Prospective mathematics secondary teachers' perceptions of their pedagogical content knowledge: An approach in the educational context of Zambia

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Abstract

The purpose of the current study was to contribute to a better understanding of Zambian prospective mathematics secondary teachers' pedagogical content knowledge of mathematics (M-PCK) as part of a larger project aimed at characterising their subject matter and pedagogical content knowledge related to the function concept. K-means Cluster analysis was used to derive profiles based on teachers´ perception of knowledge of instructional strategies, knowledge of subject specific language and symbols used in the study of mathematics, knowledge of conceptions and misconception of learners, knowledge of learner characteristics and knowledge of the mathematics curriculum. Three profiles related to M-PCK perceptions of prospective teachers emerged from the clustering process namely the (i) self-doubting (cluster 1), anti-M-PCK (cluster 2) and (iii) confident and enthusiastic (cluster 3). A One-way ANOVA test was performed to determine the existence of differences in prospective teachers' perceptions between Clusters based on the five M-PCK sub-factors. The test revealed significant differences between Clusters of prospective teachers' perception of all sub-factors. Prospective teachers in Clusters 1 and 2 showed low confidence in their M-PCK perceptions while those in Cluster 3 generally showed moderate confidence. Results have implications for teacher training in Zambia.

Keywords: perceptions; prospective mathematics secondary teachers; pedagogical content knowledge

Introduction

Mathematics teachers' perceptions about teaching and learning have a significant influence on how effective their instruction would be. Akarsu (1975) in Bukova-Guzel et al. (2013) defined perception as an awareness of one's mind resulting from an individual as he/she makes sense of or interprets the stimulus that occurs in his/her mind when exposed to a concept. Therefore, it is important for teacher educators to conduct studies that are focused on perceptions of teachers in relation to their mathematical pedagogical content knowledge (M-PCK) (Teo, 2010). This is because prospective teachers' M-PCK development that goes beyond their own perceptions provides insight about the kind of teachers they would turn out to be in future (Carr-Chellman and Dyer, 2000). This implies that prospective teachers' perceived M-PCK does not limit the extent to which they can be helped to develop desired M-PCK. While some prospective teachers may perceive their M-PCK to be low, university education can provide opportunities for them to develop their M-PCK to high levels by the time they graduate. Thus, the context in which prospective teachers are trained adds a dimension that further helps to understand their M-PCK perceptions. Considering that local context is relevant for a better understanding of teachers' practices (Shulman, 1987), as well as to provide useful recommendations to achieve higher educational quality standards by means of teacher training programs, this study contributes to fill a gap in the literature on prospective mathematics teachers' M-PCK perceptions with no evidence of previous research on this topic concerning the situation in Zambia.

Teachers' pedagogical content knowledge

The concept of pedagogical content knowledge was first conceptualised by Shulman (1986). In his desire to identify special knowledge that mathematics teachers needed for them to teach effectively, Shulman observed an increasing imbalance between teachers' knowledge of content and their knowledge of pedagogical strategies and argued for the need to bridge the gap (missing paradigm) between the two types of knowledge if teachers were to teach effectively. This, according to Shulman (1987), necessitated the introduction of a third type he referred to as pedagogical content knowledge (PCK). Since it was first introduced over 25 years ago, PCK has earned its place as a useful construct in conceptualising the unique nature and development of knowledge needed by teachers for instructional purposes, and research in teacher education focusing on PCK has rapidly increased (Berry et al., 2016).

After Shulman's (1986) PCK model, several other models emerged that emphasised knowledge of curriculum and its implementation, knowledge of assessment, knowledge of student characteristics, and knowledge of context (for example, Grossman, 1990; Magnusson, 1999; Shulman 1987). While these models do not directly emphasise PCK perceptions, they help us, for example, to analyse teachers' perception of the curriculum, its assessment and the context in which teachers implement it and learner contexts of how learning takes places (Hashweh, 2005).

These somewhat general PCK models gave rise to more domain specific models like the mathematics knowledge for teaching (MKT) framework (Hill et al., 2008) and mathematics teacher's specialised knowledge (MTSK) framework (Carrillo et al., 2013). The MTSK framework was inspired by the MKT and incorporates beliefs as a central element that permeates the whole model. Carrillo et al. (2013) argued that the emphasis on teachers' beliefs in their model is because they are influential on one's knowledge for teaching mathematics. We argue that perceptions like beliefs have great influence on prospective teachers' M-PCK. This makes our study relevant in the wider field of mathematics education knowledge especially because when discussions about teachers' M-PCK arise, beliefs are usually more pronounced than perceptions. At this point, we state that in the remainder of this paper PCK related to the field of mathematics shall be referred to as M-PCK to mean mathematical pedagogical content knowledge. This is important for the reader to differentiate general PCK from domain specific M-PCK.

Several studies in the literature that have investigated teachers' M-PCK have either focused on their general M-PCK (Norton, 2018), topic specific M-PCK (Danisman & Tanisli, 2017; Depaepe et al., 2015; Tröbst et al., 2019), M-PCK for developing mathematics curriculum (Thompson, 2018) and M-PCK related to problem solving (Lee et al., 2018). While these studies demonstrate what has been the major focus of M-PCK research, little has been done in relation to teachers' M-PCK perceptions. Determining prospective teachers' perceptions related to their subject matter knowledge and their knowledge of teaching strategies must be seen as an important issue for research (Bukova-Guzel et al., 2013) since perceptions act as a support to prospective teachers' own learning because individuals learn through their perception of phenomena. By identifying M-PCK domains in which prospective mathematics teachers perceive themselves weak or strong (Bukova-Guzel et al., 2013), studies focusing on perceptions would act as support for their knowledge development and would provide important information for teacher education providers.

To end this section, let us recall and highlight that the main goal of this study was to determine and understand perceptions of Zambian prospective mathematics secondary teachers related to their PCK. The primary question, which sought answers from this study, was: How do Zambian prospective secondary mathematics teachers perceive their M-PCK? To accomplish this, Cluster profiles that revealed interesting M-PCK characteristics exhibited by prospective mathematics teachers in relation to mathematics instruction were derived. Clusters enabled us to identify the existence of differences among the M-PCK perceptions of the prospective teachers when studied by means of Cluster analysis as well as determining how such Clusters perceive the influence of M-PCK factors on their M-PCK.

Methodology

Participants

To accomplish the main goals of the study stated above, data were collected from 104 male and female students from two public universities in Zambia. Our sample comprised male and female prospective teachers in the second semester of their third and fourth years of university training. All participants had completed the compulsory teaching practice experience at a government school of their choice and all mathematics education courses. At this stage, participants had also completed most of the mathematics content courses. It was important for us to sample from prospective teachers who had school teaching experience because their perceptions would include the real classroom experiences.

Zambia operates an education system that allows for a compulsory period of seven years at primary school (Grades 1 to 7) with student average age of 7-13 years and five years secondary school (grades 8 to 12). Grades 8 and 9 form junior secondary level while grades 10, 11 and 12 form the senior secondary level. The student average ages for secondary school are from 13 to 17 years. Thus, participants of the current study are being prepared to teach students in grades 8 to 12. Gender differences mirror the enrolment levels at the two universities. On average, these universities enrol more male than female students for two main reasons – (i) there are more male applicants to mathematics related programmes than females, and (ii) there are more male candidates who pass mathematics at grade 12 level. A good grade in mathematics at Grade 12 level is a pre-requisite for entry into the mathematics programme at the two universities.

Instrumentation

As a data collection tool, a survey designed for measuring prospective mathematics teachers' PCK perceptions developed by Bukova-Guzel et al. (2013) was adapted to the Zambian context in this study. The scale is a five factor, five-point Likert scale. The adapted survey consisted of seventeen items distributed across five knowledge domains. The knowledge domains included teachers' knowledge of language used in the teaching of mathematics together with symbols (KMLS), knowledge of appropriate teaching strategies (KTS), knowledge of learners' misconceptions (KoLM), knowledge of learner characteristics (KoLC), and curriculum knowledge (CK). The full instrument has been included in the appendix. For the discussion in this sub-section, knowledge domains and example items have been extracted for readers' ease of reference (see Table 1).

Table 1. M-PCK knowledge domains and item examples

Knowledge of Teaching Strategies (KTS)
I can design appropriate activities to present mathematical concepts.
I can relate mathematical concepts to daily life in instruction.
Knowledge of Mathematical Language and Symbols (KMLS)
I can use mathematical language properly when presenting mathematical concepts.
I can use mathematical symbols properly.
Knowledge of Misconceptions (KM)
I can anticipate students' possible difficulties about a topic
I can design activities that will not cause students to develop misconceptions about the topic.
Knowledge of Learners (KL)
I know students' prior knowledge about a topic
I can choose appropriate examples for students' developmental levels in my lessons.
Knowledge of Curriculum (KC)
I have knowledge about the purposes of the mathematics curriculum.
I plan my lessons so as to relate the purposes of the mathematics curriculum with students' needs.

The adaptation process of the M-PCK survey included performing a confirmatory factor analysis (CFA) using SmartPLS 3 to check the model fit in a Zambian context. This means that the survey was pilot tested in Zambia to determine whether psychometric indices generated from the data obtained in Zambia were satisfactory for its use. Results of the CFA revealed an SRMR value of 0.094. A detailed discussion of this index has been provided in a sub-section on “discriminant validity’ below.

Cronbach alpha reliability of the survey was established for all M-PCK factors and for the entire M-PCK adapted scale using SPSS 23. The reliability coefficients for the M-PCK factors of the adapted scale were found to be KC = 0.65, KM = 0.66, KMLS = 0.75, KTS = 0.83 and KL = 0.88. Regarding the Cronbach alpha reliability, the rule of thumb states that Cronbach’s alpha reliability coefficient of greater than 0.7 is acceptable (George & Mallery, 2003). Our instrument’s Cronbach value of 0.76 is acceptable. This value was affected by the low reliability coefficients of the KC and KM variables. This could have been due to the number of items in each of these M-PCK sub-factors. For example, KM only consisted of three items. It was difficult to remove one or two items to improve the reliability coefficient. In addition, while KC consisted of seven items, the entire instrument was made up of only seventeen items. Thus, we could not reduce this number of items. Considering that we obtained an acceptable reliability coefficient for the entire instrument we agreed to use it for data collection. We invite readers to be mindful of this limitation as they read results of our study.

Adaptation of the M-PCK self-concept survey (Reproduced from Sintema & Marbán, 2020)

To adapt the M-PCK survey, a CFA was performed to calculate the fit indices of the instrument. In this section, we highlight part of the adaptation and validation process by discussing (i) Internal consistency reliability, (ii) Convergent validity and (iii)

Discriminant validity of the instrument.

Internal consistency reliability

Internal consistency has commonly been measured using Cronbach's alpha. This has presented challenges such that some scholars suggested the use of composite reliability in PLS-SEM as a preferred measure of internal consistency (Bagozzi & Yi, 1988; Hair et al., 2012; Hair et al., 2014). A composite reliability of at least 0.7 is preferred but a minimum of 0.6 would be acceptable to achieve internal consistency. Examining Table 2, it can be seen that all the latent variables KTS, KMLS, KL, KM and KC had composite reliability greater than 0.7. Thus, all the latent variables recorded high internal consistency.

Table 2 . Composite reliability and average variance extracted (AVE) values

Latent variable	Composite reliability	AVE
KTS	0.897	0.748
KMLS	0.886	0.796
KM	0.818	0.601
KL	0.941	0.889
KC	0.773	0.328

Convergent validity

Convergent validity is established by considering the Average Variance Extracted (AVE) of each latent variable and an AVE of 0.5 and higher is acceptable (Bagozzi & Yi, 1988; Henseler et al., 2009). The latent variables KTS, KMLS, KL and KM all had AVE higher than the acceptable minimum of 0.5. However, the variable KC had the AVE of 0.328, which was far below the minimum. This could have been caused by the poor indicator reliability of the indicators of KC and we conclude that it does not capture the intended construct – curriculum knowledge - well. The next analysis about discriminant validity will provide insight about how the issues surrounding KC were overcome.

Discriminant validity

Discriminant validity referred to how variance in the indicators is able to explain variance in the latent variables (De Sousa Magalhaes et al., 2012). Table 3 shows indices of the Fornell-Larcker criterion for checking discriminant validity. Fornell and Larcker (1981) proposed that discriminant validity could be achieved by finding the square root of the AVE of each latent variable.

Table 3. Fornell-Larcker criterion for checking discriminant validity

	KC	KL	KM	KMLS	KTS	M-PCK
KC	**0.573**					
KL	0.060	**0.943**				
KM	0.158	0.314	**0.775**			
KMLS	0.302	0.126	0.322	**0.892**		
KTS	0.235	0.064	0.267	0.355	**0.865**	
M-PCK	0.752	0.336	0.594	0.581	0.605	**1.000**

According to Fornell-Larcker (1981) and Chin (1998), if correlation values (Table 4) of other latent variables are less than the square root of the AVE then discriminant validity is achieved. Examining Table 4, it can be seen that all the latent variables except KC had the AVE values larger than the correlations in their columns. Discriminant validity was also checked using the HTMT to resolve the challenge presented by the latent variable KC.

Table 4. Correlation of latent variables

	KC	KL	KM	KML	KTS	M-PCK
KC	1.000					
KL	0.060	1.000				
KM	0.158	0.314	1.000			
KML	0.302	0.126	0.322	1.000		
KTS	0.235	0.064	0.267	0.355	1.000	
M-PCK	0.752	0.336	0.594	0.581	0.605	1.000

The Heterotrait-Monotrait (HTMT) ratio for determining discriminant validity is said to be more efficient. Henseler et al. (2015) proposed that the HTMT ratio of less than 1.0 meets the threshold for the establishment of discriminant validity. From Table 5, all the constructs had their HTMT ratios below 1.0. Thus, discriminant validity was achieved. However, other scholars have proposed even a lower threshold with Gold et al., (2001) and Teo et al., (2008) proposing a 0.90 threshold. A threshold of 0.85 was suggested by Kline (2011).

Table 5. Heterotrait-Monotrait ratio (HTMT)

	KC	KL	KM	KML	KTS
KC					
KL	0.195				
KM	0.353	0.421			
KML	0.400	0.164	0.433		
KTS	0.321	0.075	0.358	0.451	
M-PCK	0.916	0.355	0.726	0.660	0.671

One of the most important indices for determining a good model fit is the standardized root mean residual (SRMR) which is responsible for measuring the approximate model fit by taking into consideration the difference between the observed correlation matrix and the model implied correlation matrix (Garson, 2016, p. 68). A SRMR cut-of-point of less than 0.10" is considered a good measure of model fit (Garson, 2016, p. 68; Kante, 2018; Petrescu-Mag et al., 2022). Joo (2020) reported that "… SRMR indicates an acceptable fit when it produces a value of smaller than 0.10 …" (p. 350). The SRMR index for the estimated model was 0.094, which falls within the 0.10 cut-off.

Data analysis

Data analysis was conducted using IBM SPSS 23. Data were analysed using the following techniques: K-Means Cluster analysis and descriptive statistics. Descriptive statistics were analysed in terms of the mean and standard deviations. K-Means Cluster analysis was adopted for this study because of its ability of naturally finding groupings of data which consist of categories of objects that are similar and of those that are dissimilar (Berkhin, 2006) and Cluster analysis was considered in the sense provided by Verma (2013, p. 318) who contended that

> *Cluster analysis as a multivariate statistical technique is best suited for the purpose of identifying and placing objects into groups known as Clusters. the technique allocates homogeneous objects to the same Cluster and those that dissimilar to other Clusters. the Clusters are purely derived from the nature of the data and characteristics of the subjects that constitute the data to be analysed [...]*

Considering that the main goal of this study was to identify prospective mathematics secondary teachers' profiles based on their perceptions of M-PCK, Cluster analysis served as the main analysis technique. In accomplishing the goal of the study, we were able to derive Clusters of participants' M-PCK perceptions using this technique. Cronbach's alpha was used to ensure the reliability of the M-PCK perceptions survey that was used for data collection. Reliability of the data collection instrument was key to the success of the study and thus contributed to achieving the goal of the study. It was also important to know the variability of participants' M-PCK perceptions. Thus, standard deviation was calculated for the sample.

Results and Discussion

K-Means Cluster analysis derived three Clusters profiling prospective secondary mathematics teachers' perceptions of their M-PCK from the data obtained from 104 male and female prospective teachers. Cluster characteristics were defined according to the five M-PCK factors (KTS, KMLS, KM, KL, KC). While gender and year of study have been used to give an overview of prospective teachers' characteristics in each cluster, they were excluded from the Clustering procedure. The data were handled under strict ethical practices of confidentiality and anonymity. Participants were advised not to provide any identifying information like their names. This was to ensure that they participated without fear of being identified by either the researcher or the reader of the research output.

The Cluster quality was determined by the silhouette measure of cohesion and separation. This is an important validation technique when performing a two-step Cluster analysis because it measures the goodness-of-fit of the Cluster structure (Kaufman & Rousseeuw, 2009), and the relationship of variables within and between Clusters, with a silhouette value of greater than 0.2 being indicative of a fair separation distance between Clusters (Tkaczynski, 2017). The silhouette measure of cohesion and separation for the Cluster structure in our study is 0.4 which indicates a fair Cluster structure and implies that the measured distance between the three derived Clusters was fair and acceptable for the analysis to continue. We begin by presenting a correlation analysis of the M-PCK sub-factors before delving into profiles derived from the clustering algorithm.

Correlations among M-PCK sub-factors of prospective teachers

One of the pre-clustering steps that were considered for this study was analysing the relationship between the five M-PCK sub-factors that were selected for the K-Means cluster analysis procedure. One of the critical aspects of cluster analysis is that clustering variables are supposed to be unique in order to obtain meaningful and distinct groupings. This also implies that clustering variables should exhibit correlations between each other. For example, it is expected that a teacher who exhibits high-perceived knowledge of learners is also expected to have high perceived knowledge of teaching strategies. In this way, the teacher is more likely to be confident about selecting suitable teaching strategies that fit the diversity of students in their class. However, the correlations should not be so high that they result into collinearity. Analysing Table 6, there was no statistically significant relationship between prospective teachers' KL and their KTS ($r = 0.062$, $p > 0.05$) as well as their KMLS ($r = 0.130$, $p > 0.05$). Similarly, prospective teachers' KC did not significantly relate with their KM ($r = 0{,}144$, $p > 0.05$) and their KL ($r = 0.046$, $r > 0.05$). The rest of the clustering variables showed significant relationships. The significant correlations or lacks of them are will be visible in the teacher profiles. Considering that the absolute values of all significant correlations are less than 0.90, it is less likely that they will influence the formation of clusters (c.f. Sarstedi & Mooi, 2014).

Table 6. **Correlations of M-PCK variables**

	KTS	KMLS	KM	KL	KC
KTS	1				
KMLS	$.355^{**}$	1			
KM	$.262^{**}$	$.301^{**}$	1		
KL	.062	.130	$.325^{**}$	1	
KC	$.241^{*}$	$.280^{**}$	.144	.046	1

Correlation: $^{**}p < 0.01$, $^{*}p < 0.05$

Cluster analysis results of prospective teachers' M-PCK

Cluster analysis was seen as useful in the descriptive context of this study under the assumption that the study group is not homogeneous. For this study, clustering was seen as a powerful tool to help classify data into structures that might be more easily understood and manipulated, providing at the same time a starting point for deeper analysis of prospective teachers' M-PCK. We start by presenting demographic characteristics of clusters before delving into a deeper analysis of participants' profiles.

Demographic characteristics of pre-service teachers' clusters

Demographic characteristics of each cluster were defined according to Cluster size, gender of participants, year of study of participants, and age of participants (see Table 7). While gender and year of study were used to give an overview of prospective teachers' characteristics in each cluster, they were excluded from the clustering procedure.

Table 7. Demographic characteristics of pre-service teachers' clusters (n = 104)

Demographic characteristic		Cluster 1 (n = 31)		Cluster 2 (n = 10)		Cluster 3 (n = 63)	
		n	%	*n*	%	*n*	%
Gender	Male	21	67.74	8	80	49	77.78
	Female	10	32.26	2	20	14	22.22
Age	18-22	7	22.58	1	10	5	7.94
	23-26	18	58.06	8	80	49	77.78
	27-32	6	19.36	1	10	8	12.69
	32+	0	0	0	0	1	1.59
Year of study	3rd Year	18	58.06	5	50	35	55.56
	4th Year	13	41.94	5	50	28	44.44

At the time of their participation in this study, all participants had completed their teaching practice at various schools across the country. Majority of prospective teachers in each cluster were the age range 23-26 years old with males dominating each cluster. On average, Zambian students are expected to graduate from a 4-year degree program at 24 years considering that most of them complete high school at 19 years old and have a year or two to wait for university entry. It is not surprising that male prospective teachers dominated all three clusters. This is because male entrants usually dominate mathematics courses in Zambian universities. At Grade 12 level, we see males outperform females in mathematics. Thus, gender issues manifest when choosing courses in university.

Analysing Table 7, cluster 3 was the largest cluster with 63 (60.58%) prospective teachers while cluster 2 was the smallest with 10 (9.62%) prospective teachers. The cluster with 10 prospective teachers provides an interesting perspective of their M-PCK. This will be elaborated later on, as we will be discussing profiles of all participants. Overall, the distribution of participants according to the year of study was balanced across clusters.

Cluster profiles of prospective teachers' M-PCK

Five M-PCK factors KTS, KMLS, KM, KL and KC of prospective teachers were included when running the Cluster analysis. This was to ensure that demographic variables like gender and year of study did not affect the formation of clusters. Variables of interest for this study were M-PCK sub-factors. As a first step in the pre-processing of our data for clustering, we conducted a z-standardisation of al M-PCK sub-factors. Standardisation *helps to make the relative weight of each variable equal.* In addition, this was to mitigate the effect of variations in the number of items for each M-PCK sub-factor. Some sub-factors like KMLS had two items while KC had seven items. We standardised these sub-factors so that their means were all at zero. The data were handled under strict ethical practices of confidentiality and anonymity. Participants were advised not to provide any identifying information like their names. This was to ensure that they participated without fear of being identified by either the researcher or the reader of the research output.

To obtain our clusters, data were subjected to an iteration process, which involved 10 iterations. This was aimed at determining the number of iterations that led to the convergence of cluster centers. A two-cluster solution did not converge even after the maximum number of iterations. This implied that it was not the best cluster solution for meaningful clustering. We then iterated a three-cluster solution, which converged at nine iterations. Thus, a cluster solution with three clusters was the best representation of the data for the current study. Table 8 shows final cluster centers. Cluster centers help in determining which clusters were far apart based on the distances between them. Overall, Table 8 shows that in terms of M-PCK variables, cluster 3 was far from clusters 1 and 2. This can be seen from the positive values recorded for all clustering variables. Clusters 1 and 2 posted negative values except for KTS in cluster 1. In spite of the negative values, the two clusters were far apart. A close analysis of M-PCK variables shows that in terms of KTS, cluster 2 was far from clusters 1 and 3, which were almost the same. For the rest of the M-PCK variables, cluster 3 was far from clusters 1 and 2.

Table 8. Final cluster centers for prospective teachers' M-PCK

	Cluster		
	1	2	3
KTS	0.222	-2.192	0.239
KMLS	-0.267	-1.620	0.389
KM	-0.692	-0.970	0.494
KL	-0.914	-0.075	0.461
KC	-0.308	-0.599	0.247

To obtain cluster profiles of prospective teachers' M-PCK (Figure 1), we used standardised score. Three profiles emerged from the clustering process namely the (i) self-doubting (cluster 1), anti-MPCK (cluster 2) and (iii) confident and enthusiastic (cluster 3), according to Figure 1,

Profile 1: *Self-doubting*: This profile includes prospective teachers in cluster 1 who have moderate perceived knowledge of teaching strategies. This group of teachers are likely to be innovative in the choice of teaching approaches and will be confident in using them in their future classes. However, they have low perceived knowledge of their learners as well as learner misconceptions. This implies that they are not likely to be confident in their anticipation and identification of learner misconceptions. In addition, they are less confident in their use of appropriate mathematical language and symbols owing to their low perceived knowledge of this domain. This group might have been facing difficulties in their pedagogical university courses. This might have contributed to them becoming less confident in their perceived knowledge of M-PCK. Prospective teachers in this profile clearly exhibit gaps in the overall M-PCK. This is like the finding of Depaepe et al. (2015), whose study also revealed gaps in teacher PCK.

Profile 2: *Anti-MPCK*: This profile refers to prospective teachers in cluster 2. These prospective teachers have an overall negative view of their overall M-PCK and therefore do not perceive themselves as confident in the use of all M-PCK factors. They have a very low perception of their knowledge of teaching strategies and consider that they will be less confident in their perceived knowledge and use of mathematical language and symbols in their future classrooms. In addition, they consider that they will not be confident in identifying student misconceptions based on their low perceived knowledge of misconceptions. Based on their perceived knowledge, these teachers will be less confident of the use of curriculum materials for teaching mathematics. This group of students present an interesting case considering that they were only 10 out of a total of 104 participants. These prospective teachers might have had a shaky background in their methods courses. While their counterparts in profile 1 also showed insufficient perceived M-PCK knowledge, theirs was much lower (see Figure 1). Results related to prospective teachers in profiles 1 and 2 might suggest the need for revisiting at the curriculum for methods courses in their universities. With reference to previous studies that investigated similar phenomena, characteristics of prospective teachers in profile 2 are consistent with finding of Danisman and Tanisli (2017), and Lee et al. (2018). Both of these studies reported insufficient PCK knowledge of high school mathematics teachers.

Profile 3: *Confident and enthusiastic*: This profile includes teachers who are moderately confident about their perceived knowledge of teaching strategies. They are expected to be innovative in their choice of instructional approaches in their future classrooms by choosing strategies that maximise academic benefits for their students. However, they are likely to be less confident their perceived knowledge of student mathematical needs. Their low perceived knowledge of misconceptions implies that they will not be confident in anticipating their students' misconceptions about the task they prepare for their classes. In addition, they will not be confident in using appropriate mathematical language and symbols owing to their low perceived knowledge of this domain. While this group did not show very high levels of their M-PCK, they have the potential to develop more confidence as novice teachers. This result is consistent with findings of Trobst et al. (2019) who found that participants in their study exhibited high levels of PCK.

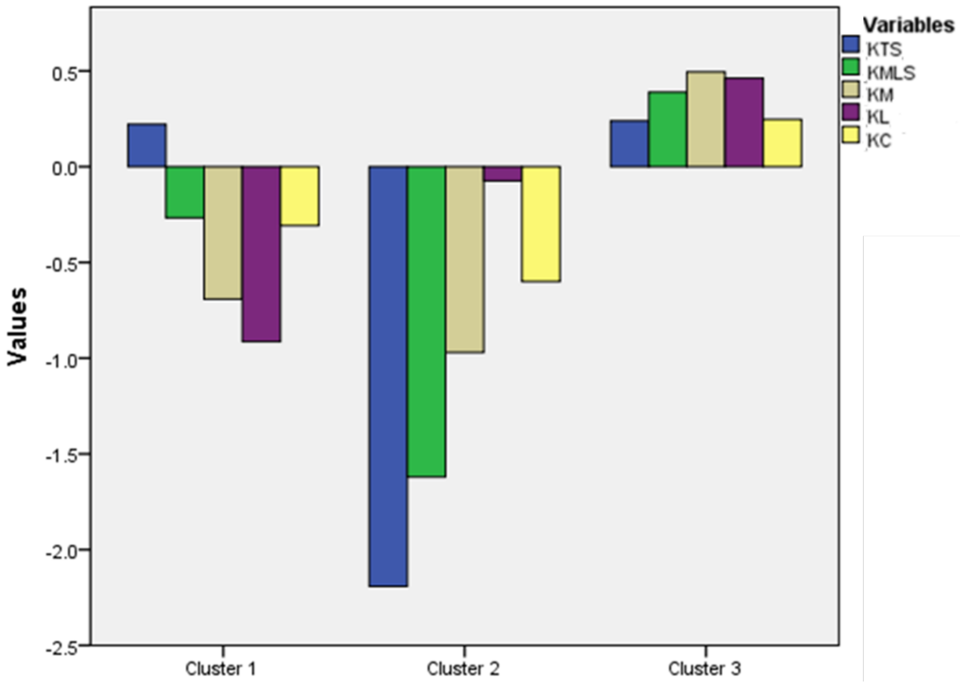

Figure 1. Cluster profiles of prospective teachers' M-PCK

As a follow-up to the K-Means clustering results, we conducted a one-way analysis of variance (ANOVA) to validate the clusters by determining that they were all distinguished from each other. We used a Bonferroni post-hoc test that identifies the actual differences between clusters lie. Overall, ANOVA results show that there were significant differences. For teachers' KTS perceptions, there were significant pairwise differences between cluster 1 – cluster 2 [F(53.835, $p < .001$)] and cluster 2 – cluster 3 [F(53.835, $p < .001$)]. However, no significant differences were found for KTS perceptions between clusters 1 and 3. This is visible by inspecting Figure 1 and examination of final cluster centres in Table 8. As regards prospective teachers' KMLS perceptions, all clusters were significantly different [F(29. 518, $p < .001$)].

In addition, Table 9 shows significant pairwise differences in prospective teachers' KM perceptions between cluster 1 – cluster 3 [F(31.616, $p < .001$)] and cluster 2 – cluster 3 [F(31.616, $p < .001$)]. However, prospective teachers in clusters 1 and 2 did not have significant differences in their KM perceptions (see also Figure 1 and Table 3). In terms of their KL perceptions, significant pairwise differences were only evident in between cluster 1 – cluster 2 [F(31.231, $p < .05$)] and cluster 1 – cluster 3 [F(31.231, $p < .001$)]. Further, significant pairwise differences were found for prospective teachers' KC perceptions between cluster 1 – cluster 3 [F(5.654, $p < .05$)] and cluster 2 – cluster 3 [F(5.654, $p < .05$)]

Table 9. Results of the one-way ANOVA of cluster centroids

	Sum of Squares	df	Mean Square	F	p	Bonferroni post-hoc Pairwise comparison	p
KTS	53.146	2	26.573	53.835	.000	Cluster 1 – Cluster 2	.000
						Cluster 2 – Cluster 3	.000
KMLS	37.996	2	18.998	29.518	.000	Cluster 1 – Cluster 2	.000
						Cluster 1 - Cluster 3	.001
						Cluster 2 - Cluster 3	.000
KM	39.657	2	19.828	31.616	.000	Cluster 1 - Cluster 3	.000

						Cluster 2 - Cluster 3	.000
KL	39.359	2	19.679	31.231	.000	Cluster 1 - Cluster 2	.013
						Cluster 1 - Cluster 3	.000
KC	10.371	2	5.185	5.654	.005	Cluster 1 - Cluster 3	.029
						Cluster 2 - Cluster 3	.032

Research limitations and implications

Considering characteristics of the sample and the mathematics education course which prospective teachers are exposed to by their institutions, findings of this study may be generalised to other university contexts in Zambia only. Firstly, most prospective teachers in various universities in Zambia are admitted based on their performance in the national Grade 12 examinations. At admission, they are expected to have passed mathematics with a credit (60%) or better. Thus, they have similar knowledge pre-requisites for the programme. Secondly, in almost all universities mathematics education courses are offered in third and fourth years. These courses are mainly teaching methods courses, teaching practice and a mathematics education project.

As such, all the results should be limited to the Zambian context because the instrument used in the study was adapted to the Zambian context and should not be assumed to be representative of all prospective teachers outside Zambia. These preliminary findings can be further investigated on a larger scale and in great depth. For example, which M-PCK factors have the greatest impact on pre-service teachers' M-PCK perceptions of each of the three Clusters? Results of this study are based on prospective teachers' self-reported perceptions of their M-PCK. Authors of this paper are cognizant of the fact that survey data alone cannot offer an in-depth understanding of teacher characteristics as they relate to their M-PCK perceptions. Thus, with the results of this study as starting point and with extra data, this can be achieved by administering in-depth interviews or focus group discussions as a way of further confirming the findings of the current study. Results presented in this study can be confirmed further by involving teachers in professional development programs (CPDs) and then measure their M-PCK perceptions before and after the CPDs.

Conclusion and policy recommendations

This study examined the M-PCK perceptions of mathematics prospective secondary teachers. Findings revealed moderate to low levels of self-perception of prospective teachers' M-PCK. This implies moderate to low confidence of the prospective teachers in their M-PCK, which can impact the teaching in a way. Those with moderate confidence are likely to motivate their future students to learn mathematics in their classroom, while not so much with teachers with low perceptions of their M-PCK. This has implications for the teacher-training programme that these participants undertook. It might be that the course content needs to be tailored more to the needs of the students. M-PCK perceptions of prospective teachers in profiles 1 and 2 seem to indicate that they might not be getting the best out of the pedagogical courses in their academic programme.

We propose that the structure of the mathematics education programme in which participating prospective teachers were enrolled may be revised to align with the pedagogical needs of students. There is a clear imbalance in the number of mathematics education and mathematics courses in Zambian universities. The programme is dominated by mathematics content course with few pedagogical courses which are only introduced at third year. Prospective teachers enrol for a course in mathematics teaching methods and for a teaching practice course at third year. They spend three months – a full term – teaching in a secondary school of their choice to put into practice skills they acquired from the mathematics teaching methods course. At fourth year, they register for a course in mathematics education and another course in which they write and present a mathematics education project. We are of the view that the few courses in mathematics education could have contributed to the moderately low levels of their M-PCK perceptions.

The aspects covered in this study have not previously been examined among mathematics preservice teachers in Zambia. There have been studies on the general PCK of prospective teachers but there is a gap in studies focusing on PCK perceptions of prospective teachers of mathematics. This study serves as a good opportunity for policy makers and teacher educators at various levels in Zambia to consider prospective teachers' perceptions when designing teacher education curricula, and when designing professional development activities for in-service teachers of mathematics.

The teaching council of Zambia, which is charged with a responsibility of ensuring that there is quality teaching in all schools and colleges of education should take keen interest in the findings of this study. Hughes (2003), who argued that for all teacher education providers and professional bodies charged with the responsibility of regulating the teaching profession, it is important to pay attention and discover more about the teachers whom we work with before we engage in teaching, guiding, and collaborating with them, echoes this. This is because the provision of external incentives or stimuli, or even opportunities are just one perspective from which individual teachers can be engaged in a learning experience.

Ethical consideration

The ethics committee of the Ministry of General Education, Zambia, approved the study. We wrote a letter to the Ministry seeking permission to involve university students in this study. They responded with a positive answer, and we went on to seek permission from the universities involved, which was granted. Considering that the study involved participation of human subjects, informed consent was obtained from all participants that took part in the study. Subjects were further informed that participation in the study was voluntary. Thus, they were free to withdraw from the study at any stage.

Data Availability Statement

The data that support the findings of this study are available on request from author

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References

Bagozzi, R. P., & Yi, Y. (1988). On the evaluation of structural equation models. *Journal of Academy of Marketing Science*, *16*(1), 74-94. https://doi.org/10.1007/BF02723327

Berkhin, P. (2006). A survey of clustering data mining techniques. In J. Kogan, C. Nicholas & M. Teboulle (Eds.). *Grouping multidimensional data: Recent advances in clustering* (pp. 25-71). Berlin, Heidelberg: Springer. https://doi.org/10.1007/3-540-28349-8_2

Berry, A., Depaepe, F., & van Driel, J. (2016). Pedagogical content knowledge in teacher education. In J. Loughran & M. L. Hamilton. *International handbook of teacher education, Volume 1* (pp. 347- 386). Springer. https://doi.org/10.1007/978-981-10-0366-0_9

Bukova-Güzel, E., Cantürk-Günhan, B., Kula, S., Özgür, Z., & Elçí, A. N. (2013). Scale development for pre-service mathematics teachers' perceptions related to their pedagogical content knowledge. *South African Journal of Education*, *33*(2), 1-21. https://doi.org/10.15700/saje.v33n2a690

Carr-Chellman, A. A., & Dyer, D. (2000). The pain and ecstasy: Pre-service teacher perceptions on changing teacher roles and technology. *Journal of Educational Technology & Society*, *3*(2), 96-105.

Carrillo, J., Climent, N., Contreras, L. C., & Muñoz-Catalán, M. C. (2013). Determining specialised knowledge for mathematics teaching. In B. Ubuz, Ç. Haser & M. Mariotti (Eds.), *Proceedings of the Eighth Congress of the European Society for Research in Mathematics Education* (pp. 2985-2994). ERME.

Chin, W. W. (1998). The partial least squares approach to structural equation modeling. *Modern methods for business research*, *295*(2), 295-336.

Danisman, S & Tanisli, D. (2017). Examination of mathematics teachers' pedagogical content knowledge of probability. *Malaysian Online Journal of Educational Sciences*, *5*(2), 16-34

de Sousa Magalhaes, S., Fernandes Malloy-Diniz, L., & Cavalheiro Hamdan, A. (2012). Validity convergent and reliability test-retest of the rey auditory Verbal Learning Test. *Clinical Neuropsychiatry*, *9*(3), 129-137.

Depaepe, F., Torbeyns, J., Vermeersch, N., Janssens, D., Janssen, R., Kelchtermans, G., ... & Van Dooren, W. (2015). Teachers' content and pedagogical content knowledge on rational numbers: A comparison of prospective elementary and lower secondary school teachers. *Teaching and teacher education*, *47*, 82-92. https://doi.org/10.1016/j.tate.2014.12.009

Fornell, C. G., & Larcker, D. F. (1981). Evaluating structural equation models with unobservable variables and measurement error. *Journal of Marketing Research*, *18*(1), 38-50.

Garson, G. D. (2016). *Partial least squares: regression & structural equation models*. Statistical Associates Publishing.

George, D., & Mallery, P. (2003). *SPSS for Windows step by step: A simple guide and reference*(4th ed.). Allyn & Bcon.

Gold, A. H., Malhotra, A., & Sergas, A. H. (2001). Knowledge management: an organizational capabilities perspective. *Journal of Management Information Systems*, *18*(1), 185-214.

Grossman, P. L. (1990). The making of a teacher: Teacher knowledge and teacher education. *Teachers College Press*, *91*, 192-208

Hair J. F., Sarstedt, M., Pieper, T. M., & Ringle, C. M. (2012). The use of partial least squares structural equation modeling in strategic management research: a review of past practices and recommendations for future applications. *Long Range planning*, *5*(6), 320-340.

Hair Jr, J. F., Sarstedt, M., Hopkins, L., & Kuppelwieser, V. G. (2014). Partial least squares structural equation modeling (PLS-SEM): An emerging tool in business research. *European business review*, *26*(2), 106-121.

Hashweh, M. Z. (2005). Teacher pedagogical constructions: a reconfiguration of pedagogical content knowledge. *Teachers and Teaching*, *11*(3), 273-292. https://doi.org/10.1080/13450600500105502

Henseler, J., Ringle, C. M., & Sarstedt, M. (2015). A new criterion for assessing discriminant validity in variance-based structural equation modeling. *Journal of Academy of Marketing Science*, *42*(1), 115-135

Henseler, J., Ringle, C. M., & Sinkovics, R. R. (2009). The use of partial least squares path modeling in international marketing, *Advances in International Marketing*, *20*, 277-320.

Hill, H. C., Ball, D. L., & Schilling, S. G. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. *Journal for research in mathematics education*, 372-400.

Hughes, J. E. (2003). Toward a model of teachers' technology learning. *Action in teacher education*, *24*(4), 10-17. https://doi.org/10.1080/01626620.2003.10463274

Joo, J. (2020). Customers' psychological ownership toward the third place. *Service Business*, *14*(3), 333-360. https://doi.org/10.1007/s11628-020-00418-5

Kante, M., Chepken, C., & Oboko, R. (2018). Partial least square structural equation modelling's use in information systems: An updated guideline of practices in exploratory settings. *Journal of Research & Innovation*, *6*(1). 49-67.

Kaufman, L., & Rousseeuw, P. J. (2009). *Finding groups in data: an introduction to cluster analysis*. John Wiley & Sons.

Kline, R. B. (2011). *Principles and practice of structural equation modeling* (93rd ed.), Guilford Press.

Lee, Y., Capraro, R. M., & Capraro, M. M. (2018). Mathematics teachers' subject matter knowledge and pedagogical content knowledge in problem posing. *International Electronic Journal of Mathematics Education*, *13*(2), 75-90. https://doi.org/10.12973/iejme/2698

Magnusson, S., Krajcik, J., & Borko, H. (1999). Nature, sources, and development of pedagogical content knowledge for science teaching. In J. Gess-Newsome & N. G. Leaderman (Eds.). *Examining pedagogical content knowledge* (pp. 95-132). Springer. https://doi.org/10.1007/0-306-47217-1_4

Norton, S. (2018). Middle School Pre-Service Teachers' Mathematics Content Knowledge and Mathematical Pedagogy Content Knowledge: Assessing and Relating. In J. Hunter, P. Perger & L. Darraph (Eds.), *Proceedings of the 41st annual conference of the Mathematics Education Research Group of Australasia* (pp. 599-606). MERGA.

Petrescu-Mag, R. M., Rastegari Kopaei, H., & Petrescu, D. C. (2022). Consumers' acceptance of the first novel insect food approved in the European Union: Predictors of yellow mealworm chips consumption. *Food Science & Nutrition*, *10*(3), 846-862. https://doi.org/10.1002/fsn3.2716

Sarstedt, M., & Mooi, E. (2014). *A concise guide to market research. The Process, Data, and, Methods Using IBM SPSS Statistics*. Springer.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational researcher*, *15*(2), 4-14.

Shulman, L. S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review*, *57*(1), 1-23.

Sintema, E. J., & Marbán, J. M. (2020). Pre-service secondary teachers' mathematical pedagogical content knowledge self-concept related to their content knowledge of functions and students. *International Electronic Journal of Mathematics Education, 15*(3), em0598. https://doi.org/10.29333/iejme/8327

Teo T. S., Srivastava, S. C., & Jiang, L. (2008). Trust and electronic government success: an empirical study. *Journal of Management Information Systems*, *25*(3), 99-132.

Teo, T. (2010). A path analysis of pre-service teachers' attitudes to computer use: applying and extending the technology acceptance model in an educational context. *Interactive Learning Environments*, *18*(1), 65-79.

Thompson, R. (2018). Pedagogical Content Knowledge in a Mathematics Adolescent Education Master of Arts Program: A Case Study. In M Harford, S. M. Uzzo, S. B. Graves, E. Shay & R. Thompson (Eds.). *Pedagogical Content Knowledge in STEM: research to* Practice (pp. 145-155). Springer.

Tkaczynski, A. (2017). Segmentation using two-step cluster analysis. *Segmentation in social marketing: Process, methods and application*, 109-125. https://doi.org/10.1007/978-981-10-1835-0_8

Tröbst, S., Kleickmann, T., Depaepe, F., Heinze, A., & Kunter, M. (2019). Effects of instruction on pedagogical content knowledge about fractions in sixth-grade mathematics on content knowledge and pedagogical knowledge. *Unterrichtswissenschaft*, *47*(1), 79-97. https://doi.org/10.1007/s42010-019-00041-y

Verma, J. P. (2013). *Data analysis in management with SPSS software*. Springer.

Appendix

Scale for measuring Zambian pre-service mathematics teachers' perceptions related to their pedagogical content knowledge

Programme: ………………………………………………… **Grade:…….**

Gender:………….. **Age:………..**

University/College:**……………………………………………………………….**

Instructions

Indicate with a Tick (✓) in the corresponding box the option you believe best defines your level of competence with respect to each of the skills listed:

		Never	**Rarely**	**Undecided**	**Usually**	**Always**
	Knowledge of Teaching Strategies (KTS)	1	2	3	4	5
1	I can design appropriate activities to present mathematical concepts.					
2	I can relate mathematical concepts to daily life in instruction.					
3	I can use analogies to mathematical concepts in instruction.					
	Knowledge of Mathematical Language and Symbols (KMLS)	1	2	3	4	5
4	I can use mathematical language properly when presenting mathematical concepts.					
5	I can use mathematical symbols properly.					
	Knowledge of Misconceptions (KM)	1	2	3	4	5
6	I can anticipate students' possible difficulties about a topic.					
7	I know students' possible misconceptions about a topic.					
8	I can design activities that will not cause students to develop					

	misconceptions about the topic.					
	Knowledge of Learners (KL)	1	2	3	4	5
9	I know students' prior knowledge about a topic					
10	I can choose appropriate examples for students' developmental levels in my lessons.					
	Knowledge of Curriculum (KC)	1	2	3	4	5
11	I have knowledge about the purposes of the mathematics curriculum.					
12	I can design a lesson plan for a topic.					
13	I plan my lessons so as to relate the purposes of the mathematics curriculum with students' needs.					
14	When designing my lesson plans, I consider the goals of the topic.					
15	I can use the assessment tools presented in the mathematics curriculum.					
16	I can evaluate the effectiveness of the activities I use in the class for students' conceptual understanding.					
17	I can draw on the results of my evaluations in designing and adjusting the instruction.					