vec{S} = 2ES_L$$

$$Q_{enS} = S_L \int_{-a}^a dz \rho_0 \left(1 - \frac{z^2}{a^2} \right) = 2S_L \rho_0 \left(a - \frac{a^3}{3a^2} \right) = 2S_L \rho_0 a \left(1 - \frac{1}{3} \right)$$

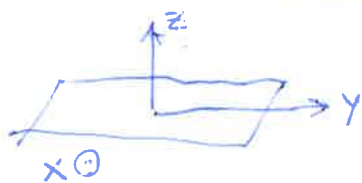
$$= 2S_L \rho_0 a \frac{2}{3} = \frac{4S_L \rho_0 a}{3}$$

$$Q_{enS} \Rightarrow 2ES_L = 4S_L \rho_0 a / 3\epsilon_0 \Rightarrow E = \frac{2\rho_0 a}{3\epsilon_0}$$

$$\vec{E} = \begin{cases} \vec{E}_1 & z > a \\ \vec{E}_2 & z < -a \end{cases} = \begin{cases} E \vec{u}_z & z > a \\ -E \vec{u}_z & z < -a \end{cases} = E \frac{z}{|z|} \vec{u}_z =$$

$$= \boxed{\frac{2\rho_0 a}{3\epsilon_0} \frac{z}{|z|} \vec{u}_z \quad |z| > a}$$

(b)



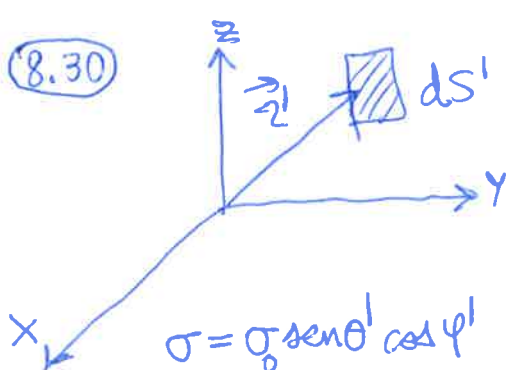
$$\vec{E}_{\sigma_0} = \frac{\sigma_0}{2\epsilon_0} \frac{z}{|z|} \vec{u}_z \Rightarrow \vec{E}_{\sigma_0, z>a} = \frac{\sigma_0}{2\epsilon_0} \vec{u}_z$$

$$\vec{E}_{z>a} = \frac{2\rho_0 a}{3\epsilon_0} \vec{u}_z$$

$$\vec{E}_{\sigma_0, z>a} = \vec{E}_{z>a} \Rightarrow \frac{\sigma_0}{2\epsilon_0} = \frac{2\rho_0 a}{3\epsilon_0} \Rightarrow \boxed{\sigma_0 = \frac{4\rho_0 a}{3}}$$

Autor: João Carlos Álvares

(8.30)



$$dS' = r'^2 \sin \theta' d\theta' d\varphi'$$

(8.32)

$$\vec{r}' = r' \sin \theta' \cos \varphi' \vec{u}_x + r' \sin \theta' \sin \varphi' \vec{u}_y + r' \cos \theta' \vec{u}_z$$

$$\vec{r} = 0$$

$$\vec{r} - \vec{r}' = -\vec{r}'$$

$$|\vec{r} - \vec{r}'| = r'$$

$$r' = R$$

$$\sigma = \sigma_0 \sin \theta' \cos \varphi'$$

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int_{S'} \frac{\sigma dS' (\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3} = \frac{1}{4\pi\epsilon_0} \int_0^{\pi/2} d\varphi' \int_0^{\pi/2} d\theta' \frac{\sin \theta' r'^2 \sigma_0 \sin \theta' \cos \varphi' (\vec{r} - \vec{r}')}{r'^3}$$

$$= \frac{\sigma_0}{4\pi\epsilon_0} \int_0^{\pi/2} d\varphi' \int_0^{\pi/2} d\theta' (\sin \theta')^2 \cos \varphi' (-\sin \theta' \cos \varphi' \vec{u}_x - \sin \theta' \sin \varphi' \vec{u}_y - \cos \theta' \vec{u}_z)$$

$$= \frac{\sigma_0}{4\pi\epsilon_0} \left[- \int_0^{\pi/2} d\varphi' (\cos \varphi')^2 \int_0^{\pi/2} d\theta' (\sin \theta')^3 \vec{u}_x - \int_0^{\pi/2} d\varphi' \cos \varphi' \sin \varphi' \int_0^{\pi/2} d\theta' (\sin \theta')^3 \vec{u}_y - \int_0^{\pi/2} d\varphi' \cos \varphi' \int_0^{\pi/2} d\theta' (\sin \theta')^2 \cos \theta' \vec{u}_z \right]$$

$$= \frac{\sigma_0}{4\pi\epsilon_0} \left[- \int_0^{\pi/2} d\varphi' (\cos \varphi')^2 \int_0^{\pi/2} d\theta' (\sin \theta')^3 \vec{u}_x - \int_0^{\pi/2} d\varphi' \cos \varphi' \sin \varphi' \int_0^{\pi/2} d\theta' (\sin \theta')^3 \vec{u}_y - \int_0^{\pi/2} d\varphi' \cos \varphi' \int_0^{\pi/2} d\theta' (\sin \theta')^2 \cos \theta' \vec{u}_z \right]$$

$$\int_0^{\pi/2} d\varphi' \cos \varphi' = \sin \varphi' \Big|_0^{\pi/2} = \sin \frac{\pi}{2} - \sin 0 = 1 - 0 = 1$$

$$\int_0^{\pi/2} d\varphi' (\cos \varphi')^2 = \frac{\varphi'}{2} \Big|_0^{\pi/2} + \frac{\sin 2\varphi'}{4} \Big|_0^{\pi/2} = \frac{\pi}{4} + \frac{(\sin \pi - \sin 0)}{4} = \frac{\pi}{4}$$

$$\int_0^{\pi/2} d\theta' (\sin \theta')^3 = \frac{\cos 3\theta'}{12} \Big|_0^{\pi/2} - \frac{3\cos \theta'}{4} \Big|_0^{\pi/2} = \frac{(\cos \frac{3\pi}{2} - \cos 0)}{12} - \frac{3}{4}(\cos \frac{\pi}{2} - \cos 0) = \frac{(0-1)}{12} - \frac{3}{4}(0-1) = -\frac{1}{12} + \frac{3}{4} = \frac{(-1+9)}{12} = \frac{8}{12} = \frac{2}{3}$$

$$\int_0^{\pi/2} d\theta' (\sin \theta')^2 \cos \theta' = \frac{(\sin \theta')^3}{3} \Big|_0^{\pi/2} = \frac{(\sin \frac{\pi}{2})^3 - (\sin 0)^3}{3} = \frac{1}{3}$$

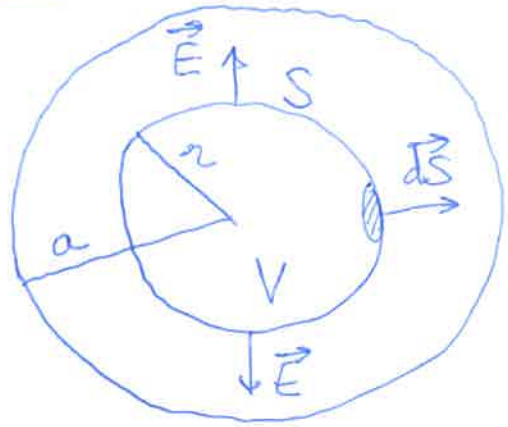
$$\int_0^{\pi/2} d\varphi' \cos \varphi' \sin \varphi' = -\frac{\cos 2\varphi'}{4} \Big|_0^{\pi/2} = \frac{(-\cos \pi + \cos 0)}{4} = \frac{2}{4} = \frac{1}{2}$$

$$\vec{E}(\vec{r}) = -\frac{\sigma_0}{4\pi\epsilon_0} \left[\frac{\pi}{4} \frac{2}{3} \vec{u}_x + \frac{1}{2} \frac{2}{3} \vec{u}_y + 1 \cdot \frac{1}{3} \vec{u}_z \right]$$

$$= -\frac{\sigma_0}{12\pi\epsilon_0} (\frac{\pi}{2} \vec{u}_x + \vec{u}_y + \vec{u}_z)$$

Autor: Juan Cabria Flores

8.31 $z < a$ $\vec{E} = E\vec{u}_r$ $d\vec{S} = dS\vec{u}_r$ 8.33

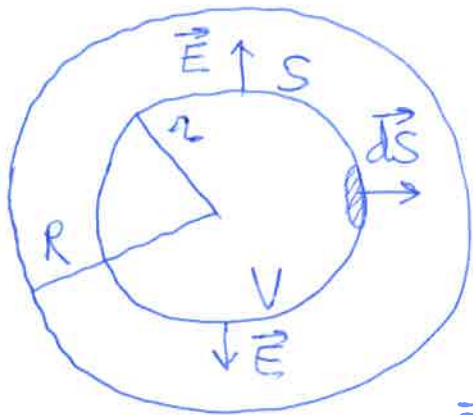


$$\vec{E} \cdot d\vec{S} = E dS \Rightarrow \oint_S \vec{E} \cdot d\vec{S} = \oint_S E dS = E \oint_S dS = E 4\pi z^2$$

$$Q_{\text{enc}} = \int_V \rho dV = \int_V \rho_0 dV = \rho_0 \int_V dV = \rho_0 \frac{4\pi}{3} z^3$$

$$\oint_S \vec{E} \cdot d\vec{S} = \frac{Q_{\text{enc}}}{\epsilon_0} \Rightarrow E 4\pi z^2 = \frac{\rho_0 4\pi z^3}{3\epsilon_0} \Rightarrow \boxed{\vec{E} = \frac{\rho_0 z}{3\epsilon_0} \vec{u}_r \quad z < a}$$

$a < z < R$ $\vec{E} = E\vec{u}_r$ $d\vec{S} = dS\vec{u}_r$



$$\vec{E} \cdot d\vec{S} = E dS \Rightarrow \oint_S \vec{E} \cdot d\vec{S} = \oint_S E dS = E \oint_S dS = E 4\pi z^2$$

$$Q_{\text{enc}} = \int_V \rho dV = \int_0^{2\pi} d\phi \int_0^\pi \sin\theta d\theta \int_0^z dr r^2 \rho(r) = 4\pi \int_0^z dr r^2 \rho(r) = 4\pi \left[\int_0^a dr r^2 \rho_0 + \int_a^z dr r^2 \rho_0 e^{-(r-a)/r_0} \right]$$

$$= 4\pi \left[\rho_0 \frac{a^3}{3} + \rho_0 e^{a/r_0} \int_a^z dr r^2 e^{-r/r_0} \right] = 4\pi \rho_0 \left[\frac{a^3}{3} + e^{a/r_0} \int_a^z dr r^2 e^{-r/r_0} \right]$$

$$\int_a^z dr r^2 e^{-r/r_0} = \left\{ \begin{array}{l} r = r_0 s \\ dr = r_0 ds \end{array} \right\} = \int_{a/r_0}^{z/r_0} r_0^3 ds s^2 e^{-s} = r_0^3 \int_{a/r_0}^{z/r_0} ds s^2 e^{-s} \Rightarrow \int ds s^2 e^{-s} = -e^{-s}(s^2 + 2s + 2) = I(s)$$

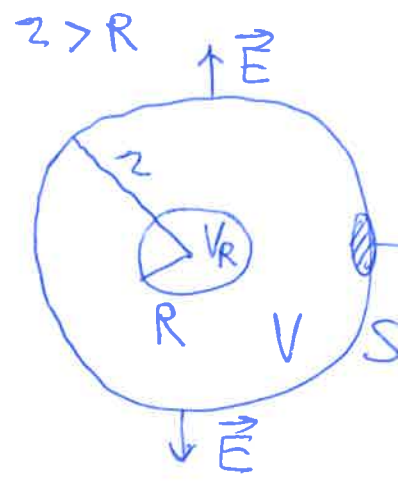
$$\Rightarrow \int_a^z dr r^2 e^{-r/r_0} = r_0^3 I(s) \Big|_{s=a/r_0}^{s=z/r_0} = r_0^3 (I(z/r_0) - I(a/r_0))$$

$$Q_{\text{enc}} = 4\pi \rho_0 \left[\frac{a^3}{3} + e^{a/r_0} r_0^3 (I(z/r_0) - I(a/r_0)) \right]$$

Profesor: Juan Carlos Álvarez

$$\oint_S \vec{E} \cdot d\vec{S} = \frac{Q_{enS}}{\epsilon_0} \Rightarrow E 4\pi r^2 = \frac{4\pi \rho_0}{\epsilon_0} \left[\frac{a^3}{3} + e^{a/r_0} r_0^3 (I(r/r_0) - I(a/r_0)) \right] \Rightarrow$$

$$\Rightarrow \vec{E} = \frac{\rho_0}{\epsilon_0 r^2} \left[\frac{a^3}{3} + r_0^3 e^{a/r_0} (I(r/r_0) - I(a/r_0)) \right] \vec{u}_r \quad a < r < R$$



$$\vec{E} = E \vec{u}_r \quad d\vec{S} = dS \vec{u}_r \quad \rho = 0 \text{ en } r > R$$

$$\oint_S \vec{E} \cdot d\vec{S} = \oint_S E dS = E \oint_S dS = E 4\pi r^2$$

$$Q_{enS} = \int_V \rho dV = \int_{V_R} \rho dV = \int_0^{2\pi} d\phi \int_0^\pi \sin\theta d\theta \int_0^R dr r^2 \rho(r) =$$

$$= 4\pi \int_0^R dr r^2 \rho(r) = 4\pi \left[\int_0^a dr r^2 \rho_0 + \int_a^R dr r^2 \rho_0 e^{-(r-a)/r_0} \right] =$$

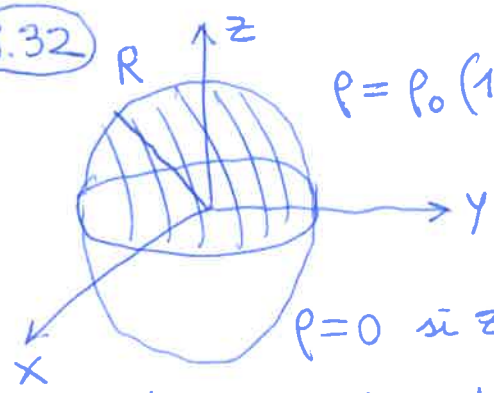
$$= 4\pi \rho_0 \left[\frac{a^3}{3} + e^{a/r_0} \int_a^R dr r^2 e^{-r/r_0} \right] = 4\pi \rho_0 \left[\frac{a^3}{3} + e^{a/r_0} r_0^3 (I(R/r_0) - I(a/r_0)) \right]$$

$$\oint_S \vec{E} \cdot d\vec{S} = \frac{Q_{enS}}{\epsilon_0} \Rightarrow E 4\pi r^2 = \frac{4\pi \rho_0}{\epsilon_0} \left[\frac{a^3}{3} + e^{a/r_0} r_0^3 (I(R/r_0) - I(a/r_0)) \right] \Rightarrow$$

$$\Rightarrow \vec{E} = \frac{\rho_0}{\epsilon_0 r^2} \left[\frac{a^3}{3} + r_0^3 e^{a/r_0} (I(R/r_0) - I(a/r_0)) \right] \vec{u}_r \quad r > R$$

8.32

8.35



$$\rho = \rho_0 \left(1 + \frac{z}{R}\right) \text{ si } z > 0 \quad \vec{r} = 0$$

$$\vec{r}' = z' \text{sen } \theta' \cos \varphi' \vec{u}_x + z' \text{sen } \theta' \text{sen } \varphi' \vec{u}_y + z' \cos \theta' \vec{u}_z$$

$$\rho = 0 \text{ si } z < 0 \quad \vec{r} - \vec{r}' = -\vec{r}' \quad |\vec{r} - \vec{r}'| = z'$$

$$dV' = z'^2 dz' \text{sen } \theta' d\theta' d\varphi'$$

$$\phi(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int_V \frac{\rho(\vec{r}') dV'}{|\vec{r} - \vec{r}'|} = \frac{1}{4\pi\epsilon_0} \int_0^{2\pi} d\varphi' \int_0^\pi d\theta' \text{sen } \theta' \int_0^R dz' z'^2 \frac{\rho(z', \theta')}{z'}$$

$$\rho(z', \theta') = \begin{cases} \rho_0 \left(1 + \frac{z}{R}\right) \text{sen } \theta & \text{si } z > 0, \text{ es decir, si } 0 < \theta' < \frac{\pi}{2} \\ 0 & \text{si } z < 0, \text{ es decir, si } \frac{\pi}{2} < \theta' < \pi \end{cases}$$

$$\begin{aligned} \phi(\vec{r}) &= \frac{2\pi}{4\pi\epsilon_0} \int_0^\pi d\theta' \text{sen } \theta' \int_0^R dz' z' \rho(z', \theta') = \frac{1}{2\epsilon_0} \left[\int_0^{\pi/2} d\theta' \text{sen } \theta' \int_0^R dz' z' \rho_0 \left(1 + \frac{z'}{R}\right) \times \right. \\ &\quad \left. + \int_{\pi/2}^\pi d\theta' \text{sen } \theta' \int_0^R dz' z' 0 \right] = \frac{1}{2\epsilon_0} \int_0^{\pi/2} d\theta' (\text{sen } \theta')^2 \int_0^R dz' z' \rho_0 \left(1 + \frac{z'}{R}\right) \end{aligned}$$

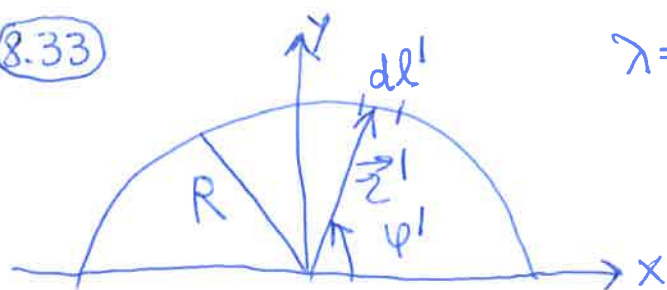
$$\int_0^{\pi/2} d\theta' (\text{sen } \theta')^2 = \frac{\theta'}{2} \Big|_0^{\pi/2} - \frac{\text{sen } 2\theta'}{4} \Big|_0^{\pi/2} = \frac{\pi}{4} - \frac{(\text{sen } \pi - \text{sen } 0)}{4} = \frac{\pi}{4}$$

$$\int_0^R dz' z' \left(1 + \frac{z'}{R}\right) = \frac{z'^2}{2} \Big|_0^R + \frac{z'^3}{3R} \Big|_0^R = \frac{R^2}{2} + \frac{R^3}{3R} = R^2 \left(\frac{1}{2} + \frac{1}{3}\right) = R^2 \frac{5}{6}$$

$$\phi(\vec{r}) = \frac{1}{2\epsilon_0} \frac{\pi}{4} \rho_0 R^2 \frac{5}{6} = \boxed{\frac{\rho_0 \pi R^2 5}{48 \epsilon_0}}$$

tutor: Iván Cabrera Álvarez

(8.33)



$$\lambda = \lambda_0 \sin \varphi'$$

$$dl' = r' d\varphi'$$

(8.36)

$$r' = R$$

$$dq' = \lambda dl' = \lambda_0 \sin \varphi' r' d\varphi' = \lambda_0 \sin \varphi' R d\varphi'$$

$$\begin{aligned} a) \quad Q &= \int_L dq' = \int_L \lambda dl' = \int_0^\pi \lambda_0 \sin \varphi' R d\varphi' = \lambda_0 R \int_0^\pi \sin \varphi' d\varphi' = \\ &= -\lambda_0 R \cos \varphi' \Big|_0^\pi = -\lambda_0 R (\cos \pi - \cos 0) = -\lambda_0 R (-1 - 1) = \boxed{2\lambda_0 R} \end{aligned}$$

$$b) \quad \vec{r} = 0 \quad \vec{r}' = R \cos \varphi' \vec{u}_x + R \sin \varphi' \vec{u}_y \quad r' = R$$

$$\vec{r} - \vec{r}' = -R \cos \varphi' \vec{u}_x - R \sin \varphi' \vec{u}_y \quad |\vec{r} - \vec{r}'| = R$$

$$\vec{E}(\vec{r}) = \int \frac{\lambda dl' (\vec{r} - \vec{r}')}{4\pi \epsilon_0 |\vec{r} - \vec{r}'|^3} = \frac{\lambda_0 R}{4\pi \epsilon_0} \int_0^\pi \frac{\sin \varphi' d\varphi' (-R \cos \varphi' \vec{u}_x - R \sin \varphi' \vec{u}_y)}{R^3} =$$

$$= -\frac{\lambda_0 R^2}{4\pi \epsilon_0 R^3} \int_0^\pi (\sin \varphi' \cos \varphi' \vec{u}_x + (\sin \varphi')^2 \vec{u}_y) d\varphi' =$$

$$= -\frac{\lambda_0}{4\pi \epsilon_0 R} \left[\int_0^\pi \sin \varphi' \cos \varphi' d\varphi' \vec{u}_x + \int_0^\pi (\sin \varphi')^2 d\varphi' \vec{u}_y \right]$$

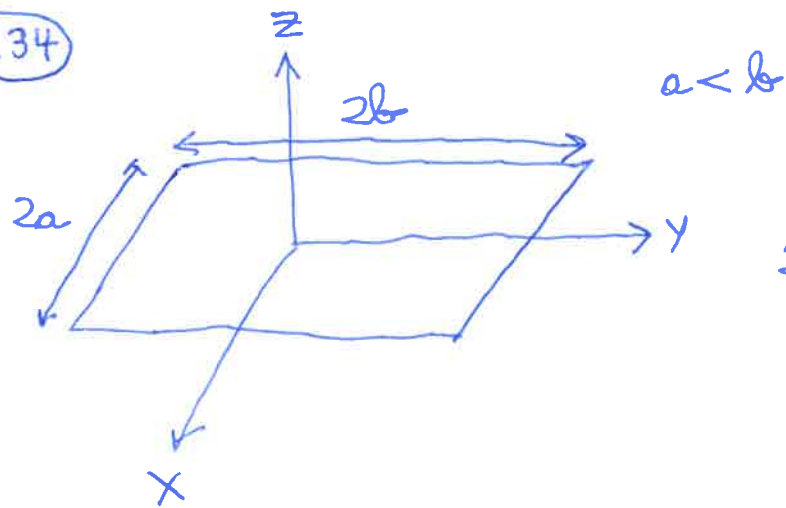
$$\int_0^\pi \sin \varphi' \cos \varphi' d\varphi' = -\frac{\cos 2\varphi'}{4} \Big|_0^\pi = -\frac{(\cos 2\pi - \cos 0)}{4} = 0$$

$$\int_0^\pi (\sin \varphi')^2 d\varphi' = \frac{\varphi'}{2} \Big|_0^\pi - \frac{\sin 2\varphi'}{4} \Big|_0^\pi = \frac{\pi}{2}$$

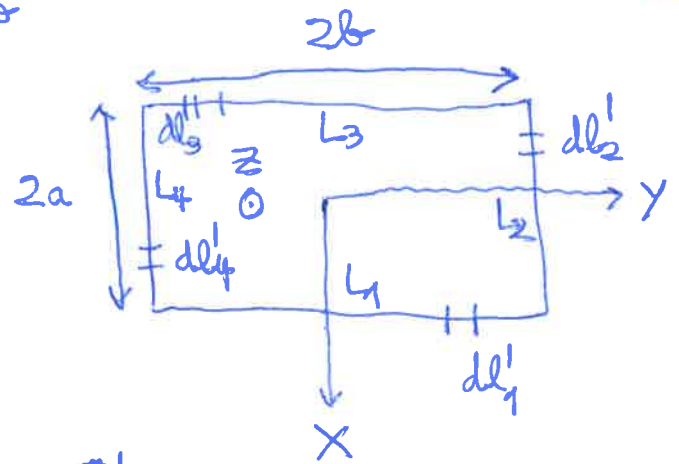
$$\vec{E}(\vec{r}) = -\frac{\lambda_0}{4\pi \epsilon_0 R} \frac{\pi}{2} \vec{u}_y = \boxed{-\frac{\lambda_0}{8\epsilon_0 R} \vec{u}_y}$$

Autor: Juan Carlos Álvarez

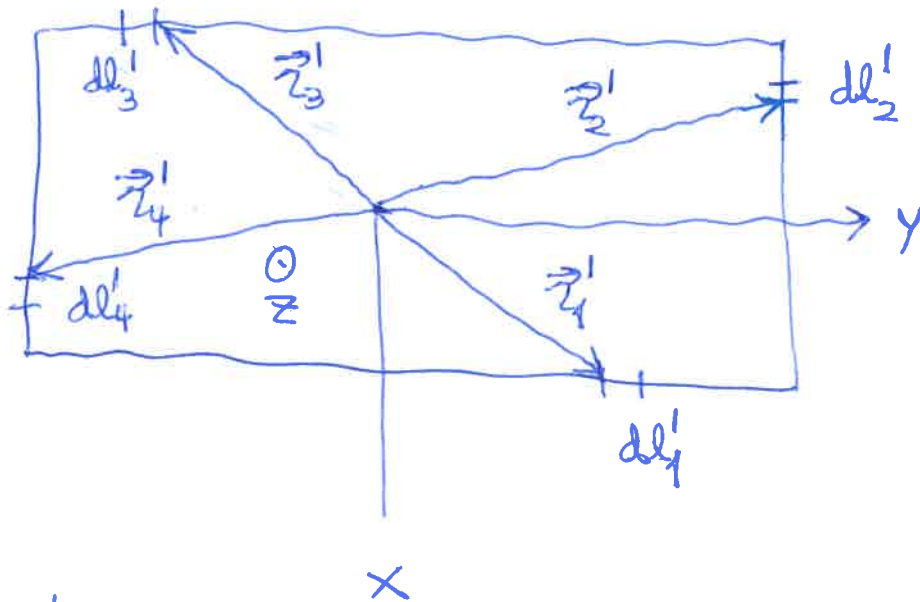
8.34



8.37



Región L_1 : $\vec{r}_1' = a\vec{u}_x + y'\vec{u}_y$ $d\vec{l}_1' = dy'$
 Región L_2 : $\vec{r}_2' = -a\vec{u}_x + y'\vec{u}_y$ $d\vec{l}_2' = dy'$
 Región L_3 : $\vec{r}_3' = x'\vec{u}_x + b\vec{u}_y$ $d\vec{l}_3' = dx'$
 Región L_4 : $\vec{r}_4' = x'\vec{u}_x - b\vec{u}_y$ $d\vec{l}_4' = dx'$



$$\vec{E}_i = \int_{L_i} \frac{\lambda d\vec{l}_i' (\vec{r} - \vec{r}_i')}{4\pi\epsilon_0 |\vec{r} - \vec{r}_i'|^3}$$

$$i = 1-4 \quad \vec{r} = z\vec{u}_z$$

El campo total $\vec{E}$ es: $\vec{E} = \sum_{i=1}^4 \vec{E}_i$

$$\int \frac{du}{(u^2 + c^2)^{3/2}} = \frac{u}{c^2 \sqrt{u^2 + c^2}}$$

$$\vec{r} - \vec{r}_1' = -a\vec{u}_x - y'\vec{u}_y + z\vec{u}_z$$

$$\vec{r} - \vec{r}_2' = +a\vec{u}_x - y'\vec{u}_y + z\vec{u}_z$$

$$\vec{r} - \vec{r}_3' = -x'\vec{u}_x - b\vec{u}_y + z\vec{u}_z$$

$$\vec{r} - \vec{r}_4' = -x'\vec{u}_x + b\vec{u}_y + z\vec{u}_z$$

Autor: Iván Galvis Alvarado

$$I_{abz} \equiv \int_{-a}^a \frac{dx'}{(x'^2 + b^2 + z^2)^{3/2}} = \left\{ \begin{array}{l} u = x' \\ c^2 = b^2 + z^2 \end{array} \right\} = 2 \int_0^a \frac{du}{(u^2 + c^2)^{3/2}} =$$

$$= \frac{2u}{(b^2 + z^2) \sqrt{u^2 + b^2 + z^2}} \Big|_0^a = \frac{2a}{(b^2 + z^2) \sqrt{a^2 + b^2 + z^2}}$$

$$I_{ba\bar{z}} \equiv \int_{-b}^b \frac{dy'}{(a^2 + y'^2 + z^2)^{3/2}} = \left\{ \begin{array}{l} u = y' \\ c^2 = a^2 + z^2 \end{array} \right\} = 2 \int_0^b \frac{du}{(u^2 + c^2)^{3/2}} =$$

$$= \frac{2u}{(a^2 + z^2) \sqrt{a^2 + u^2 + z^2}} \Big|_0^b = \frac{2b}{(a^2 + z^2) \sqrt{a^2 + b^2 + z^2}}$$

$$J(a, b, z) \equiv \int_{-a}^a \frac{x' dx'}{(x'^2 + b^2 + z^2)^{3/2}} = 0 \quad J(b, a, z) = 0$$

$$\vec{E}_1 = \frac{\lambda}{4\pi\epsilon_0} \int_{-b}^b \frac{dy' (-a\vec{u}_x - y'\vec{u}_y + z\vec{u}_z)}{(a^2 + y'^2 + z^2)^{3/2}} = \frac{\lambda}{4\pi\epsilon_0} \left[-a\vec{u}_x I(b, a, z) \right.$$

$$\left. - \vec{u}_y J(b, a, z) + z\vec{u}_z I(b, a, z) \right] = \frac{\lambda}{4\pi\epsilon_0} I(b, a, z) (-a\vec{u}_x + z\vec{u}_z)$$

$$\vec{E}_2 = \frac{\lambda}{4\pi\epsilon_0} \int_{-a}^a \frac{dx' (-x'\vec{u}_x - b\vec{u}_y + z\vec{u}_z)}{(x'^2 + b^2 + z^2)^{3/2}} = \frac{\lambda}{4\pi\epsilon_0} \left[-\vec{u}_x J(a, b, z) \right.$$

$$\left. + (-b\vec{u}_y + z\vec{u}_z) I(a, b, z) \right] = \frac{\lambda}{4\pi\epsilon_0} I(a, b, z) (-b\vec{u}_y + z\vec{u}_z)$$

$$\vec{E}_3 = \frac{\lambda}{4\pi\epsilon_0} \int_{-b}^b \frac{dy' (a\vec{u}_x - y'\vec{u}_y + z\vec{u}_z)}{(a^2 + y'^2 + z^2)^{3/2}} = \frac{\lambda}{4\pi\epsilon_0} \left[(a\vec{u}_x + z\vec{u}_z) I(b, a, z) \right.$$

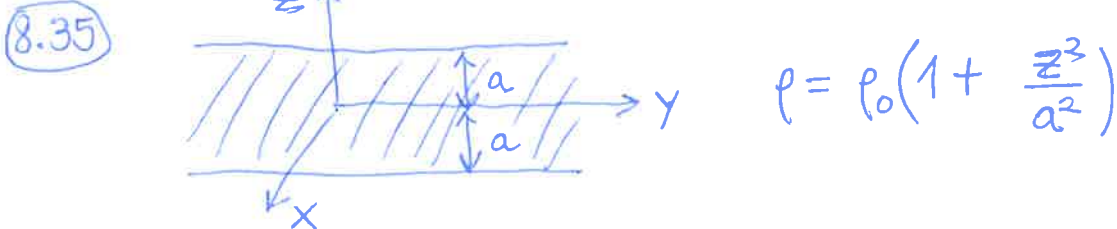
$$\left. - \vec{u}_y J(b, a, z) \right] = \frac{\lambda}{4\pi\epsilon_0} I(b, a, z) (a\vec{u}_x + z\vec{u}_z)$$

$$\vec{E}_4 = \frac{\lambda}{4\pi\epsilon_0} \int_{-a}^a \frac{dx' (-x'\vec{u}_x + b\vec{u}_y + z\vec{u}_z)}{(x'^2 + b^2 + z^2)^{3/2}} = \frac{\lambda}{4\pi\epsilon_0} \left[-\vec{u}_x J(a, b, z) \right.$$

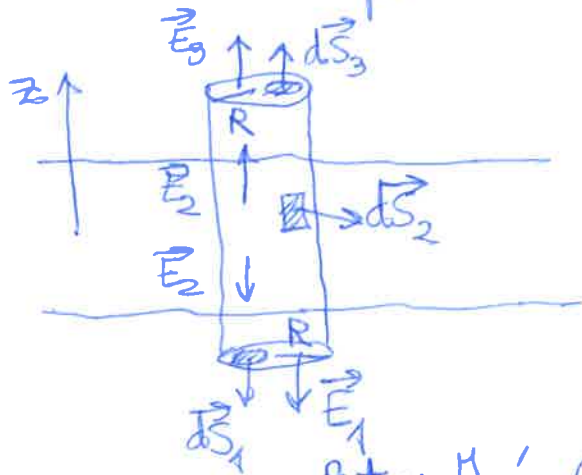
$$\left. + (b\vec{u}_y + z\vec{u}_z) I(a, b, z) \right] = \frac{\lambda}{4\pi\epsilon_0} I(a, b, z) (b\vec{u}_y + z\vec{u}_z)$$

Autor: Iván Gabrila Álvarez

$$\begin{aligned}
\vec{E} &= \sum_{i=1}^4 \vec{E}_i = \frac{\lambda}{4\pi\epsilon_0} \left[I(b, a, z) (-a\vec{u}_x + z\vec{u}_z) + \right. \\
&+ I(a, b, z) (-b\vec{u}_y + z\vec{u}_z) + I(b, a, z) (a\vec{u}_x + z\vec{u}_z) + \\
&+ \left. I(a, b, z) (b\vec{u}_y + z\vec{u}_z) \right] = \\
&= \frac{\lambda}{4\pi\epsilon_0} \left[I(b, a, z) (-a\vec{u}_x + z\vec{u}_z + a\vec{u}_x + z\vec{u}_z) + \right. \\
&+ \left. I(a, b, z) (-b\vec{u}_y + z\vec{u}_z + b\vec{u}_y + z\vec{u}_z) \right] = \\
&= \frac{\lambda}{4\pi\epsilon_0} \left[2z I(b, a, z) \vec{u}_z + 2z I(a, b, z) \vec{u}_z \right] = \\
&= \frac{\lambda z}{2\pi\epsilon_0} \vec{u}_z \left[I(b, a, z) + I(a, b, z) \right] = \\
&= \frac{\lambda z \vec{u}_z}{2\pi\epsilon_0} \left[\frac{2b}{(a^2+z^2)\sqrt{a^2+b^2+z^2}} + \frac{2a}{(b^2+z^2)\sqrt{a^2+b^2+z^2}} \right] = \\
&= \boxed{\frac{\lambda z \vec{u}_z}{\pi\epsilon_0 \sqrt{a^2+b^2+z^2}} \left(\frac{b}{(a^2+z^2)} + \frac{a}{(b^2+z^2)} \right)}
\end{aligned}$$



Calculamos el campo eléctrico en $z > a$ y en $z < -a$:



$$\vec{E}_2 \perp d\vec{S}_2 \Rightarrow \vec{E}_2 \cdot d\vec{S}_2 = 0$$

$$\vec{E}_3 \cdot d\vec{S}_3 = \vec{E}_1 \cdot d\vec{S}_1 = E_3 dS_3 = E_1 dS_1$$

$$\oint_S \vec{E} \cdot d\vec{S} = \int_{S_1} \vec{E}_1 \cdot d\vec{S}_1 + \int_{S_2} \vec{E}_2 \cdot d\vec{S}_2 + \int_{S_3} \vec{E}_3 \cdot d\vec{S}_3$$

$$\vec{E}_1 = -E_1 \vec{u}_z$$

$$\vec{E}_3 = +E_3 \vec{u}_z$$

$$E_1 = E_3$$

Autor: Iván Cabria Alvarro

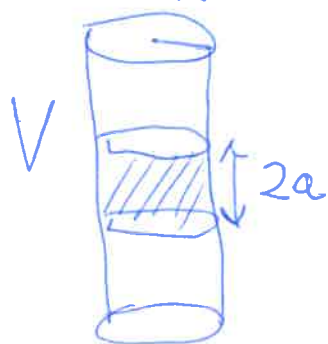
$$\left. \begin{aligned} \int_{S_1} \vec{E} \cdot d\vec{S}_1 &= \int_{S_1} E_1 dS_1 = \int_{S_1} E_3 dS_3 = \int_{S_3} E_3 dS_3 \\ \int_{S_3} \vec{E}_3 \cdot d\vec{S}_3 &= \int_{S_3} E_3 dS_3 \end{aligned} \right\} \Rightarrow \oint_S \vec{E} \cdot d\vec{S} = 2 \int_{S_3} E_3 dS_3$$

$S_1 = S_3$

$$\vec{E}_2 \cdot d\vec{S}_2 = 0 \Rightarrow \int_{S_2} \vec{E}_2 \cdot d\vec{S}_2 = 0$$

$$\int_{S_3} E_3 dS_3 = E_3 \int_{S_3} dS_3 = E_3 \pi R^2 \Rightarrow \oint_S \vec{E} \cdot d\vec{S} = 2 E_3 \pi R^2$$

$$Q_{\text{en } S} = \int_V \rho(z) dV = \int_{\text{cilindro de radio } R \text{ y altura } 2a} \rho(z) dV = \int_0^R \rho \int_0^{2\pi} d\varphi \int_{-a}^a dz \rho(z) \rho =$$



$$dV = \rho d\rho d\varphi dz = \frac{R^2}{2} 2\pi \int_{-a}^a dz \rho(z) =$$

$$= \pi R^2 \rho_0 \int_{-a}^a dz \left(1 + \frac{z^2}{a^2}\right) =$$

$$= \pi R^2 \rho_0 \left[z + \frac{z^3}{3a^2} \right]_{-a}^a = \pi R^2 \rho_0 \left(a + \frac{a^3}{3a^2} - \left(-a - \frac{a^3}{3a^2} \right) \right) =$$

$$= \pi R^2 \rho_0 \left(a + \frac{a}{3} + a + \frac{a}{3} \right) = \pi R^2 \rho_0 2 \left(a + \frac{a}{3} \right) = \frac{8\pi R^2 \rho_0 a}{3}$$

$$\oint_S \vec{E} \cdot d\vec{S} = \frac{Q_{\text{en } S}}{\epsilon_0} \Rightarrow 2 E_3 \pi R^2 = \frac{8\pi R^2 \rho_0 a}{3\epsilon_0} \Rightarrow E_3 = \frac{4\rho_0 a}{3\epsilon_0} \Rightarrow$$

$$\Rightarrow \vec{E}_3 = \frac{4\rho_0 a}{3\epsilon_0} \vec{u}_z \Rightarrow \vec{E}_1 = -\frac{4\rho_0 a}{3\epsilon_0} \vec{u}_z$$

$\vec{E}_3$ es el campo eléctrico en $z > a \Rightarrow$

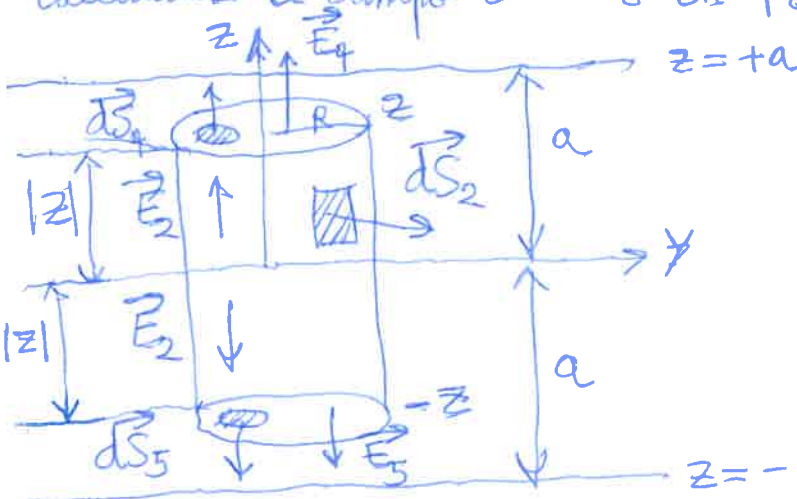
$$\boxed{\vec{E} = \frac{4\rho_0 a}{3\epsilon_0} \vec{u}_z \quad z > a}$$

$\vec{E}_1$ es el campo eléctrico en $z < -a \Rightarrow$

$$\boxed{\vec{E} = -\frac{4\rho_0 a}{3\epsilon_0} \vec{u}_z \quad z < -a}$$

Calculamos el campo eléctrico en $|z| < a$:

(8.41)



$$\oint_S \vec{E} \cdot d\vec{S} = \int_{S_2} \vec{E}_2 \cdot d\vec{S}_2 + \int_{S_4} \vec{E}_4 \cdot d\vec{S}_4 +$$

$$+ \int_{S_5} \vec{E}_5 \cdot d\vec{S}_5$$

$$\vec{E}_4 = E_4 \vec{u}_z \quad \vec{E}_3 = -|E_5| \vec{u}_z$$

$$E_4 = E_5$$

$$\vec{E}_2 \perp d\vec{S}_2 \Rightarrow \vec{E}_2 \cdot d\vec{S}_2 = 0 \Rightarrow \int_{S_2} \vec{E}_2 \cdot d\vec{S}_2 = 0$$

$$|\vec{E}_4| = |\vec{E}_5|$$

$$\left. \begin{aligned} \vec{E}_4 \cdot d\vec{S}_4 = \vec{E}_5 \cdot d\vec{S}_5 = E_4 dS_4 = E_5 dS_5 \\ S_4 = S_5 \end{aligned} \right\} \Rightarrow \int_{S_4} \vec{E}_4 \cdot d\vec{S}_4 = \int_{S_5} \vec{E}_5 \cdot d\vec{S}_5$$

$$\Rightarrow \oint_S \vec{E} \cdot d\vec{S} = 0 + 2 \int_{S_4} \vec{E}_4 \cdot d\vec{S}_4 = 2 E_4 \int_{S_4} dS_4 = 2 E_4 \pi R^2$$

$$Q_{\text{en } S} = \int_{V_{R,2z}} \rho(z) dV = \int_0^R \rho dz \int_0^{2\pi} d\varphi \int_{-z}^z dz \rho_0 \left(1 + \frac{z^2}{a^2}\right) \Rightarrow$$

$V_{R,2z}$: volumen del cilindro de radio R y altura 2z

$$\Rightarrow Q_{\text{en } S} = \frac{R^3}{2} 2\pi \int_{-z}^z dz \rho_0 \left(1 + \frac{z^2}{a^2}\right) = \pi R^2 \rho_0 \int_{-z}^z dz \left(1 + \frac{z^2}{a^2}\right) =$$

$$= \pi R^2 \rho_0 \left[z + \frac{z^3}{3a^2} \right]_{-z}^z = \pi R^2 \rho_0 \left(z + \frac{z^3}{3a^2} - \left(-z - \frac{z^3}{3a^2} \right) \right) =$$

$$= \pi R^2 \rho_0 2 \left(z + \frac{z^3}{3a^2} \right) = 2\pi R^2 \rho_0 z \left(1 + \frac{z^2}{3a^2} \right)$$

$$\oint_S \vec{E} \cdot d\vec{S} = \frac{Q_{\text{en } S}}{\epsilon_0} \Rightarrow 2 E_4 \pi R^2 = 2\pi R^2 \rho_0 z \left(1 + \frac{z^2}{3a^2} \right) \Rightarrow E_4 = \frac{\rho_0}{\epsilon_0} z \left(1 + \frac{z^2}{3a^2} \right)$$

$$\vec{E}_4 = E_4 \vec{u}_z \quad 0 < z < a$$

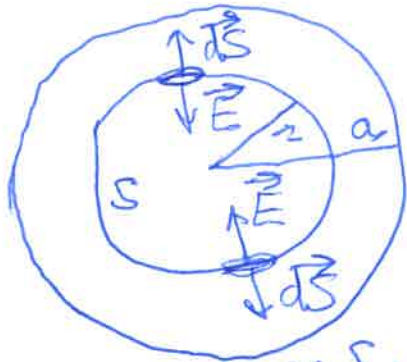
$$\vec{E}_5 = -|E_4| \vec{u}_z \quad -a < z < 0$$

$$\Rightarrow \vec{E} = \frac{\rho_0}{\epsilon_0} z \left(1 + \frac{z^2}{3a^2} \right) \vec{u}_z \quad |z| < a$$

Autor: Iván Cabria Álvarez

8.36 Calculamos $\vec{E}$ en $r \leq a$:

8.42



$$\vec{E} = -E(r)\vec{u}_r \quad d\vec{S} = dS\vec{u}_r$$

$$\oint_S \vec{E} \cdot d\vec{S} = - \oint_S E(r) dS = - \int_0^{2\pi} d\varphi \int_0^\pi d\theta \sin\theta E(r) r^2$$

$$= -4\pi E(r) r^2$$

El campo $\vec{E}$ en S apunta hacia dentro de la esfera de radio r , porque la densidad de carga es negativa. Si colocáramos una carga q positiva en la superficie S , la carga negativa de la esfera de radio a atraería dicha carga. La fuerza sería hacia dentro de la esfera. $\vec{F} = q\vec{E}$ $q > 0 \Rightarrow \vec{E}$ apunta hacia dentro de la esfera.

$$Q_{\text{enc. en } S} = \int_{\text{enc. } S} dq = \int_V \rho dV = - \int_0^{2\pi} d\varphi \int_0^\pi d\theta \sin\theta \int_0^r r^2 dr \rho_0 \frac{r^3}{a^3} =$$

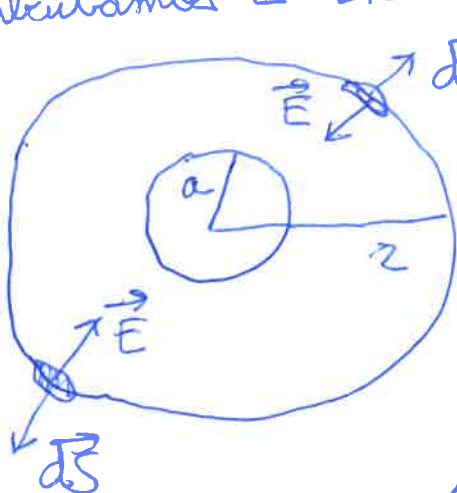
$$= - \frac{4\pi \rho_0}{a^3} \int_0^r r^5 dr = - \frac{4\pi \rho_0 r^6}{a^3 6}$$

$$\text{Ley de Gauss} \Rightarrow \oint_S \vec{E} \cdot d\vec{S} = \frac{Q_{\text{enc. } S}}{\epsilon_0} \Rightarrow -4\pi r^2 E(r) = - \frac{4\pi \rho_0 r^6}{a^3 6 \epsilon_0}$$

$$\Rightarrow E(r) = \frac{\rho_0 r^4}{6a^3 \epsilon_0} \Rightarrow \boxed{\vec{E} = - \frac{\rho_0 r^4}{6a^3 \epsilon_0} \vec{u}_r}$$

$$r < a$$

Calculamos $\vec{E}$ en $a \leq r \leq b$:



$$\vec{E} = -E(r)\vec{u}_r \quad d\vec{S} = dS\vec{u}_r$$

$$\oint_S \vec{E} \cdot d\vec{S} = -4\pi E(r) r^2$$

$$Q_{\text{enc. } S} = \int_0^{2\pi} d\varphi \int_0^\pi d\theta \sin\theta \int_0^r \rho(r) r^2 dr$$

Autor: Iván Gabrín Álvarez

$$Q_{\text{enc. S}} = 4\pi \int_0^r \rho(z) z^2 dz = 4\pi \left[- \int_0^a \rho_0 \frac{z^5}{a^3} dz - \int_a^r \rho_0 z^2 dz \right] \quad (8.43)$$

$$= -4\pi \rho_0 \left[\int_0^a \frac{z^5 dz}{a^3} + \int_a^r z^2 dz \right] = -4\pi \rho_0 \left[\frac{a^6}{6a^3} + \frac{z^3}{3} - \frac{a^3}{3} \right] =$$

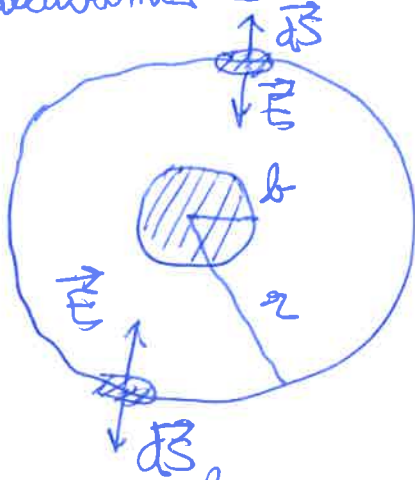
$$= -4\pi \rho_0 \left(\frac{z^3}{3} - \frac{a^3}{6} \right)$$

Ley de Gauss $\Rightarrow \oint_S \vec{E} \cdot d\vec{S} = \frac{Q_{\text{enc. S}}}{\epsilon_0} \Rightarrow -4\pi r^2 E(r) = -\frac{4\pi \rho_0}{\epsilon_0} \left(\frac{r^3}{3} - \frac{a^3}{6} \right)$

$$\Rightarrow E(r) = \frac{\rho_0}{\epsilon_0} \left(\frac{r}{3} - \frac{a^3}{6r^2} \right) \Rightarrow \boxed{\vec{E} = -\frac{\rho_0}{\epsilon_0} \left(\frac{r}{3} - \frac{a^3}{6r^2} \right) \vec{u}_r}$$

$a < r < b$

Calcularmos $\vec{E}$ en $b < r$:



$$\vec{E} = -E(r) \vec{u}_r \quad d\vec{S} = dS \vec{u}_r$$

$$\oint_S \vec{E} \cdot d\vec{S} = -4\pi r^2 E(r)$$

$$Q_{\text{enc. S}} = \int_0^a d\varphi \int_0^\pi d\theta \sin\theta \int_0^b \rho(z) z^2 dz =$$

$$= 4\pi \int_0^b \rho(z) z^2 dz = 4\pi \left[- \int_0^a \rho_0 \frac{z^5}{a^3} dz - \int_a^b \rho_0 z^2 dz \right] =$$

$$= -4\pi \rho_0 \left[\frac{a^6}{6a^3} + \frac{(b^3 - a^3)}{3} \right] = -4\pi \rho_0 \left(\frac{a^3}{6} + \frac{b^3}{3} - \frac{2a^3}{6} \right)$$

$$= -4\pi \rho_0 \left(\frac{b^3}{3} - \frac{a^3}{6} \right)$$

Ley de Gauss $\Rightarrow -4\pi r^2 E(r) = -\frac{4\pi \rho_0}{\epsilon_0} \left(\frac{b^3}{3} - \frac{a^3}{6} \right) \Rightarrow$

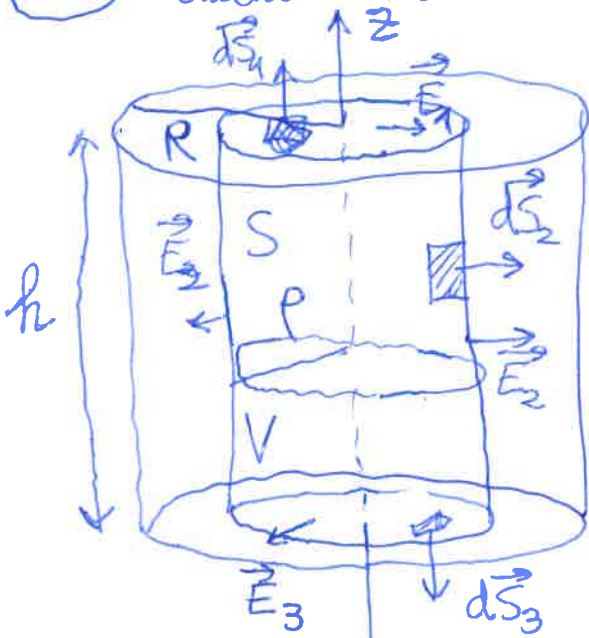
$$\Rightarrow E(r) = \frac{\rho_0}{\epsilon_0 r^2} \left(\frac{b^3}{3} - \frac{a^3}{6} \right) \Rightarrow \boxed{\vec{E} = -\frac{\rho_0}{\epsilon_0 r^2} \left(\frac{b^3}{3} - \frac{a^3}{6} \right) \vec{u}_r}$$

$b < r$

Autor: Iván Cabria Álvarez

8.37) Calculamos $\vec{E}$ en $\rho < R$:

8.44



$$\vec{E}_1 \cdot d\vec{S}_1 = 0$$

$$\vec{E}_3 \cdot d\vec{S}_3 = 0$$

$$\vec{E}_i = E \vec{u}_\rho \quad i=1,2,3$$

$$d\vec{S}_2 = dS \vec{u}_\rho$$

$$E_1 = E_2 = E_3 = E$$

$$\vec{E}_2 \cdot d\vec{S}_2 = E_2 dS_2 = E dS_2$$

$$\oint_S \vec{E} \cdot d\vec{S} = \sum_{i=1}^3 \int_{S_i} \vec{E}_i \cdot d\vec{S}_i = \int_{S_2} \vec{E}_2 \cdot d\vec{S}_2 =$$

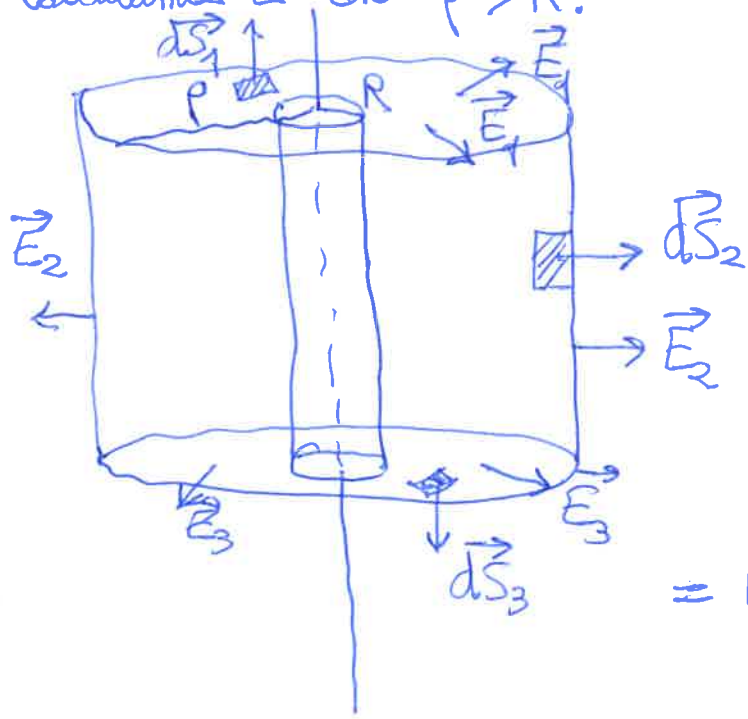
$$= E 2\pi \rho h$$

$$Q_{enc.} \text{ en } S = \int_V dq = \int_0^h \lambda(z) dz = \int_0^h \lambda_0 dz = \lambda_0 h$$

$$\text{Ley de Gauss} \Rightarrow E 2\pi \rho h = \frac{\lambda_0 h}{\epsilon_0} \Rightarrow E = \frac{\lambda_0}{2\pi \epsilon_0 \rho} \Rightarrow$$

$$\Rightarrow \boxed{\vec{E} = \frac{\lambda_0}{2\pi \epsilon_0 \rho} \vec{u}_\rho \quad \rho < R}$$

Calculamos $\vec{E}$ en $\rho > R$:



$$\vec{E}_i = E \vec{u}_\rho \quad i=1,2,3$$

$$\vec{E}_1 \cdot d\vec{S}_1 = 0 \quad \vec{E}_3 \cdot d\vec{S}_3 = 0$$

$$\vec{E}_2 \cdot d\vec{S}_2 = E_2 dS_2 = E dS_2$$

$$\oint_S \vec{E} \cdot d\vec{S} = \sum_{i=1}^3 \int_{S_i} \vec{E}_i \cdot d\vec{S}_i = \int_{S_2} \vec{E}_2 \cdot d\vec{S}_2 =$$

$$= E 2\pi \rho h$$

Autor: Iván Caloria Álvarez

$$\Phi_{enc. S} = \int_V dq = \int_0^h \lambda(z) dz + \int_S \sigma dS$$

$$\int_S \sigma dS = \int_S \sigma_0 dS = \sigma_0 2\pi R h$$

$$\text{Ley de Gauss} \Rightarrow E 2\pi \rho h = \frac{\lambda h + 2\pi R h \sigma_0}{\epsilon_0} \Rightarrow$$

$$\Rightarrow \boxed{\vec{E} = \frac{\lambda + 2\pi R \sigma_0}{2\pi \epsilon_0 \rho} \vec{u}_\rho \quad \rho > R}$$

Calculamos el potencial eléctrico:

$$\vec{E} = -\vec{\nabla} \phi = -\frac{\partial \phi}{\partial \rho} \vec{u}_\rho \Rightarrow$$

$$\Rightarrow -\frac{\partial \phi}{\partial \rho} = \begin{cases} \frac{\lambda}{2\pi \epsilon_0 \rho} & \rho < R \\ \frac{\lambda + 2\pi R \sigma_0}{2\pi \epsilon_0 \rho} & \rho > R \end{cases} \Rightarrow$$

$$\Rightarrow \phi = \begin{cases} -\frac{\lambda}{2\pi \epsilon_0} \ln \rho + C & \rho \leq R \\ -\frac{(\lambda + 2\pi R \sigma_0)}{2\pi \epsilon_0} \ln \rho & \rho \geq R \end{cases} \quad \begin{array}{l} \rho \text{ está en} \\ \text{unidades u.} \\ \text{y } \ln \rho = \ln\left(\frac{\rho}{\rho_0}\right) \\ \rho_0 = 1 \text{ u.} \end{array}$$

Continuidad de ϕ en $\rho = R \Rightarrow$

$$\Rightarrow -\frac{\lambda}{2\pi \epsilon_0} \ln R + C = -\frac{(\lambda + 2\pi R \sigma_0)}{2\pi \epsilon_0} \ln R \Rightarrow$$

$$\Rightarrow C = -\frac{R \sigma_0 \ln R}{\epsilon_0} \Rightarrow$$

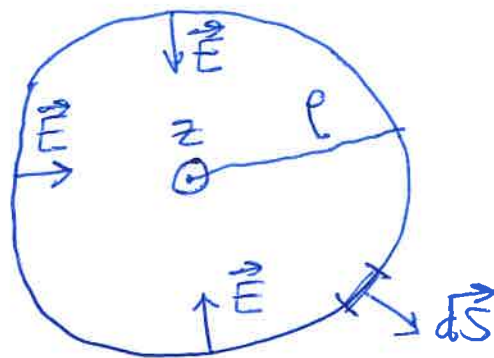
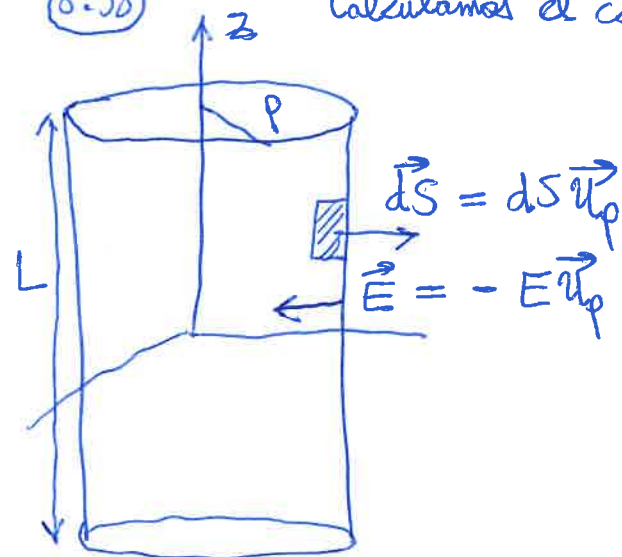
Autor: Iván Cabrera Álvarez

$$\Rightarrow \phi = \begin{cases} -\frac{\lambda}{2\pi \epsilon_0} \ln \rho - \frac{R \sigma_0 \ln R}{\epsilon_0} & \rho \leq R \\ -\frac{\lambda}{2\pi \epsilon_0} \ln \rho - \frac{R \sigma_0}{\epsilon_0} \ln \rho & \rho \geq R \end{cases}$$

(8.38)

Calculamos el campo en $\rho < a$:

(8.46)



$$\vec{E} \cdot d\vec{S} = -E dS = E_\rho dS$$

$$E = |\vec{E}| \geq 0 \quad E_\rho = -E \leq 0$$

$$\vec{E} = -E \vec{u}_\rho = E_\rho \vec{u}_\rho$$

$$\oint_S \vec{E} \cdot d\vec{S} = E_\rho \int_S dS = E_\rho 2\pi \rho L$$

$$\begin{aligned} \Phi_{\text{encerrada en } S} &= \int_V dq' = \int_V dV(\rho') dV' = \int_0^{2\pi} d\phi' \int_0^L dz' \int_0^\rho dV(\rho') \rho' d\rho' = \\ &= 2\pi L \int_0^\rho dV(\rho') \rho' d\rho' = -2\pi L \int_0^\rho \frac{D}{a} \rho'^2 d\rho' = -\frac{2\pi L D}{a} \frac{\rho^3}{3} \end{aligned}$$

$$\text{Ley de Gauss} \Rightarrow E_\rho 2\pi \rho L = -\frac{2\pi L D \rho^3}{3a\epsilon_0} \Rightarrow E_\rho = -\frac{D\rho^2}{3a\epsilon_0} \Rightarrow$$

$$\Rightarrow \boxed{\vec{E} = -\frac{D\rho^2}{3a\epsilon_0} \vec{u}_\rho \quad \rho < a}$$

Comprobamos que la dimensión de $D\rho^2/3a\epsilon_0$ es correcta:

$$[D] = \text{C L}^{-3}$$

$$[\epsilon_0] = \text{C}^2 \text{L}^{-3} \text{M}^{-1} \text{T}^2$$

$$[a] = \text{L}$$

$$[\rho] = \text{L}$$

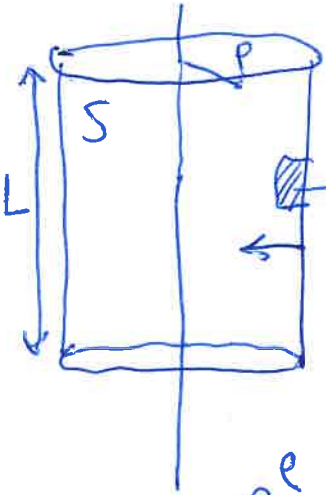
$$[3] = 1$$

$$\begin{aligned} \Rightarrow \left[\frac{D\rho^2}{3a\epsilon_0} \right] &= \frac{\text{C L}^{-3} \text{L}^2}{\text{L C}^2 \text{L}^{-3} \text{M}^{-1} \text{T}^2} = \\ &= \text{L M T}^{-2} \text{C}^{-1} = \frac{\text{L M T}^{-2}}{\text{C}} = \frac{[F]}{[q]} = \end{aligned}$$

$$= [E] \Rightarrow \frac{D\rho^2}{3a\epsilon_0} \text{ tiene dimensión de campo eléctrico} \Rightarrow \text{tiene la dimensión correcta.}$$

Autor: Iván Cabrera Álvarez

Calculamos el campo en $a < \rho < b$:



$$\oint_S \vec{E} \cdot d\vec{S} = E_\rho 2\pi \rho L$$

$$Q_{\text{encerrada en } S} = \int_0^{2\pi} d\varphi' \int_0^L dz' \int_0^e dv(\rho') \rho' d\rho' =$$

$$= 2\pi L \int_0^e dv(\rho') \rho' d\rho' = 2\pi L \left[\int_0^a dv(\rho') \rho' d\rho' + \int_a^e dv(\rho') \rho' d\rho' \right] =$$

$$\int_0^a dv(\rho') \rho' d\rho' = -\frac{D}{a} \int_0^a \rho'^2 d\rho' = -\frac{Da^3}{3a} = -\frac{Da^2}{3}$$

$$\int_a^e dv(\rho') \rho' d\rho' = -D \int_a^e \rho' d\rho' = -D \frac{\rho'^2}{2} \Big|_a^e = -\frac{D}{2} (\rho^2 - a^2)$$

$$Q_{\text{encerrada en } S} = 2\pi L \left[-\frac{Da^2}{3} - \frac{D}{2} (\rho^2 - a^2) \right]$$

$$\text{Ley de Gauss} \Rightarrow E_\rho 2\pi \rho L = 2\pi L \left[-\frac{Da^2}{3\epsilon_0} - \frac{D}{2} \frac{(\rho^2 - a^2)}{\epsilon_0} \right] \Rightarrow$$

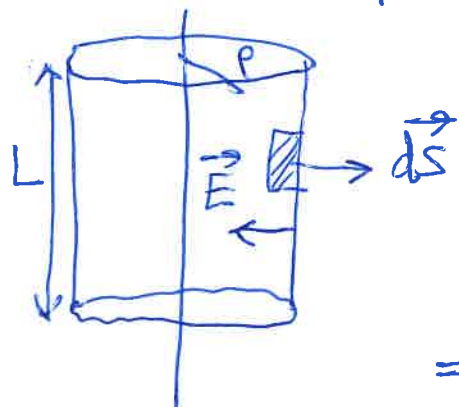
$$\Rightarrow E_\rho = \frac{-Da^2}{3\epsilon_0} - \frac{D}{2\epsilon_0} \left(\rho - \frac{a^2}{\rho} \right) = \frac{Da^2}{6\epsilon_0} - \frac{D\rho}{2\epsilon_0} \Rightarrow$$

$$\Rightarrow \boxed{\vec{E} = \left(\frac{Da^2}{6\epsilon_0} - \frac{D\rho}{2\epsilon_0} \right) \vec{u}_\rho \quad a < \rho < b}$$

Tutor: Iván Gabrila Álvarez

Calculamos el campo en $\rho > b$:

(8.18)



$$\oint_S \vec{E} \cdot d\vec{S} = E_\rho 2\pi \rho L$$

$$Q_{encerrada \text{ en } S} = \int_0^{2\pi} d\varphi' \int_0^L dz' \int_0^\rho dv(\rho') \rho' d\rho' =$$

$$= 2\pi L \int_0^\rho dv(\rho') \rho' d\rho' =$$

$$= 2\pi L \left[-\int_0^a \frac{D}{a} \rho'^2 d\rho' - \int_a^b D \rho' d\rho' \right] = -2\pi L D \left[\int_0^a \frac{\rho'^2 d\rho'}{a} + \int_a^b \rho' d\rho' \right] =$$

$$= -2\pi L D \left[\frac{2a^2}{6} + \frac{b^3}{2} - \frac{3a^3}{6} \right] = -2\pi L D \left(\frac{b^3}{2} - \frac{a^3}{6} \right)$$

$$\text{Ley de Gauss} \Rightarrow E_\rho 2\pi \rho = \frac{-2\pi L D}{\epsilon_0} \left(\frac{b^3}{2} - \frac{a^3}{6} \right) \Rightarrow$$

$$\Rightarrow E_\rho = -\frac{D}{\epsilon_0} \left(\frac{b^3}{2} - \frac{a^3}{6} \right) \Rightarrow \boxed{\vec{E} = -\frac{D}{\rho \epsilon_0} \left(\frac{b^3}{2} - \frac{a^3}{6} \right) \vec{u}_\rho \quad \rho > b}$$

Calculamos el potencial eléctrico:

$$\vec{E} = -\vec{\nabla} \phi = -\frac{\partial \phi}{\partial \rho} \vec{u}_\rho \Rightarrow$$

$$\Rightarrow -\frac{\partial \phi}{\partial \rho} = \begin{cases} -\frac{D \rho^3}{3a\epsilon_0} & \rho < a \\ -\frac{D}{\epsilon_0} \left(\frac{\rho}{2} - \frac{a^3}{6\rho} \right) & a < \rho < b \\ -\frac{D}{\rho \epsilon_0} \left(\frac{b^3}{2} - \frac{a^3}{6} \right) & b < \rho \end{cases} \Rightarrow$$

Autor: Iván Cabria Álvarez

$$\Rightarrow \phi(\rho) = \begin{cases} \frac{D \rho^3}{9a\epsilon_0} & \rho \leq a \\ \frac{D \rho^2}{4\epsilon_0} - \frac{Da^2}{6\epsilon_0} \ln \rho + C_1 & a \leq \rho \leq b \\ \frac{D}{\epsilon_0} \left(\frac{b^3}{2} - \frac{a^3}{6} \right) \ln \rho + C_2 & b \leq \rho \end{cases}$$

ρ está en unidades de a
y $\ln \rho = \ln \left(\frac{\rho}{\rho_0} \right)$, $\rho_0 = 1$.

ϕ debe ser continua en $\rho=a \Rightarrow$

$$\Rightarrow \frac{Da^3}{9a\epsilon_0} = \frac{Da^2}{4\epsilon_0} - \frac{Da^2}{6\epsilon_0} \ln a + C_1 \Rightarrow$$

$$\Rightarrow C_1 = \frac{4Da^2}{36\epsilon_0} - \frac{9Da^2}{36\epsilon_0} + \frac{Da^2}{6\epsilon_0} \ln a = -\frac{5Da^2}{36\epsilon_0} + \frac{Da^2}{6\epsilon_0} \ln a =$$

$$= \boxed{\frac{Da^2}{6\epsilon_0} \left(\ln a - \frac{5}{6} \right)}$$

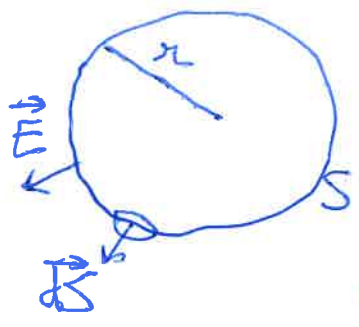
ϕ debe ser continua en $\rho=b \Rightarrow$

$$\Rightarrow \frac{Db^2}{4\epsilon_0} - \frac{Da^2}{6\epsilon_0} \ln b + C_1 = \frac{b}{\epsilon_0} \left(\frac{b^2}{2} - \frac{a^2}{6} \right) \ln b + C_2 \Rightarrow$$

$$\Rightarrow C_2 = \frac{Db^2}{4\epsilon_0} - \frac{Da^2}{6\epsilon_0} \ln b + C_1 - \frac{Db^2}{2\epsilon_0} \ln b + \frac{Da^2}{6\epsilon_0} \ln b =$$

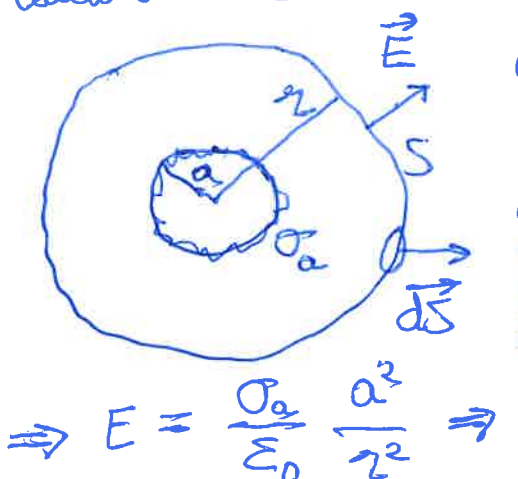
$$= \boxed{\frac{Db^2}{4\epsilon_0} - \frac{Db^2}{2\epsilon_0} \ln b + \frac{Da^2}{6\epsilon_0} \ln a - \frac{5Da^2}{36\epsilon_0}}$$

(8.39) Calculamos $\vec{E}$ en $r < a$:



$$\left. \begin{array}{l} Q_{enc} S = 0 \\ \oint_S \vec{E} \cdot d\vec{S} = E 4\pi r^2 \\ \vec{E} = E \vec{u}_r \\ \text{Teor. Gauss} \end{array} \right\} \Rightarrow E = 0 \Rightarrow \vec{E} = \vec{0} \quad r < a$$

Calculamos $\vec{E}$ en $a < r < b$:

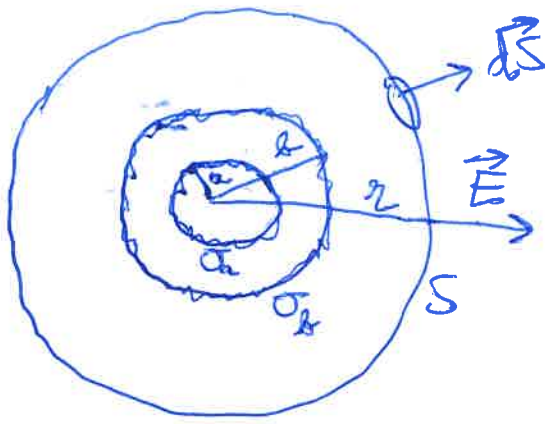


$$\left. \begin{array}{l} Q_{enc} S = \int_V \rho dV = \sigma_a 4\pi a^2 \\ \oint_S \vec{E} \cdot d\vec{S} = E 4\pi r^2 \\ \vec{E} = E \vec{u}_r \\ \text{Teor. Gauss} \end{array} \right\} \Rightarrow E 4\pi r^2 = \frac{\sigma_a 4\pi a^2}{\epsilon_0} \Rightarrow$$

$$\Rightarrow E = \frac{\sigma_a}{\epsilon_0} \frac{a^2}{r^2} \Rightarrow \vec{E} = \frac{\sigma_a}{\epsilon_0} \frac{a^2}{r^2} \vec{u}_r \quad a < r < b$$

Autor: Juan Cabria Álvarez

Calculamos $\vec{E}$ en $b < r$:



(8.50)

$$\begin{aligned}
 Q_{enc S} &= \int_V \rho dV = \sigma_a 4\pi a^2 + \sigma_b 4\pi b^2 = \\
 &= \sigma_a 4\pi a^2 - |\sigma_b| 4\pi b^2 = \\
 &= \sigma_a 4\pi a^2 - \sigma_a \frac{a^3}{b^2} 4\pi b^2 = 0 \\
 \oint_S \vec{E} \cdot d\vec{S} &= E 4\pi r^2 \\
 \vec{E} &= E \vec{u}_r \\
 &\text{Teor. Gauss}
 \end{aligned}
 \Rightarrow$$

$$\Rightarrow E = 0 \Rightarrow \vec{E} = \vec{0} \quad b < r$$

Calculamos ϕ :

$$\vec{E} = -\vec{\nabla}\phi \Rightarrow \vec{E} = -\frac{\partial\phi}{\partial r} \vec{u}_r \Rightarrow \frac{\partial\phi}{\partial r} = \begin{cases} 0 & r < a \\ -\frac{\sigma_a}{\epsilon_0} \frac{a^2}{r^2} & a < r < b \\ 0 & b < r \end{cases}$$

$$\Rightarrow \phi = \begin{cases} C_1 & r \leq a \\ + \frac{\sigma_a}{\epsilon_0} \frac{a^2}{r} & a \leq r \leq b \\ C_2 & b \leq r \end{cases}$$

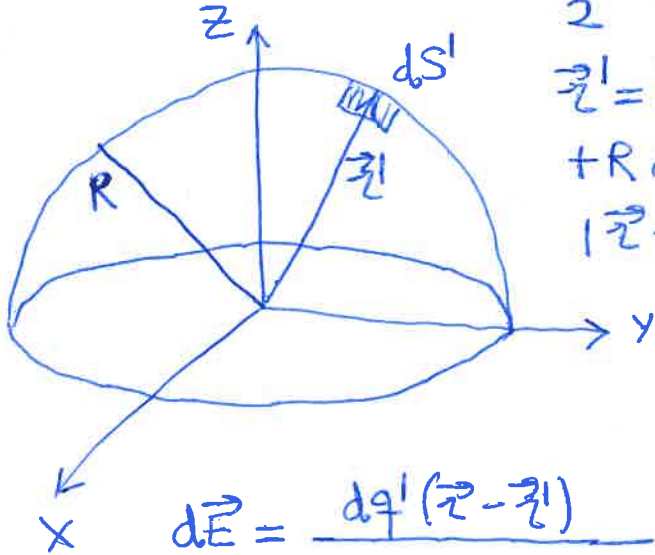
$$\phi \text{ continua en } r=a \Rightarrow C_1 = \frac{\sigma_a}{\epsilon_0} \frac{a^2}{a} = \boxed{\frac{\sigma_a a}{\epsilon_0}}$$

$$\phi \text{ continua en } r=b \Rightarrow \boxed{C_2 = \frac{\sigma_a}{\epsilon_0} \frac{a^2}{b}}$$

(8.40)

$$\sigma = \begin{cases} \sigma_0 \sin \theta & \text{si } 0 \leq \theta \leq \pi/2 \\ 0 & \text{si } \frac{\pi}{2} < \theta \leq \pi \end{cases}$$

(8.51)



$$\vec{r}' = R \cos \varphi' \sin \theta' \vec{u}_x + R \sin \varphi' \sin \theta' \vec{u}_y + R \cos \theta' \vec{u}_z$$

$$\vec{r} = \vec{0}$$

$$|\vec{r} - \vec{r}'| = R$$

$$\vec{r} - \vec{r}' = 0 - \vec{r}'$$

$$dq' = \sigma dS' = \sigma R^2 \sin \theta' d\theta' d\varphi' = \sigma_0 R^2 (\sin \theta')^2 d\theta' d\varphi'$$

$$d\vec{E} = \frac{dq' (\vec{r} - \vec{r}')}{4\pi \epsilon_0 |\vec{r} - \vec{r}'|^3} = \frac{\sigma_0 R^2 (\sin \theta')^2 d\theta' d\varphi'}{4\pi \epsilon_0 R^3} (\vec{r} - \vec{r}') =$$

$$= - \frac{\sigma_0 (\sin \theta')^2}{4\pi \epsilon_0 R} d\theta' d\varphi' (R \cos \varphi' \sin \theta' \vec{u}_x + R \sin \varphi' \sin \theta' \vec{u}_y + R \cos \theta' \vec{u}_z) =$$

$$= - \frac{\sigma_0}{4\pi \epsilon_0} d\theta' d\varphi' (\cos \varphi' (\sin \theta')^3 \vec{u}_x + \sin \varphi' (\sin \theta')^3 \vec{u}_y + \cos \theta' (\sin \theta')^2 \vec{u}_z)$$

$$\text{si } 0 \leq \theta \leq \frac{\pi}{2} \quad d\vec{E} = \vec{0} \quad \text{si } \frac{\pi}{2} < \theta \leq \pi$$

$$\vec{E} = \int d\vec{E} = - \frac{\sigma_0}{4\pi \epsilon_0} \left[\int_0^{2\pi} d\varphi' \cos \varphi' \int_0^{\pi/2} (\sin \theta')^3 d\theta' \vec{u}_x + \int_0^{2\pi} d\varphi' \sin \varphi' \int_0^{\pi/2} (\sin \theta')^3 d\theta' \vec{u}_y + \int_0^{2\pi} d\varphi' \int_0^{\pi/2} \cos \theta' (\sin \theta')^2 d\theta' \vec{u}_z \right]$$

$$I \equiv \int_0^{\pi/2} (\sin \theta')^3 d\theta' \quad J \equiv \int_0^{\pi/2} \cos \theta' (\sin \theta')^2 d\theta' = \frac{(\sin \theta')^3}{3} \Big|_0^{\pi/2} = \frac{1}{3}$$

$$K \equiv \int_0^{2\pi} d\varphi' \cos \varphi' = \int_0^{2\pi} d\varphi' \sin \varphi' = 0 \quad \int_0^{2\pi} d\varphi' = 2\pi$$

$$\vec{E} = - \frac{\sigma_0}{4\pi \epsilon_0} [K I \vec{u}_x + K I \vec{u}_y + 2\pi J \vec{u}_z] = - \frac{\sigma_0}{4\pi \epsilon_0} \frac{2\pi}{3} \vec{u}_z$$

$$\boxed{\vec{E} = - \frac{\sigma_0}{6\epsilon_0} \vec{u}_z}$$

Autor: Iván Gabrila Álvarez

Calculamos el potencial en $\vec{r} = \vec{0}$:

(8.52)

$$d\phi = \frac{dq'}{4\pi\epsilon_0 |\vec{r} - \vec{r}'|} = \frac{\sigma_0 \sin\theta' R^2 \sin\theta' d\theta' d\varphi'}{4\pi\epsilon_0 R} = \frac{\sigma_0 R}{4\pi\epsilon_0} (\sin\theta')^2 d\theta' d\varphi'$$

si $0 \leq \theta \leq \frac{\pi}{2}$

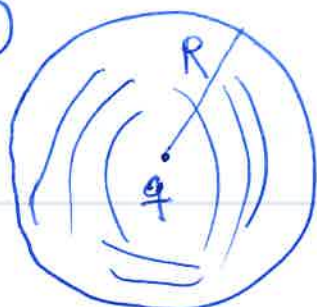
$$d\phi = 0 \text{ si } \frac{\pi}{2} < \theta \leq \pi$$

$$\phi = \int d\phi = \frac{\sigma_0 R}{4\pi\epsilon_0} \int_0^{\pi/2} (\sin\theta')^2 d\theta' \int_0^{2\pi} d\varphi' = \frac{\sigma_0 R}{4\pi\epsilon_0} K 2\pi = \frac{\sigma_0 R K}{2\epsilon_0}$$

$$K \equiv \int_0^{\pi/2} (\sin\theta')^2 d\theta' = \left[\frac{\theta'}{2} - \frac{\sin 2\theta'}{4} \right]_0^{\pi/2} = \frac{\pi}{4}$$

$$\phi = \frac{\sigma_0 R}{2\epsilon_0} \frac{\pi}{4} = \boxed{\frac{\sigma_0 R \pi}{8\epsilon_0}}$$

(8.41)



$$q = +Ze$$

El conjunto está formado por una carga puntual q en el centro y una distribución continua de carga dentro de la esfera de radio R .

Primera forma de resolver el problema:

Según el principio de superposición, el potencial será la

suma de los potenciales debidos a las dos distribuciones de carga:

$$\phi(\vec{r}) = \phi_{\text{con}}(\vec{r}) + \phi_p(\vec{r})$$

$\phi_{\text{con}}(\vec{r})$: El potencial debido a la distribución continua de carga.

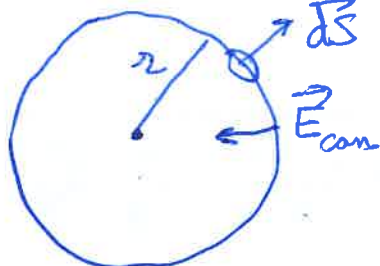
$\phi_p(\vec{r})$: El potencial debido a la carga puntual q .

Calculamos $\phi_{\text{con}}(\vec{r})$: Calculamos primero $\vec{E}_{\text{con}}(\vec{r})$ y luego, mediante

$$\vec{E}_{\text{con}} = -\vec{\nabla} \phi_{\text{con}}, \text{ calculamos } \phi_{\text{con}}(\vec{r}).$$

Calculamos $\vec{E}_{\text{con}}$ para $r < R$:

$$\vec{E}_{\text{con}} = E \vec{u}_r \quad E < 0$$



$$\rho(r) = \begin{cases} -\frac{Ze}{\pi R^4} r & \text{si } 0 < r \leq R \\ 0 & \text{si } r > R \end{cases}$$

$$Q_{\text{enc},S} = \int_{V_z} \rho(r) dV = \int_0^z 4\pi \rho(r) r^2 dr = -\frac{Ze}{\pi R^4} 4\pi \int_0^z r^3 dr =$$

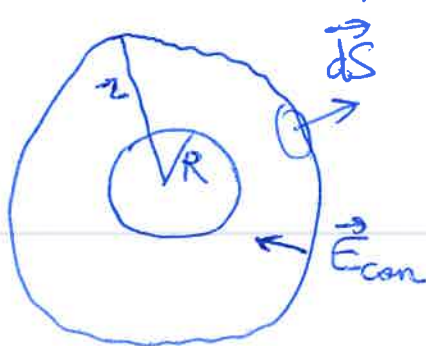
$$= -\frac{4Ze}{R^4} \frac{r^4}{4} \Big|_0^z = -\frac{Ze r^4}{R^4}$$

$$\oint_{S_z} \vec{E}_{\text{con}} \cdot d\vec{S} = +E 4\pi r^2 \quad E < 0$$

$$\text{Teor. de Gauss} \Rightarrow +E 4\pi r^2 = -\frac{Ze r^4}{\epsilon_0 R^4} \Rightarrow E = -\frac{Ze r^2}{4\pi \epsilon_0 R^4} < 0$$

$$\Rightarrow \vec{E}_{\text{con}} = -\frac{Ze r^2}{4\pi \epsilon_0 R^4} \vec{u}_r \quad r < R$$

Calculamos $\vec{E}_{\text{con}}$ para $R < r$:



$$\vec{E}_{\text{con}} = E \vec{u}_r \quad E < 0$$

$$Q_{\text{enc},S} = \int_{V_z} \rho(r) dV = \int_{V_R} \rho(r) dV = -\frac{Ze R^4}{R^4} = -Ze$$

$$Q_{\text{enc},S} = -Ze < 0$$

$$\vec{E}_{\text{con}} \cdot d\vec{S} = E \vec{u}_r \cdot dS \vec{u}_r = E dS \quad E < 0$$

$$\oint_{S_z} \vec{E} \cdot d\vec{S} = +E 4\pi r^2$$

$$\text{Teor. de Gauss} \Rightarrow +E 4\pi r^2 = -\frac{Ze}{\epsilon_0} \Rightarrow E = -\frac{Ze}{4\pi \epsilon_0 r^2} < 0 \Rightarrow$$

$$\Rightarrow \vec{E}_{\text{con}} = -\frac{Ze}{4\pi \epsilon_0 r^2} \vec{u}_r \quad R < r$$

Calculamos ϕ_{con} mediante $\vec{E}_{\text{con}} = -\vec{\nabla} \phi_{\text{con}}$:

$$\vec{E}_{\text{con}} = -\vec{\nabla} \phi_{\text{con}} \Rightarrow -\frac{\partial \phi_{\text{con}}}{\partial r} = \begin{cases} -\frac{Ze r^2}{4\pi \epsilon_0 R^4} & r < R \\ -\frac{Ze}{4\pi \epsilon_0 r^2} & R < r \end{cases} \Rightarrow$$

$$\Rightarrow \phi_{\text{con}}(z) = \begin{cases} + \frac{Ze z^3}{12\pi\epsilon_0 R^4} + C & z \leq R \\ - \frac{Ze}{4\pi\epsilon_0 z} & R \leq z \end{cases}$$

$$\phi_{\text{con}} \text{ debe ser continua en } z=R \Rightarrow + \frac{Ze R^3}{12\pi\epsilon_0 R^4} + C = - \frac{Ze}{4\pi\epsilon_0 R} \Rightarrow$$

$$\Rightarrow C = - \frac{Ze}{4\pi\epsilon_0 R} - \frac{Ze}{12\pi\epsilon_0 R} = - \frac{Ze}{3\pi\epsilon_0 R}$$

$$\text{Calculamos } \phi_p(z): \quad \phi_p(z) = \frac{q}{4\pi\epsilon_0 z} = + \frac{Ze}{4\pi\epsilon_0 z} \quad z > 0$$

$$\phi(z) = \phi_{\text{con}}(z) + \phi_p(z) = \begin{cases} \frac{Ze z^3}{12\pi\epsilon_0 R^4} + \frac{Ze}{4\pi\epsilon_0 z} - \frac{Ze}{3\pi\epsilon_0 R} & z \leq R \\ 0 & R \leq z \end{cases}$$

La solución más general:

$$\Rightarrow \phi_{\text{con}}(z) = \begin{cases} + \frac{Ze z^3}{12\pi\epsilon_0 R^4} + C_1 & z \leq R \\ - \frac{Ze}{4\pi\epsilon_0 z} + C_2 & R \leq z \end{cases}$$

$$\phi_{\text{con}} \text{ debe ser continua en } z=R \Rightarrow \frac{Ze R^3}{12\pi\epsilon_0 R^4} = - \frac{Ze}{4\pi\epsilon_0 R} + C_2 - C_1$$

$$C_2 - C_1 = + \frac{Ze}{3\pi\epsilon_0 R}$$

$$\phi(z) = \phi_{\text{con}}(z) + \phi_p(z) = \begin{cases} + \frac{Ze z^3}{12\pi\epsilon_0 R^4} + \frac{Ze}{4\pi\epsilon_0 z} + C_1 & z \leq R \\ C_2 & R \leq z \end{cases}$$

Segunda manera de resolver el problema: Calculamos primero $\vec{E}_{con}$. Después calculamos $\vec{E}_p$. El campo debido a todo el conjunto es $\vec{E} = \vec{E}_{con} + \vec{E}_p$ según el principio de superposición. Finalmente calculamos ϕ a partir de $\vec{E} = -\vec{\nabla}\phi$.

Ya nos calculado anteriormente $\vec{E}_{con}(\vec{r})$. Calculamos $\vec{E}_p(\vec{r})$:

$$\vec{E}_p(\vec{r}) = \frac{q}{4\pi\epsilon_0 r^2} \vec{u}_r = \frac{Ze}{4\pi\epsilon_0 r^2} \vec{u}_r \quad r > 0$$

$$\vec{E}(\vec{r}) = \vec{E}_{con}(\vec{r}) + \vec{E}_p(\vec{r}) = \begin{cases} -\frac{Ze r^2}{4\pi\epsilon_0 R^4} \vec{u}_r + \frac{Ze}{4\pi\epsilon_0 r^2} \vec{u}_r & r < R \\ -\frac{Ze}{4\pi\epsilon_0 r^2} \vec{u}_r + \frac{Ze}{4\pi\epsilon_0 r^2} \vec{u}_r & R < r \end{cases} \Rightarrow$$

$$\Rightarrow \vec{E}(\vec{r}) = \begin{cases} -\frac{Ze r^2}{4\pi\epsilon_0 R^4} \vec{u}_r + \frac{Ze}{4\pi\epsilon_0 r^2} \vec{u}_r & r < R \\ \vec{0} & R < r \end{cases}$$

$$\vec{E}(\vec{r}) = -\vec{\nabla}\phi \Rightarrow -\frac{\partial\phi}{\partial r} = \begin{cases} -\frac{Ze r^2}{4\pi\epsilon_0 R^4} + \frac{Ze}{4\pi\epsilon_0 r^2} & r < R \\ 0 & R < r \end{cases} \Rightarrow$$

$$\Rightarrow \phi = \begin{cases} +\frac{Ze r^3}{12\pi\epsilon_0 R^4} + \frac{Ze}{4\pi\epsilon_0 r} & r \leq R \\ C & R \leq r \end{cases}$$

$$\phi \text{ debe ser continua en } r=R \Rightarrow \frac{Ze R^3}{12\pi\epsilon_0 R^4} + \frac{Ze}{4\pi\epsilon_0 R} = C \Rightarrow$$

$$\Rightarrow C = \frac{4Ze}{12\pi\epsilon_0 R} = \frac{Ze}{3\pi\epsilon_0 R} \Rightarrow$$

Autor: Iván Gabría Álvarez

$$\Rightarrow \phi(\vec{r}) = \begin{cases} \frac{Ze r^3}{12\pi\epsilon_0 R^4} + \frac{Ze}{4\pi\epsilon_0 r} & r \leq R \\ \frac{Ze}{3\pi\epsilon_0 R} & R \leq r \end{cases}$$

La solución más general:

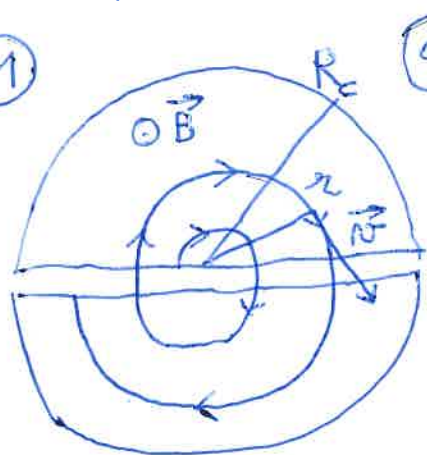
$$\vec{E}(z) = -\vec{\nabla}\phi(z) \Rightarrow -\frac{\partial\phi}{\partial z} = \begin{cases} -\frac{ze z^3}{4\pi\epsilon_0 R^4} + \frac{ze}{4\pi\epsilon_0 z^2} & z < R \\ 0 & R < z \end{cases} \quad (8.56) \Rightarrow$$

$$\Rightarrow \phi(z) = \begin{cases} +\frac{ze z^3}{12\pi\epsilon_0 R^4} + \frac{ze}{4\pi\epsilon_0 z} + C_1 & z \leq R \\ C_2 & R \leq z \end{cases}$$

$\phi(z)$ debe ser continua en $z=R \Rightarrow$

$$\Rightarrow \frac{ze R^3}{12\pi\epsilon_0 R^4} + \frac{ze}{4\pi\epsilon_0 R} + C_1 = C_2 \Rightarrow \boxed{C_2 - C_1 = +\frac{ze}{3\pi\epsilon_0 R}}$$

9.1



$$(a) \vec{F} = q(\vec{v} \times \vec{B}) \Rightarrow \left. \begin{aligned} F &= qvB \\ F &= \frac{mv^2}{r} \end{aligned} \right\} \Rightarrow \left. \begin{aligned} qB &= \frac{mv}{r} \\ v &= \omega r \end{aligned} \right\} \Rightarrow$$

$$\Rightarrow qB = m \frac{\omega r}{r} \Rightarrow \omega = \frac{qB}{m}$$

$$\omega = 2\pi\nu \Rightarrow \nu = \frac{\omega}{2\pi} = \frac{qB}{2\pi m}$$

$$\nu = \frac{1.6020 \cdot 10^{-19} \text{ C} \cdot 1.5 \text{ T}}{2\pi \cdot 1.67 \cdot 10^{-27} \text{ kg}} = 22.9 \cdot 10^6 \text{ s}^{-1} = \boxed{22.9 \text{ MHz}}$$

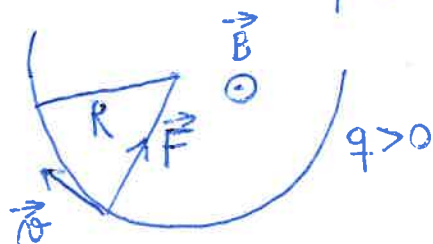
(b) La energía cinética de un protón al salir del ciclotrón es la energía cinética de un protón con una trayectoria cuyo radio de curvatura es R_c , el radio del ciclotrón

$$v = \omega R_c \Rightarrow T = \frac{mv^2}{2} = \frac{m}{2} \omega^2 R_c^2 = \frac{m}{2} (2\pi\nu)^2 R_c^2 = m 2\pi^2 \nu^2 R_c^2$$

$$= 1.67 \cdot 10^{-27} \text{ kg} \cdot 2\pi^2 (22.9 \cdot 10^6 \text{ s}^{-1})^2 (0.5 \text{ m})^2 = \boxed{4.32 \cdot 10^{-12} \text{ J}}$$

9.2 $\vec{r} = r_x \vec{u}_x + r_y \vec{u}_y$

$$\vec{F} = q(\vec{r} \times \vec{B}) = q \begin{vmatrix} \vec{u}_x & \vec{u}_y & \vec{u}_z \\ r_x & r_y & 0 \\ 0 & 0 & B_0 \end{vmatrix} = q r_y B_0 \vec{u}_x - q r_x B_0 \vec{u}_y \quad q > 0$$



$\vec{F} = q(\vec{r} \times \vec{B})$ es normal a la trayectoria en $xy \Rightarrow \vec{F}$ sólo tiene componente normal a la trayectoria en xy

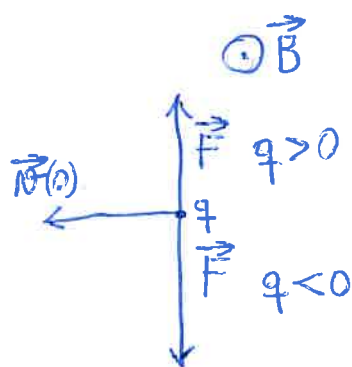
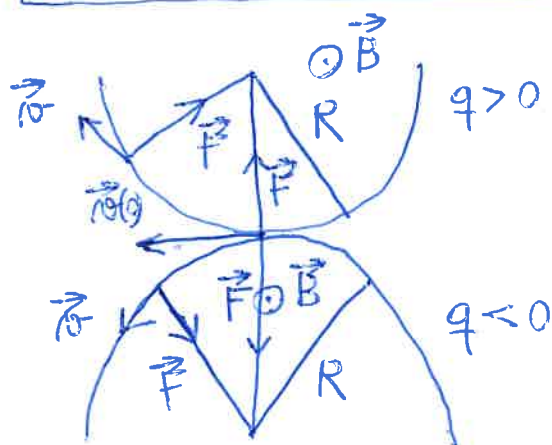
$$\left. \begin{aligned} \vec{F} &= m\vec{a} \\ \vec{a} &= \vec{a}_t + \vec{a}_n \end{aligned} \right\} \Rightarrow \left. \begin{aligned} \vec{F} &= m(\vec{a}_t + \vec{a}_n) \\ \vec{F} &\text{ sólo tiene componente normal} \end{aligned} \right\} \Rightarrow \vec{a}_t = 0 \Rightarrow v = |\vec{v}| \text{ constante}$$

$$\left. \begin{aligned} \vec{F} &= m\vec{a} = m\vec{a}_n \\ \vec{F} &= q(\vec{r} \times \vec{B}) \\ \vec{r} &\perp \vec{B} \end{aligned} \right\} \Rightarrow \left. \begin{aligned} qvB &= ma_n \\ a_n &= \frac{qvB}{m} \end{aligned} \right\} \Rightarrow v \text{ constante}$$

$$\Rightarrow \left. \begin{aligned} a_t &= 0 \\ a_n &\text{ constante} \end{aligned} \right\} \Rightarrow \boxed{\text{mov. circular uniforme en } xy}$$

Autor: Iván Cabria Álvarez

Si $q < 0$, entonces el mov. también es circular uniforme con la misma velocidad $v \equiv |\vec{v}|$, am y radio de curvatura $R = \frac{v^2}{a_m}$, pero centrado en otro punto.



Si $\vec{v}$ tuviera una componente en z :

$$\vec{v} = v_x \vec{u}_x + v_y \vec{u}_y + v_z \vec{u}_z$$

$$\vec{F} = q(\vec{v} \times \vec{B}) = q \begin{vmatrix} \vec{u}_x & \vec{u}_y & \vec{u}_z \\ v_x & v_y & v_z \\ 0 & 0 & B_0 \end{vmatrix} = q v_y B_0 \vec{u}_x - q v_x B_0 \vec{u}_y \Rightarrow \text{mov. circular uniforme en } XY$$

$$\vec{F} = m \vec{a} \Rightarrow 0 = m \frac{dv_z}{dt} \Rightarrow v_z(t) = v_z(0) \text{ constante} \Rightarrow$$

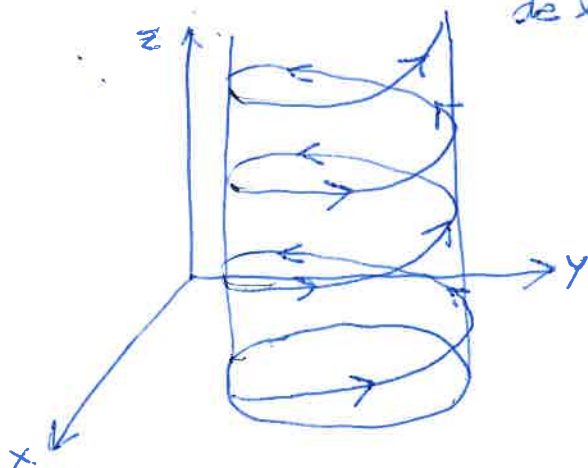
$$\Rightarrow z(t) = z(0) + v_z(0)t \Rightarrow$$

mov. ^{lineal} uniforme en z
y circular uniforme en $XY \Rightarrow$

$\Rightarrow$ mov. en hélice,
el eje de la hélice

con z

El sentido de giro de la hélice
depende de $\vec{v}(0)$ y del signo
de la carga q



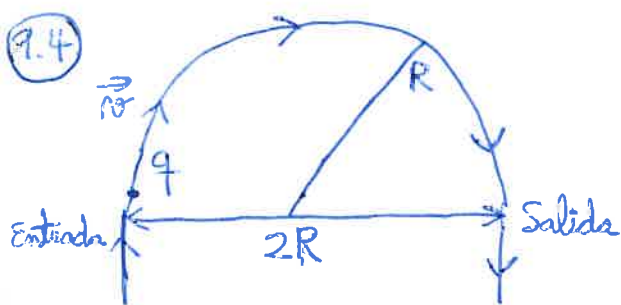
9.3
$$\left. \begin{aligned} E_x = T_x + V_x &= 0 + e\Delta\Phi \\ E_y = T_y + V_y &= \frac{m_e}{2} v^2 + 0 \\ E_x &= E_y \end{aligned} \right\} \Rightarrow v = \sqrt{\frac{2e\Delta\Phi}{m_e}} = \sqrt{\frac{2 \cdot 1.602 \cdot 10^{-19} \text{ C} \cdot 10^4 \text{ V}}{9.1 \cdot 10^{-31} \text{ kg}}}$$

$v = 5.93 \cdot 10^7 \text{ m/s} \approx 0.2 c$

$c = \text{velocidad de la luz en el vacío} = 300\,000 \text{ km/s} = 3 \cdot 10^8 \text{ m/s}$

$$\left. \begin{aligned} F &= e v B_0 \\ F &= m_e \frac{v^2}{R} \end{aligned} \right\} \Rightarrow R = \frac{m_e v^2}{e v B_0} = \frac{m_e v}{e B_0} = \frac{9.1 \cdot 10^{-31} \text{ kg} \cdot 5.93 \cdot 10^7 \text{ m/s}}{1.602 \cdot 10^{-19} \text{ C} \cdot 1 \text{ T}}$$

$$R = 0.00034 \text{ m}$$

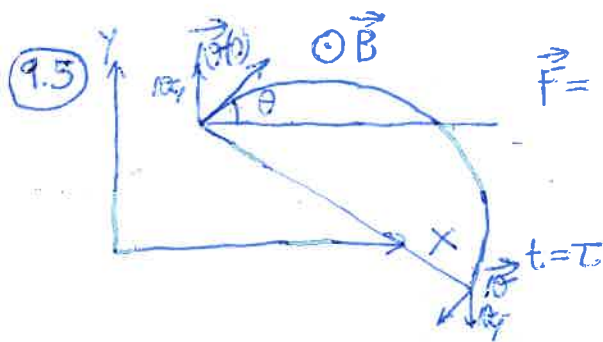


$$\left. \begin{aligned} F &= |q| v B_0 \\ F &= \frac{m v^2}{R} \end{aligned} \right\} \Rightarrow \frac{m v^2}{R} = |q| v B_0 \Rightarrow$$

$$\Rightarrow v = \frac{|q| R B_0}{m}$$

$$T_x + V_x = T_y + V_y \Rightarrow 0 + |q| \Delta\Phi = \frac{m}{2} v^2 + 0 \Rightarrow v = \sqrt{\frac{2|q|\Delta\Phi}{m}}$$

$$\Rightarrow \frac{|q| R B_0}{m} = \sqrt{\frac{2|q|\Delta\Phi}{m}} \Rightarrow \frac{|q|^2 R^2 B_0^2}{m^2} = \frac{2|q|\Delta\Phi}{m} \Rightarrow \boxed{\frac{|q| R^2 B_0^2}{2\Delta\Phi} = m}$$



$$\vec{F} = q(\vec{v} \times \vec{B}) = q \begin{vmatrix} \vec{u}_x & \vec{u}_y & \vec{u}_z \\ v \cos \theta & v \sin \theta & 0 \\ 0 & 0 & B_0 \end{vmatrix} =$$

$$= q v B_0 \sin \theta \vec{u}_x - q v B_0 \cos \theta \vec{u}_y \Rightarrow$$

$$\Rightarrow F_y = -q v B_0 \cos \theta \Rightarrow m \frac{dv_y}{dt} = -q v B_0 \cos \theta$$

$$v_y = v \sin \theta \Rightarrow \frac{dv_y}{dt} = v \cos \theta \frac{d\theta}{dt}$$

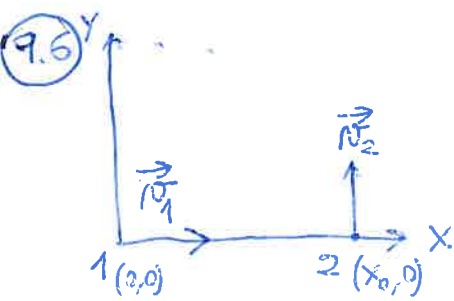
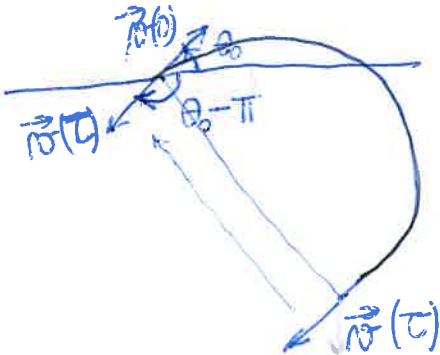
Autor: Juan Cabria Álvarez

$$\Rightarrow m r \cos \theta \frac{d\theta}{dt} = -q r B \cos \theta \Rightarrow$$

$$\Rightarrow m \frac{d\theta}{dt} = -q B_0 \Rightarrow -\frac{m}{q B_0} d\theta = dt \Rightarrow$$

$$\Rightarrow -\frac{m}{q B_0} \int_{\theta_0+0}^{\theta_0-\pi} d\theta = \int_0^T dt \Rightarrow -\frac{m}{q B_0} [\theta]_{\theta_0+0}^{\theta_0-\pi} = T \Rightarrow$$

$$\Rightarrow T = \frac{\pi m}{B_0 q} = \frac{\pi}{7.3 \text{ T } 91 \text{ C/kg}} = \boxed{0.0047 \text{ s}}$$



$$\Rightarrow \boxed{\vec{F}_{1 \rightarrow 2} = 0}$$

$$\vec{r}_1 = \vec{0}$$

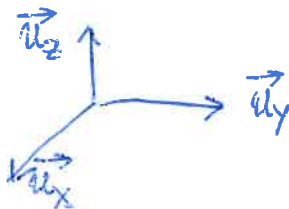
$$\vec{r}_2 = x_0 \vec{u}_x$$

$$\vec{F}_{1 \rightarrow 2} = q_2 (\vec{r}_2 \times \vec{B}_{1 \rightarrow 2}) = -e (\vec{r}_2 \times \vec{B}_{1 \rightarrow 2})$$

$$\vec{r} = \vec{r}_2 \quad \vec{r}' = \vec{r}_1 \quad \vec{r} - \vec{r}' = x_0 \vec{u}_x$$

$$q' = q_1 \quad \vec{r}' = \vec{r}_1$$

$$\vec{B}_{1 \rightarrow 2} = \frac{\mu_0}{4\pi} q_1 \frac{(\vec{r}_1 \times (\vec{r} - \vec{r}'))}{|\vec{r} - \vec{r}'|^3} = \frac{\mu_0 q_1}{4\pi} \frac{\vec{r}_1 \vec{u}_x \times x_0 \vec{u}_x}{x_0^3} = 0$$



$$\vec{F}_{2 \rightarrow 1} = q_1 (\vec{r}_1 \times \vec{B}_{2 \rightarrow 1})$$

$$\vec{r} = \vec{r}_1 \quad \vec{r}' = \vec{r}_2 \quad \vec{r} - \vec{r}' = \vec{r}_1 - \vec{r}_2 = -x_0 \vec{u}_x \quad q' = q_2 \quad \vec{r}' = \vec{r}_2$$

$$\vec{B}_{2 \rightarrow 1} = \frac{\mu_0}{4\pi} q_2 \frac{(\vec{r}_2 \times (\vec{r} - \vec{r}'))}{|\vec{r} - \vec{r}'|^3} = \frac{\mu_0 q_2}{4\pi} \frac{\vec{r}_2 \times (-x_0 \vec{u}_x)}{x_0^3} = -\frac{\mu_0 q_2}{4\pi} \frac{\vec{r}_2 \vec{u}_y \times \vec{u}_x}{x_0^2}$$

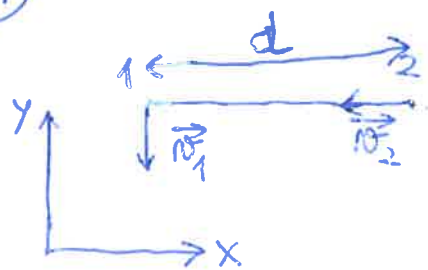
$$= +\frac{\mu_0 q_2 \vec{r}_2}{4\pi x_0^2} \vec{u}_z = -\frac{\mu_0 e \vec{r}_2}{4\pi x_0^2} \vec{u}_z$$

Autor: Iván Calvira Álvarez

$$\vec{F}_{2 \rightarrow 1} = q_1 (\vec{r}_1 \times \vec{B}_{2 \rightarrow 1}) = -e (\vec{r}_1 \vec{u}_x \times \vec{B}_{2 \rightarrow 1}) = -e \vec{r}_1 (\vec{u}_x \times (-\frac{\mu_0 e \vec{r}_2}{4\pi x_0^2} \vec{u}_z))$$

$$= \frac{e^2 \vec{r}_1 \vec{r}_2 \mu_0}{4\pi x_0^2} \vec{u}_x \times \vec{u}_z = \boxed{-\frac{\mu_0 e^2 \vec{r}_1 \vec{r}_2}{4\pi x_0^2} \vec{u}_y}$$

(9.7)



$$\vec{r}_1 = y\vec{u}_y \quad \vec{r}_2 = d\vec{u}_x + y\vec{u}_y$$

$$1 \rightarrow 2: \quad \vec{r} = \vec{r}_2 \quad \vec{r}' = \vec{r}_1 \quad q' = q_1 = -e$$

$$\vec{B}_{1 \rightarrow 2} = \frac{\mu_0}{4\pi} q' \frac{(\vec{r}' \times (\vec{r} - \vec{r}'))}{|\vec{r} - \vec{r}'|^3} = \frac{\mu_0 e \rho_1}{4\pi d^2} (\vec{u}_y \times \vec{u}_x)$$

$$e > 0$$

$$= \frac{\mu_0 q_1}{4\pi} \frac{(-\rho_1 \vec{u}_y \times d\vec{u}_x)}{d^3} = \frac{\mu_0 e \rho_1}{4\pi d^2} (\vec{u}_y \times \vec{u}_x)$$

$$= -\frac{\mu_0 e \rho_1}{4\pi d^2} \vec{u}_z$$

$$\vec{F}_{1 \rightarrow 2} = q_2 (\vec{r}_2 \times \vec{B}_{1 \rightarrow 2}) = -e \left((-\rho_2 \vec{u}_x) \times \left(-\frac{\mu_0 e \rho_1}{4\pi d^2} \vec{u}_z \right) \right) =$$

$$= -\frac{\mu_0 e^2 \rho_1 \rho_2}{4\pi d^2} (\vec{u}_x \times \vec{u}_z) = \boxed{+\frac{\mu_0 e^2 \rho_1 \rho_2}{4\pi d^2} \vec{u}_y}$$

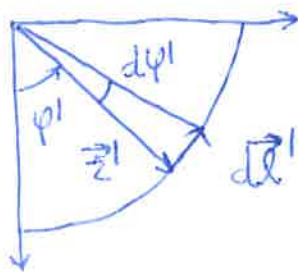
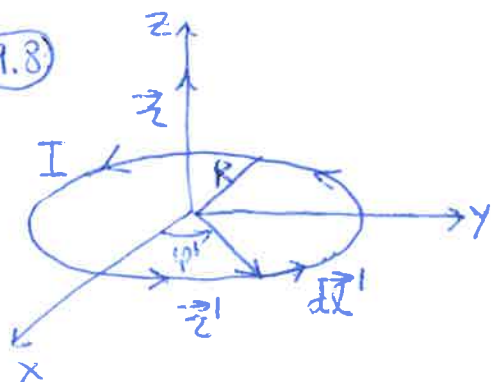
$$2 \rightarrow 1: \quad \vec{r} = \vec{r}_1 \quad \vec{r}' = \vec{r}_2 \quad q' = q_2 = -e \quad \vec{r} - \vec{r}' = \vec{r}_1 - \vec{r}_2 = -d\vec{u}_x$$

$$\vec{B}_{2 \rightarrow 1} = \frac{\mu_0}{4\pi} q' \frac{(\vec{r}' \times (\vec{r} - \vec{r}'))}{|\vec{r} - \vec{r}'|^3} = \frac{\mu_0 e}{4\pi} \frac{((- \rho_2 \vec{u}_x) \times (-d\vec{u}_x))}{d^3} =$$

$$= -\frac{\mu_0 e \rho_2}{4\pi d^2} (\vec{u}_x \times \vec{u}_x) = 0$$

$$\vec{F}_{2 \rightarrow 1} = q_1 (\vec{r}_1 \times \vec{B}_{2 \rightarrow 1}) = \boxed{0}$$

(9.8)



$$\vec{r}' = R \cos \varphi' \vec{u}_x + R \sin \varphi' \vec{u}_y$$

$$d\vec{r}' = \frac{d\vec{r}'}{d\varphi'} d\varphi' =$$

$$= R d\varphi' (-\sin \varphi' \vec{u}_x + \cos \varphi' \vec{u}_y)$$

$$\vec{r} = z \vec{u}_z$$

$$|\vec{r} - \vec{r}'| = \sqrt{R^2 + z^2}$$

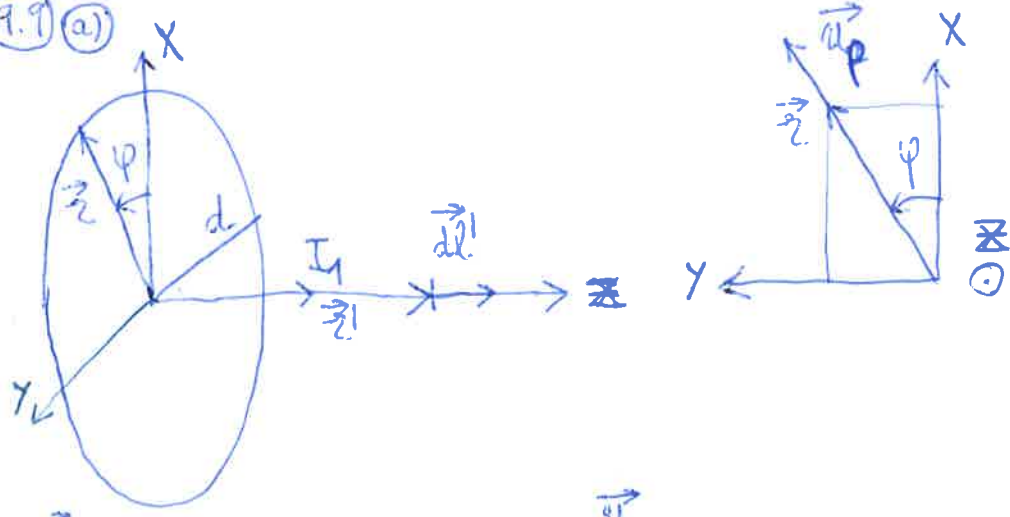
$$\vec{r} - \vec{r}' = -R \cos \varphi' \vec{u}_x - R \sin \varphi' \vec{u}_y + z \vec{u}_z$$

$$d\vec{r}' \times (\vec{r} - \vec{r}') = \begin{vmatrix} \vec{u}_x & \vec{u}_y & \vec{u}_z \\ -\sin \varphi' & \cos \varphi' & 0 \\ -R \cos \varphi' & -R \sin \varphi' & z \end{vmatrix} R d\varphi' = R d\varphi' (\cos \varphi' z \vec{u}_x + \sin \varphi' z \vec{u}_y + R \vec{u}_z)$$

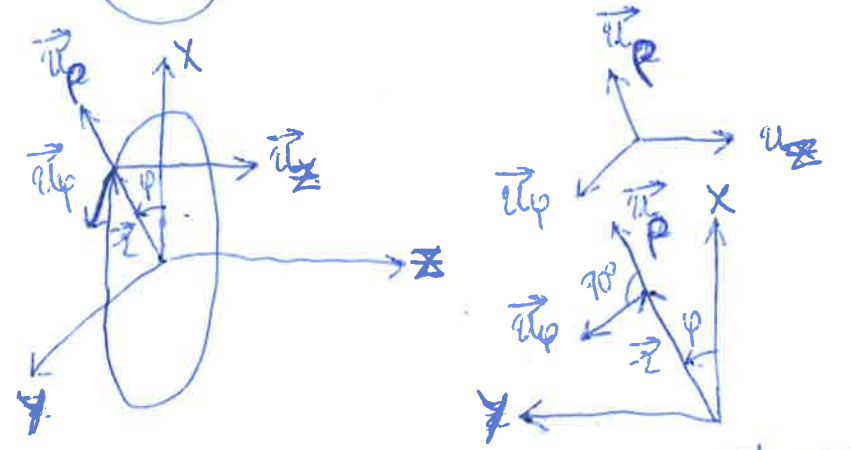
Autor: Iván Cabrera Álvarez

$$\begin{aligned}\vec{B} &= \frac{\mu_0 I}{4\pi} \int_0^{2\pi} d\varphi' \frac{\vec{r} \times (\vec{z} - \vec{z}')}{|\vec{z} - \vec{z}'|^3} = \frac{\mu_0 I R}{4\pi} \int_0^{2\pi} d\varphi' \frac{(\cos\varphi' z \vec{u}_x + \sin\varphi' z \vec{u}_y + R \vec{u}_z)}{(R^2 + z^2)^{3/2}} \\ &= \frac{\mu_0 I R}{4\pi (R^2 + z^2)^{3/2}} \left[z \vec{u}_x \int_0^{2\pi} \cos\varphi' d\varphi' + z \vec{u}_y \int_0^{2\pi} \sin\varphi' d\varphi' + R \vec{u}_z \int_0^{2\pi} d\varphi' \right] \\ &= \frac{\mu_0 I R}{4\pi (R^2 + z^2)^{3/2}} \left[0 + 0 + R \vec{u}_z 2\pi \right] = \boxed{\frac{\mu_0 I R^2}{2 (R^2 + z^2)^{3/2}} \vec{u}_z}\end{aligned}$$

9.9 a)



$$\begin{aligned}\vec{z} &= d \vec{u}_p \\ \vec{z}' &= z \vec{u}_z \\ d\vec{l} &= d\vec{u}_z \\ \vec{z} - \vec{z}' &= -z \vec{u}_z + d \vec{u}_p \\ |\vec{z} - \vec{z}'| &= \sqrt{z^2 + d^2}\end{aligned}$$



$$\begin{aligned}\vec{u}_z \times \vec{u}_p &= \vec{u}_\varphi \\ \vec{u}_\varphi \times \vec{u}_z &= \vec{u}_p\end{aligned}$$

$$\begin{aligned}\text{Ley de Biot-Savart: } d\vec{B} &= \frac{\mu_0 I}{4\pi} \frac{d\vec{l} \times (\vec{z} - \vec{z}')}{|\vec{z} - \vec{z}'|^3} = \frac{\mu_0 I}{4\pi} \frac{d\vec{u}_z \times (-z \vec{u}_z + d \vec{u}_p)}{(z^2 + d^2)^{3/2}} \\ &= \frac{\mu_0 I d \vec{u}_p}{4\pi (z^2 + d^2)^{3/2}} \left[\vec{u}_z \times \vec{u}_z + d (\vec{u}_z \times \vec{u}_p) \right] = \frac{\mu_0 I d \vec{u}_p}{4\pi (z^2 + d^2)^{3/2}} [0 + d \vec{u}_\varphi] \\ &= \frac{\mu_0 I d d \vec{u}_\varphi}{4\pi (z^2 + d^2)^{3/2}}\end{aligned}$$

$$\vec{B} = \lim_{a \rightarrow \infty} \int_{-a}^a d\vec{B} = \frac{\mu_0 I d}{4\pi} \vec{u}_\varphi \lim_{a \rightarrow \infty} \int_{-a}^a \frac{d\vec{u}_\varphi}{(z^2 + d^2)^{3/2}}$$

Autor: Iván Caloria Álvarez

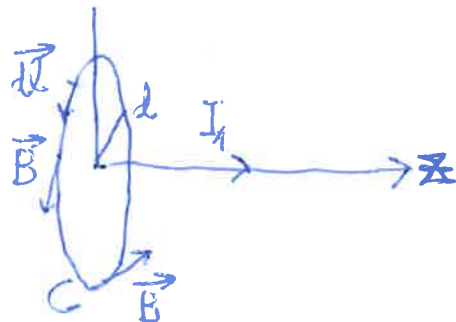
$$\int_{-a}^a \frac{dx'}{(x'^2 + d^2)^{3/2}} = 2 \int_0^a \frac{dx'}{(x'^2 + d^2)^{3/2}} = \frac{2x'}{d^2 \sqrt{x'^2 + d^2}} \bigg|_{x'=0}^{x'=a} = \frac{2a}{d^2 \sqrt{a^2 + d^2}} =$$

$$\vec{B} = \frac{\mu_0 I_1 d}{4\pi} \vec{u}_y \lim_{a \rightarrow \infty} \frac{2}{d^2 \sqrt{1 + (d/a)^2}} = \frac{\mu_0 I_1 d}{4\pi} \vec{u}_y \frac{2}{d^2} = \frac{2}{d^2 \sqrt{1 + (d/a)^2}}$$

$$\boxed{\vec{B} = \frac{\mu_0 I_1}{2\pi d} \vec{u}_y}$$

Usando la ley de Ampère:

$$\oint_C \vec{B} \cdot d\vec{l} = \mu_0 I_1$$

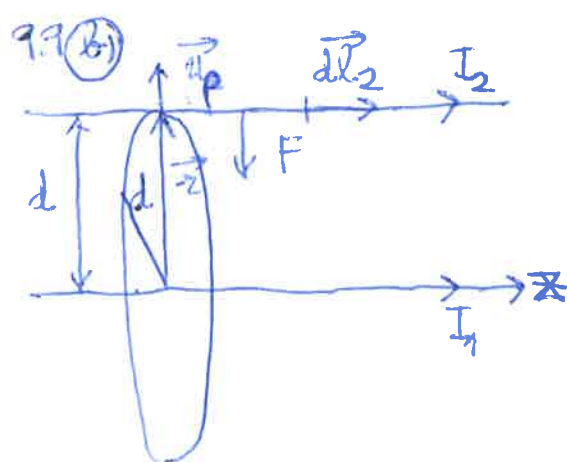


$$\vec{B} = B \vec{u}_y \quad d\vec{l} = dl \vec{u}_y$$

$$\oint_C \vec{B} \cdot d\vec{l} = \oint_C B dl = B \oint_C dl = B 2\pi d \Rightarrow B 2\pi d = \mu_0 I_1 \Rightarrow$$

$$\Rightarrow B = \frac{\mu_0 I_1}{2\pi d} \Rightarrow \boxed{\vec{B} = \frac{\mu_0 I_1}{2\pi d} \vec{u}_y}$$

Circunferencia de radio d



$$|\vec{r}| = d \quad d\vec{l}_2 = dx \vec{u}_x \quad \vec{B}_{1 \rightarrow 2} = \frac{\mu_0 I_1}{2\pi d} \vec{u}_y$$

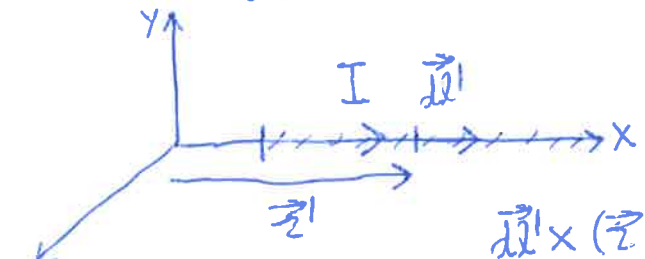
$$\frac{\vec{F}}{l} = \frac{1}{l} \int_{l/2}^{3l/2} I_2 d\vec{l}_2 \times \vec{B}_{1 \rightarrow 2} = \frac{I_2}{l} \int_{l/2}^{3l/2} dx \vec{u}_x \times \frac{\mu_0 I_1}{2\pi d} \vec{u}_y$$

$$= \frac{\mu_0 I_1 I_2}{2\pi d l} \int_{l/2}^{3l/2} dx (\vec{u}_x \times \vec{u}_y) = -\frac{\mu_0 I_1 I_2}{2\pi d l} \int_{l/2}^{3l/2} dx \vec{u}_z$$

$$= -\frac{\mu_0 I_1 I_2}{2\pi d l} l \vec{u}_z = \boxed{-\frac{\mu_0 I_1 I_2}{2\pi d} \vec{u}_z}$$

9.10 Hay tres campos magnéticos:

Calculamos $\vec{B}_3$:



$$\vec{r} = 0 \quad \vec{r}' = x' \vec{u}_x$$

$$\vec{r} - \vec{r}' = -x' \vec{u}_x$$

$$d\vec{l}' = dx' \vec{u}_x$$

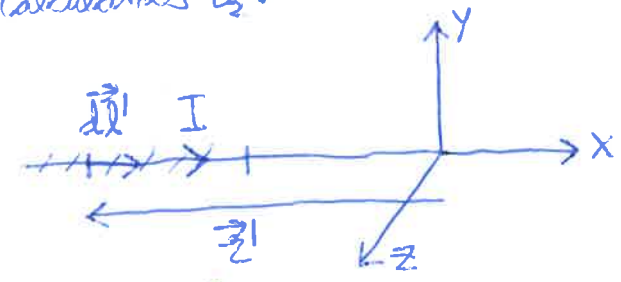
$$d\vec{l}' \times (\vec{r} - \vec{r}') = -dx' x' (\vec{u}_x \times \vec{u}_x) = 0$$

⇒ También se puede resolver de la siguiente manera:

$d\vec{l}'$ y $(\vec{r} - \vec{r}')$ son antiparalelos (misma dirección y sentido contrario) $\Rightarrow d\vec{l}' \times (\vec{r} - \vec{r}') = 0$

$$\Rightarrow \vec{B}_3 = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{l}' \times (\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3} = 0$$

Calculamos $\vec{B}_2$:

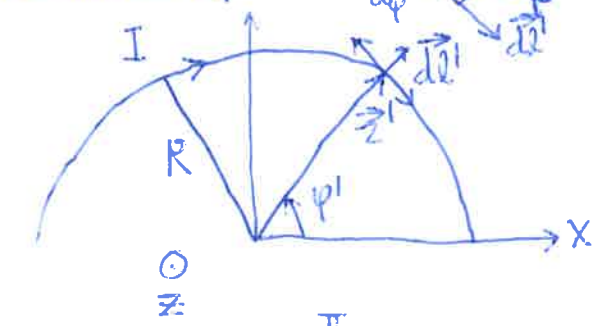


$$\vec{r} = 0 \quad \vec{r}' = -x' \vec{u}_x \quad d\vec{l}' = dx' \vec{u}_x$$

$d\vec{l}'$ y $(\vec{r} - \vec{r}')$ son paralelos $\Rightarrow d\vec{l}' \times (\vec{r} - \vec{r}') = 0 \Rightarrow$

$$\Rightarrow \vec{B}_2 = 0$$

Calculamos $\vec{B}_1$:



$$\vec{u}_p \times \vec{u}_\phi = \vec{u}_z \quad d\vec{l}' = -R d\phi' \vec{u}_\phi$$

$$\vec{r}' = R \vec{u}_p \quad \vec{r} = 0$$

$$\vec{r} - \vec{r}' = -R \vec{u}_p \quad |\vec{r} - \vec{r}'| = R$$

$$d\vec{l}' \times (\vec{r} - \vec{r}') = R^2 d\phi' \vec{u}_\phi \times \vec{u}_p = -R^2 d\phi' \vec{u}_z$$

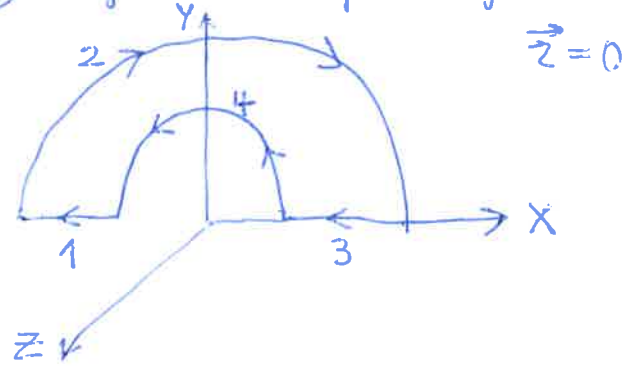
$$\vec{B}_1 = \frac{\mu_0 I}{4\pi} \int_0^\pi \frac{d\vec{l}' \times (\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3} = \frac{\mu_0 I}{4\pi} \int_0^\pi \frac{(-\vec{u}_z R^2 d\phi')}{R^3} = -\frac{\mu_0 I}{4\pi R} \vec{u}_z \int_0^\pi d\phi'$$

$$= -\frac{\mu_0 I}{4\pi R} \vec{u}_z \pi = -\frac{\mu_0 I}{4R} \vec{u}_z$$

$$\vec{B} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3 = \boxed{-\frac{\mu_0 I}{4R} \vec{u}_z}$$

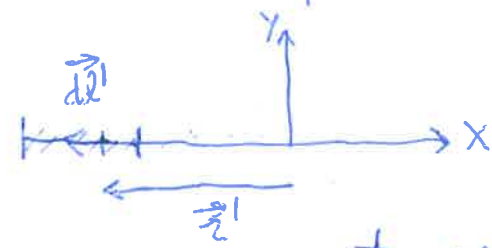
Autor: Iván Cabria Álvarez

9.11 Hay cuatro campos magnéticos



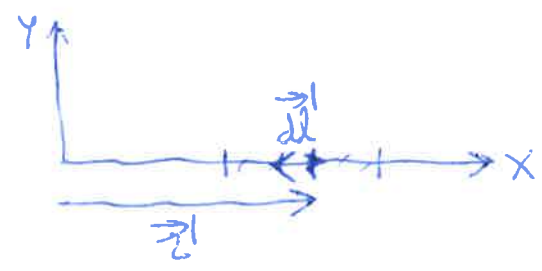
$$\vec{z} = 0$$

Calculamos $\vec{B}_1$:



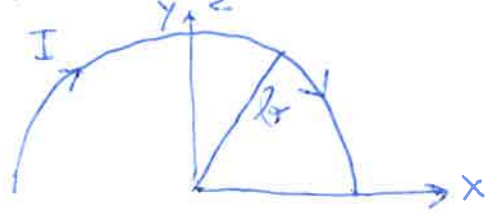
$\vec{dl}'$ y $(\vec{r}-\vec{r}')$ son ^{ant} paralelos $\Rightarrow$
 $\Rightarrow \vec{dl}' \times (\vec{r}-\vec{r}') = 0 \Rightarrow \vec{B}_1 = 0$

Calculamos $\vec{B}_3$:



$\vec{dl}'$ y $(\vec{r}-\vec{r}')$ son paralelos $\Rightarrow \vec{B}_3 = 0$

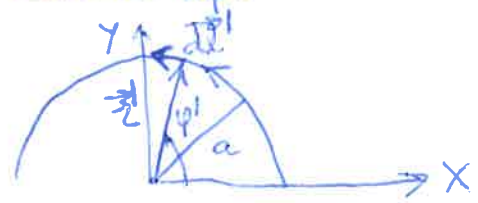
Calculamos $\vec{B}_2$:



$$\vec{B}_2 = - \frac{\mu_0 I}{4b} \vec{u}_z$$

Es el campo $\vec{B}_1$ del problema 9.10, con radio b

Calculamos $\vec{B}_4$:

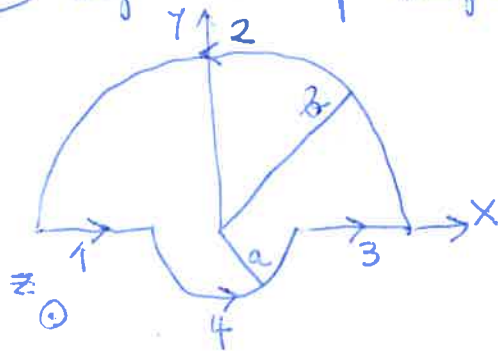


$$\begin{aligned} \vec{B}_4 &= \frac{\mu_0 I}{4\pi} \int_0^\pi \frac{\vec{dl}' \times (\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3} = \frac{\mu_0 I}{4\pi} \int_0^\pi \frac{\vec{u}_z d\phi'}{a} = \\ &= + \frac{\mu_0 I}{4a} \vec{u}_z \end{aligned}$$

$$\vec{B} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3 + \vec{B}_4 = \boxed{\frac{\mu_0 I}{4} \left(\frac{1}{a} - \frac{1}{b} \right) \vec{u}_z}$$

9.12) Hay cuatro campos magnéticos

9.10



$\vec{B}_1 = 0$ y $\vec{B}_3 = 0$ como en los problemas 9.10 y 9.11

$$\vec{B}_2 = + \frac{\mu_0 I}{4b} \vec{u}_z$$

$$\vec{B}_4 = + \frac{\mu_0 I}{4a} \vec{u}_z$$

$$\vec{B} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3 + \vec{B}_4 = \frac{\mu_0 I}{4} \left(\frac{1}{a} + \frac{1}{b} \right) \vec{u}_z$$

9.13

a) Es posible que no sientan una fuerza magnética. Si la carga se mueve ~~por~~ con velocidad paralela o antiparalela a $\vec{B}$, entonces no sentirá una fuerza magnética. Si la corriente del hilo es paralela o antiparalela a $\vec{B}$, entonces el hilo no sentirá una fuerza magnética.

b) La fuerza de Lorentz es $\vec{F} = q(\vec{v} \times \vec{B})$. Su módulo es $F = qvB \sin \alpha$, donde α es el ángulo que forman $\vec{v}$ y $\vec{B}$. F será máxima cuando α sea $\frac{\pi}{2}$.

c) $F = qvB \sin \alpha$ F será nula cuando α sea 0 ó 180 grados

9.14 a) Ciento. $\vec{F} = q(\vec{v} \times \vec{B}) \Rightarrow \vec{F} \perp \vec{v}$ por definición del producto vectorial $\vec{v} \times \vec{B}$

b) Ciento. $\vec{F} = q(\vec{v} \times \vec{B}) \Rightarrow \vec{F} \perp \vec{B}$ || || || ||

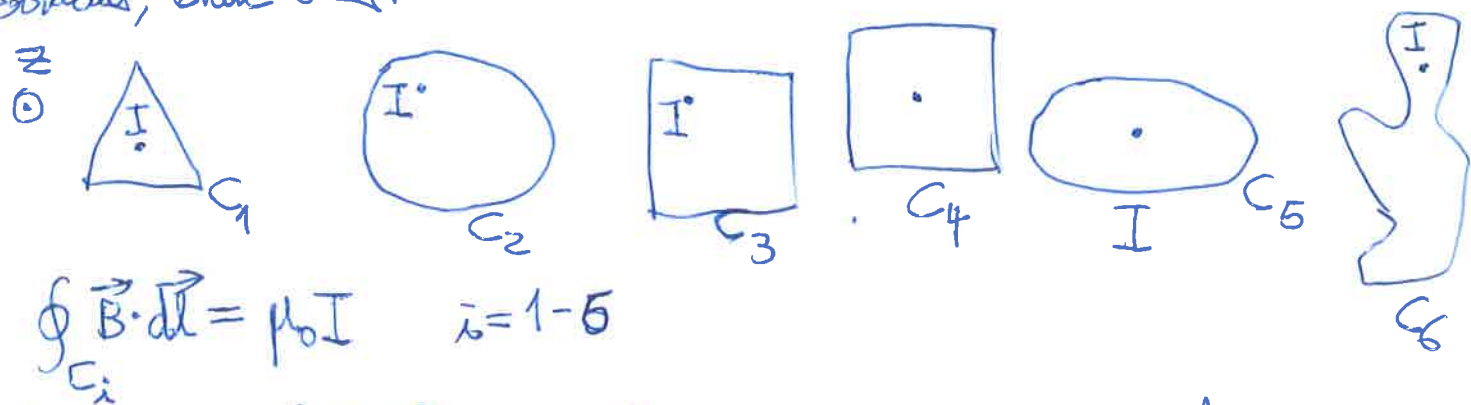
c) Ciento, si $\vec{v}$ no es paralela ni antiparalela a $\vec{B}$ "

d) Falso. $\vec{F} = q(\vec{v} \times \vec{B}) \Rightarrow \vec{F} \perp \vec{v}$ $\vec{v}$ paralela a la trayectoria } $\Rightarrow \vec{F} \perp$ a la trayectoria

Autor: Juan Gabrín Álvarez

9.15 a) Falso. Se cumple para cualquier tipo de hilo de corriente, finito o infinito, rectilíneo o curvo.

b) Falso. Se cumple para cualquier tipo de ^{curva} cerrada C , no sólo para circunferencias. ^{Por ejemplo} Se cumple para las siguientes curvas cerradas, entre otras:



$$\oint_{C_i} \vec{B} \cdot d\vec{l} = \mu_0 I \quad i=1-6$$

La integral $\oint_{C_i} \vec{B} \cdot d\vec{l}$ tiene el mismo valor, $\mu_0 I$, en estos seis casos: $i=1-6$

Las curvas C_2, C_3 y C_6 no están centradas en el hilo de corriente I

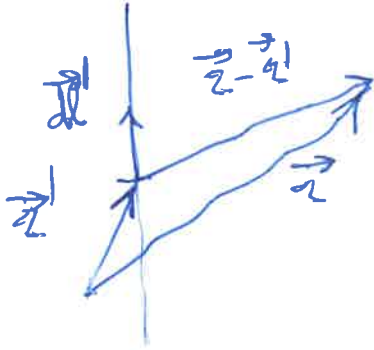
c) Falso. Se cumple para cualquier curva cerrada C en 2d y en 3d

d) Cierto.

e) Falso. Se cumple para campos magnéticos con y sin simetría.

9.16 a) Falso. Según la ley de Biot-Savart, el campo $d\vec{B}$ debido a un elemento de corriente $d\vec{l}$ es:

$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{l} \times (\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3}$$



$$d\vec{B} \perp d\vec{l}$$

$d\vec{l}$ no es paralelo a $d\vec{B}$

El enunciado es falso

Autor: Iván Cabús Alvarado

(b) Cierta. La fuerza de Lorentz debida al campo magnetico no cambia el modulo de la velocidad, $v = |\vec{v}|$, y por tanto no cambia la energia cinetica $T = \frac{m}{2} v^2$

(c) Cierta, solo si la velocidad inicial es perpendicular al campo magnetico el movimiento sera circular y uniforme:

$$\vec{v} = \vec{v}_{||} + \vec{v}_{\perp} \quad \vec{v}_{||} \parallel \vec{B}$$

$$\vec{v} \times \vec{B} = \vec{v}_{\perp} \times \vec{B} + \vec{v}_{||} \times \vec{B} = \vec{v}_{\perp} \times \vec{B}$$

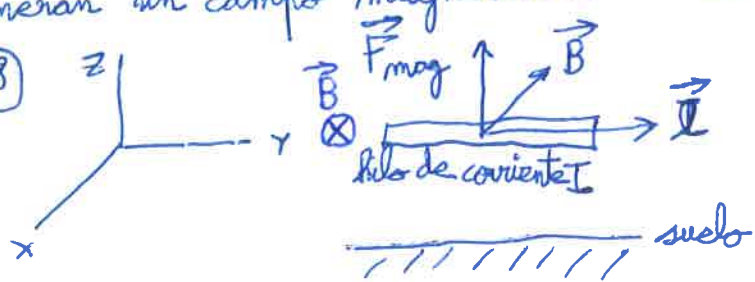
$$\left. \begin{aligned} \vec{F} &= m \frac{d\vec{v}}{dt} \\ \vec{F} &= q(\vec{v} \times \vec{B}) \end{aligned} \right\} \Rightarrow m \frac{d\vec{v}_{\perp}}{dt} = q(\vec{v}_{\perp} \times \vec{B}) \Rightarrow \text{mov. circular y uniforme en el plano } \perp \text{ a } \vec{B}$$

$$m \frac{d\vec{v}_{||}}{dt} = 0 \Rightarrow \text{mov. lineal uniforme en el eje paralelo a } \vec{B}$$

$\Rightarrow$ Si $\vec{v}$ tiene $\vec{v}_{||}$, entonces el movimiento es en forma de helice. Si $\vec{v}$ no tiene $\vec{v}_{||}$, entonces " " " " circular y uniforme. El enunciado es cierto.

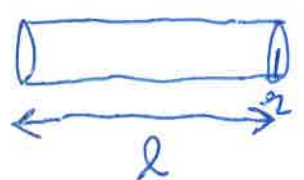
(9.17) Cualquiera partícula en movimiento no genera necesariamente un campo magnetico a su alrededor. Solo las partículas con carga electrica y en movimiento generan un campo magnetico a su alrededor.

(9.18)



$$\begin{aligned} \vec{F}_{mag} &= I \vec{l} \times \vec{B} = I l B \vec{u}_z \\ \vec{F}_g &= -mg \vec{u}_z \\ B > 0 \quad I > 0 \end{aligned}$$

$$\vec{F}_{mag} + \vec{F}_g = 0 \Rightarrow I l B - mg = 0 \Rightarrow I = \frac{mg}{lB} \quad m: \text{masa del hilo}$$



$$m = \rho V = \rho S l = \rho \pi r^2 l$$

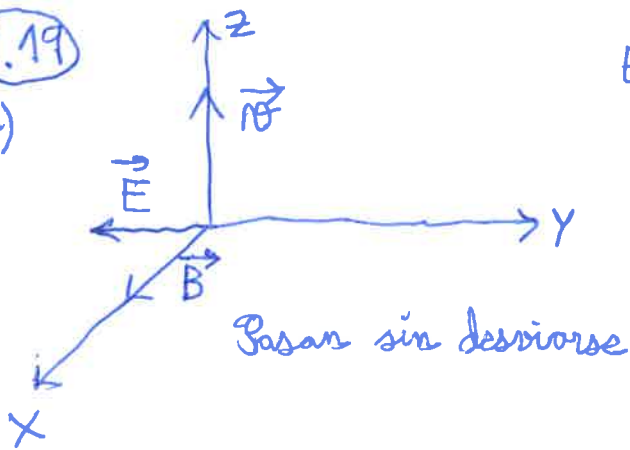
$$I = \frac{mg}{lB} = \frac{\rho \pi r^2 g}{B} = \frac{8960 \text{ Kg/m}^3 \pi (0.0004 \text{ m})^2 9.8 \text{ m/s}^2}{0.02 \text{ T}} = \boxed{2.2 \text{ A}}$$

$$1 \text{ T} = 1 \frac{\text{Kg}}{\text{As}^2}$$

Si B tuviera la intensidad del campo magnetico terrestre, 0.25 gauss, entonces $I = 1765 \text{ A}$. Esta intensidad de corriente produciría suficiente temperatura como para fundir el hilo de cobre.
Autor: Iván Cabris Álvarez

9.19

a)



$$E > 0 \quad B > 0 \quad n > 0$$

$$\vec{F} = q\vec{E} + q(\vec{n} \times \vec{B}) \Rightarrow \vec{F} = 0$$

$$\Rightarrow q\vec{E} + q(\vec{n} \times \vec{B}) = 0$$

$$\vec{n} \times \vec{B} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{vmatrix} nB = nB \vec{j} \Rightarrow (0, -E, 0) + (0, nB, 0) = 0 \Rightarrow$$

$$\vec{E} = E(0, -1, 0)$$

$$\Rightarrow E = nB = 1000 \frac{m}{s} 0.5 T = \boxed{500 \frac{N}{C}}$$

$$b) \Delta\Phi = 5000 V \quad 2R = 2.0436 m \Rightarrow R = 0.010218 m$$

$$\frac{|q|}{m} = \frac{2\Phi}{R^2 B^2} = \frac{2 \cdot 5000 V}{(0.010218 m)^2 (1 T)^2} = \boxed{95\,778\,538 \frac{C}{kg}}$$

$$\frac{|q|_{proton}}{m_{proton}} = \frac{1.602 \cdot 10^{-19} C}{1.6726 \cdot 10^{-27} kg} = 95\,779\,027 \frac{C}{kg}$$

Autor: Iván Cabrera Álvarez

9.20 $\vec{F}_{1 \rightarrow 2} = q_2 (\vec{N}_2 \times \vec{B}_{1 \rightarrow 2})$

Calculamos $\vec{B}_{1 \rightarrow 2}$:

$$\vec{B} = \frac{\mu_0}{4\pi} q' \frac{\vec{N}' \times (\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3}$$

$$\vec{r}' = \vec{r}_1 = a(0, 1, 1)$$

$$\vec{N} = \vec{N}_1 = N_0(1, 0, 1)$$

$$q' = q_1 = -e$$

$$\vec{r} = \vec{r}_2 = a(1, 1, 0)$$

$$\vec{r} - \vec{r}' = a(1, 1, 0) - a(0, 1, 1) = (a-0, a-a, 0-a) = a(1, 0, -1)$$

$$|\vec{r} - \vec{r}'| = a\sqrt{2} \quad |\vec{r} - \vec{r}'|^3 = a^3 2\sqrt{2}$$

$$\vec{B}_{1 \rightarrow 2} = \frac{\mu_0}{4\pi} (-e) \frac{N_0(1, 0, 1) \times a(1, 0, -1)}{a^3 2\sqrt{2}} = -\frac{\mu_0 e N_0}{8\pi a^2 \sqrt{2}} (1, 0, 1) \times (1, 0, -1)$$

$$(1, 0, 1) \times (1, 0, -1) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & 1 \\ 1 & 0 & -1 \end{vmatrix} = -\vec{j} \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} = +2\vec{j}$$

$$\vec{B}_{1 \rightarrow 2} = -\frac{\mu_0 e N_0}{4\pi a^2 \sqrt{2}} \vec{j}$$

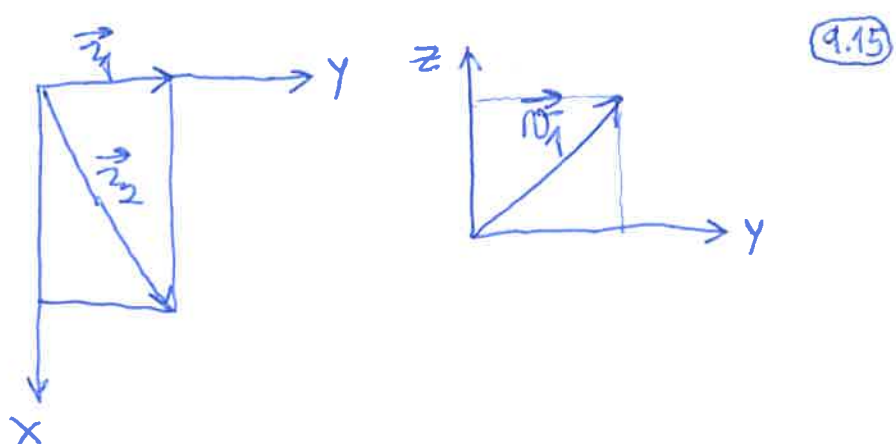
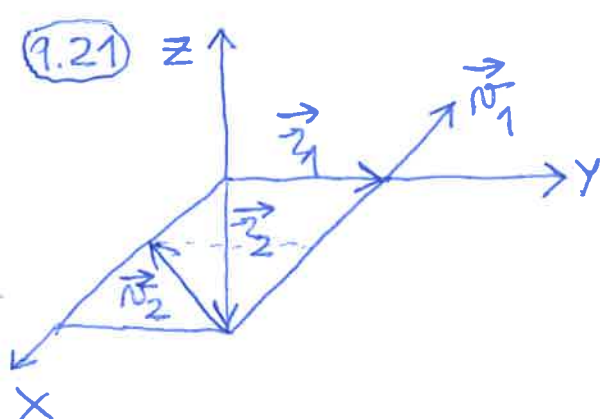
Ahora calculamos $\vec{F}_{1 \rightarrow 2}$:

$$\vec{F}_{1 \rightarrow 2} = e N_0(0, 1, 1) \times \left(-\frac{\mu_0 e N_0}{4\pi a^2 \sqrt{2}} \right) (0, 1, 0)$$

$$(0, 1, 1) \times (0, 1, 0) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 1 & 1 \\ 0 & 1 & 0 \end{vmatrix} = \vec{i} \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} = -\vec{i}$$

$$\vec{F}_{1 \rightarrow 2} = -\frac{\mu_0 e^2 N_0^2}{4\pi a^2 \sqrt{2}} (0, 1, 1) \times (0, 1, 0) = \boxed{+\frac{\mu_0 e^2 N_0^2}{4\pi a^2 \sqrt{2}} \vec{i}}$$

Autor: Iván Gabrín Álvarez



$$q_1 = q_2 = +e > 0$$

$$\vec{r}_1 = (0, a, 0) \quad \vec{r}_1 = (0, r_0, r_0) \quad \vec{r}_2 = (2a, a, 0) \quad \vec{r}_2 = (-r_0, -r_0, 0)$$

Calculamos $\vec{F}_{1 \rightarrow 2}$: $\vec{F}_{1 \rightarrow 2} = q_2 (\vec{r}_2 \times \vec{B}_{1 \rightarrow 2}) = e (\vec{r}_2 \times \vec{B}_{1 \rightarrow 2})$

Calculamos $\vec{B}_{1 \rightarrow 2}$: $\vec{B} = \frac{\mu_0}{4\pi} \frac{q' (\vec{r}' \times (\vec{r} - \vec{r}'))}{|\vec{r} - \vec{r}'|^3}$

$$\vec{r} = \vec{r}_2 = a(2, 1, 0) \quad \vec{r}' = \vec{r}_1 = a(0, 1, 0) \quad q' = q_1 = +e > 0$$

$$\vec{r}' = \vec{r}_1 = r_0(0, 1, 1)$$

$$\vec{r} - \vec{r}' = a(2, 1, 0) - a(0, 1, 0) = a(2, 0, 0) = 2a \vec{u}_x \quad |\vec{r} - \vec{r}'| = 2a$$

$$\vec{B}_{1 \rightarrow 2} = \frac{\mu_0}{4\pi} \frac{e r_0 ((0, 1, 1) \times a(2, 0, 0))}{8a^3}$$

$$(0, 1, 1) \times (2, 0, 0) = (\vec{u}_y + \vec{u}_z) \times 2\vec{u}_x = 2(\vec{u}_y \times \vec{u}_x + \vec{u}_z \times \vec{u}_x) = 2(-\vec{u}_z + \vec{u}_y)$$

$$\vec{B}_{1 \rightarrow 2} = \frac{\mu_0 e r_0 2a}{4\pi 8a^3} (-\vec{u}_z + \vec{u}_y) = \frac{\mu_0 e r_0}{16\pi a^2} (-\vec{u}_z + \vec{u}_y)$$

$$\vec{F}_{1 \rightarrow 2} = e (\vec{r}_2 \times \vec{B}_{1 \rightarrow 2}) = e r_0 \frac{\mu_0 e r_0}{16\pi a^3} \begin{vmatrix} \vec{u}_x & \vec{u}_y & \vec{u}_z \\ -1 & -1 & 0 \\ 0 & 1 & -1 \end{vmatrix}$$

$$\begin{vmatrix} \vec{u}_x & \vec{u}_y & \vec{u}_z \\ -1 & -1 & 0 \\ 0 & 1 & -1 \end{vmatrix} = \vec{u}_x \begin{vmatrix} -1 & 0 \\ 1 & -1 \end{vmatrix} - \vec{u}_y \begin{vmatrix} -1 & 0 \\ 0 & -1 \end{vmatrix} + \vec{u}_z \begin{vmatrix} -1 & -1 \\ 0 & 1 \end{vmatrix} = \vec{u}_x - \vec{u}_y - \vec{u}_z$$

$$\boxed{\vec{F}_{1 \rightarrow 2} = \frac{\mu_0 e^2 r_0^2}{16\pi a^2} (\vec{u}_x - \vec{u}_y - \vec{u}_z)}$$

Autor: João Gabriel Alvaro

Calculamos $\vec{F}_{2 \rightarrow 1}$: $\vec{F}_{2 \rightarrow 1} = q_1 (\vec{r}_1 \times \vec{B}_{2 \rightarrow 1}) = e (\vec{r}_1 \times \vec{B}_{2 \rightarrow 1})$

(9.16)

Calculamos $\vec{B}_{2 \rightarrow 1}$:

$\vec{r} = \vec{r}_1 = a(0, 1, 0)$ $\vec{r}_1 = \vec{r}_2 = a(2, 1, 0)$ $q_1 = q_2 = +e > 0$

$\vec{r}_1 = \vec{r}_2 = r_0(-1, -1, 0)$

$\vec{r} - \vec{r}_1 = a(0, 1, 0) - a(2, 1, 0) = -2a\vec{u}_x$ $|\vec{r} - \vec{r}_1| = 2a$

$\vec{B}_{2 \rightarrow 1} = \frac{\mu_0 e r_0}{4\pi} \frac{((-1, -1, 0) \times a(-2, 0, 0))}{8a^3}$

$(-1, -1, 0) \times (-2, 0, 0) = (-\vec{u}_x - \vec{u}_y) \times (-2)\vec{u}_x = 2(\vec{u}_x + \vec{u}_y) \times \vec{u}_x =$
 $= 2((\vec{u}_x \times \vec{u}_x) + (\vec{u}_y \times \vec{u}_x)) = 2(0 - \vec{u}_z) = -2\vec{u}_z$

$\vec{B}_{2 \rightarrow 1} = -\frac{\mu_0 e r_0}{16\pi a^2} \vec{u}_z$

$\vec{F}_{2 \rightarrow 1} = e(\vec{r}_1 \times \vec{B}_{2 \rightarrow 1}) = e r_0 \frac{\mu_0 e r_0}{16\pi a^2} \begin{vmatrix} \vec{u}_x & \vec{u}_y & \vec{u}_z \\ 0 & 1 & 1 \\ 0 & 0 & -1 \end{vmatrix}$

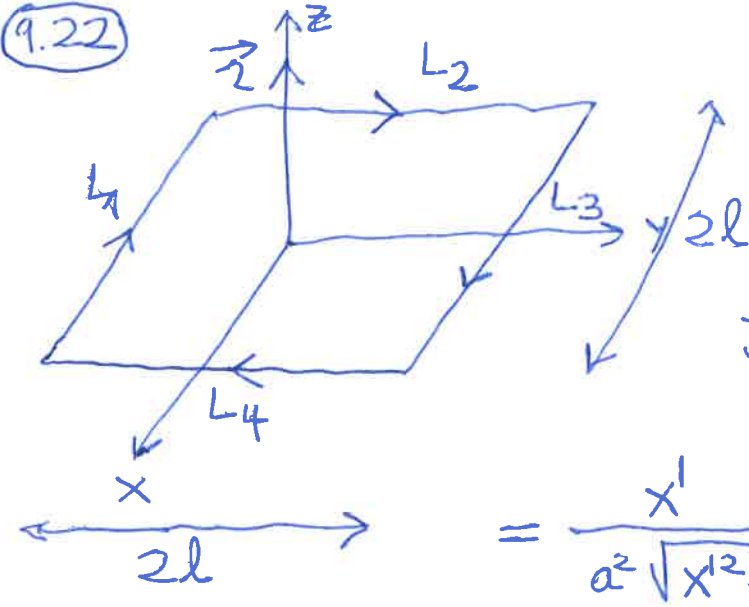
$\begin{vmatrix} \vec{u}_x & \vec{u}_y & \vec{u}_z \\ 0 & 1 & 1 \\ 0 & 0 & -1 \end{vmatrix} = \vec{u}_x \begin{vmatrix} 1 & 1 \\ 0 & -1 \end{vmatrix} - \vec{u}_y \begin{vmatrix} 0 & 1 \\ 0 & -1 \end{vmatrix} + \vec{u}_z \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} = -\vec{u}_x$

$$\vec{F}_{2 \rightarrow 1} = -\frac{\mu_0 e^2 r_0^2}{16\pi a^2} \vec{u}_x$$

Autor: Juan Gabriel Álvarez

9.22

9.17



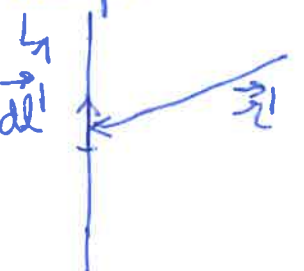
$$\vec{r} = z \vec{u}_z$$

$$\vec{B} = \int_L d\vec{B} = \int_L \frac{\mu_0 I}{4\pi} \frac{d\vec{\ell}' \times (\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3}$$

$$J(l, z) \equiv \int_{-l}^l \frac{dx'}{(x'^2 + l^2 + z^2)^{3/2}} =$$

$$= \frac{x'}{a^2 \sqrt{x'^2 + a^2}} \Big|_{x'=-l}^{x'=l} = \frac{2l}{a^2 \sqrt{l^2 + a^2}} \quad a^2 \equiv l^2 + z^2$$

Campo debido a L_1 :



$$\vec{r}' = x' \vec{u}_x - l \vec{u}_y \quad d\vec{\ell}' = -dx' \vec{u}_x$$

$$\vec{r} - \vec{r}' = (-x', l, z) \quad |\vec{r} - \vec{r}'| = \sqrt{x'^2 + l^2 + z^2}$$

$$d\vec{\ell}' \times (\vec{r} - \vec{r}') = -dx' \vec{u}_x \times (-x', l, z) =$$

$$= -dx' ((\vec{u}_x \times \vec{u}_y) l + (\vec{u}_x \times \vec{u}_z) z) =$$

$$= -dx' (\vec{u}_z l - \vec{u}_y z)$$

$$\vec{B}_1 = \int_{L_1} d\vec{B} = - \int_{-l}^l \frac{\mu_0 I}{4\pi} \frac{dx' (\vec{u}_z l - \vec{u}_y z)}{(x'^2 + l^2 + z^2)^{3/2}} = - \frac{\mu_0 I}{4\pi} (\vec{u}_z l - \vec{u}_y z) J(l, z)$$

Campo debido a L_2 :



$$\vec{r}' = -l \vec{u}_x + y' \vec{u}_y \quad d\vec{\ell}' = +dy' \vec{u}_y$$

$$\vec{r} - \vec{r}' = (l, -y', z) \quad |\vec{r} - \vec{r}'| = \sqrt{l^2 + y'^2 + z^2}$$

$$d\vec{\ell}' \times (\vec{r} - \vec{r}') = dy' \vec{u}_y \times (l, -y', z) =$$

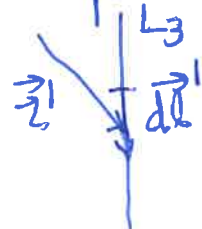
$$= dy' ((\vec{u}_y \times \vec{u}_x) l + (\vec{u}_y \times \vec{u}_z) z) = dy' (-\vec{u}_z l + \vec{u}_x z)$$

$$\vec{B}_2 = \int_{L_2} d\vec{B} = \int_{-l}^l \frac{\mu_0 I}{4\pi} \frac{dy' (-\vec{u}_z l + \vec{u}_x z)}{(l^2 + y'^2 + z^2)^{3/2}} = \frac{\mu_0 I}{4\pi} (-\vec{u}_z l + \vec{u}_x z) J(l, z)$$

Autor: Iván Cabrera Álvarez

Campo debido a L_3 : $\vec{r}_1 = x' \vec{u}_x + l \vec{u}_y$

$$d\vec{l} = + dx' \vec{u}_x$$



$$\vec{r} - \vec{r}_1 = (-x', -l, z)$$

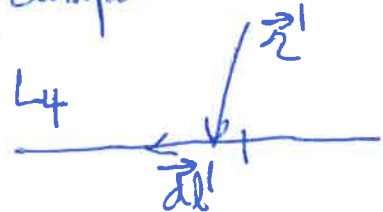
$$|\vec{r} - \vec{r}_1| = \sqrt{x'^2 + l^2 + z^2}$$

$$d\vec{l} \times (\vec{r} - \vec{r}_1) = dx' (1, 0, 0) \times (-x', -l, z) = dx' (-\vec{u}_z l - \vec{u}_y z)$$

$$\vec{B}_3 = \int_{L_3} d\vec{B} = \int_{-l}^l \frac{\mu_0 I}{4\pi} \frac{dx' (-\vec{u}_z l - \vec{u}_y z)}{(x'^2 + l^2 + z^2)^{3/2}} = \frac{\mu_0 I}{4\pi} (-\vec{u}_z l - \vec{u}_y z) J(l, z)$$

Campo debido a L_4 : $\vec{r}_1 = l \vec{u}_x + y' \vec{u}_y$

$$d\vec{l} = -dy' \vec{u}_y$$



$$\vec{r} - \vec{r}_1 = (-l, -y', z)$$

$$|\vec{r} - \vec{r}_1| = \sqrt{l^2 + y'^2 + z^2}$$

$$d\vec{l} \times (\vec{r} - \vec{r}_1) = -dy' (0, 1, 0) \times (-l, -y', z) =$$

$$= -dy' (-(\vec{u}_y \times \vec{u}_x) l + (\vec{u}_y \times \vec{u}_z) z) = -dy' (\vec{u}_z l + \vec{u}_x z)$$

$$\vec{B}_4 = \int_{L_4} d\vec{B} = \int_{-l}^l \frac{\mu_0 I}{4\pi} \frac{(-dy') (\vec{u}_z l + \vec{u}_x z)}{(l^2 + y'^2 + z^2)^{3/2}} = -\frac{\mu_0 I}{4\pi} (\vec{u}_z l + \vec{u}_x z) J(l, z)$$

$$\vec{B} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3 + \vec{B}_4 = \frac{\mu_0 I}{4\pi} J(l, z) (-\vec{u}_z l + \vec{u}_y z - \vec{u}_z l + \vec{u}_x z - \vec{u}_z l - \vec{u}_y z - \vec{u}_z l - \vec{u}_x z) = \frac{\mu_0 I}{4\pi} J(l, z) (-4\vec{u}_z l) \Rightarrow$$

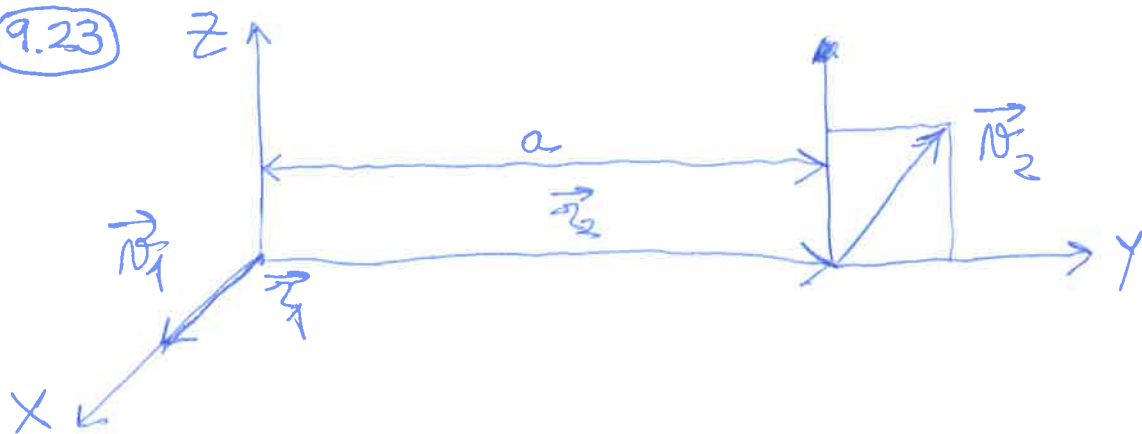
$$\Rightarrow \boxed{\vec{B} = -\frac{\mu_0 I l}{\pi} J(l, z) \vec{u}_z}$$

$$J(l, z) = \frac{2l}{(l^2 + z^2) \sqrt{2l^2 + z^2}}$$

Autor: Iván Caloria Álvarez

(9.23)

(9.19)



$$\vec{r}_1 = \vec{0} \quad q_1 = +e$$

$$\vec{B}_1 = N_0 \vec{u}_x$$

$$\vec{r}_2 = a \vec{u}_y \quad q_2 = +e$$

$$a > 0$$

$$\vec{B}_2 = \frac{N_0}{\sqrt{2}} (\vec{u}_y + \vec{u}_z)$$

$$N_0 > 0$$

Calculamos $\vec{F}_{1 \rightarrow 2}$: $\vec{F}_{1 \rightarrow 2} = q_2 (\vec{B}_2 \times \vec{B}_{1 \rightarrow 2})$

Calculamos $\vec{B}_{1 \rightarrow 2}$: $\vec{B} = \frac{\mu_0}{4\pi} q' \frac{(\vec{B}' \times (\vec{r} - \vec{r}'))}{|\vec{r} - \vec{r}'|^3}$

$$q' = q_1$$

$$\vec{r}' = \vec{r}_1$$

$$\vec{B}' = \vec{B}_1$$

$$\vec{r} = \vec{r}_2$$

$$\vec{r} - \vec{r}' = \vec{r}_2 - \vec{r}_1 = a \vec{u}_y - \vec{0} = a \vec{u}_y$$

$$\vec{B}' = \vec{B}_1 = N_0 \vec{u}_x$$

$$q' = q_1 = +e$$

$$\vec{B}' \times (\vec{r} - \vec{r}') = N_0 \vec{u}_x \times a \vec{u}_y = N_0 a \vec{u}_z$$

$$\vec{u}_x \times \vec{u}_y = \vec{u}_z$$

$$|\vec{r} - \vec{r}'|^3 = a^3$$

$$\vec{B}_{1 \rightarrow 2} = \frac{\mu_0 q_1}{4\pi} \frac{N_0 a \vec{u}_z}{a^3} = \frac{\mu_0 e N_0}{4\pi a^2} \vec{u}_z$$

$$\vec{F}_{1 \rightarrow 2} = q_2 (\vec{B}_2 \times \vec{B}_{1 \rightarrow 2}) = e (\vec{B}_2 \times \vec{B}_{1 \rightarrow 2}) = \frac{e N_0}{\sqrt{2}} ((\vec{u}_y + \vec{u}_z) \times \frac{\mu_0 e N_0}{4\pi a^2} \vec{u}_z) =$$

$$= \frac{\mu_0 e^2 N_0^2}{4\pi a^2 \sqrt{2}} ((\vec{u}_y + \vec{u}_z) \times \vec{u}_z)$$

$$(\vec{u}_y + \vec{u}_z) \times \vec{u}_z = \vec{u}_y \times \vec{u}_z + \vec{u}_z \times \vec{u}_z = \vec{u}_x + \vec{0} = \vec{u}_x \quad \Rightarrow$$

$$\Rightarrow \boxed{\vec{F}_{1 \rightarrow 2} = \frac{\mu_0 e^2 N_0^2}{4\pi a^2 \sqrt{2}} \vec{u}_x}$$

Profesor: Juan Carlos Alvarado

Calculamos $\vec{F}_{2 \rightarrow 1}$:

$$\vec{F}_{2 \rightarrow 1} = q_1 (\vec{r}_1 \times \vec{B}_{2 \rightarrow 1})$$

(9.20)

Calculamos $\vec{B}_{2 \rightarrow 1}$:

$$\vec{B} = \frac{\mu_0}{4\pi} q' \frac{(\vec{r}' \times (\vec{r} - \vec{r}'))}{|\vec{r} - \vec{r}'|^3}$$

$$q' = q_2 \quad \vec{r}' = \vec{r}_2 \quad \vec{r}' = \vec{r}_2 \quad \vec{r} = \vec{r}_1$$

$$\vec{r} - \vec{r}' = \vec{r}_1 - \vec{r}_2 = \vec{0} - a\vec{u}_y = -a\vec{u}_y$$

$$\vec{r}' = \vec{r}_2 = \frac{r_0}{\sqrt{2}} (\vec{u}_y + \vec{u}_z) \quad q' = q_2 = +e$$

$$\vec{r}' \times (\vec{r} - \vec{r}') = \frac{r_0}{\sqrt{2}} (\vec{u}_y + \vec{u}_z) \times (-a\vec{u}_y) = -\frac{ar_0}{\sqrt{2}} (\vec{u}_y \times \vec{u}_y + \vec{u}_z \times \vec{u}_y) =$$

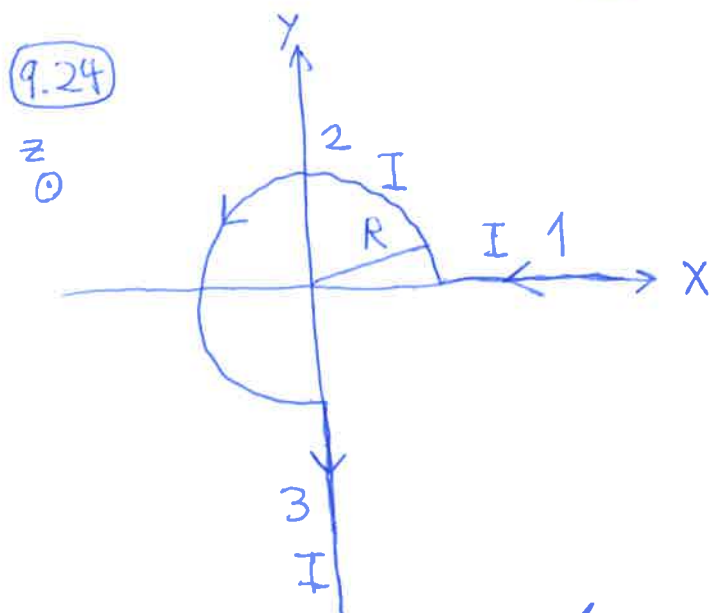
$$= -\frac{ar_0}{\sqrt{2}} (\vec{0} - \vec{u}_x) = +\frac{ar_0}{\sqrt{2}} \vec{u}_x$$

$$|\vec{r} - \vec{r}'| = a^3$$

$$\vec{B}_{2 \rightarrow 1} = \frac{\mu_0 q_2}{4\pi} \frac{ar_0 \vec{u}_x}{\sqrt{2} a^3} = \frac{\mu_0 e r_0}{4\pi \sqrt{2} a^2} \vec{u}_x$$

$$\vec{F}_{2 \rightarrow 1} = q_1 (\vec{r}_1 \times \vec{B}_{2 \rightarrow 1}) = e (\vec{r}_1 \times \vec{B}_{2 \rightarrow 1}) = e (r_0 \vec{u}_x \times \frac{\mu_0 e r_0}{4\pi \sqrt{2} a^2} \vec{u}_x) =$$

$$= \frac{\mu_0 e^2 r_0^2}{4\pi \sqrt{2} a^2} (\vec{u}_x \times \vec{u}_x) \quad \left. \begin{array}{l} \vec{u}_x \times \vec{u}_x = \vec{0} \end{array} \right\} \Rightarrow \boxed{\vec{F}_{2 \rightarrow 1} = \vec{0}}$$

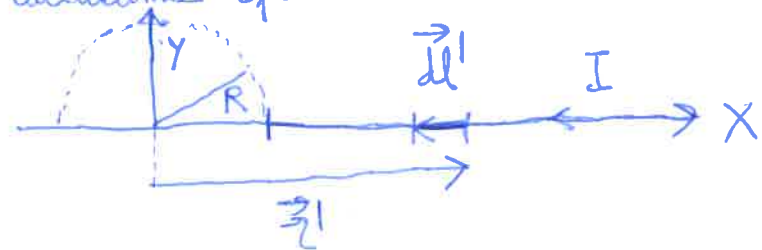


$$\vec{z} = \vec{0}$$

$$\vec{B} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3$$

$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{l}' \times (\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3}$$

Autor: Iván Cabrera Álvarez

Calculamos $\vec{B}_1$:

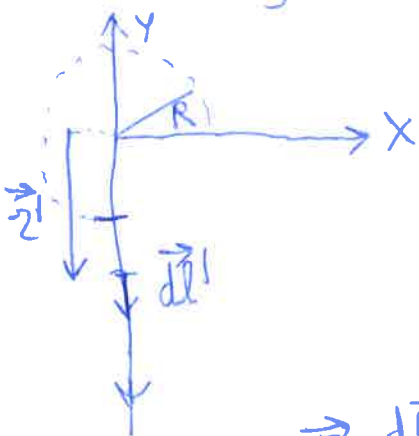
$$d\vec{l}' = -dx' \vec{u}_x$$

$$\vec{r}_1 = +x' \vec{u}_x$$

$$\vec{r} - \vec{r}_1 = -x' \vec{u}_x$$

$$d\vec{l}' \times (\vec{r} - \vec{r}_1) = -dx' \vec{u}_x \times (-x' \vec{u}_x) = +x' dx' (\vec{u}_x \times \vec{u}_x) = \vec{0} \Rightarrow$$

$$\Rightarrow d\vec{B}_1 = \frac{\mu_0 I}{4\pi} \frac{d\vec{l}' \times (\vec{r} - \vec{r}_1)}{|\vec{r} - \vec{r}_1|^3} = \vec{0} \Rightarrow \vec{B}_1 = \int_{\text{región o curva}} d\vec{B}_1 = \vec{0}$$

Calculamos $\vec{B}_3$:

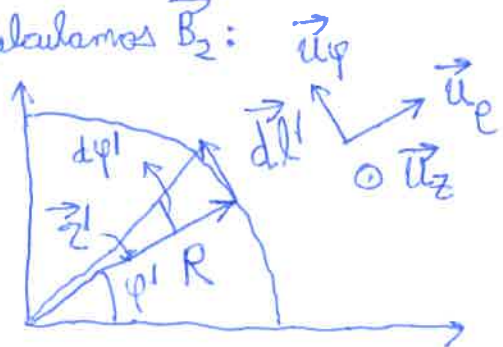
$$d\vec{l}' = -dy' \vec{u}_y$$

$$\vec{r}_1 = -y' \vec{u}_y$$

$$\vec{r} - \vec{r}_1 = +y' \vec{u}_y$$

$$d\vec{l}' \times (\vec{r} - \vec{r}_1) = -dy' \vec{u}_y \times (y' \vec{u}_y) = -y' dy' (\vec{u}_y \times \vec{u}_y) = \vec{0} \Rightarrow$$

$$\Rightarrow d\vec{B}_3 = \frac{\mu_0 I}{4\pi} \frac{d\vec{l}' \times (\vec{r} - \vec{r}_1)}{|\vec{r} - \vec{r}_1|^3} = \vec{0} \Rightarrow \vec{B}_3 = \int_{\text{región o curva}} d\vec{B}_3 = \vec{0}$$

Calculamos $\vec{B}_2$:

$$d\vec{l}' = R d\phi' \vec{u}_\phi$$

$$\vec{r}_1 = R \cos \phi' \vec{u}_x + R \sin \phi' \vec{u}_y = R \vec{u}_\rho$$

$$\vec{r} - \vec{r}_1 = -R \vec{u}_\phi$$

$$|\vec{r} - \vec{r}_1| = R$$

$$d\vec{l}' \times (\vec{r} - \vec{r}_1) = R d\phi' \vec{u}_\phi \times (-R \vec{u}_\phi) = -R^2 d\phi' (\vec{u}_\phi \times \vec{u}_\phi) =$$

$$= +R^2 d\phi' (\vec{u}_\rho \times \vec{u}_\phi)$$

$$\vec{u}_\rho \times \vec{u}_\phi = \vec{u}_z$$

$$\Rightarrow d\vec{l}' \times (\vec{r} - \vec{r}_1) = +R^2 d\phi' \vec{u}_z$$

Autor: Juan Cabera Álvarez

$$d\vec{B}_2 = \frac{\mu_0 I}{4\pi} \frac{d\vec{l}' \times (\vec{r} - \vec{r}_1)}{|\vec{r} - \vec{r}_1|^3} = \frac{\mu_0 I}{4\pi} \frac{R^2 d\phi' \vec{u}_z}{R^3} = \frac{\mu_0 I d\phi' \vec{u}_z}{4\pi R}$$

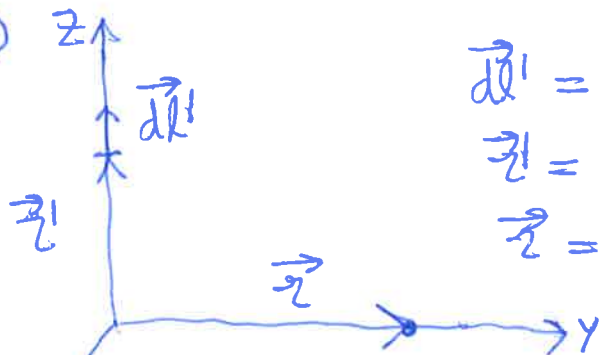
$$\vec{B}_2 = \int_{\text{región o curva 2}} d\vec{B}_2 = \int_0^{3\pi/2} \frac{\mu_0 I d\phi'}{4\pi R} \vec{u}_z = \frac{\mu_0 I}{4\pi R} \frac{3\pi}{2} \vec{u}_z = \frac{3\mu_0 I}{8R} \vec{u}_z$$

$$\vec{B} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3 = \boxed{\frac{3\mu_0 I}{8R} \vec{u}_z}$$

(9.25)

a)

(9.22)



$$d\vec{l}' = dz' \vec{u}_z$$

$$\vec{r}' = z' \vec{u}_z$$

$$\vec{r} = a \vec{u}_y$$

$$\vec{r} - \vec{r}' = a \vec{u}_y - z' \vec{u}_z$$

$$|\vec{r} - \vec{r}'| = \sqrt{a^2 + z'^2}$$

$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{l}' \times (\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3}$$

$$d\vec{l}' \times (\vec{r} - \vec{r}') = dz' \vec{u}_z \times (a \vec{u}_y - z' \vec{u}_z) =$$

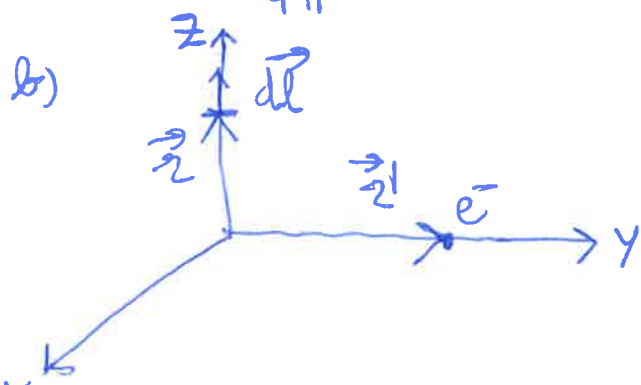
$$= dz' [a(\vec{u}_z \times \vec{u}_y) - z'(\vec{u}_z \times \vec{u}_z)] = -a dz' \vec{u}_x$$

$$\vec{B}_{\text{hilo}} = \int_0^L d\vec{B} = - \int_0^L \frac{\mu_0 I}{4\pi} \frac{a dz' \vec{u}_x}{(a^2 + z'^2)^{3/2}} = - \frac{\mu_0 I a}{4\pi} J(a, L) \vec{u}_x$$

$$J(a, L) \equiv \int_0^L \frac{dz'}{(a^2 + z'^2)^{3/2}} = \frac{z'}{a^2 \sqrt{z'^2 + a^2}} \Big|_{z'=0}^{z'=L} = \frac{L}{a^2 \sqrt{L^2 + a^2}}$$

$$\vec{F}_{\text{hilo} \rightarrow e^-} = -e(\vec{v} \times \vec{B}_{\text{hilo}}) = -e v (\vec{u}_y \times (-\frac{\mu_0 I a}{4\pi} J(a, L) \vec{u}_x))$$

$$= + \frac{e v \mu_0 I a}{4\pi} J(a, L) (\vec{u}_y \times \vec{u}_x) = \boxed{- \frac{e v \mu_0 I L \vec{u}_z}{4\pi a \sqrt{L^2 + a^2}}}$$



$$d\vec{l} = dz \vec{u}_z$$

$$\vec{r}' = a \vec{u}_y$$

$$\vec{r} = z \vec{u}_z$$

$$\vec{r} - \vec{r}' = z \vec{u}_z - a \vec{u}_y$$

$$|\vec{r} - \vec{r}'| = \sqrt{a^2 + z^2}$$

$$\vec{r}' = r \vec{u}_y \quad q' = -e$$

$$\text{Campo debido a } e^-, \text{ en } \vec{r}: \vec{B}_{e^-} = \frac{\mu_0 q'}{4\pi} \frac{\vec{r}' \times (\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3}$$

$$\vec{r}' \times (\vec{r} - \vec{r}') = r \vec{u}_y \times (-a \vec{u}_y + z \vec{u}_z) = r z (\vec{u}_y \times \vec{u}_z) =$$

$$= r z \vec{u}_x$$

Autor: Iván Cabrera Álvarez

$$\vec{B}_e = - \frac{\mu_0 e}{4\pi} \frac{v z \vec{u}_x}{(a^2 + z^2)^{3/2}}$$

$$d\vec{F}_e = I d\vec{l} \times \vec{B}_e = -I dz \frac{\mu_0 e v z (\vec{u}_z \times \vec{u}_x)}{4\pi (a^2 + z^2)^{3/2}}$$

$$d\vec{F}_e = - \frac{\mu_0 e v I z}{4\pi (a^2 + z^2)^{3/2}} dz \vec{u}_y$$

$$\vec{F}_{e \rightarrow \text{fido}} = \int_0^L d\vec{F}_e = \left[- \frac{\mu_0 e v I}{4\pi} \int_0^L \frac{z dz}{(a^2 + z^2)^{3/2}} \vec{u}_y \right]$$

$$\left[\int_0^L \frac{z dz}{(a^2 + z^2)^{3/2}} = - \frac{1}{\sqrt{a^2 + z^2}} \Big|_0^L = - \frac{1}{\sqrt{a^2 + L^2}} + \frac{1}{a} \right]$$

9.26 a)

$$F = |q| |\vec{v} \times \vec{B}| = q v B_0 \sin 90^\circ = q v B_0 = e v B_0$$

La carga del protón es positiva $\Rightarrow |q| = q = e > 0$

$$F = e v B_0$$

$$F = \frac{m v^2}{R}$$

$$\left. \begin{array}{l} F = e v B_0 \\ F = \frac{m v^2}{R} \end{array} \right\} \Rightarrow e v B_0 = \frac{m v^2}{R} \Rightarrow R = \frac{m v}{e B_0}$$

$$R = \frac{m v}{e B_0}$$

$$\frac{m v^2}{2} = e \Delta \Phi \Rightarrow v = \sqrt{\frac{2 e \Delta \Phi}{m}} \Rightarrow R = \frac{m}{e B_0} \sqrt{\frac{2 e \Delta \Phi}{m}} \Rightarrow$$

$$\Rightarrow R = \sqrt{\frac{m}{e}} \frac{1}{B_0} \sqrt{2 \Delta \Phi} = \sqrt{\frac{1}{95779027 \text{ C}}} \frac{1}{1 \text{ T}} \sqrt{2 \cdot 2 \cdot 10^4 \text{ V}} =$$

$$= \boxed{0.0204359 \text{ m}}$$

b)

$$\left. \begin{array}{l} \vec{F} = q \vec{E} + q (\vec{v} \times \vec{B}) \\ \vec{F} = \vec{0} \end{array} \right\} \Rightarrow \vec{E} = -(\vec{v} \times \vec{B})$$

$$\vec{v} \times \vec{B} = -v B_0 \vec{u}_y$$

$$\vec{E} = -(\vec{v} \times \vec{B})$$

$$\left. \begin{array}{l} \vec{v} \times \vec{B} = -v B_0 \vec{u}_y \\ \vec{E} = -(\vec{v} \times \vec{B}) \end{array} \right\} \Rightarrow \vec{E} = +v B_0 \vec{u}_y$$

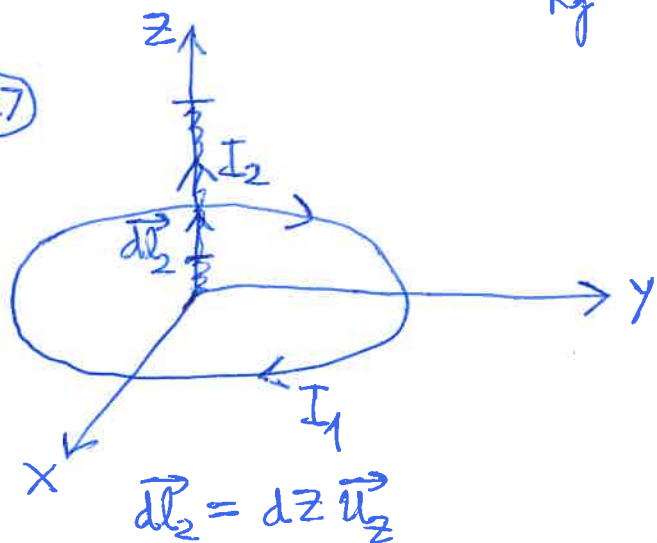
Autor: Juan Carlos Álvarez

$$\left. \begin{aligned} \vec{E} &= E \vec{u}_y \\ \vec{E} &= \omega B_0 \vec{u}_y \end{aligned} \right\} \Rightarrow E = \omega B_0 = \sqrt{\frac{2e \Delta \Phi}{m}} B_0 =$$

$$= \sqrt{2 \cdot 2 \cdot 10^4 \text{ V} \cdot 95779027 \frac{\text{C}}{\text{kg}}} \quad 1\text{T} = \boxed{1957335.20 \frac{\text{N}}{\text{C}}}$$

(9.24)

(9.27)



El campo magnético en el punto $(0, 0, z)$ debido a la corriente que circula por la circunferencia es:

$$\vec{B}_{1 \rightarrow 2} = - \frac{\mu_0 I_1 R^2}{2(z^2 + R^2)^{3/2}} \vec{u}_z$$

Este campo es el campo calculado en el problema 9.8, cambiando de signo.

$$d\vec{F} = I_2 d\vec{l}_2 \times \vec{B}_{1 \rightarrow 2} = I_2 dz \vec{u}_z \times \left(- \frac{\mu_0 I_1 R^2}{2(z^2 + R^2)^{3/2}} \vec{u}_z \right) = \vec{0} \Rightarrow$$

$$\Rightarrow \vec{F} = \int_L d\vec{F} = \boxed{\vec{0}}$$