Effects of Initial Spacing on Height Development of Loblolly Pine

Clara Antón-Fernández, Harold E. Burkhart, Mike Strub, and Ralph L. Amateis

Abstract: The relationship between dominant height and age is the base of site index, the most widely used measure of site quality. In applying the site index concept, one typically assumes that height development is not affected by stand density or thinning treatment. This assumption has been challenged by recent studies on loblolly pine. A detailed data set with initial densities ranging from 6,730 to 750 trees/ha and covering ages 1 through 25 after plantation establishment was used to study and model the effect of initial spacing on height development of loblolly pine. Dominant height was found to be dependent on initial spacing. Height-age models are proposed that take into account the effect of spacing on average and dominant height. The differences among plantation densities are evident from age 6 and are consistent to the end of the 25-year period of study. Previous studies in other conifers have reported an early advantage in terms of height growth in denser stands that tend to disappear with age, producing a crossover of the growth trajectories. No evidence of this crossover effect in height was found. FOR. SCI. 57(3):201–211.

Keywords: site index, stand density, growth and yield, *Pinus taeda*

I N PLANNING FORESTRY OPERATIONS, reliable estimates of future growth and yield are critical. One of the main factors affecting stand dynamics, and, hence, defining the response of the stand to different silvicultural treatments and the outcomes of such interventions is site productivity. The most widely used method for assessing site quality, site index (SI), is based on the dominant height-age relationship.

In applying the SI concept, one typically assumes that height development is not affected by stand density or thinning treatment. Data from a number of studies support this notion for shade-intolerant conifers. For example, Pienaar and Shiver (1984) found no consistent effect between dominant height and spacing [1] in a study on slash pine (*Pinus elliottii* Engelm.). Harms et al. (1994, 2000) found no significant differences in dominant height in two spacing trials with loblolly pine, one in Hawaii and one in South Carolina. However, there is also experimental evidence that shows that dominant height is not independent of stand density (such as Curtis and Reukema 1970, MacFarlane et al. 2000, and Sharma et al. 2002a).

In studies of Western conifers SI corrections for stand density were proposed at least as early as the 1950s. For example, Lynch (1958) proposed a correction for SI for ponderosa pine (*Pinus ponderosa* Dougl. ex Laws.) because it was observed that dominant height in the stands decreased with increased stand density. Alexander et al. (1967) found similar results for lodgepole pine (*Pinus contorta* Dougl.) and also proposed a density correction for SI. Both studies used data from temporary plots to quantify the effect of density on dominant height.

If we consider the more abundant studies on the effect of spacing on average height, the results are even more inconsistent. Most authors have described increasing average height with increasing spacing (Harms and Lloyd 1981,

Zhang et al. 1996, MacFarlane et al. 2000, Sharma et al. 2002a), some have found negligible effects (Harms et al. 1994, 2000), and others observed even decreasing average height at very wide spacings (Pienaar and Shiver 1993). All of these studies have been basically descriptive, contributing to the body of evidence indicating that the assumption of independence is, at best, doubtful; and the investigations that went beyond description, modeling the effect of spacing on dominant height, assumed, rather than investigated, how spacing affected height (e.g., Sharma et al. 2002a).

The effect of initial spacing on height at early ages has also been the focus of several studies. Some of them have found a positive response to initial spacing in terms of height for juvenile conifer plantings (Scott et al. 1998, Knowe and Hibbs 1996, Woodruff et al. 2002). This positive response to density is reversed when competition begins, producing the crossover of the height curves, and, thus, it has been called the "crossover effect" (Scott et al. 1998). If a crossover effect in height was present and not accounted for, inaccuracies in SI estimation could result. Juvenile loblolly pine response to density has been investigated in several studies, but only a few of them have reported a potential crossover effect in average or dominant height. Among these studies, Land et al. (1991) noted higher, statistically significant, average heights in loblolly pine plantations at ages 3 and 5 in the 5×5 ft (2.32 m²/tree, 4,305 tph, where tph is trees/ha) spacing compared with the wider spacings of 8×8 ft (5.95 m²/tree) and 10×10 ft (9.29 m²/tree). Nance et al. (1983) also reported higher average heights for loblolly pine at age 7 in closer spacings in a trial where initial planting densities ranged from 6×6 ft (3.34 m²/tree) to 10×10 ft (9.29 m²/tree). Pienaar and Shiver (1993) observed higher average height for the plots with the two highest densities (800 tpa [4.04 m²/tree] and

Clara Antón-Fernández, This work was completed when a postdoctoral research associate at Virginia Tech. Current address Norwegian Forest and Landscape Institute, Postboks 115, 1431 Ås, Norway—Phone: +47 64948990; caf@skogoglandskap.no. Harold E. Burkhart, Virginia Tech—burkhart@vt.edu. Mike Strub, Weyerhaeuser Company—mike.strub@weyerhaeuser.com. Ralph L. Amateis, Virginia Tech—ralph@vt.edu.

1000 tpa [5.05 m^2/tree], where tpa is trees/acre) at ages 3, 5, and 8. Harms et al. (2000) found no significant effect of spacing on average stand height in a study of the growth and development of loblolly pine in a spacing trial in Hawaii that included early age (4–7 years) data, and Sharma et al. (2002a) found no statistical significance of spacing on average height until year 9.

These studies (Nance et al. 1983, Land et al. 1991, Pienaar and Shiver 1993, Harms et al. 2000, Sharma et al. 2002a) did not specifically test for the crossover effect, and all of them, except for Sharma et al. (2002a) and Land et al. (1991), lack detailed measurements during the initial years after plantation establishment. Some authors (Woodruff et al. 2002) have opined that this lack of detailed information might be one of the reasons that the crossover effect has not been detected yet in other species. It has also been argued that, because the early positive response to density is not expected, some authors might have not reported it even though it was observed.

We present a study of the effects of initial spacing on the dynamics of dominant and average height on loblolly pine from year 1 after plantation establishment to SI base age, year 25. This study extends that of MacFarlane et al. (2000), but differs from that study in that we have additional data that now extend to SI base age 25 years, and we present a height-age model that takes into account the effect of spacing on dominant height. The objectives of this analysis were to study, test, and model the effect of initial spacing on
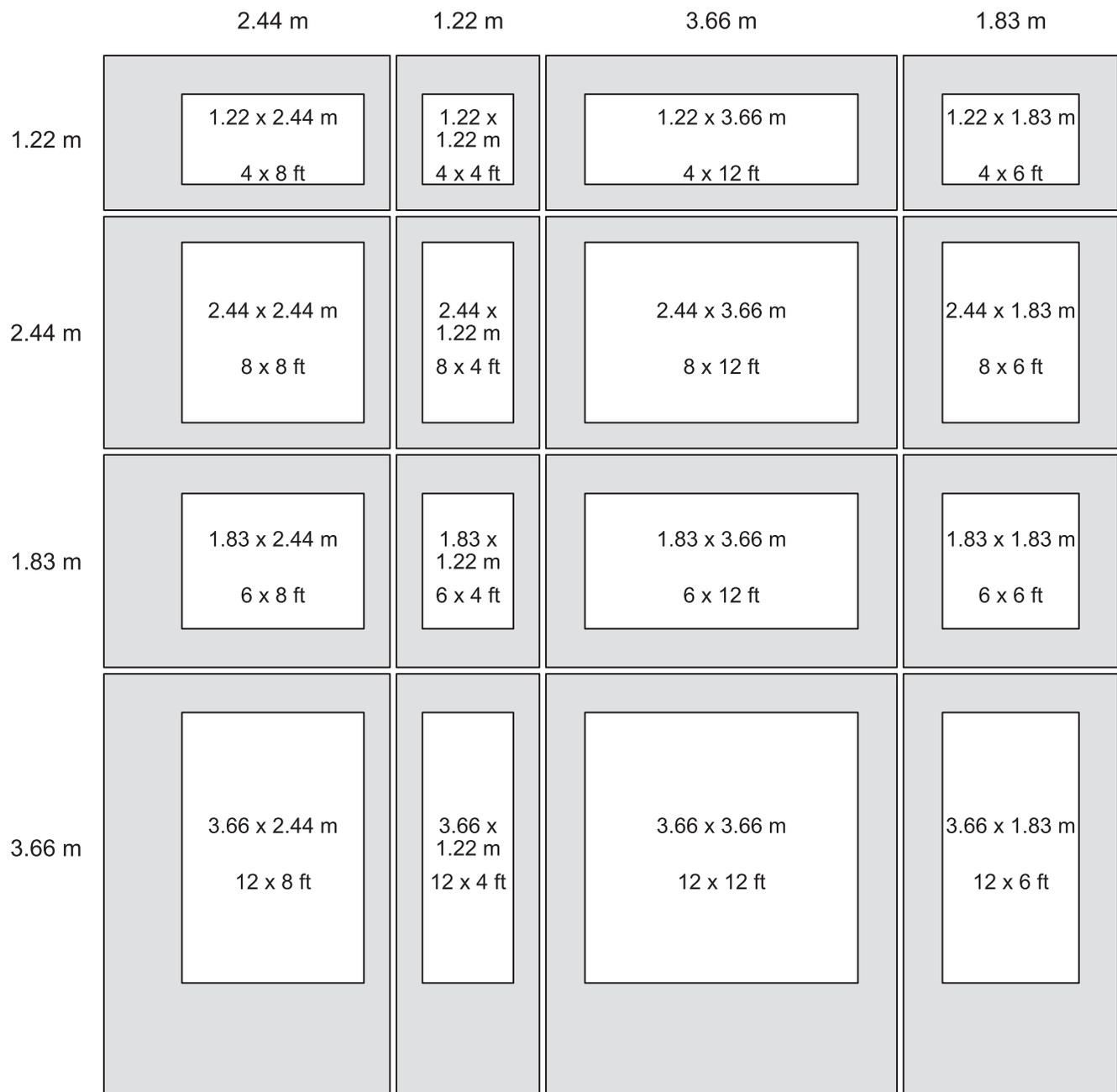

Figure 1. Design of a representative block where rows and columns are randomly assigned. The gray areas indicate the border zones.

average and dominant height, examine the effect of varying definitions of dominant height on modeling results, and test for the presence of a crossover effect at early stand stages in dominant height and average height development.

Data

Data for this research come from a spacing trial established by the Forest Modeling Research Cooperative at Virginia Tech. The study design (Figure 1) is nonsystematic, allowing the spacing to be varied in two dimensions on a factorial basis with a constant number of trees per plot (Lin and Morse 1975). Data were available from four locations (Figure 2), two in the Coastal Plain (C1 and C2) and two in the Piedmont (P1 and P2) physiographic regions (Amateis et al. 1988). At each of the four locations three replications (blocks) were initially available, each containing 16 initial planting designs (Table 1). Thus, a total of 192 plots were established (4 locations × 3 blocks × 16 planting designs), each including 49 trees. All measurements were taken in English units. Exact conversions were used for the analysis (Thompson and Taylor 2008), but for the sake of simplicity we only report two significant digits. Competing vegetation was chemically controlled up to age 3. Trees were measured annually for diameter and height until age 10 and annually for diameter and biannually for height thereafter. Diameter was measured at ground level from age 1 to age 5 and at breast height (4.5 ft, approximately 1.4 m) from age 5 to age 25. Several plots were damaged at different ages by southern pine beetle (*Dendroctonus frontalis* Zimm.), ice storms, or anthropogenic factors. A severe ice storm heavily damaged the plots at P1 at age 11 (Amateis and Burkhart 1996); data from those plots after the ice storm were not used. One of the plots at location

P2, spacing 4.46 m²/tree (6 × 8 ft), was abandoned at age 12 because of a southern pine beetle outbreak, and at age 19 the whole location was discarded because of thinning in the adjacent stand. At location C2 an ice storm after growing season 15 resulted in broken tops, which affected height. Data for age 16 were not used at that location for any analyses.

We have considered spacing as the initial growing space (area) available per tree, independent of the rectangularity (Table 1). The design of the study was intended to examine the effect of spacing and rectangularity on development of loblolly pine stands, and, thus, the degree of rectangularity varied among plots. Because the effect of rectangularity in this spacing trial has proven negligible for the main tree variables, including height (Sharma et al. 2002a, 2002b), we have modeled spacing effects based on the average space available per tree regardless of the configuration of that space.

Methodology

We used the Chapman-Richards equation as a base model to describe the height development of the stands. This equation is a well-known flexible growth function with biologically interpretable coefficients (Pienaar and Turnbull 1973) and is usually expressed in the integral form

$$H = \alpha(1 - e^{-\beta A})^{\gamma}, \qquad (1)$$

where H is stand height (m), α is the maximum height achievable (upper asymptote), β is the growth rate-related parameter, A is the stand age (years), and γ determines the shape of the curve near the origin.

Because of the hierarchical nature of the data and the advantages of using mixed-effects modeling on unbalanced and incomplete repeated data and its higher power to detect
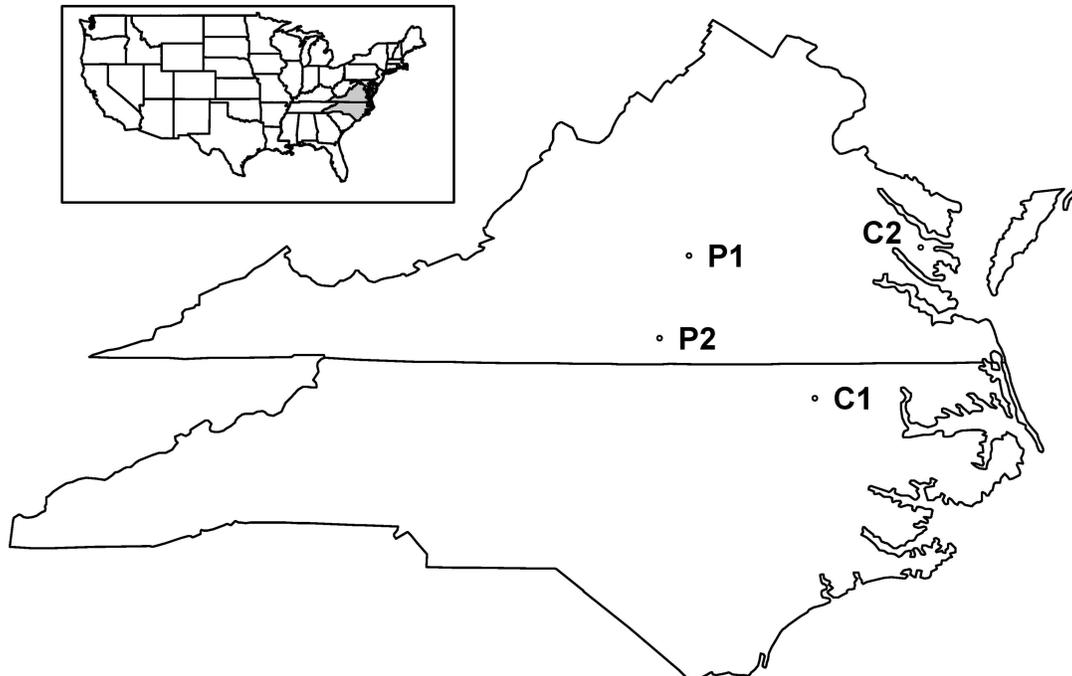

Figure 2. Geographical location of the spacing trial blocks (three blocks at each of the four locations: P1 is Piedmont site 1, P2 is Piedmont site 2, C1 is coastal plain site 1, and C2 is Coastal Plain site 2).

Table 1. Number of initial plots per spacing in each plot of each location and total number of trees used in the study at ages 1 and 25

Spacing (m²/tree)	Tree arrangements (m)	tph	ft²	tpa	No. plots	No. trees Age 1	No. trees Age 25
1.49	1.22 × 1.22	6,730	16	2,725	1	588	49
2.23	1.22 × 1.83; 1.83 × 1.22	4,485	24	1,815	2	1,176	196
2.97	1.22 × 2.44; 2.44 × 1.22	3,365	32	1,360	2	1,176	227
3.34	1.83 × 1.83	2,990	36	1,210	1	588	122
4.46	1.22 × 3.66, 3.66 × 1.22; 1.83 × 2.44, 2.44 × 1.83	2,240	48	910	4	2,352	575
5.95	2.44 × 2.44	1,680	64	680	1	588	179
6.69	1.83 × 3.66; 3.66 × 1.83	1,495	75	605	2	1,176	346
8.92	2.44 × 3.66; 3.66 × 2.44	1,120	96	455	2	1,176	417
13.38	3.66 × 2.44	747	144	305	1	588	230
Total					16	9,408	2,341

relationships (Zhao et al. 2005), we have used mixed-effects models to account for random effects related with the location of the plots. Because autocorrelation and heteroscedasticity commonly exist in forest growth and yield data from permanent plots (Lappi and Bailey 1988, Fang and Bailey 2001) and they were also present in our data set, we adopted a nonlinear mixed-effects model with random effects at the location, block, and plot level, allowing heteroscedastic and correlated within-group errors. All models were fitted using maximum likelihood, which allows direct comparison between models with different fixed effects (Pinheiro and Bates 2000). We tested different correlation structures and variance functions to model, respectively, the dependence among the within-plot errors and the variance heterogeneity. Models were compared using likelihood ratio tests (LRTs) when nested and the Akaike information criterion (Akaike 1974) and the Bayesian information criterion (Schwarz 1978) otherwise. The significance of the fixed-effects parameters was assessed with conditional t tests and F tests (Pinheiro and Bates 2000). The assumptions about the within-group errors and random effects were assessed graphically and analytically using LRTs and numerical summaries including intervals of the estimated parameters. We assessed the need and adequacy of autocorrelation functions using plots of the empirical autocorrelation function of the normalized residuals. To assess the assumption of constant variance of the within-group errors, plots of the residuals versus the fitted responses from the model were used. The assumption of normality for the within-group errors was assessed with normal probability plots of the residuals. The model assumptions on the random effects were assessed using several diagnostic plots. Normal probability plots of the estimated random effects were used to check marginal normality. Scatter plot matrices of the estimated random effects were used to check the assumptions of homogeneity of the random-effects covariance matrix.

Let y_{ijkl} be the height (average or dominant) of the lth measurement in the kth tree arrangements of the jth block

in the ith location. Then the general form of the Chapman-Richards nonlinear mixed effects model is

$$y_{ijkl} = (\phi_{1ijk})(1 - e^{-(\phi_{2ijk})A})^{\phi_{3ijk}} + \varepsilon_{ijkl}$$

$$\phi_{1ijk} = \alpha + b_{1i} + b_{1ij} + b_{1ijk}$$

$$\phi_{2ijk} = \beta + b_{2i} + b_{2ij} + b_{2ijk}$$

$$\phi_{3ijk} = \gamma + b_{3i} + b_{3ij} + b_{3ijk}$$

$$\begin{pmatrix} b_{1i} \\ b_{2i} \\ b_{3i} \end{pmatrix} \sim \mathrm{N}(\mathbf{0}, \boldsymbol{\psi}_1)$$

$$\begin{pmatrix} b_{1ij} \\ b_{2ij} \\ b_{3ij} \end{pmatrix} \sim \mathrm{N}(\mathbf{0}, \boldsymbol{\psi}_2)$$

$$\begin{pmatrix} b_{1ijk} \\ b_{2ijk} \\ b_{3ijk} \end{pmatrix} \sim \mathrm{N}(\mathbf{0}, \boldsymbol{\psi}_3)$$

$$\varepsilon_{ijkl} \sim \mathrm{N}(\mathbf{0}, \sigma^2 \boldsymbol{\Lambda}_{ijkl}) \qquad (2)$$

where b_{1i}, b_{2i}, and b_{3i} are random effects at the location level, b_{1ij}, b_{2ij}, and b_{3ij} are random effects at the block level, b_{1ijk}, b_{2ijk}, and b_{3ijk} are random effects at the plot (tree arrangement) level, $\boldsymbol{\Psi}_1$, $\boldsymbol{\Psi}_2$, and $\boldsymbol{\Psi}_3$ are the variance–covariance matrices for the random effects at the location, block, and plot level, respectively, and $\sigma^2 \boldsymbol{\Lambda}_{ijkl}$ jointly defines the variance–covariance matrix for the within-group errors, where σ^2 is a scaling factor and $\boldsymbol{\Lambda}_{ijkl}$ are positive-definite matrices parametrized by a fixed $\boldsymbol{\lambda}$ vector. The within-group errors ε_{ijkl} are assumed to be independent of the random effects, and the random effects at each level are assumed to be independent of each other. The $\boldsymbol{\Lambda}_{ijkl}$ matrices can be decomposed into two independent components: a variance structure component and a correlation structure component (Pinheiro and Bates 2000). The general variance function model for the within-group errors is $\mathrm{Var}(\varepsilon_{ijkl}|b_i, b_{ij}, b_{ijk}) = \sigma^2 g^2(\mu_{ijkl}, v_{ijkl}, \boldsymbol{\delta})$, where $\mu_{ijkl} = \mathrm{E}[y_{ijkl}|b_i, b_{ij}, b_{ijk}]$, v_{ijkl} is a vector of variance covariates, $\boldsymbol{\delta}$ is an unrestricted

vector of variance parameters, and $g(\cdot)$ is the variance function. Optionally, it is possible to specify one or several stratification variables for δ, with a different δ being used for each stratum. Common variance functions are the power variance model $\mathrm{Var}(\varepsilon_{ijkl}) = \sigma^2|v_{ijkl}|^{2\delta}$ and the exponential variance model $\mathrm{Var}(\varepsilon_{ijkl}) = \sigma^2 e^{2\delta}|v^{ijkl}|$.

Modeling Dominant and Average Height

Dominant height is commonly defined in the loblolly pine growing region of the United States as the average height of undamaged dominant and codominant trees (e.g., Amateis et al. 2006, Sharma et al. 2006). However, crown class information is sometimes not recorded. An alternative when crown class data are not available is to use the average height of undamaged trees with diameter greater than the quadratic mean diameter. In this study, crown class was available only for age 25. Hence, we used the average height of undamaged trees with diameter greater than the quadratic mean diameter (DHQ) as one of the definitions of dominant height considered. Because the definition of dominant height in the literature varies and definitions can affect results (see Sharma et al. 2002c, for a discussion on stability of different definitions of dominant height under thinning treatments and on performance of alternative definitions of dominant height for SI estimation), we repeated the analysis using three other definitions of dominant height. MacFarlane et al. (2000) used the average height of the seven tallest trees to define dominant height. In our data set some of the plots with the closest spacing had only 5 trees alive at age 25. Thus, we defined DH5 as the 5 tallest trees per plot. As a highly restrictive definition of dominant height we choose DH2, defined as the average height of the 2 tallest trees per plot. Table 2 shows a summary of average values and average number of trees used to calculate these definitions of dominant height. A common definition of dominant height is the average height of the 100 thickest trees/ha. When applied to the closest spacing plots (area of 72.84 m^2), this definition corresponds to 0.72 trees/plot. A definition that would result in averaging the height of the two thickest trees in the smallest plot corresponds to 275 trees/ha. Because the position of each tree in each plot was available, we defined a plot equal in size to the smallest plot (area of 72.84 m^2) and placed it at the center of each plot.

The height of the two largest trees in diameter was then used to calculate DHCA as a measure of dominant height with a constant number of trees per unit area. That is, DHCA is defined as the average height of the two thickest trees per plot, where plot in this case is defined as a 8.5m $\times$ 8.5m area centered in each original plot. For all definitions, diameter was at ground level for ages 1 to 4 and at breast height for ages 5 to 25. The fitting process followed conforms, in general, to the procedure described in detail by Zhao et al. (2005).

To model the effect of spacing on the height/age relationship our initial model included a spacing effect (S, in m^2/tree) in all three parameters: $\alpha = f(S)$, $\beta = f(S)$, and $\gamma = f(S)$. One of our objectives was to explore the effect of spacing on height, making as few assumptions as possible, so we considered different transformations and combinations of spacing for $f(S)$, including S, S^2, S^{-1}, $\ln(S)$, and $\sqrt{S}$, and the combinations of S with its transformations. The combination of S, $\sqrt{S}$ was found to fit the data best according to the log-likelihood.

We considered random effects at the location, block, and plot levels. We modeled the heteroscedasticity by fitting a power or exponential function with age as a variance covariate. Both functions, power and exponential, can model cases where the variance increases with the variance covariate. We modeled autocorrelation with mixed autoregressive moving average models (ARMAs) (Box et al. 1994). A full specification of the models for average height, DH2, DH5, and DHCA is given in the Appendix.

Testing the Crossover Effect

Preliminary graphical analysis of the data showed noticeable differences between the close and wide spacings starting at age 6, with closer spacings having lower dominant and average height than the wider spacings. Results from Sharma et al. (2002a) suggested statistically significant differences in height between spacings, with closer spacings having lower height than the wider spacings, starting at age 9. Hence, if present, the crossover would occur before age 10. The first step for examining the possibility of a crossover effect was to test the significance of spacing as an explanatory variable in the context of a linear mixed-effects model for ages 1 through 10, testing at each age

Table 2. Number of trees used to calculate the four definitions of dominant height by spacing and average values of average height AVG, and dominant heights DHQ, DH2, DH5, and DHCA for the different spacings at age 25

Spacing (m^2/tree)	No. trees					Average at age 25				
	DHQ[a]	DHQ[b]	DH2	DH5	DHCA	AVG	DHQ	DH2	DH5	DHCA
1.49	19	3	2	5	2	17.7	19.0	19.6	18.5	19.2
2.23	20	6	2	5	2	18.1	19.6	20.6	19.9	20.1
2.97	21	7	2	5	2	18.6	20.1	21.3	20.7	20.4
3.34	20	8	2	5	2	18.5	19.9	21.0	20.4	20.6
4.46	21	9	2	5	2	19.4	20.8	21.9	21.4	21.1
5.95	22	12	2	5	2	20.1	21.4	22.6	22.2	21.8
6.69	22	13	2	5	2	19.8	21.1	22.3	21.9	21.8
8.92	22	16	2	5	2	20.4	21.5	22.7	22.4	21.5
13.38	23	17	2	5	2	20.9	22.0	23.1	22.8	22.1

[a] Average number of trees per plot used to calculate DHQ for all ages.
[b] Average number of trees per plot used to calculate DHQ for age 25.

Table 3. Maximum likelihood fixed-effects parameter estimates and fit statistics for DHQ, DH2, DH5, and DHCA dominant height models and average height model

Parameter	DHQ		DH2		DH5		DHCA		Average height	
	Estimates	SE	Estimates	SE	Estimates	SE	Estimates	SE	Estimates	SE
α_0	20.8473[a]	1.3651	19.2388[a]	1.4867	13.4171[a]	2.0435	18.0439[a]	1.1571	17.4124[a]	1.0373
α_1	n.s.		−0.9550[a]	0.1703	−1.4032[a]	0.3558	n.s.		n.s.	
α_2	2.4685[a]	0.3544	6.7298[a]	0.8197	10.8118[a]	1.6323	3.9076[a]	0.3584	9.1438[a]	0.3434
β_0	0.0619[a]	0.0069	0.0671[a]	0.0028	0.0920[a]	0.0052	0.0823[a]	0.0038	0.0722[a]	0.0083
β_1	−0.0033[a]	0.0006	n.s.		0.0026[b]	0.0009	−0.0017[a]	0.0003	−0.0362[a]	0.0008
β_2	0.0135[a]	0.0038	n.s.		−0.0189[a]	0.0042	n.s.		0.0421[b]	0.0047
γ_0	1.2440[a]	0.1034	1.2113[a]	0.0706	1.4230[a]	0.0336	1.2804[a]	0.0743	1.3374[b]	0.1266
γ_1	−0.0445[a]	0.0121	−0.0200[b]	0.0084	n.s.		−0.0419[a]	0.0104	−0.4841[a]	0.0143
γ_2	0.2480[a]	0.0600	0.1438[a]	0.0408	n.s.		0.1976[a]	0.0462	0.8463[a]	0.0716

n.s., not significant.
[a] Significant at $\alpha = 0.001$.
[b] Significant at $\alpha = 0.05$.

separately and using the anova.lme (Wald test) and lme functions implemented in the nlme package (Pinheiro and Bates 2000). Initially, we considered random effects at the location, block, and plot levels. Preliminary analyses indicated that random effects were needed (according to LRTs) at the location and block levels only. Thus, the final models have the form

$$H_{Amijk} = \mu_{Am} + a_i + a_{ij} + \varepsilon_{Amijk}, \qquad (3)$$

where μ_{Am} is the average or dominant height for the mth spacing ($m = 1, …, 9$ spacings) at age A ($A = 1, …, 10$ years of age); a_i and a_{ij} are the random effects at the location ($i = 1$ [P1], 2 [P2], 3 [C1], 4 [C2]) and block level ($j = 1, …, 3$), respectively; $k = 1, …, n_{Am}$ plots with spacing m at age A; and ε_{Amijk} is the within-group error term where $\varepsilon_{Amijk} \sim N(0, \sigma^2)$. These mixed-effects analysis of variance tests provide insight into the age at which different spacings start to depart from a common curve.

The second step was to model dominant and average height for the first 10 years of stand development by selecting a set of transformations that would describe the data accurately and allow expression of the crossover effect, if present, namely A, A^2, and A, A^{-1}. The best model according to the Akaike information criterion and Bayesian information criterion was obtained using A, A^{-1}. Because dominant height for closer spacings starts to be distinctly shorter than in wider spacings around age 6 for DHQ, a crossover effect involving the closer spacings, if present, would occur before age 6. Hence, we tested the crossover effect in all spacings for ages 1–5 and ages 1–10 for spacings greater than 3 m^2/tree. The model fitted was

$$H_{ijkl} = \alpha_0 + \alpha_1 S + b_{0i} + b_{0ij} + (\beta_0 + \beta_1 S + b_{1i} + b_{1ij})A$$

$$+ (\beta_2 + \beta_3 S + b_{2i})A^{-1} + \varepsilon_{ijkl},$$

$$\begin{pmatrix} b_{0i} \\ b_{1i} \\ b_{2i} \end{pmatrix} \sim N(\mathbf{0}, \mathbf{\Psi}_1),$$

$$\begin{pmatrix} b_{0ij} \\ b_{1ij} \end{pmatrix} \sim N(\mathbf{0}, \mathbf{\Psi}_2), \qquad (4)$$

where b_{0i}, b_{1i}, b_{2i}, and b_{0ij}, b_{1ij} denote the random effects for location (i) and block (ij) levels and $\varepsilon_{ijkl} \sim N(0, \sigma^2 \Lambda_{ijkl})$.

We modeled the heteroscedasticity by fitting a power function with age as a variance covariate. Autocorrelation was modeled using an autoregressive model of order 1.

Regression analysis allows use of all of the measurement data and thus increases the power to detect the presence of a potential crossover effect. Because of the small range of years studied in this case, linear mixed-effects models (rather than nonlinear) were used to test the hypotheses and model the possible crossover effect.

For the analyses reported here, the open-source statistical environment R (R Development Core Team 2009) and the nlme package (Pinheiro and Bates 2000) were used.

Results
Modeling Dominant and Average Height

All parameters of the Chapman-Richards equation (α, β, γ) were affected by initial spacing ($\alpha = 0.05$) at the fixed-effects level, regardless of the definition of dominant height used (Table 3). Details are presented here for the DHQ model. A full specification of the models for DH2, DH5, DHCA, and average height is given in the Appendix. The final model for dominant height DHQ has the form

$$H_{ijkl} = (\alpha_0 + \alpha_2 S^{0.5} + b_{1i} + b_{1ij} + b_{1ijk})$$

$$\times (1 - e^{-(\beta_0 + \beta_1 S + \beta_2 S^{0.5} + b_{2i} + b_{2ij})A})^{\gamma_0 + \gamma_1 S + \gamma_2 S^{0.5} + b_{3i} + b_{3ijk}} + \varepsilon_{ijkl}$$

$$\begin{pmatrix} b_{1i} \\ b_{2i} \\ b_{3i} \end{pmatrix} \sim N(\mathbf{0}, \mathbf{\Psi}_1)$$

$$\begin{pmatrix} b_{1ij} \\ b_{2ij} \end{pmatrix} \sim N(\mathbf{0}, \mathbf{\Psi}_2)$$

$$\begin{pmatrix} b_{1ijk} \\ b_{3ijk} \end{pmatrix} \sim N(\mathbf{0}, \mathbf{\Psi}_3) \qquad (5)$$

where b_{1i}, b_{2i}, b_{3i}, b_{1ij}, b_{2ij}, and b_{1ijk}, b_{3ijk} denote the random effects at the location ($i = 1$[P1], 2[P2], 3[C1], 4[C2]), block ($j = 1, …, 3$), and plot ($k = 1, …, 16$) levels. The structure of the within-group error ε_{ijkl} is modeled for the dominant height models with an ARMA(1, 1), with autoregressive parameter estimate = 0.5373, moving average parameter = 0.1551, and a power variance model with age as
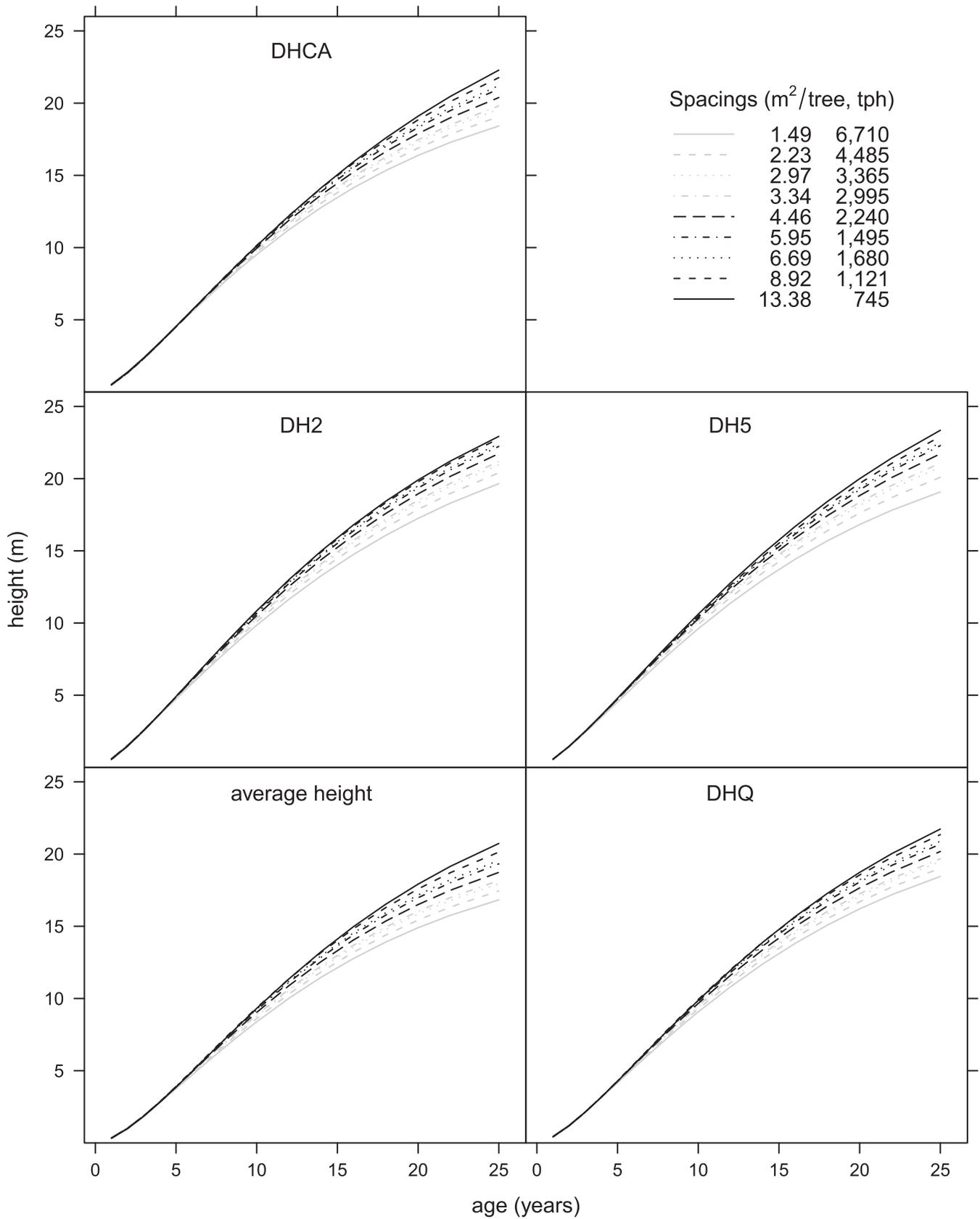
Figure 3. Fixed-effects (all random effects = 0) model predictions for all spacings included in the study for all definitions of dominant height and average height.

a variance covariate and parameter δ varying by location $\hat{\delta}_i$ = (0.1193, 0.0921, 0.0955, 0.0846). Estimates and standard errors of the fixed-effects parameters for the DHQ, DH2,

DH5, and DHCA dominant height models, and average height model are shown in Table 3.

The fixed-effects parameter estimates for the dominant

height (DHQ) model are given in Table 3, and the variance-covariance matrices for the random effects of the DHQ model are

$$\hat{\Psi}_1 = \begin{pmatrix} 2.1271 & 0 & 0 \\ 0 & 0.0082 & 0 \\ 0 & 0 & 0.1490 \end{pmatrix},$$

$$\hat{\Psi}_2 = \begin{pmatrix} 0.6511 & 0 \\ 0 & 0.0032 \end{pmatrix},$$

$$\hat{\Psi}_3 = \begin{pmatrix} 0.5172 & 0 \\ 0 & 0.0421 \end{pmatrix}. \tag{6}$$

All definitions of dominant height are affected by spacing, and differences among spacings increase with age for all definitions. DHQ and DH2 were affected by spacing to a lesser extent than DH5, DHCA, and average height (Figure 3), although at least two spacing covariates were statistically significant at $\alpha = 0.05$ for all the definitions of dominant height and for average height. The effect of spacing on dominant height tends to be larger at the closest spacings and to decrease at wider spacings (Figure 3).

Testing the Crossover Effect

Wald tests (Table 4) show at $\alpha = 0.05$ no difference in dominant height among different spacings until age 7 for DHQ and DH5, until age 6 for DH2, and until age 9 for DHCA. Wald F tests for the joint significance of spacing variables in different models that fit the data adequately and would be able to model the crossover effect, if present, indicate that there is no statistical significance ($\alpha = 0.05$) of the spacing variables in any of the proposed models (Table 5). It is worth noting that in a few instances in the spacing trial used here a crossover effect for DHQ was exhibited in certain locations and for certain combinations of spacings. However, there was no discernible consistent crossover effect for the trial overall.

Table 4. Wald test P values for the significance of the spacing effect in model 4 using the four definitions of dominant height and average height by age

| Age | Height | | | | |
	DHQ[a]	DH2[a]	DH5[a]	DHCA[a]	Average
1	0.348	0.910	0.906	0.306	0.499
2	0.835	0.913	0.917	0.313	0.932
3	0.856	0.656	0.691	0.098	0.900
4	0.831	0.875	0.849	0.915	0.888
5	0.687	0.391	0.516	0.902	0.664
6	0.074	0.041	0.074	0.705	0.115
7	0.003	0.011	0.004	0.114	0.003
8	<0.001	<0.001	<0.001	0.055	<0.001
9	<0.001	<0.001	<0.001	0.028	<0.001
10	<0.001	<0.001	<0.001	<0.001	<0.001

[a] DHQ, average height of undamaged trees with diameter larger than the quadratic mean diameter of the stand; DH2, average height of the 2 tallest trees per plot; DH5, average height of the 5 tallest trees per plot; DHCA, average height of the two thickest trees in a square plot of size 8.5 × 8.5 m, corresponding to 275 trees/ha.

Discussion and Conclusions

Previous reports on the effects of initial spacing on dominant height of even-aged loblolly pine have given diverse answers to the question of how spacing affects height development. Results have ranged from a positive or neutral to negative effect on height for increasing initial spacing. In the present study we used a detailed data set from a spacing trial in the southeastern United States to explore and model the effect of different spacings on dominant and average height from year 1 after plantation establishment to age 25. The data set encompasses a wide range of spacings and ages, which allowed us to study the effect of spacing as a continuum, rather than merely testing differences at several ages. Our results have shown that spacing has an obvious effect on dominant and average height that decreases at the wider spacings included in this study. The differences between dominant heights at different spacings are maintained and do not tend to disappear with age, which results in different dominant heights at index age.

Most recent studies on the effects of spacing on dominant height have reported increasing dominant height with increasing spacing. However, the range of responses across the literature varies from significant positive effects on dominant height development in loblolly pine plantations (Zhang et al. 1996) to cases of negative effects (e.g., Nance et al. 1983). The studies found in the literature on this topic are quite diverse in terms of range of spacings and ages considered, the interspecific competition control applied, the number of replicates, the size of the data set, and other design characteristics of the trials (e.g., randomization). Differences in the results induced by this variability in design, installation, and maintenance of trials are expected.

Different definitions of dominant height are sensitive to initial spacing to different degrees. For those definitions that are directly comparable, that is, DH2 and DH5, it is obvious that the more restrictive definition (DH2) is affected to a lesser degree by initial spacing (Figure 3). Our results also suggest that gains in dominant height accrue at a decreasing rate as initial spacing increases. The increase in dominant height seems to diminish rapidly after approximately 9 m^2/tree [2] (1,110 tph). This finding suggests an upper limit to the effect of spacing on dominant height. Spacings wider than 9 m^2/tree, are not common, even for research plots, and the results published with such spacings are scarce. The Virginia Department of Forestry established a study that focused on wide spacings in loblolly pine with spacings from 20 to 10 m^2/tree (500–1,000 tph). Further results from this trial for ages 14 through 17 (Virginia Department of Forestry 2007) corroborate the results for the same trial reported by Scrivani and Bowman (2004) for age 15 in finding no effect of planting density on height at those spacings. Differences among wider spacings might appear at older ages, but given the usual rotation age for loblolly pine in the region, they would not materially affect growth and yield estimations. Because this upper limit is not intrinsically modeled by the proposed equation, we suggest that for spacings wider than 13.38 m^2/tree, instead of using the equation directly with the spacing of interest, the user should use the 13.38 m^2/tree height predictions to avoid

Table 5. Wald F test P values for the joint test that the spacing variables are zero in models that would be able to capture a potential crossover effect

Model	Age 1–5	Age 6–10[a]
$DHQ_{ijkl} = \alpha_0 + \alpha_1 S + \phi_0 + (\phi_1)A + \beta_1 SA + (\phi_2)A^{-1} + \beta_3 SA^{-1} + \varepsilon_{ijkl}$	0.6794	0.2330
$DHQ_{ijkl} = \alpha_0 + \alpha_1 S + \phi_0 + (\phi_1)A + \beta_1 SA + (\phi_2)A^{-1} + \varepsilon_{ijkl}$	0.7027	0.1373
$DHQ_{ijkl} = \alpha_0 + \alpha_1 S + \phi_0 + (\phi_1)A + (\phi_2)A^{-1} + \beta_3 SA^{-1} + \varepsilon_{ijkl}$	0.9435	0.4771

$\phi_0 = b_{0i} + b_{0ij}$; $\phi_1 = \beta_0 + b_{1i} + b_{1ij}$; $\phi_2 = \beta_2 + b_{2i}$.
[a] The models for ages 6–10 include only data for spacings greater than 3 m^2/tree.

extrapolation and the possibility of anomalous results. Closer spacings than the ones studied here are not common in commercial stands, but those individuals who use the proposed model to predict dominant height for spacings closer than those present in the data set should take the predicted heights with caution, as when using any empirical model beyond the limits of the original data.

After variations in site quality through the mixed-effects modeling approach were accounted for, these trials in loblolly pine exhibited differences in SI originating from differences in initial planting density of approximately 4 m (more than 13 ft) between the closest spacing and the widest spacing. This finding means that if forest managers want to have realistic estimates of future growth and yield they should use different SI values for the same site depending on the initial planting density in loblolly pine plantations in the southeastern United States. One cannot compare the site productivity of two sites by simply comparing the observed height; the effect of density on dominant height development and, thus, on SI should be included, especially if close spacings are involved.

In conclusion, initial spacing has an impact on the height development of loblolly pine. The pattern of impact was similar for average height and for the four definitions of dominant height considered. Deviation in height development first occurred at approximately age 6 and was maintained to the final measurement age of 25 years. For this trial, there was no evidence of increased height development at closer spacings at early ages.

Endnotes

[1] Spacing is defined as the growing space (area) available per tree.
[2] 1 m^2/tree = 10.7639104 ft^2 = 10,000 tph = 4,047 tpa.

Literature Cited

AKAIKE, H. 1974. A new look at the statistical model identification. *IEEE Trans. Autom. Cont.* 19(6):716–723.

ALEXANDER, R.R., D. TACKLE, AND W.G. DAHMS. 1967. *Site indexes for lodgepole pine, with corrections for stand density: Methodology.* US For. Serv. Res. Paper RMRS-RP-29. Fort Collins, CO.

AMATEIS, R.L., AND H.E. BURKHART. 1996. Impact of heavy glaze in a loblolly pine spacing trial. *South. J. Appl. For.* 20(3): 151–155.

AMATEIS, R.L., H.E. BURKHART, AND S.M. ZEDAKER. 1988. *Experimental design and early analyses for a set of loblolly pine spacing trials.* US For. Serv. Gen. Tech. Rep. NC-120.

AMATEIS, R.L., S.P. PRISLEY, H.E. BURKHART, AND J. LIU. 2006. The effect of physiographic region and geographic locale on predicting the dominant height and basal area of loblolly pine plantations. *South. J. Appl. For.* 30(3):147–153.

BOX, G.E.P., G.M. JENKINS, AND G.C. REINSEL. 1994. *Time series analysis, forecasting and control.* 3rd ed. San Francisco: Prentice Hall.

CURTIS, R.O., AND D.L. REUKEMA. 1970. Crown development and site estimates in a Douglas-fir plantation spacing test. *For. Sci.* 16(3):287–301.

FANG, Z., AND R.L. BAILEY. 2001. Nonlinear mixed effects modeling for slash pine dominant height growth following intensive silvicultural treatments. *For. Sci.* 47(3):287–300.

HARMS, W.R., AND F.T. LLOYD. 1981. Stand structure and yield relationships in a 20-year-old loblolly pine spacing study. *South. J. Appl. For.* 5(3):162–166.

HARMS, W.R., D.S. DEBELL, AND C.D. WHITESELL. 1994. Stand and tree characteristics and stockability in *Pinus taeda* plantations in Hawaii and South Carolina. *Can. J. For. Res.* 24(3): 511–521.

HARMS, W.R., C.D. WHITESELL, AND D.S. DEBELL. 2000. Growth and development of loblolly pine in a spacing trial planted in Hawaii. *For. Ecol. Manag.* 126(1):13–24.

KNOWE, S.A., AND D.E. HIBBS. 1996. Stand structure and dynamics of young red alder as affected by planting density. *For. Ecol. Manag.* 82(1–3):69–85.

LAND, S.B., K.L. BELLI, AND H.W. DUZAN. 1991. Family, spacing, and family-by-spacing effects on loblolly pine during five years after planting. P. 744 in *Proc. of the Sixth Biennial Southern Silvicultural Research Conference,* Coleman, S.S., and D.G. Neary (eds). US For. Serv. Gen. Tech. Rep. SE-70. US For. Serv., Southeast. For. Exp. Stn., Asheville, NC.

LAPPI, J., AND R.L. BAILEY. 1988. A height prediction model with random stand and tree parameters: An alternative to traditional site index methods. *For. Sci.* 34(4):907–927.

LIN, C.S., AND P.M. MORSE. 1975. A compact design for spacing experiments. *Biometrics* 31(3):661–671.

LYNCH, D.W. 1958. *Effects of stocking on site measurement and yield of second-growth ponderosa pine in the Inland Empire.* US For. Serv. Res. Paper 56. US For. Serv., Intermtn. For. Range Exp. Stn., Odgen, UT.

MACFARLANE, D.W., E.J. GREEN, AND H.E. BURKHART. 2000. Population density influences assessment and application of site index. *Can. J. For. Res.* 30(9):1472–1475.

NANCE, W.L., S.B. LAND, AND R.F. DANIELS. 1983. Concepts for analysis of intergenotypic competition in forest trees. P. 131–145 in *Proc. of the 17th Southern forest tree improvement conference, Athens, GA, June 7–9 1983,* vol. 17. National Technical Information Service, Springfield, VA.

PIENAAR, L.V., AND B.D. SHIVER. 1984. The effect of planting density on dominant height in unthinned slash pine plantations. *For. Sci.* 30(4):1059–1066.

PIENAAR, L.V., AND B.D. SHIVER. 1993. Early results from an old-field loblolly pine spacing study in the Georgia Piedmont with competition control. *South. J. Appl. For.* 17(4):193–196.

PIENAAR, L.V., AND K.J. TURNBULL. 1973. The Chapman-Richards generalization of Von Bertalanffy's growth model for basal area growth and yield in even-aged stands. *For. Sci.* 19(4):2–22.

PINHEIRO, J.C. AND D.M. BATES. 2000. *Mixed-effects models in S*

and S-PLUS. Springer, New York.

R DEVELOPMENT CORE TEAM. 2009. *R: A language and environment for statistical computing.* Vienna, Austria.

SCHWARZ, G. 1978. Estimating the dimension of a model. *Ann. Statist.* 6:461–464.

SCOTT, W., R. MEADE, R. LEON, D. HYINK, AND R. MILLER. 1998. Planting density and tree-size relations in coast Douglas-fir. *Can. J. For. Res.* 28(1):74–78.

SCRIVANI, J.A., AND W.F. BOWMAN. 2004. *Wide-spacing plantings of loblolly pine. Age 15 results.* Occasional Report 124. Virginia Department of Forestry, Charlottesville, VA.

SHARMA, M., H.E. BURKHART, AND R.L. AMATEIS. 2002a. Modeling the effect of density on the growth of loblolly pine trees. *South. J. Appl. For.* 26(3):124–133.

SHARMA, M., H.E. BURKHART, AND R.L. AMATEIS. 2002b. Spacing rectangularity effect on the growth of loblolly pine plantations. *Can. J. For. Res.* 32(8):1451–1459.

SHARMA, M., R.L. AMATEIS, AND H.E. BURKHART. 2002c. Top height definition and its effect on site index determination in thinned and unthinned loblolly pine plantations. *For. Ecol. Manag.* 168(1–3):163–175.

SHARMA, M., M. SMITH, H.E. BURKHART, AND R.L. AMATEIS. 2006. Modeling the impact of thinning on height development of dominant and codominant loblolly pine trees. *Annals of For. Sci.* 63:349–354.

THOMPSON, A., AND B.N. TAYLOR. 2008. *Guide for the use of the international system of units (SI).* Special Publ. (NIST SP) 811.

VIRGINIA DEPARTMENT OF FORESTRY. 2007. Loblolly pine planting density study. P. 5–6 in *Forest Research Review.* Available online at www.dof.virginia.gov/research/resources/for-res-rev-2007–09.pdf; last accessed June 15, 2009.

WOODRUFF, D.R., B.J. BOND, G.A. RITCHIE, AND W. SCOTT. 2002. Effects of stand density on the growth of young Douglas-fir trees. *Can. J. For. Res.* 32(3):420–427.

ZHANG, S., H.E. BURKHART, AND R.L. AMATEIS. 1996. Modeling individual tree growth for juvenile loblolly pine plantations. *For. Ecol. Manag.* 89(1–3):157–172.

ZHAO, D., M. WILSON, AND B.E. BORDERS. 2005. Modeling response curves and testing treatment effects in repeated measures experiments: A multilevel nonlinear mixed-effects model approach. *Can. J. For. Res.* 35(1):122–132.

Appendix
DH2 Height–Age Model

$$H_{ijkl} = (\alpha_0 + \alpha_1 S + \alpha_2 S^{0.5} + b_{1i} + b_{1ijk})$$
$$\times (1 - e^{-(\beta_0 + b_{2i} + b_{2ij})A})^{\gamma_0 + \gamma_1 S + \gamma_2 S^{0.5} + b_{3i} + b_{3ij} + b_{3ijk}} + \varepsilon_{ijkl}$$

$$\begin{pmatrix} b_{1i} \\ b_{2i} \\ b_{3i} \end{pmatrix} \sim N(\mathbf{0}, \mathbf{\Psi}_1)$$

$$\begin{pmatrix} b_{2ij} \\ b_{3ij} \end{pmatrix} \sim N(\mathbf{0}, \mathbf{\Psi}_2)$$

$$\begin{pmatrix} b_{1ijk} \\ b_{3ijk} \end{pmatrix} \sim N(\mathbf{0}, \mathbf{\Psi}_3)$$

The structure of the within-group error ε_{ijkl} is modeled with an ARMA(1, 1) with autoregressive parameter estimate = 0.5457 and moving average parameter = 0.0150 and an exponential variance model with age as a variance covariate and parameter δ varying by location $\hat{\delta}_i$ = (0.1077, 0.0879,

0.0901, 0.0793). The variance–covariance matrices are

$$\hat{\mathbf{\Psi}}_1 = \begin{pmatrix} 2.2414 & 0 & 0 \\ 0 & 0.0047 & 0 \\ 0 & 0 & 0.1037 \end{pmatrix}$$

$$\hat{\mathbf{\Psi}}_2 = \begin{pmatrix} 0.0029 & 0 \\ 0 & 0.0285 \end{pmatrix}$$

$$\hat{\mathbf{\Psi}}_3 = \begin{pmatrix} 0.5223 & 0 \\ 0 & 0.0361 \end{pmatrix}$$

DH5 Height–Age Model

$$H_{ijkl} = (\alpha_0 + \alpha_1 S + \alpha_2 S^{0.5} + b_{1i})$$
$$\times (1 - e^{-(\beta_0 + \beta_1 S + \beta_2 S^{0.5} + b_{2ij} + b_{2ijk})A})^{\gamma_0 + b_{3i}} + \varepsilon_{ijkl}$$

$$\begin{pmatrix} b_{1i} \\ b_{3i} \end{pmatrix} \sim N(\mathbf{0}, \mathbf{\Psi}_1)$$

$$(b_{2ij}) \sim N(\mathbf{0}, \mathbf{\Psi}_2)$$

$$(b_{2ijk}) \sim N(\mathbf{0}, \mathbf{\Psi}_3)$$

The structure of the within-group error ε_{ijkl} is modeled with an ARMA(1, 0) with autoregressive parameter estimate = 0.7081 and a power variance with age as a variance-covariate and $\hat{\delta}$ = 0.6414. The variance–covariance matrices are

$$\hat{\mathbf{\Psi}}_1 = \begin{pmatrix} 1.9691 & 0 \\ 0 & 0.0653 \end{pmatrix}$$

$$\hat{\mathbf{\Psi}}_2 = 0.0027$$

$$\hat{\mathbf{\Psi}}_3 = 0.0018$$

DHCA Height–Age Model

$$H_{ijkl} = (\alpha_0 + a_2 S^{0.5} + b_{1i} + b_{1ij} + b_{1ijk})$$
$$\times (1 - e^{-(\beta_0 + \beta_1 S + b_{2i})A})^{\gamma_0 + \gamma_1 S + \gamma_2 S^{0.5} + b_{3i} + b_{3ij} + b_{3ijk}} + \varepsilon_{ijkl}$$

$$\begin{pmatrix} b_{1i} \\ b_{2i} \\ b_{3i} \end{pmatrix} \sim N(\mathbf{0}, \mathbf{\Psi}_1)$$

$$\begin{pmatrix} b_{1ij} \\ b_{3ij} \end{pmatrix} \sim N(\mathbf{0}, \mathbf{\Psi}_2)$$

$$\begin{pmatrix} b_{1ijk} \\ b_{3ijk} \end{pmatrix} \sim N(\mathbf{0}, \mathbf{\Psi}_3)$$

The structure of the within-group error ε_{ijkl} is modeled with an ARMA(1, 0) with autoregressive parameter estimate = 0.5295 and an exponential variance model with age as a variance covariate and parameter δ varying by location $\hat{\delta}_i$ = (0.1169, 0.0812, 0.0840, 0.0858). The variance–covariance matrices are

$$\hat{\mathbf{\Psi}}_1 = \begin{pmatrix} 1.6295 & 0 & 0 \\ 0 & 0.0062 & 0 \\ 0 & 0 & 0.1061 \end{pmatrix}$$

$$\hat{\mathbf{\Psi}}_2 = \begin{pmatrix} 0.9000 & 0 \\ 0 & 0.0311 \end{pmatrix}$$

$$\hat{\mathbf{\Psi}}_3 = \begin{pmatrix} 0.7962 & 0 \\ 0 & 0.0383 \end{pmatrix}$$

Average Height–Age Model

$$H_{ijkl} = (\alpha_0 + \alpha_2 S^{0.5} + b_{1i} + b_{1ij})$$

$$\times \left(1 - e^{-(\beta_0 + \beta_1 S + \beta_2 S^{0.5} + b_{2i} + b_{2ij} + b_{2ijk})A}\right)^{\gamma_0 + \gamma_1 S + \gamma_2 S^{0.5} + b_{3i} + b_{3ijk}}$$

$$+ \varepsilon_{ijkl}$$

$$\begin{pmatrix} b_{1i} \\ b_{2i} \\ b_{3i} \end{pmatrix} \sim N(\mathbf{0}, \boldsymbol{\Psi}_1)$$

$$\begin{pmatrix} b_{1ij} \\ b_{2ij} \end{pmatrix} \sim N(\mathbf{0}, \boldsymbol{\Psi}_2)$$

$$\begin{pmatrix} b_{2ijk} \\ b_{3ijk} \end{pmatrix} \sim N(\mathbf{0}, \boldsymbol{\Psi}_3)$$

The structure of the within-group error ε_{ijkl} is modeled with an ARMA(1, 1) with autoregressive parameter estimate $= 0.5646$ and moving average parameter $= 0.2597$ and an exponential variance model with age as a variance covariate and parameter δ varying by location $\hat{\delta}_i = (0.1200, 0.0949, 0.0989, 0.0889)$. The variance–covariance matrices are

$$\hat{\boldsymbol{\Psi}}_1 = \begin{pmatrix} 1.3456 & 0 & 0 \\ 0 & 0.0094 & 0 \\ 0 & 0 & 0.1850 \end{pmatrix}$$

$$\hat{\boldsymbol{\Psi}}_2 = \begin{pmatrix} 0.6137 & 0 \\ 0 & 0.0040 \end{pmatrix}$$

$$\hat{\boldsymbol{\Psi}}_3 = \begin{pmatrix} 0.0022 & 0 \\ 0 & 0.0455 \end{pmatrix}$$