Operational Research (2026) 26:25
https://doi.org/10.1007/s12351-025-01018-9

ORIGINAL PAPER

Relationships between the deck of cards method and the proximity measures approach

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Received: 10 June 2024 / Revised: 5 September 2025 / Accepted: 23 December 2025

Abstract
In this paper, we propose a theoretical comparison of two types of value-based methods within the field of Multiple Criteria Decision Making/Aiding. Both methods make use of qualitative information to produce a value on an interval scale for each alternative, assessed on a set of criteria, for ranking or classification purposes. The two methods are known in the literature as the deck of cards and the one based on ordinal proximity measures. The deck of cards method allows managing the intensities of preferences in a qualitative way by making pairwise comparisons to produce a value for each alternative, while the ordinal proximity measures method allows managing the proximities between the terms of ordered qualitative scales in a pure ordinal way and produces a value for each alternative. This paper provides the mathematical background on the concept of closeness between objects of a linear order, which is common to both methods and the way of assigning values or scores to the terms of ordered qualitative scales. It is presented a proof that, under certain circumstances, these two methods are equivalent. An illustrative example shows how to build an interval scale with the two methods.

Keywords Multi-criteria analysis · Deck of cards method · Ordered qualitative scales · Ordinal proximity measures

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Published online: 02 February 2026

1 Introduction

The field of multiple criteria decision-making/aiding (MCDA) became more and more important over the last few decades. It helps decision-makers (DMs) navigate the complexities of real-world decision situations with wisdom and appears as a guiding light for DMs. MCDA is not merely an approach for dealing with complex problems, it also encompasses methodologies, methods, procedures, and algorithms, i.e., tools that empower DMs to deal with the multidimensional facets of decision situations.

Let us point out that an MCDA decision situation presupposes the existence of three fundamental elements: (1) a decision-maker or her/his representative and an analyst; (2) a set of alternatives; and (3) a set of criteria. In a subsequent step, three main expected results can be stated, each one leading to different problem statements (see Greco et al. 2016): *choice* (i.e., select the best or a small set of the best alternatives), *ranking* (i.e., rank the alternatives from the best to the worst, sometimes with the possibility of incomparabilities), and *classification* (i.e., assign the alternatives to pre-defined categories or classes).

In MCDA, there are mainly three types of methods (see Greco et al. 2016): value – or scoring-based methods, relational-based methods, and rule systems-based methods. The most popular type of methodology worldwide is the first one, i.e., when a value is assigned to each alternative under assessment (see Keeney and Raiffa 1976), for ranking or classification purposes.

The panoply of value-based methods is vast. Some require great cognitive effort from DMs while others make use of qualitative preference judgments, which greatly facilitate the DMs' task. Going from qualitative information to values attributed to the actions is not a simple task, it requires the design of meaningful methods. The value-based methods encourage a thoughtful exploration of trade-offs, enabling DMs to easily compare alternatives. This is one of the main reasons they are immensely popular in the field of MCDA. They have been confirmed to be very effective for dealing with real-world decision-making situations in a vast range of areas.

The origins of the deck of cards method date back to the early nineties, when Simos (1990a, 1990b) proposed a procedure for computing the weights of criteria (on a ratio scale) for relational-based methods, more precisely for outranking methods. This procedure was later improved by Figueira and Roy (2002). Siskos and Tsotsolas (2015) enhanced the deck of cards method by adding a robustness analysis procedure and surveying the use of this method in real-world applications. Pictet and Bollinger (2008) adapted the method to determine the value functions (interval scales) of an additive multi-attribute value theory (MAVT) model (see Keeney and Raiffa 1976). We note that this method can be used in the context of MAVT to build ratio scales for the weights and value or cardinal scales for the performance assessment criteria. A comprehensive explanation is provided in Figueira et al. (2023).

Bottero et al. (2018) improved the method and adapted it to build the utility functions and determine the capacities of the Choquet integral aggregation model.

The most important theoretical developments of the deck of cards method, with the possibility to identify and deal with inconsistent judgements, were proposed by Corrente et al. (2021). Two additional applications of the deck of cards methods with some minor theoretical developments can be seen in Dinis et al. (2023) and Figueira et al. (2023).

We note that the deck of cards method can be used to construct cardinal, ratio, or interval scales. In this case it is used on difference measures and the blank cards are used to model the less or more attractiveness between pairs of successive levels in the scale. The method can be used to compare all pairs of alternatives as in Corrente et al. (2021). In this sense, it is used to build a comparison pairwise table as in AHP, but the meaning of the blank cads is different from AHP, where the scale is absolute.

Ordinal proximity measures, introduced by García-Lapresta and Pérez-Román (2015), allow for the management the proximities between the terms of ordered qualitative scales in a pure ordinal way. Some applications of ordinal proximity measures to several decision-making problems can be found in García-Lapresta and Pérez-Román (2017, 2018), García-Lapresta et al. (2018, 2021, 2025), García-Lapresta and González del Pozo (2019, 2023), González del Pozo et al. (2020), González del Pozo and García-Lapresta (2021) and García-Lapresta and Marques Pereira (2022), among others.

In this paper, we are interested in dealing with the problem of how to build a cardinal value function from ordinal judgments. Indeed, the purpose of this article is to provide a mathematical comparison between the two methods and outline the common features as well as their practical relevance when both are applied to build cardinal scales within the range 0-100. It is targeted at readers with a theoretical interest in methods for constructing interval scales from ordinal data. Since the methods are also described in a simple way, they can also be of a particular interest to practitioners in real-world applications.

In this paper, we have introduced the notion of proximity measure, which generalizes the notion of ordinal proximity measure, by allowing different intensities of proximity between the terms of ordered qualitative scales. The reason for this is to bring the approach of ordinal proximity measures closer to the the deck of cards method. We have shown that, under certain assumptions, both approaches can be considered formally equivalent.

While both the deck of cards method (adapted to the context of this paper) and the new proximity measures approach seek to capture the intensities of proximity between terms on ordered qualitative scales, there are some differences in the way individuals display their perceptions. From a practical point of view, the deck of cards method requires individuals to insert an integer number of blank cards between consecutive terms of an ordered qualitative scale. In contrast, in the proximity measures approach, individuals display their perceptions by means of ordinal degrees of proximity, either directly or through sliders (as in García-Lapresta et al. (2025)).

The paper is organized as follows. Section 2 provides the mathematical background on the concept of closeness between objects of a linear order. Section 3 is devoted to the two methods, which assign values or scores to the terms of ordered qualitative scales. Finally, Sect. 4 provides some conclusions and outlines avenues for future research.

2 Closeness between the objects of a linear order

This section presents the deck of cards method, (ordinal) proximity measures and some relationships between them.

Consider a linear order $\mathcal{L} = \{l_1, l_2, \ldots, l_g\}$, with $g \geqslant 2$, arranged from the lowest to the highest: $l_1 \prec l_2 \prec \cdots \prec l_g$, where $\prec$ means "strictly less preferred than". The terms of $\mathcal{L}$ can be alternatives, criteria, linguistic terms of an ordered qualitative scale[1] (**OQS**), etc. Without loss of generality, in what follows we will say that $\mathcal{L}$ is an OQS.

2.1 The deck of cards method

In this subsection, we provide a brief presentation of the deck of cards method adapted to the context of this paper. For more details the reader can refer to Corrente et al. (2021).

Let $e_{rs} \geqslant 0$ be the number of blank cards inserted between l_r and l_s, whenever $r \neq s$. First, insert blank cards in between consecutive terms:

$$l_1 \;\; [e_{12}] \;\; l_2 \;\; \cdots \;\; l_r \;\; [e_{r\,(r+1)}] \;\; l_{r+1} \;\; \cdots \;\; l_{g-1} \;\; [e_{(g-1)\,g}] \;\; l_g.$$

By convention, $e_{rr} = -1$, for every $r \in \{1, 2, \ldots, g\}$.

The meaning of $e_{r\,(r+1)}$ is as follows:

1. $e_{r\,(r+1)} = 0$ means that the psychological distance between l_r and l_{r+1} is the minimal between two different terms, i.e., the maximal proximity between two different terms, equal to the value of the unit (see below).
2. $e_{r\,(r+1)} = 1$ means that the psychological distance between l_r and l_{r+1} is twice the unit.
3. $e_{r\,(r+1)} = 2$ means that the psychological distance between l_r and l_{r+1} is three times the unit, and so on.

In the next step, insert blank cards in between non-consecutive pairs of the scale levels. All this information can be arranged in a *comparison table* (see Table 1).

The following consistency condition plays a relevant role in some results of the paper (Propositions 2, 3 and 8). In particular, that condition determines the number of cards between each pair of non-consecutive terms of the OQS from the number of cards between consecutive terms. This fact makes it easier for agents to provide their perceptions of the proximity intensities between the objects considered, avoiding inconsistencies.

Condition 1 *(Corrente et al. 2021, p. 741) Given a comparison table, Condition 1 requires that the following equality must be verified:*

$$e_{rs} + e_{st} + 1 = e_{rt} \tag{1}$$

[1] For instance, {'very bad', 'bad', 'fair', 'good', 'very good', 'excellent'}

Table 1 Comparison table

	l_1	$\cdots$	l_r	$\cdots$	l_s	$\cdots$	l_t	$\cdots$	l_g
l_1	■								
l_r			■		e_{rs}		e_{rt}		
l_s					■		e_{st}		
l_t							■		
l_g									■

Table 2 Comparison table of Example 1

	l_1	l_2	l_3	l_4
l_1	■	1	4	5
l_2		■	2	3
l_3			■	0
l_4				■

for all $r, s, t \in \{1, 2, \ldots, g\}$ *such that* $r < s < t$.

Remark 1 If a comparison table satisfies Condition 1, then $e_{rt} > e_{rs}$ and $e_{rt} > e_{st}$, whenever $1 \leqslant r < s < t \leqslant g$: since $r < s < t$, we have $e_{rs} \geqslant 0$ and $e_{st} \geqslant 0$. Then, by Condition 1, we obtain $e_{rt} = e_{rs} + e_{st} + 1$ and, consequently, $e_{rt} > e_{rs}$ and $e_{rt} > e_{st}$.

Example 1 Given an OQS $\mathcal{L} = \{l_1, l_2, l_3, l_4\}$, consider that 1 card is inserted between l_1 and l_2, 2 cards are inserted between l_2 and l_3, and no cards are inserted between l_3 and l_4, i.e., $e_{12} = 1$, $e_{23} = 2$ and $e_{34} = 0$. If Condition 1 is assumed, then we obtain

$$e_{13} = e_{12} + e_{23} + 1 = 4$$
$$e_{24} = e_{23} + e_{34} + 1 = 3$$
$$e_{14} = e_{12} + e_{24} + 1 = e_{13} + e_{34} + 1 = 5.$$

All this information is collected in Table 2.

2.2 Proximity measures

The notion of ordinal proximity measure, introduced by García-Lapresta and Pérez-Román (2015), requires four conditions: exhaustiveness, symmetry, maximum proximity and monotonicity.

In order to analyze the relationships between the deck of cards method and ordinal proximity measures, we now introduce the more general notion of proximity measure, by removing exhaustiveness.

A proximity measure on $\mathcal{L}$ is a mapping that assigns an ordinal degree of proximity to each pair of terms of $\mathcal{L}$. The ordinal degrees of proximity belong to a linear order $\Delta = \{\delta_1, \delta_2, \ldots, \delta_h\}$, with $\delta_1 \succ \delta_2 \succ \cdots \succ \delta_h$. Note that the members of Δ are not numbers, but ordinal degrees of proximity: δ_1, for the maximum degree of proximity; δ_2, for the second degree of proximity; and so on until δ_h, for the minimum degree of proximity.

Definition 1 A *proximity measure* (**PM**) *on* $\mathcal{L}$ *with values in* Δ is a mapping $\pi : \mathcal{L} \times \mathcal{L} \longrightarrow \Delta$, where $\pi(l_r, l_s) = \pi_{rs}$ represents the degree of proximity between l_r and l_s, satisfying the following conditions:

1. *Symmetry*: $\pi_{sr} = \pi_{rs}$, for all $r, s \in \{1, 2, \ldots, g\}$.
2. *Maximum proximity*: $\pi_{rs} = \delta_1 \;\Leftrightarrow\; r = s$, for all $r, s \in \{1, 2, \ldots, g\}$.
3. *Monotonicity*: $\pi_{rs} \succ \pi_{rt}$ and $\pi_{st} \succ \pi_{rt}$, if $1 \leqslant r < s < t \leqslant g$.

Symmetry means that the ordinal degree of proximity between two terms does not depend on the order of comparison. Maximum proximity means that the maximum degree of proximity, δ_1, is only reached when comparing a term with itself. And monotonicity means that, given three different terms of the scale arranged from the lowest to the highest, the ordinal degree of proximity between the first term and the second one is greater than the ordinal degree of proximity between the first and the third; and the ordinal degree of proximity between the second term and the third one is greater than the ordinal degree of proximity between the first and the third.

We shall assume that the minimum degree of proximity, δ_h, has been used in the pairwise comparisons: $h = \max\big\{\rho(\pi_{rs}) \mid r, s \in \{1, 2, \ldots, g\}\big\}$, where $\rho(\delta_k) = k$.

Note that, if π is a PM, by monotonicity we have $\pi_{rs} = \delta_h$ if and only if $(r, s) = (1, g)$ or $(r, s) = (g, 1)$.

Definition 2 If $\pi : \mathcal{L} \times \mathcal{L} \longrightarrow \Delta$ is a PM that satisfies *exhaustiveness*: for every $\delta \in \Delta$, there exist $l_r, l_s \in \mathcal{L}$ such that $\delta = \pi_{rs}$, we say that π is an *ordinal proximity measure* (**OPM**) on $\mathcal{L}$.

Exhaustiveness means that all the ordinal degrees of proximity are used at least once. This fact prevents the use of proximity intensities: the degrees of proximity are purely ordinal.

Definition 3 A PM $\pi : \mathcal{L} \times \mathcal{L} \longrightarrow \Delta$ is *uniform* if $\pi_{r(r+1)} = \pi_{s(s+1)}$ for all $r, s \in \{1, 2, \ldots, g-1\}$, i.e., $\pi_{12} = \pi_{23} = \cdots = \pi_{(g-1)\,g}$, and *totally uniform* if $\pi_{r(r+t)} = \pi_{s(s+t)}$ for all $r, s, t \in \{1, 2, \ldots, g-1\}$ such that $r + t, s + t \leqslant g$.

Uniformity means that the ordinal degree of proximity between consecutive terms is always the same. In turn, total uniformity means that the ordinal degree of proximity between terms with the same jump is always the same.

Remark 2 For each $g \geqslant 3$, there is one and only one totally uniform OPM (see García-Lapresta et al. 2018, Remark 6) and it is metrizable (see García-Lapresta et al. 2018, Prop. 4).

If $\pi : \mathcal{L} \times \mathcal{L} \longrightarrow \Delta$ is the totally uniform OPM on $\mathcal{L}$, then $h = g$ and $\pi_{r\,(r+s)} = \delta_{s+1}$ for all $r, s \in \{1, 2, \ldots, g-1\}$ such that $r + s \leqslant g$. In particular, we have $\pi_{r\,(r+1)} = \delta_2$, $\pi_{r1} = \delta_r$ and $\pi_{rg} = \delta_{g-r+1}$, for every $r \in \{1, 2, \ldots, g-1\}$.

Every PM $\pi : \mathcal{L} \times \mathcal{L} \longrightarrow \Delta$ can be represented by a $g \times g$ symmetric matrix with coefficients in Δ: (π_{rs}), the *proximity matrix associated with* π.

Taking into account Definition 1, all the elements in the main diagonal are δ_1. Since (π_{rs}) is symmetric, only its upper half will be shown:

$$(\pi_{rs}) = \begin{pmatrix} \delta_1 & \pi_{12} & \pi_{13} & \cdots & \pi_{1(g-1)} & \pi_{1g} \\ & \delta_1 & \pi_{23} & \cdots & \pi_{2(g-1)} & \pi_{2g} \\ & & & \cdots & \cdots & \cdots \\ & & & & \delta_1 & \pi_{(g-1)g} \\ & & & & & \delta_1 \end{pmatrix}.$$

Remark 3 The only difference between PMs and OPMs is exhaustiveness, which is only required in OPMs (Def. 2). Without exhaustiveness, there may be gaps between the ordinal degrees of proximity actually used, something that is not allowed in OPMs. This fact is illustrated in the following example.

Consider an individual that compares the closeness between Lisbon, Madrid and Milan (they are in a straight line). If this individual knows the distances between those three cities, she/he can define a PM as $\pi(\text{ Lisbon },\text{ Madrid}) = \delta_2$, $\pi(\text{ Madrid },\text{ Milan}) = \delta_4$ and $\pi(\text{ Lisbon },\text{ Milan}) = \delta_5$. But if that individual only is confident that Madrid is closer to Lisbon than to Milan, but not the real distances, then she/he only can show the ordinal proximities through the OPM defined as $\pi(\text{ Lisbon },\text{ Madrid}) = \delta_2$, $\pi(\text{ Madrid },\text{ Milan}) = \delta_3$ and $\pi(\text{ Lisbon },\text{ Milan}) = \delta_4$.

A relevant family of OPMs is the one of metrizable OPMs (**MOPM**s). They were introduced by García-Lapresta et al. (2018) and they use linear metrics on OQSs.

An individual whose perceptions about the ordinal proximities between the terms of an OQS are represented by a MOPM behaves as if she/he had in mind a linear metric on the OQS. We note that the family of linear metrics is a proper subset of the family of metrics (see García-Lapresta et al. 2018, Prop. 1).

Definition 4 (García-Lapresta et al. 2018, Def. 2). A *linear metric* on $\mathcal{L}$ is a mapping $d : \mathcal{L} \times \mathcal{L} \longrightarrow \mathbb{R}$ satisfying the following conditions for all $r, s, t \in \{1, 2, \ldots, g\}$:

1. $d(l_r, l_s) \geqslant 0$.
2. $d(l_r, l_s) = 0 \Leftrightarrow l_r = l_s$.

3. $d(l_s, l_r) = d(l_r, l_s)$.
4. $d(l_r, l_t) = d(l_r, l_s) + d(l_s, l_t)$, if $r < s < t$.

It is worth noting that every linear metric on an OQS is determined in a univocal way from the distances between consecutive terms of the OQS (see García-Lapresta et al. 2018, Remark 1).

Definition 5 (García-Lapresta et al. 2018, Def. 3). An OPM $\pi : \mathcal{L} \times \mathcal{L} \longrightarrow \Delta$ is *metrizable* if there exists a linear metric $d : \mathcal{L} \times \mathcal{L} \longrightarrow \mathbb{R}$ such that $\pi_{rs} \succ \pi_{tu} \Leftrightarrow d(l_r, l_s) < d(l_t, l_u)$, for all $r, s, t, u \in \{1, 2, \ldots, g\}$. We say that π is generated by d.

Metrizable proximity measures (**MPM**s) can be defined in the same way of MOPMs, but considering PMs instead of OPMs. Note that for $g = 3, 4, 5, 6$, the number of MOPMs is 3, 25, 473 and 18,262, respectively. However, there are infinitely many MPMs in all cases.

2.3 Relationships between deck of cards and proximity measures

In this subsection we show that the deck of cards method is very related to the PM' approach.

In the following result, we justify that each comparison table generates a PM.

Proposition 1 *Given a comparison table, the mapping* $\pi : \mathcal{L} \times \mathcal{L} \longrightarrow \Delta$ *defined as* $\rho(\pi_{rs}) = e_{rs} + 2$ *is a PM on* $\mathcal{L}$.

Proof Symmetry is satisfied by definition.

Maximum proximity: $\pi_{rs} = \delta_1 \Leftrightarrow e_{rs} = \rho(\pi_{rs}) - 2 = -1 \Leftrightarrow r = s$.
Monotonicity: By Remark 1. □

Example 2 Applying Proposition 1 to the comparison table of Example 1, we obtain the PM associated with the following proximity matrix

$$A_{342} = \begin{pmatrix} \delta_1 & \delta_3 & \delta_6 & \delta_7 \\ & \delta_1 & \delta_4 & \delta_5 \\ & & \delta_1 & \delta_2 \\ & & & \delta_1 \end{pmatrix},$$

that is one the MOPMs appearing in (García-Lapresta et al. 2018, page 154).

In the following result, we justify that each comparison table satisfying Condition 1 generates a MOPM.

Proposition 2 *Given a comparison table satisfying Condition 1, the mapping* $d : \mathcal{L} \times \mathcal{L} \longrightarrow \mathbb{R}$ *defined as* $d(l_r, l_s) = e_{rs} + 1$ *is a linear metric on* $\mathcal{L}$ *and, consequently, it defines a MOPM on* $\mathcal{L}$.

Table 3 Comparison table of Example 3

	l_1	l_2	l_3	l_4	l_5
l_1	■	2	4	5	9
l_2		■	1	2	6
l_3			■	0	4
l_4				■	3
l_5					■

Table 4 Ordinal degrees of proximity in Example 3

e_{rs}	$d(l_r, l_s)$	π_{rs}	δ
0	1	π_{34}	δ_2
1	2	π_{23}	δ_3
2	3	π_{12} π_{24}	δ_4
3	4	π_{45}	δ_5
4	5	π_{13} π_{35}	δ_6
5	6	π_{14}	δ_7
6	7	π_{25}	δ_8
9	10	π_{15}	δ_9

Proof 1. $d(l_r, l_s) \geqslant 0$, by definition.
2. $d(l_r, l_s) = 0 \Leftrightarrow e_{rs} = -1 \Leftrightarrow l_r = l_s$, by definition.
3. $d(l_s, l_r) = d(l_r, l_s)$, by definition.
4. $d(l_r, l_t) = e_{rt} + 1 = e_{rs} + e_{st} + 2 = d(l_r, l_s) + d(l_s, l_t)$, if $r < s < t$.

Taking into account (García-Lapresta et al. 2018, Prop. 2), the linear metric, d, generates an MOPM $\pi : \mathcal{L} \times \mathcal{L} \longrightarrow \Delta$ on $\mathcal{L}$, being π exhaustive such that $\pi_{rs} \succ \pi_{tu} \Leftrightarrow d(l_r, l_s) < d(l_t, l_u)$, for all $r, s, t, u \in \{1, 2, \ldots, g\}$. □

In the following example, we show how Proposition 2 can be applied. Starting from a comparison table that satisfies Condition 1, we calculate the distances between the terms of the OQS provided by the linear metric that appears in Proposition 2, and how the MOPM is generated from these distances.

Example 3 Consider the first example of (Corrente et al. 2021, 5.1) with the comparison table shown in Table 3.

In Table 4 we show the distances between the terms of the OQS $\mathcal{L}$ and the ordinal degrees of proximity between them, arranged from lowest to highest number of blank cards.

Thus, Table 3 generates the MOPM associated with the following proximity matrix:

$$\begin{pmatrix} \delta_1 & \delta_4 & \delta_6 & \delta_7 & \delta_9 \\ & \delta_1 & \delta_3 & \delta_4 & \delta_8 \\ & & \delta_1 & \delta_2 & \delta_6 \\ & & & \delta_1 & \delta_5 \\ & & & & \delta_1 \end{pmatrix}.$$

In the following result, we justify that each PM that provides a linear metric from the ordinal degrees of proximity, in a specific way, generates a comparison table that satisfies Condition 1.

Proposition 3 *Given a PM* $\pi : \mathcal{L} \times \mathcal{L} \longrightarrow \Delta$*, if the mapping* $d : \mathcal{L} \times \mathcal{L} \longrightarrow \mathbb{R}$ *defined as* $d(l_r, l_s) = \rho(\pi_{rs}) - 1$ *is a linear metric on* $\mathcal{L}$*, then the comparison table defined as* $e_{rs} = \rho(\pi_{rs}) - 2$ *satisfies Condition 1.*

Proof $e_{rs} + e_{st} + 1 = d(l_r, l_s) - 1 + d(l_s, l_t) - 1 + 1 = d(l_r, l_t) - 1 = e_{rt}$, for all $r, s, t \in \{1, 2, \ldots, g\}$ such that $r < s < t$. □

Example 4 Consider the OQS $\mathcal{L} = \{l_1, l_2, l_3, l_4\}$ is equipped with the PM

$$\pi : \mathcal{L} \times \mathcal{L} \longrightarrow \Delta = \{\delta_1, \delta_2, \ldots, \delta_{14}\}$$

with associated proximity matrix

$$\begin{pmatrix} \delta_1 & \delta_5 & \delta_8 & \delta_{14} \\ & \delta_1 & \delta_4 & \delta_{10} \\ & & \delta_1 & \delta_7 \\ & & & \delta_1 \end{pmatrix}.$$

Note that π is not an OPM, because it is not exhaustive: $\delta_2, \delta_3, \delta_6, \delta_9, \delta_{11}, \delta_{12}, \delta_{13} \notin \pi(\mathcal{L} \times \mathcal{L})$.

It is easy to see that the mapping $d : \mathcal{L} \times \mathcal{L} \longrightarrow \mathbb{R}$ defined as $d(l_r, l_s) = \rho(\pi_{rs}) - 1$ is a linear metric on $\mathcal{L}$ and the comparison table shown in Table 5, defined as $e_{rs} = \rho(\pi_{rs}) - 2$, satisfies Condition 1.

Taking into account Condition 1, we now introduce a similar condition in the setting of proximity measures.

Condition 2 *A PM* $\pi : \mathcal{L} \times \mathcal{L} \longrightarrow \Delta$ *is consistent if* $\rho(\pi_{rt}) = \rho(\pi_{rs}) + \rho(\pi_{st}) - 1$*, for all* $r, s, t \in \{1, 2, \ldots, g\}$ *such that* $r < s < t$*.*

It is obvious that for $g = 3$, the 3 MOPMs are consistent. However, for $g = 4$, only 10 out of the 25 MOPMs are consistent: A_{222}, A'_{223}, A_{224}, A_{232}, A_{233}, A_{243}, A'_{322}, A_{332}, A_{342} and A_{422} (these proximity matrices can be found in García-Lapresta et al. 2018, pp. 154-155).

Definition 6 Given two PMs $\pi : \mathcal{L} \times \mathcal{L} \longrightarrow \Delta$ and $\pi' : \mathcal{L} \times \mathcal{L} \longrightarrow \Delta'$, we say that they are *ordinally equivalent* if $\pi_{rs} \succ \pi_{tu} \Leftrightarrow \pi'_{rs} \succ \pi'_{tu}$, for all $r, s, t, u \in \{1, 2, \ldots, g\}$.

Table 5 Comparison table of Example 4

	l_1	l_2	l_3	l_4
l_1	■	3	6	12
l_2		■	2	8
l_3			■	5
l_4				■

In the following result, we justify that each proximity measure has associated a unique equivalent metrizable ordinal proximity measure.

Proposition 4 *If $\pi : \mathcal{L} \times \mathcal{L} \longrightarrow \varDelta$ is a PM, then there exists a unique MOPM $\pi' : \mathcal{L} \times \mathcal{L} \longrightarrow \varDelta'$ that is ordinally equivalent to π. We say that π' is the MOPM associated with π.*

Proof Given a PM $\pi : \mathcal{L} \times \mathcal{L} \longrightarrow \Delta$, let $d : \mathcal{L} \times \mathcal{L} \longrightarrow \mathbb{R}$ defined as $d(l_r, l_s) = \rho(\pi_{rs}) - 1$. Taking into account (García-Lapresta et al. 2018, Remark 1), let $\pi' : \mathcal{L} \times \mathcal{L} \longrightarrow \Delta'$ be the only MOPM generated by $d(l_1, l_2), d(l_2, l_3), \dots, d(l_{g-1}, l_g)$. Then, we have

$$\pi'_{rs} \succ \pi'_{tu} \;\Leftrightarrow\; d(l_r, l_s) < d(l_t, l_u) \;\Leftrightarrow\; \rho(\pi_{rs}) - 1 < \rho(\pi_{tu}) - 1 \;\Leftrightarrow\; \pi_{rs} \succ \pi_{tu}. \quad \square$$

Note that if two PMs are ordinally equivalent, then they share the same associated MOPM. This fact is illustrated if the following example. □

Example 5 Consider $\mathcal{L} = \{l_1, l_2, l_3, l_4\}$ and the PMs $\pi : \mathcal{L} \times \mathcal{L} \longrightarrow \Delta = \{\delta_1, \delta_2, \dots, \delta_{10}\}$ and $\pi' : \mathcal{L} \times \mathcal{L} \longrightarrow \Delta' = \{\delta_1, \delta_2, \dots, \delta_8\}$ with associated proximity matrices

$$\begin{pmatrix} \delta_1 & \delta_4 & \delta_5 & \delta_{10} \\ & \delta_1 & \delta_2 & \delta_7 \\ & & \delta_1 & \delta_6 \\ & & & \delta_1 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} \delta_1 & \delta_3 & \delta_4 & \delta_8 \\ & \delta_1 & \delta_2 & \delta_6 \\ & & \delta_1 & \delta_5 \\ & & & \delta_1 \end{pmatrix},$$

respectively.

Note that none of these PMs are OPMs because π and π' are not exhaustive: $\delta_3, \delta_8, \delta_9 \notin \pi(\mathcal{L} \times \mathcal{L})$ and $\delta_7 \notin \pi'(\mathcal{L} \times \mathcal{L})$.

Both PMs are ordinally equivalent, because $\pi_{23} \succ \pi_{12} \succ \pi_{13} \succ \pi_{34} \succ \pi_{24} \succ \pi_{14}$ and $\pi'_{23} \succ \pi'_{12} \succ \pi'_{13} \succ \pi'_{34} \succ \pi'_{24} \succ \pi'_{14}$.

Taking into account the distances between consecutive terms, through $d(l_r, l_s) = \rho(\pi_{rs}) - 1$, we have that the MOPM associated with both π and π' is the MOPM $\pi'' : \mathcal{L} \times \mathcal{L} \longrightarrow \Delta'' = \{\delta_1, \delta_2, \dots, \delta_7\}$ with associated proximity matrix

$$\begin{pmatrix} \delta_1 & \delta_3 & \delta_4 & \delta_7 \\ & \delta_1 & \delta_2 & \delta_6 \\ & & \delta_1 & \delta_5 \\ & & & \delta_1 \end{pmatrix}.$$

3 Assigning scores to the terms of ordered qualitative scales

In this section we show how to determine a value or score with the deck of cards method as well as with a proximity measure method. We also present a theoretical proof of the similarities between the two methods.

3.1 The deck of cards method

In what follows we briefly present the computations of the deck of cards method to assign scores to the terms of an OQS $\mathcal{L} = \{l_1, l_2, \ldots, l_g\}$, in a similar way to Corrente et al. (2021):

1. Count the number of units between l_1 and l_g:

$$c = \sum_{s=1}^{g-1} \left(e_{s\,(s+1)} + 1 \right).$$

2. Consider $S_w^{dc}(l_1) = 0$.
3. Compute the value of each unit:

$$\alpha = \frac{100}{c}.$$

4. Compute the value of the remaining scale levels as follows by taking into account the (r, s) cells of the diagonal next to the main diagonal:

$$S_w^{dc}(l_r) = \alpha \cdot \sum_{s=1}^{r-1} \left(e_{s\,(s+1)} + 1 \right), \quad r = 2, \ldots, g.$$

Note that $S_w^{dc}(l_g) = 100$.

3.2 Proximity measures

García-Lapresta and González del Pozo (2023) introduce and analyze several scoring functions in the setting of OQSs.

Definition 7 (García-Lapresta and González del Pozo 2023) Given an OQS $\mathcal{L} = \{l_1, l_2, \ldots, l_g\}$, a *scoring function on* $\mathcal{L}$ is a function $S : \mathcal{L} \longrightarrow \mathbb{R}$ satisfying the following conditions for all $r, s \in \{1, 2, \ldots, g\}$:

1. $S(l_r) < S(l_s) \;\Leftrightarrow\; r < s$.
2. If π is the totally uniform OPM on $\mathcal{L}$, then there exists $\lambda > 0$ such that $S(l_r) = S(l_1) + (r - 1) \cdot \lambda$.

We now generalize some of the scoring functions appearing in García-Lapresta and González del Pozo (2023) from the framework of MOPMs to the one of PMs, with values within the range [0, 100].

The first scoring function, S_w, is based on the comparison between each linguistic term and the **w**orst linguistic term, l_1 (see Fig. 1).

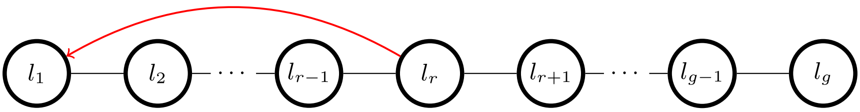

Fig. 1 Scoring function S_w (García-Lapresta and González del Pozo 2023, Fig. 2)

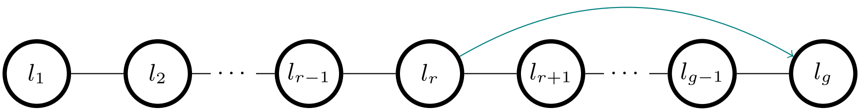

Fig. 2 Scoring function S_b (García-Lapresta and González del Pozo 2023, Fig. 1)

Proposition 5 *If* $\mathcal{L} = \{l_1, l_2, \ldots, l_g\}$ *is an OQS equipped with a PM* $\pi : \mathcal{L} \times \mathcal{L} \longrightarrow \Delta$*, then the function* $S_w : \mathcal{L} \longrightarrow \mathbb{R}$ *defined as follows*

$$S_w(l_r) = \frac{100}{h-1} \cdot (\rho(\pi_{r1}) - 1)$$

is a scoring function on $\mathcal{L}$*.*

Proof Obviously, $S_w(l_1) = 0$. By monotonicity, we have

$$r < s \;\Leftrightarrow\; \pi_{r1} \succ \pi_{s1} \;\Leftrightarrow\; \rho(\pi_{r1}) < \rho(\pi_{s1}).$$

Then, $S_w(l_r) < S_w(l_s) \;\Leftrightarrow\; r < s$ and, consequently, the first condition is satisfied.

If π is the totally uniform OPM on $\mathcal{L}$, taking into account Remark 2, we have

$$S_w(l_r) = \frac{100}{g-1} \cdot (\rho(\pi_{r1}) - 1) = (r-1) \cdot \frac{100}{g-1}.$$

Hence, the two conditions of scoring functions are satisfied. □

The second scoring function, S_b, is based on the comparison between each linguistic term and the **b**est linguistic term, l_g (see Fig. 2). Under a different approach to the present paper, these comparisons are in the basis of the group decision-making procedure introduced and analyzed by García-Lapresta and Pérez-Román (2018).

Proposition 6 *If* $\mathcal{L} = \{l_1, l_2, \ldots, l_g\}$ *is an OQS equipped with a PM* $\pi : \mathcal{L} \times \mathcal{L} \longrightarrow \Delta$*, then the function* $S_b : \mathcal{L} \longrightarrow \mathbb{R}$ *defined as*

$$S_b(l_r) = \frac{100}{h-1} \cdot (h - \rho(\pi_{rg}))$$

is a scoring function on $\mathcal{L}$*.*

Proof By monotonicity, we have

$$r < s \;\Leftrightarrow\; \pi_{sg} \succ \pi_{rg} \;\Leftrightarrow\; \rho(\pi_{sg}) < \rho(\pi_{rg}).$$

Then,

$$S_b(l_r) < S_b(l_s) \;\Leftrightarrow\; h - \rho(\pi_{rg}) < h - \rho(\pi_{sg}) \;\Leftrightarrow\; \rho(\pi_{sg}) < \rho(\pi_{rg}) \;\Leftrightarrow\; r < s.$$

Thus, the first condition is satisfied.

If π is the totally uniform OPM on $\mathcal{L}$, taking into account Remark 2, we have

$$S_b(l_r) = \frac{100}{g-1} \cdot (g - \rho(\pi_{rg})) = \frac{100}{g-1} \cdot (g - (g - r + 1)) = (r-1) \cdot \frac{100}{g-1}.$$

Hence, the two conditions of scoring functions are satisfied. □

The third scoring function, S_{bw}, is based on the comparison between each linguistic term and the **b**est and **w**orst linguistic terms, l_g and l_1 (see Fig. 3). This approach is related to the TOPSIS method (see Hwang and Yoon (1981)), discrete choice tasks (see Finn and Louviere (1992) and Marley and Louviere (2005)), voting systems (see García-Lapresta et al. (2010)) and the Best Worst Method (see Rezaei (2015)).

Proposition 7 *If* $\mathcal{L} = \{l_1, l_2, \ldots, l_g\}$ *is an OQS equipped with a PM* $\pi : \mathcal{L} \times \mathcal{L} \longrightarrow \Delta$*, then the function* $S_{bw} : \mathcal{L} \longrightarrow \mathbb{R}$ *defined as*

$$S_{bw}(l_r) = 50 \cdot \left(1 + \frac{\rho(\pi_{r1}) - \rho(\pi_{rg})}{h-1}\right)$$

is a scoring function on $\mathcal{L}$*.*

Proof First condition:

$\Rightarrow$) If $S_{bw}(l_r) < S_{bw}(l_s)$, then we have $\rho(\pi_{r1}) - \rho(\pi_{rg}) < \rho(\pi_{s1}) - \rho(\pi_{sg})$ and, consequently, $\rho(\pi_{r1}) + \rho(\pi_{sg}) < \rho(\pi_{s1}) + \rho(\pi_{rg})$. Obviously, $r \neq s$. Suppose, by way of contradiction, that $r > s$. By monotonicity, $\pi_{s1} \succ \pi_{r1}$ and $\pi_{rg} \succ \pi_{sg}$. Then, $\rho(\pi_{s1}) < \rho(\pi_{r1})$ and $\rho(\pi_{rg}) < rho(\pi_{sg})$. Consequently, we have $\rho(\pi_{s1}) + \rho(\pi_{rg}) < \rho(\pi_{r1}) + \rho(\pi_{sg})$, which is a contradiction.

$\Leftarrow$) If $r < s$, we have $\pi_{r1} \succ \pi_{s1}$ and $\pi_{sg} \succ \pi_{rg}$. Then, $\rho(\pi_{r1}) < \rho(\pi_{s1})$ and $\rho(\pi_{sg}) < \pi_{rg}$. Consequently, we have $\rho(\pi_{r1}) - \rho(\pi_{rg}) < \rho(\pi_{s1}) - \rho(\pi_{sg})$. Thus, $S_{bw}(l_r) < S_{bw}(l_s)$.

Second condition: Let π be the totally uniform OPM on $\mathcal{L}$. Note that $S_{bw}(l_1) = 0$. By Remark 2, we have

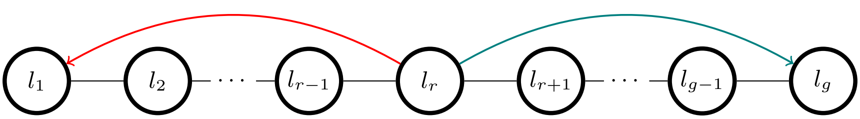

Fig. 3 Scoring function S_{bw} (García-Lapresta and González del Pozo 2023, Fig. 3)

Table 6 Scores

	l_1	l_2	l_3	l_4	l_5
S_w	0	10	50	80	100
S_b	0	10	30	70	100
S_{bw}	0	10	40	75	100

$$S_{bw}(l_r) = 50 \cdot \left(1 + \frac{\rho(\delta_r) - \rho(\delta_{g-r+1})}{g-1}\right) = 50 \cdot \left(1 + \frac{r-g+r-1}{g-1}\right) = (r-1) \cdot \frac{100}{g-1}.$$

Hence, the two conditions of scoring functions are satisfied. □

Note that $S_w(l_g) = S_b(l_g) = S_{bw}(l_g) = 100$ and, consequently, $S_w(l_r), S_b(l_r), S_{bw}(l_r) \in [0, 100]$ for every $r \in \{1, 2, \ldots, g\}$. We also note that $S_{bw}(l_r) = \dfrac{S_b(l_r) + S_w(l_r)}{2}$ for every $r \in \{1, 2, \ldots, g\}$.

Remark 4 The three scoring functions S_w, S_b and S_{bw} are different. For example, consider the OQS $\mathcal{L} = \{l_1, l_2, l_3, l_4, l_5\}$ equipped with the MOPM that has the following associated proximity matrix

$$\begin{pmatrix} \delta_1 & \delta_2 & \delta_6 & \delta_9 & \delta_{11} \\ & \delta_1 & \delta_3 & \delta_7 & \delta_{10} \\ & & \delta_1 & \delta_5 & \delta_8 \\ & & & \delta_1 & \delta_4 \\ & & & & \delta_1 \end{pmatrix}.$$

This MOPM can be visualized in Fig. 4.

Table 6 shows the scores obtained by the five terms of $\mathcal{L}$ according to the scoring functions S_w, S_b and S_{bw}.

Note that the three scoring functions provide different scores to l_3 and l_4. Since S_{bw} is the average of S_w and S_b, it can be considered more balanced than the other two.

In the following result we justify that, under some assumptions, the scores given by the deck of cards and the ones provided by the scoring functions S_w, S_b and S_{bw} coincide.

Proposition 8 *Given a comparison table satisfying Condition 1, if* $\pi : \mathcal{L} \times \mathcal{L} \longrightarrow \Delta$ *is the PM defined as* $\rho(\pi_{rs}) = e_{rs} + 2$, *then* $S_w^{dc} = S_w = S_b = S_{bw}$.

Proof It is easy to check that π is a PM on $\mathcal{L}$.

By Condition 1, we have $\sum_{s=1}^{r-1} \left(e_{s\,(s+1)} + 1\right) = e_{1r} + 1 = \rho(\pi_{1r}) - 1$. In particular,

$$c = \sum_{s=1}^{g-1} \left(e_{s\,(s+1)} + 1\right) = e_{1g} + 1 = \rho(\pi_{1g}) - 1 = h - 1.$$

Then,

$$S_w^{dc}(l_r) = \alpha \cdot \sum_{s=1}^{r-1} \big(e_{s\,(s+1)} + 1\big) = \frac{100}{h-1} \cdot (\rho(\pi_{r1}) - 1) = S_w(l_r).$$

On the other hand,

$$S_w^{dc}(l_r) = \alpha \cdot \sum_{s=1}^{r-1} \big(e_{s\,(s+1)} + 1\big) = \alpha \cdot \left(\sum_{s=1}^{g-1} \big(e_{s\,(s+1)} + 1\big) - \sum_{s=r}^{g-1} \big(e_{s\,(s+1)} + 1\big) \right) =$$

$$\alpha \cdot (h - 1 - (e_{rg} + 1)) = \alpha \cdot (h - 1 - (\rho(\pi_{rg}) - 1)) = \frac{100}{h-1} \cdot (h - \rho(\pi_{rg})) = S_b(l_r).$$

Since $S_w^{dc}(l_r) = S_w(l_r) = S_b(l_r)$, we also have $S_w^{dc}(l_r) = S_{bw}(l_r)$. □

The previous result shows the importance of Condition 1. Considering the way a PM is generated from a comparison table provided by Proposition 1, in Proposition 8 we have shown that Condition 1, introduced by Corrente et al. (2021), ensures that the four scores considered in this paper coincide.

4 Concluding remarks

The theoretical comparison presented in this paper between the deck of cards method and the proximity measures method represents a significant contribution to the field of Multiple Criteria Decision Analysis (MCDA). By focusing on the construction of an interval scale within the range of 0-100, we have demonstrated that, under certain fundamental assumptions, the scores or values derived from both methods are the same. This result underscores the robustness and reliability of these methods in quantifying preferences and facilitating decision-making processes within complex decision environments.

Moreover, our findings underscore the pivotal role of the deck of cards method and the proximity measures method within the MCDA framework, particularly in the context of building scoring functions. These methods serve as suitable tools for decision-makers seeking to systematically evaluate alternatives across multiple criteria and arrive at informed decisions that align with their preferences. In particular, the additive Multi-Attribute Value Theory (MAVT) model (Keeney and Raiffa 1976) stands to benefit significantly from the insights provided by these methods, offering a structured approach, which can be used into a coherent decision-making framework. An application of the deck of cards method to design a MAVT procedure can be seen in Figueira et al. (2023).

Looking ahead, several promising avenues for future research emerge from our investigation. First, there is a compelling need to explore the design and implementation of a proximity measure method for computing criteria weights, thereby establishing a coherent linkage with the weights obtained through the deck of cards method.

This endeavour promises to enhance the consistency of decision-making processes, fostering greater transparency and accountability.

Furthermore, the development of an additive MAVT model grounded in proximity measures represents a fertile area for future inquiry, offering new perspectives and methodologies for addressing complex decision problems characterized by diverse and often conflicting objectives. By leveraging the insights gleaned from the theoretical comparison between the deck of cards method and the proximity measures method, we can unlock new pathways for refinement within the realm of MCDA.

Additionally, our intention to explore the comparison of the two methods under conditions where more comprehensive information is available than that provided by the diagonal of the deck of cards method holds significant promise for advancing our understanding of their respective strengths and limitations. By delving deeper into the nuances of these methods and their interactions with varying degrees of informational granularity, we can glean valuable insights into their applicability across diverse decision contexts and stakeholder preferences.

In summary, the conclusions drawn from our theoretical comparison not only deepen our understanding of the deck of cards method and the proximity measures method but also catalyze further inquiry and innovation within the field of MCDA in those aspects where agents must show their perceptions about the closeness between objects in a non-numerical way.

Acknowledgments The authors are grateful to two anonymous reviewers for their useful comments and suggestions. The financial support of the research project PID2021-122506NB-I00 funded by MICIU/AEI /10.13039/501100011033 and ERDF, EU, is acknowledged. Jos\'e Rui Figueira has been financed by Portuguese funds through the FCT - Foundation for Science and Technology, I.P., under project UID/97/2025 (CEGIST).

Funding Open access funding provided by FEDER European Funds and the Junta de Castilla y León under the Research and Innovation Strategy for Smart Specialization (RIS3) of Castilla y León 2021-2027.

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