Influence of Ductile Damage Evolution on the Collapse Load of Frames

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Abstract

In this note we analyze the influence of four damage models on the collapse load of a structure. The models considered here have been developed using the hypothesis based on the concept of effective stress and the principle of strain equivalence and they were proposed by Lemaitre and Chaboche, Wang, Chandrakanth and Bonora. The differences between them consist mainly in the form of the dissipative potential from which the kinetic law of damage is derived, and also in the assumptions made about some parameters of the material.

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1 Introduction

Continuum Damage Mechanics (CDM) is a new approach through which the material degradation can be quantified as a measurable parameter called the *damage variable*. It is considered as an internal variable in the framework of thermodynamics, and it is a measure of the degradation of the material. The constitutive model of a postulated damage parameter should be a function of the local stress, strain, strain rate, etc. Integrating over the loading history, the damage law will predict the material failure dynamically [1, 2]. The dissipation potential function (ϕ) is a scalar function of all the *observable variables* (elastic strain tensor ε_{ij} and temperature T; their associated variables are the stress tensor σ_{ij} and the entropy s) and the *internal variables* (accumulated plastic strain p and damage variable D; their associated variables are the increment of yield surface R and the damage strain energy release rate Y) as parameters. [2–4]. The differences between many ductil models are mainly based on the form of this potential.

2 Materials and methods

The analysis of frames considering damaged material can be done using the same concepts of equivalence of stress and strain as in Continuum Mechanics. Therefore, if we consider a 2D beam element of a frame between nodes 1 and 2 (Fig. 1), generalized damage, stresses and displacements can be defined respectively at the beam-ends, as

$$\{D\} = \{D_1, D_2\} \tag{1}$$

$$\{dF\} = \{dN_{x1}, dV_{y1}, dM_{z1}, dN_{x2}, dV_{y2}, dM_{z2}\} \tag{2}$$

$$\{du^{ep}\} = \left\{u_{x1}^{ep}, v_{y1}^{ep}, \theta_1^{ep}, u_{x2}^{ep}, v_{y2}^{ep}, \theta_2^{ep}\right\} \tag{3}$$

A constitutive model for the 2D beam element can be defined as the set of equations that relates the generalized stress with the history of generalized displacements. For the elastic case, considering that the variation of the elastoplastic displacement du^{ep} at the beam-ends can be split into its elastic (du^e) and its plastic (du^p) component, in a vectorial form $\{du^{ep}\} = \{du^e\} + \{du^p\}$

$$\{dF\} = [K]\{du^e\} = [K](\{du^{ep}\} - \{du^p\}) \tag{4}$$

where $[K]$ is the elastic stiffness matrix for the 2D beam element.

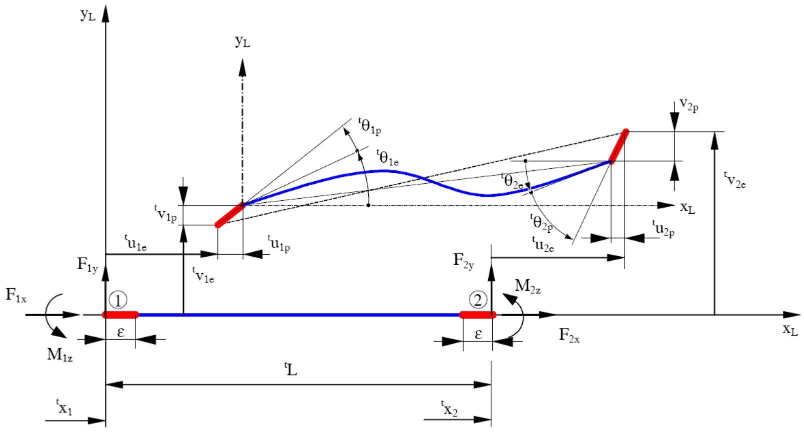


Figure 1: Beam element with plasticity and damage at its ends

The variation of plastic displacement $\{du^p\}$, taking into account the laws in the case of associated flow, can be expressed through

$$\{du^p\} = \{d\lambda\} \left\{ \frac{dZ}{dF} \right\} \tag{5}$$

where Z is the yield function for the beam element and $\{d\lambda\}$ is a 2×1 column vector of so-called plastic multipliers $d\lambda_1, d\lambda_2$ that measure the total plastic flow of the beam-ends.

The classic CDM formulation from Chaboche and Lemaitre, in the form specifically related to damage evolution, can be generically expressed as

$$\{dD\} = \{d\lambda\} \left\{ \frac{-\partial\phi*}{\partial Y} \right\} \qquad \text{and} \qquad \dot{p} = \frac{\dot{\lambda}}{(1-D)} \tag{6}$$

Equation (6), the kinetic law of damage evolution, shows the coupling between the damage rate and the effective accumulated plastic strain rate $\dot{p}$ by means of the plastic multiplier. ϕ^* is the damage dissipation potential.

From the plastic consistency condition, we can write

$$\dot{Z} = \left\{ \frac{\partial Z}{\partial F} \right\} \{dF\} + \left\{ \frac{\partial Z}{\partial D} \right\} \{dD\} = 0 \tag{7}$$

Substituting Eqs. (4)-(6) into Eq. (7), isolating $\{d\lambda\}$ and substituting it in Eqs. (6) and Eq. (4)

$$\{dF\} = [K] \left[1 - \frac{[K] \left\{ \frac{\partial Z}{\partial F} \right\} \left\{ \frac{\partial Z}{\partial F} \right\}}{\left\{ \frac{\partial Z}{\partial F} \right\} [K] \left\{ \frac{\partial Z}{\partial F} \right\} + \left\{ \frac{\partial Z}{\partial D} \right\} \left\{ \frac{\partial \phi*}{\partial Y} \right\}} \right] \{du^{ep}\} = [K^{ep}]\{du^{ep}\} \tag{8}$$

where $[K^{ep}]$ is the elastoplastic degradation stiffness matrix for the 2D beam element.

To determine the elastoplastic degradation stiffness matrix, it is necessary to evaluate the potential derivate with respect to the damage strain energy Y.

$$Y = -\frac{\sigma_{eq}^2}{2E(1-D)^2} f\left(\frac{\sigma_m}{\sigma_{eq}}\right) \quad \text{where} \quad f\left(\frac{\sigma_m}{\sigma_{eq}}\right) = \left[\frac{2}{3}(1+\nu) + 3(1-2\nu)\left(\frac{\sigma_m}{\sigma_{eq}}\right)^2 \right] \tag{9}$$

where σ_m is the hydrostatic stress, σ_{eq} is the von Mises equivalent stress, ν is the Poisson's ratio, E is the Young's modulus.

The next step consists in deriving the damage dissipation potential (Table 1) with respect to Y for obtaining the damage evolution law. Now we describe the procedure for Lemaitre's model [5].

For a ductile material, the effective equivalent von Mises stress can be written as a function of the accumulated plastic strain, using Ramberg-Osgood power law, as follows

$$\frac{\sigma_{eq}}{1-D} = \kappa p^n, \qquad p^n = \frac{\sigma_{eq}}{\kappa(1-D)} \qquad \text{or} \qquad \sigma_{eq} = \kappa p^n (1-D) \tag{10}$$

Table 1: Damage evolution law for different approaches

Bonora [3]	$\phi^* = \left[\frac{1}{2}\left(-\frac{Y}{S_o}\right)^2 \frac{S_o}{1-D}\right] \frac{(D_{cr}-D)^{\alpha_B - 1/\alpha_B}}{p^{(2+n)/n}}$
Lemaitre [5]	$\phi^* = \left[\frac{1}{2}\left(-\frac{Y}{S_L}\right)^2 \frac{S_L}{1-D}\right]$
Wang [1]	$\phi^* = \left[\frac{1}{2}\left(-\frac{Y}{S_W}\right)^2 \frac{S_W}{1-D}\right] \frac{(p_{cr}-p)^{\alpha_W - 1}}{p^{2n}}$
Chandrakanth [2]	$\phi^* = \left[\frac{1}{2}\left(-\frac{Y}{S_C}\right)^2 \frac{S_C}{1-D}\right] \frac{1}{D^{\alpha_C/n} \cdot p^{2/n}}$

D_{cr} and p_{cr} are the damage and deformation at failure initiation, respectively, the terms $S_o, S_L, S_W, S_C, \alpha_B, \alpha_W, \alpha_C$ are material constants, and n is the hardening constant of the material.

where κ is a material constant. Then, substituting Eqs. (9) and (10) into Lemaitre's damage evolution law (Table 1), we get

$$\frac{\partial \phi^*}{\partial Y} = -\left[\frac{\kappa^2}{2ES_L} f\left(\frac{\sigma_m}{\sigma_{eq}}\right)\right] \frac{p^{2n}}{1-D} = -[B_0]\frac{p^{2n}}{1-D} \tag{11}$$

In the case of proportional loading, the ratio σ_m/σ_{eq} can be considered as constant with respect to time so, for simplicity, the term $\left[\frac{\kappa^2}{2ES_L} f\left(\frac{\sigma_m}{\sigma_{eq}}\right)\right]$ is renamed as $[B_0]$.

Substituting Eq. (11) in the damage evolution law given in Eq. (6), we obtain

$$\dot{D} = [B_0]p^{2n}\dot{p} \qquad \text{or} \qquad \frac{dD}{dp} = [B_0]p^{2n} \tag{12}$$

Then, we integrate Eq. (12) between the initial condition $D = D_o$, and $D = D_{cr}$. The damage process remains inactivated (i.e. $D = D_o$) until the effective accumulated plastic strain p reaches a threshold strain p_{th} (i.e. $dD = 0$ and $D = 0$ or $D = D_o$). When $p = p_{th}$, nucleation is the

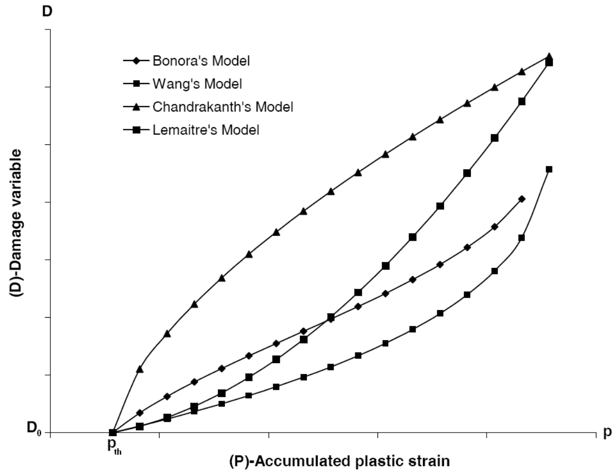


Figure 2: Damage (D) vs accumulated plastic strain (p)

dominating void growth state [3]. When $D = D_{cr}$, coalescence dominates the void growth process and the effective accumulated plastic strain p reaches the critical value p_{cr} for which failure occurs.

$$D_{cr} - D_o = [B_0]\left(\frac{p_{cr}^{2n+1} - p_{th}^{2n+1}}{2n+1}\right) \qquad \text{and} \qquad D_{cr} - D = [B_0]\left(\frac{p_{cr}^{2n+1} - p^{2n+1}}{2n+1}\right) \tag{13}$$

Eliminating $[B_0]$ in these equations, we obtain a general integrated evolution law for ductile plastic damage

$$p^{2n+1} = p_{cr}^{2n+1} - \left(\frac{D_{cr} - D}{D_{cr} - D_o}\right)\left(p_{cr}^{2n+1} - p_{th}^{2n+1}\right) \tag{14}$$

Substituting the same term into Eq. (11) we obtain the derivative of the dissipation potential with respect to Y.

$$\frac{\partial \phi *}{\partial Y} = -\frac{p^{2n}}{1-D}\left(\frac{D_{cr} - D_o}{p_{cr}^{2n+1} - p_{th}^{2n+1}}\right)(2n+1) \tag{15}$$

Similar procedures are used for obtaining the expressions for other models (Table 2). Figure 2 shows the evolution law for ductile plastic damage of the models considered. The material coefficients are taken from reference [3].

The next step is to determine $\left\{\dfrac{\partial Z}{\partial F}\right\}$ and $\left\{\dfrac{\partial Z}{\partial D}\right\}$. It is necessary to define the yield function Z for the beam element in function of the stress and the damage of the material. For the following

Table 2: $\left\{\dfrac{\partial \phi *}{\partial Y}\right\}$ and ductile plastic damage evolution law for different models

Bonora $(PA_{MNVD})B$	$\dfrac{\partial \phi *}{\partial Y} = -\alpha \left(\dfrac{(D_{cr}-D_o)^{1/\alpha}}{\ln(p_{cr}/p_{th})}\right)\dfrac{(D_{cr}-D)^{\alpha-1/\alpha}}{(1-D)\,p}$ $D = D_o + (D_{cr}-D_o)\left[1-\left(1-\dfrac{\ln(p/p_{th})}{\ln(p_{cr}/p_{th})}\right)^{\alpha}\right]$ $p = e^A$, where e is the base of the neperian logarithm $A = \ln(p_{cr}) - \ln(p_{cr}/p_{th})\left(\dfrac{D_{cr}-D}{D_{cr}-D_o}\right)^{1/\alpha}$
Lemaitre $(PA_{MNVD})L$	$\dfrac{\partial \phi *}{\partial Y} = -\dfrac{p^{2n}}{1-D}\left(\dfrac{D_{cr}-D_o}{p_{cr}^{2n+1}-p_{th}^{2n+1}}\right)(2n+1)$ $D = D_{cr} - (D_{cr}-D_o)\left(\dfrac{p_{cr}^{2n+1}-p^{2n+1}}{p_{cr}^{2n+1}-p_{th}^{2n+1}}\right)$ $p^{2n+1} = p_{cr}^{2n+1} - \left(\dfrac{D_{cr}-D}{D_{cr}-D_o}\right)\left(p_{cr}^{2n+1}-p_{th}^{2n+1}\right)$
Wang $(PA_{MNVD})W$	$\dfrac{\partial \phi *}{\partial Y} = -\alpha\left(\dfrac{D_{cr}-D_o}{(p_{cr}-p_{th})^{\alpha}}\right)\dfrac{(p_{cr}-p)^{\alpha-1}}{1-D}$ $D = D_{cr} - (D_{cr}-D_o)\left(\dfrac{p_{cr}-p}{p_{cr}-p_{th}}\right)^{\alpha}$ $p = p_{cr} - (p_{cr}-p_{th})\left(\dfrac{D_{cr}-D}{D_{cr}-D_o}\right)^{1/\alpha}$
Chandrakanth $(PA_{MNVD})C$	$\dfrac{\partial \phi *}{\partial Y} = -\dfrac{1}{\alpha_n}\left(\dfrac{D_{cr}-D_o}{p_{cr}-p_{th}}\right)\dfrac{1}{D^{\alpha/n}(1-D)}$ $D = \left[D_o^{\alpha_n} + (D_{cr}^{\alpha_n}-D_o^{\alpha_n})\left(\dfrac{p-p_{th}}{p_{cr}-p_{th}}\right)\right]^{1/\alpha_n}$ $p = p_{th} + (p_{cr}-p_{th})\left(\dfrac{D^{\alpha_n}-D_o^{\alpha_n}}{D_{cr}^{\alpha_n}-D_o^{\alpha_n}}\right)$ $\alpha_n = \dfrac{\alpha}{n} + 1$

assumptions (material nonlinearity simulated by the formation of plastic zones of zero length at the beam-ends, effect of strain hardening not considered and rectangular cross sections $b \times h$) [6]

$$Z = \frac{|M_z|}{M_p} + \left(\frac{N_x}{N_p}\right)^2 \frac{1}{1-D} + \frac{1}{3}\left(\frac{V_y}{V_p}\right)^2 \frac{1}{(1-D)^3} - (1-D) = 0 \tag{16}$$

M_z, N_x and V_y are the stresses on the cross section of the beam, and M_p, N_p and V_p are the plastic bending moment, plastic axial force and plastic shear force, respectively, that cause the full yielding of the cross section of the beam.

Considering the yielding function of Eq. (16), we get

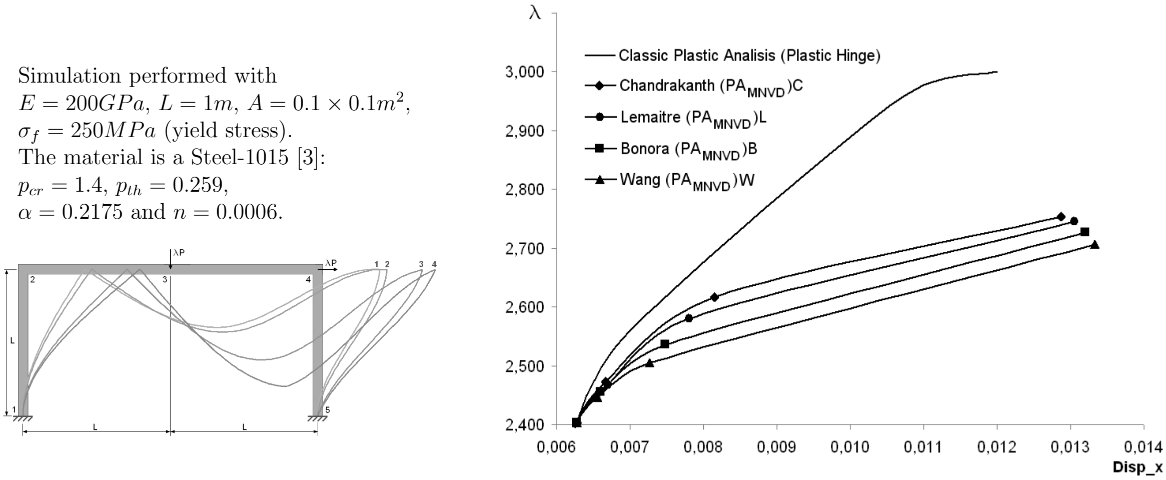
$$\left\{\frac{\partial Z}{\partial F}\right\} = \begin{bmatrix} A_1 & B_1 & \frac{1}{M_p} & 0 & 0 & 0 \\ 0 & 0 & 0 & A_2 & B_2 & \frac{1}{M_p} \end{bmatrix}^T \text{where } A_i = \frac{2N_{xi}}{N_p^2(1-D_i)}, B_i = \frac{2}{3}\frac{V_{yi}}{V_p^2(1-D_i)^3} \tag{17}$$

$$\left\{\frac{\partial Z}{\partial D}\right\} = \begin{bmatrix} C_1 & 0 \\ 0 & C_2 \end{bmatrix} \text{where } C_i = \left(\frac{N_{xi}}{N_p}\right)^2 \frac{1}{(1-D_i)^2} + \left(\frac{V_{yi}}{V_p}\right)^2 \frac{1}{(1-D_i)^4} + 1 \tag{18}$$

3 Results and result analysis

The accuracy of the model is verified by simulating one experiment for which data was available in the literature [7]. We can conclude that the model is simple but it still represents accurately the behavior of the structure.

After this validation, we apply the method to compare the collapse load of the 2D frame shown in Fig. 3(a). We consider yielding by bending moment and axial and shear forces. The loads are proportionally increased from zero to their collapse values, using an incremental and iterative procedure. Within each load increment, the equilibrium equations are solved by Newton-Raphson methods. Figure 3(a) shows the accumulated deformed shape of the frame for different load factors



(a) Frame example (b) Load factor vs horizontal displacement of node 4 (in m)

Figure 3: Test on a 2D frame

(λ) for the model $(PA_{MNVD})B$. In all the models, the sequence of the cross section yielding is 5, 4, 3 and 1, and the collapse load is $P = \lambda \cdot M_p$.

The response curves for the classic plastic analysis and the elastoplastic degradation analysis are shown in Fig. 3(b). They were obtained considering the material nonlinear effect and the elastoplastic damage model proposed, using the hypothesis of strain equivalence and dissipative potential from which the kinetic law of damage is derived. The curves of damage models are below the curve of the plastic analysis model due to the loss of stiffness of some sections: the load factor is lower and the displacements are higher. The effective accumulated plastic strain plays an important role on the damage evolution law. The evolution of the damage variable is much greater with Wang's model than with the other models. Therefore, the progressive reduction of material ductility is much higher.

4 Conclusions

The damage model shows a nonlinear variation with respect to plastic strain and it can be identified with a quantitative evaluation of the parameters D_{cr}, D_o, p_{th}, p_{cr} and also the hardening parameter, which defines the real stress-strain curve. The effects of axial and shear forces and bending moment have been taken into account for determining the yielding of the cross section of the beam.

The results lead to a more accurate prediction of the load that causes the yielding of the sections of the beam until the mechanism of collapse is formed. We can observe that the transmission of the load state among all the beams of a system is affected by the behavior of the plastic material and the accumulation of plastic strain, which leads to damage in the section and to the subsequent decreasing in the load-bearing capacity of the structure.

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