The Impacts of Environmental Policy on Industrial Allocation: A Transboundary Pollution Dynamic Game

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Abstract

We examine the impact of environmental policy on industrial location between two trading regions dealing with transboundary pollution. Firstly, we study how the distribution of firms and trade costs affect the governments' environmental policies, particularly, the issuance of emission permits. Secondly, we study how the resulting environmental policies alter the allocation of the industry. The microeconomic behavior of the agents is framed within the Economic Geography literature, through a linear Footloose Capital (FC) model. The macroeconomic model that arises is a transboundary pollution dynamic game. When regions have different industrial shares, we find that if pollution damage is low, the more industrialized region adopts environmentally irresponsible behavior by increasing the offer of emission permits, which reduces their price. Firms benefit from lower production costs, ultimately attracting more firms (agglomeration force). However, due to transport costs, as the share of firms in a region increases, the benefits decrease (dispersion force). The final spatial distribution of the industry between the regions depends on the balance between agglomeration and dispersion forces. This agglomerative force and the governments' strategic behavior, absent in the FC model, could lead to industrial activity fully concentrating in a core region. As pollution damage increases, agglomerative power loses strength.

Keywords Economic geography · Dynamic game · Transboundary pollution · Emission permits · Transport cost

JEL classification C73 · F12 · F18 · Q58 · R12

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1 Introduction

Some pollutants, such as greenhouse gases (GHGs), produced in a particular region, have negative impacts on the quality of the global environment. Moreover, the damage is not limited to the present time, but will continue to be suffered in the future. Many efforts have already been made within the framework of the United Nations to seek measures to reduce pollutant emissions. EU countries are leading some of the most ambitious actions, such as the Paris Agreement 2015, which seeks to keep the average global temperature increase below 2 Celsius degrees above pre-industrial levels. Nevertheless, world CO_2 emissions are continuously rising as Fig. 1 shows.[1]

Figure 1 also shows the dates of some of the subsequent environmental actions promoted by the United Nations, European Union and other OECD countries. Different emission trends are observed for different groups of countries. Emissions from EU countries remain under control, slightly decreasing during the last decade. The same behavior is observed in OECD countries, although at higher levels. Nevertheless, emissions from Non-OECD countries keep rising above the level of the OECD countries since the beginning of the century. The decline observed in 2020 is due to the impact of COVID on all economies, but it is already known that emission levels have increased again in the later years.

Emission reductions often imply an increase in production costs, which reduces the chances of attracting foreign capital and fuels footloose capital. Firms may reallocate to regions with laxer environmental policies and the reduction of emissions in one region leads to an increase in emissions elsewhere. Despite relocation, international trade ensures that the demand for these products continues to be met in the countries of origin. The phenomenon is known as *carbon leakage*, which means that global emissions are not reducing but reallocating. The concept of *carbon leakage* is somehow related to the idea formalized by Copeland and Taylor (2004) and known as the *pollution haven hypothesis* that establishes that a reduction of trade barriers encourages a relocation of polluting industries from countries with strict environmental policy to those with less stringent policies. Both concepts are related to relocation of polluting industries but from different perspectives. *Carbon leakage* focusses on the global impact of climate policies, while the *pollution haven hypothesis* centers on firms' location choice.[2] Figure 2 shows the ratio between CO_2 consumption-based emissions[3] over emissions derived from production for different groups of countries. Periods of increments in this ratio are periods where emissions generated by the consumed goods (imported and domestically produced goods) grow at a faster rate than emissions generated by domestic production.

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[1] Source OECD Greenhouse Gas emissions Footprints Indicators (Figs. 1 and 2): https://data-explorer.oecd.org/?tm=DF_ICIO_GHG_MAIN_2023%26snb=5. EU ETS stands for the European Union Emissions Trading System and WCI denotes the Western Climate Initiative of a group of American states and Canadian provinces.

[2] The literature has provided mixed empirical evidence for the *pollution haven hypothesis*, being the main argument against this hypothesis that environmental policy alone cannot justify firms' relocation in an international trade context. However, more theoretical papers, as for example, Petrakis and Xepapadeas (2003) analyze the relocation decisions of a monopolist under two alternative policy regimes dependent on the regulator's commitment and show that the monopolist will relocate more often under time-consistent emission taxes.

[3] Regional emissions from production minus emissions embedded in exports, plus emissions embedded in imports.
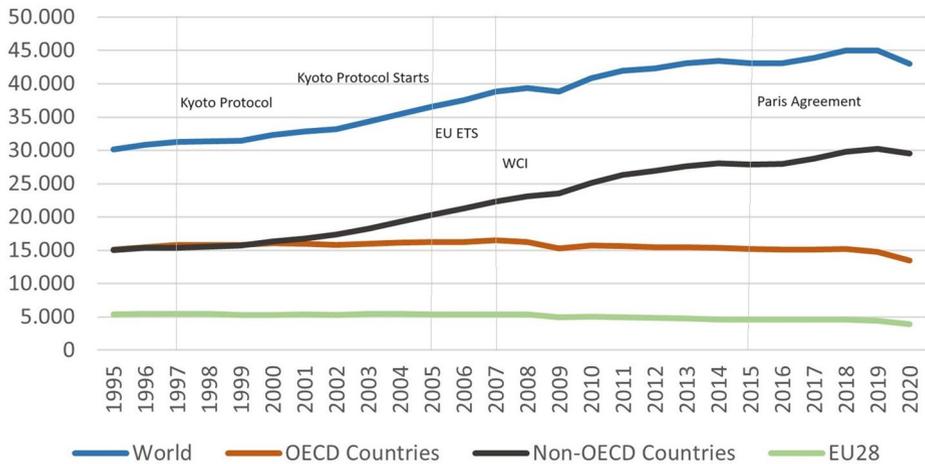
Fig. 1 CO_2 emissions based on production (million tones)

It is noteworthy that EU countries are increasing the ratio since the Paris Agreement, which could mean that polluting production is transferred to other countries. The behavior of this ratio in Non-OECD and OECD countries is the opposite. The first ratio is always below 1, thus emissions generated by domestic production are higher than those due to domestic consumption. Some of the pollutant emissions are used to produce goods that are exported to OECD countries, whose ratio is always above 1. Some countries find it difficult to comply with international agreements because companies, in order to avoid costs, might relocate, which would hinder economic growth and lead to unemployment, so they opt for free-rider behavior. Achieving a cooperative solution is proving difficult. This raises the natural question of whether unilateral non-cooperative measures could be effective in reducing global emissions. We address the effectiveness of these unilateral, non-cooperative measures using the issuance of emission permits by governments in a context of international capital mobility.

The use of emission permit markets is gaining relevance in the mainstream environmental policy. Well-known examples of such systems are the European Union Emissions Trading System (EU ETS), the UK Emissions Trading Scheme (UK ETS), that replaced the UK's participation in the EU ETS, and the Western Climate Initiative (WCI) of a group of American states and Canadian provinces, that developed a system for capping and trading GHG emission allowances. When each country or region chooses non-cooperatively the number of emission permits to issue so as to maximize its own social welfare, what are the implications on domestic welfare and on the global stock of pollution? Addressing this question involves taking into account three dimensions: interdependencies, time, and strategic behavior.

These three characteristics make the problem very suitable to be modelled and analyzed as a dynamic game.[4] The literature on transboundary pollution dynamic games has its origins in the seminal papers by Van der Ploeg and De Zeeuw (1992) and Dockner and Long (1993). Since then, the original models have been modified and enriched to study

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[4] See, for example, the survey papers by Jørgensen et al. (2010), and De Zeeuw, Chapter 16 in the Handbook of Dynamic Game Theory (Başar and Zaccour 2018).
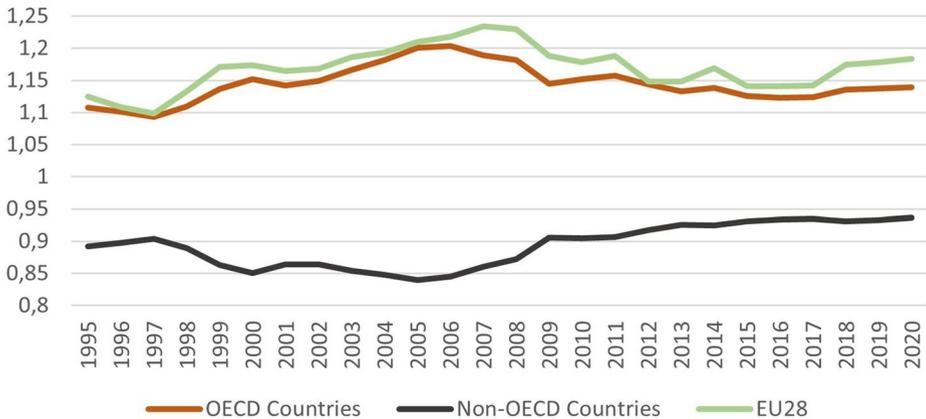
Fig. 2 Ratio between CO_2 emissions embodied in domestic final demand and emissions based on production

different questions with a special emphasis on the effects of pollution control instruments such as command-and-control instruments (for example, standards and quotas) or market-based instruments (for example, emission taxes or subsidies).[5] In this literature on environmental regulation with polluting firms in the context of stock dynamics, the seminal work of Benchekroun and Long (1998) stands out. Other more recent articles that consider different environmental policy instruments in a dynamic context are Yanase (2009), Benchekroun and (2011), Martín-Herrán and Rubio (2018, 2024, 2025) and Dragone et al. (2022). However, most of this literature focuses on closed economies and disregards that countries and regions trade with each other. The literature on dynamic games and trade mainly focuses on dynamic games among firms. However, there are some papers that shift the focus and analyze dynamic games between governments that choose the level of their trade policy instruments to maximize regional/national welfare, taking into account the dynamics of the state variable (for example, capital, natural resources, etc.).[6] In contrast to these contributions, which typically abstract from explicit microeconomic foundations, our paper contributes to this literature by analyzing a two-region trade model, enriched with the fundamental drivers of interregional trade (taste for variety, transport costs and scale economies), in the tradition of the New Economic Geography. The model incorporates the dynamics of the global stock of pollution and takes into account the strategic behavior of the two regional governments when choosing the emission permits.

To the best of our knowledge, there are no previous studies that consider agents who behave both dynamically and strategically in the context of a New Economic Geography model. This paper fills also this gap in the literature and characterizes the linear feedback Nash equilibrium of an intertemporal transboundary pollution dynamic game, where two regional governments maximize regional social welfare by determining their emission permits taking into account the pollution dynamics. Our model is grounded in microeconomic foundations that reflect the structural relationships arising from each region's environmental

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[5] See, for example, Long (2010), Chapter 2 for a survey of these models.

[6] Long (2010), Chapter 4, surveyed these papers up to 2010.

policy, trade interactions, and capital mobility. This microfoundation gives rise to a dual linkage between the players (regions), through both the welfare functions and the dynamics of transboundary pollution. Within this framework, transport costs and environmental damage emerge as key parameters for understanding regions' strategic decisions. Interestingly, the microfounded structure enables us to identify firm-level profits (an insight that would not be attainable otherwise) which in turn allows for a stability analysis and the characterization of the distribution of firms at steady states.

One of the key characteristics of the differential game played by the two regional governments is that the welfare in each region depends on the emissions of both regions. This feature arises from the trade relationship between the regions and reflects the interaction between their environmental policies. Studies that analyze models of environmental policies in open economies usually assume that the instantaneous payoff of each region only depends on its own emission level, independent of the emission level in the other region. Yanase (2005, 2007, 2009, 2010, 2012) are exceptions and analyze the effects of international trade on environmental policies in stock pollutant problems where the instantaneous payoff of each region depends on the emission tax rates of both regions. However, unlike our approach, Yanase incorporates this interdependence through an ad hoc assumption. Moreover, like Fujiwara (2009) and Nkuiya and Plantinga (2021), all the analyses are carried out in partial equilibrium frameworks, while we present a general equilibrium model. All these papers, except Yanase (2010), assume that there are no costs associated with trade, and most of the analyses are carried out under the assumption that the trading countries or regions are completely symmetric. In contrast, our model explicitly incorporates transportation costs related to industrial goods and accounts for asymmetries between countries, reflecting a more nuanced and realistic representation of economic interactions.

To the best of our knowledge, Yanase and Kamei (2022) is the only paper where the market equilibria that determine the prices of the traded polluting goods and pollution permits are explicitly considered by the decision-makers of the differential game when taking environmental policy decisions. Comparing with previous studies that consider international trade in a partial equilibrium framework, this paper introduces the structure of general oligopolistic equilibrium into the transboundary pollution differential game. We also consider the structure of the market. However, our paper differs from Yanase and Kamei (2022) in the following important aspects. First, Yanase and Kamei (2022) focuses on the comparison of autarky versus bilateral free trade of polluting goods under three different scenarios: international cooperation, non-cooperation under open-loop strategies and non-cooperation under linear feedback strategies. Our focus is completely different. On the one hand, the focus is on the effects on the environmental policies and welfare of the distribution of firms between the two trading regions. Additionally, attention is paid to the characterization of different symmetric and agglomeration equilibria for the final spatial distribution of the industry between the two regions, as well as their stability depending on the degree of environmental damage suffered by the regions and the amount of transport costs. Second, Yanase and Kamei (2022) focuses on the case of symmetric countries, implying among other things that both countries share identical firm distributions of all industries. However, as previously pointed out, one of our main interests relies on which is the long-run equilibrium of the distribution of the firms. Third, most of the results derived in Yanase and Kamei (2022) are obtained under the assumption that trade is frictionless in the sense that there are no trade costs. However,

trade costs are one of the main ingredients in our model and play an important role in all the results Martin and Rogers (1995).

The role of pollution and environmental policies in the New Economic Geography models has been studied in few papers (Pflüger 2001; Zeng and Zhao 2009; Forslid et al. 2017; Ishikawa and Okubo 2017). There are however some important differences between these previous studies and our proposal. For example, in his seminal work, Pflüger (2001) focuses on the case of two governments that choose an emission tax and that face local and instantaneous pollution. Ishikawa and Okubo (2017) study the causes of carbon leakage, by developing an asymmetric economic geography model with the relocation of capital where only the larger region imposes an emission tax (or quota). The authors also consider instantaneous pollution, but in this case, one of their main novelties comes from incorporating the transboundary feature of this negative externality. However, their formulation does not give rise to any interregional game. Forslid et al. (2017) employ a monopolistic competitive framework (Martin and Rogers 1995) with endogenous determination of the number of firms, like Ishikawa and Okubo (2017). In their setup, transboundary pollution is considered, but it does not accumulate, and both regional governments impose an emission tax. Our goal is to further enrich this growing literature by incorporating a market of emission permits, which is used by many countries around the world, and by considering the dynamic component of pollution; that is, allowing the pollutant emissions to accumulate over time, and consequently, allowing the regional governments to behave both dynamically and strategically.

The dynamic framework we use allows the regional governments to decide their environmental policies depending on the stock of pollution, and therefore, dependent on how serious the environmental problem is. Furthermore, these policies, whether more or less demanding, influence the long-term location choices of firms. A remarkable feature of the emerging dynamic game is that it does not have symmetrical players, because the underlying New Economic Geography model considers regions with different industrial shares. The complexity of the model we propose leads us to study it in two stages. Our procedure makes the problem analytically tractable. In the first stage, the regional governments play a differential game and decide their emission permits that correspond with their Nash equilibrium strategies. They do so by considering the evolution of the pollution stock over time while taking into account a given distribution of firms. That is, regions are myopic with respect to the time evolution of the share of firms in each region. In the second stage, taking as inputs the equilibrium decisions of the governments from the first stage, firms relocate and the long-run industrial distribution is determined. In this second stage we propose an evolutionary process to describe how industrial distribution reaches a stable equilibrium in which the pollution stock level and the distribution of firms across regions stabilize and no longer change. It is important to note that in this process, the evolution of firms' profits over time depends not only on the dynamics of emissions and pollution, but also on how the share of firms in each region evolves. This is due to the presence of agglomeration and dispersion effects, which are an inherent aspect of the New Economic Geography model. Our model is able to capture different long-run situations.

A first justification for this two-stage procedure is that the movement of firms from one region to another takes time. Even though both pollution and the distribution of firms evolve continuously over time, firms make relocation decisions on a different time scale. A second purely technical justification is that if governments take into account the movement of firms

when making their decisions, the differential game would lose its linear-quadratic structure. The analytical characterization of the equilibrium strategies of the new differential game would not be feasible. Instead, it would require the development of numerical algorithms to approximate the equilibrium strategies in a differential game with two state variables and two totally asymmetric players. The same type of two-stage procedure has been previously used in the literature that analyzes international environmental agreements using a dynamic framework (see, for example, Breton et al. (2010) and Breton and Sbragia (2023), that analyze time-discrete dynamic games in a infinite horizon setting).

Our analysis reveals that the strategic behavior of regions depends on the damage of the pollution stock on social welfare. If this damage is high, the more industrialized region behaves responsibly by reducing the emission permits per firm, while the less industrialized region will try to attract firms by increasing these permits. This situation, which we refer to as responsible-freerider behavior, removes the incentives for firms to agglomerate, and the industry will end up equally distributed between the regions. However, if pollution damage is low, our findings indicate the emergence of an irresponsible-hostage behavior, which leads to the agglomeration of industry in one core and the deindustrialization of the other region. The strategic interaction between the two regional governments gives rise to agglomeration equilibria, which is completely new in the standard Footloose Capital model. In this case, the region with the higher share of firms irresponsibly allows firms to increase their emissions, while the smaller region is forced to reduce emission permits per firm. As a consequence, more firms will be willing to move to the region with the larger share of the industry, triggering an agglomeration process. If the damage of the stock of pollution is intermediate, partial agglomeration equilibria appear. The equilibria that appear in our model help to explain the observations in Figs. 1 and 2, where polluting production was progressively shifting to Non-OECD countries during the first decade of this century, while OECD countries were increasing their imports of these products. In the second decade, more converging trajectories can be observed, although the Non-OECD group still has a lower demand-based emissions per production emission. In our model, the existence of trade costs is essential for regions to pursue different strategies, which cannot be explained solely by variations in their levels of industrialization. Moreover, positive trade costs make the welfare in each region depends on the share of firms settled in that region. Freeness of trade will favor agglomeration.

This paper is organized as follows. In Sect. 2, we present the different agents and elements of our linear Footloose Capital model with cumulative and non-cumulative pollution, and we look for the market solution of the model. In Sect. 3, we study the problem of regional governments, which act as social planners who seek to maximize the social welfare in their regions by choosing the equilibrium emission permits. Both planners take into account the evolution of cumulative pollution over time and the strategic interaction between the governments is formulated as a dynamic game between them. We characterize the feedback Nash equilibrium in linear strategies of the differential game where the distribution of firms between regions may or may not differ. In the long run, firms can move from one region to the other looking for the greatest nominal rewards. In Sect. 4, the spatial distribution of firms at the steady states and their stability are analyzed. Section 5 concludes the study. The proofs are presented in the online appendix.

2 The Model

We begin with an extension of the linear Footloose Capital (FC) model presented by Ottaviano et al. (2002). There are two regions indexed by $j = 1, 2$, two sectors (industrial and homogeneous) and two productive factors: capital and labor. Worldwide supplies of capital (K^w) and labor (L^w) are fixed and evenly distributed between the two regions, and we normalize the world endowment to one ($L^w + K^w = 1$). By K^w this model means both the number of capitalists and the capital endowment. Labor is immobile between regions, capital moves internationally as in the linear FC model.

Interregional trade of industrial goods incurs in transport costs (τ), while the homogeneous good is freely tradable.[7] The industrial sector in region j consists of a continuum of firms $[0, n_j], j = 1, 2$, characterized by increasing returns to scale and monopolistic competition. We assume that a typical industrial firm requires one unit of capital to operate (fixed cost), while all variable costs involve only labor. As a consequence of this, each firm produces a single variety of industrial good and the total number of firms in the world n^w must be equal to the world capital endowment ($n_1 + n_2 = n^w = K^w$). Since our main focus is industrial location, we make assumptions that keep the homogeneous sector as simple as possible: this sector produces under constant returns to scale, and one unit of labor produces one unit of homogeneous good. Labor can move freely between the two sectors, which makes wages in both sectors equal.

As a consequence of perfect competition in the homogeneous sector, the price of the homogeneous good and the wage are equal. Moreover, since the homogeneous good is freely tradable between regions, its price equals in both regions. Therefore, the price of the homogeneous good (p_{H_1} and p_{H_2}) and labor wages (w_1 and w_2) are equal to each other and equal between the two regions ($p_{H_1} = p_{H_2} = w_1 = w_2 = w$). We take labor as numéraire, that is, $w = 1$.

A new element in our model is that industrial production releases emissions into the environment which accumulate over time in a common pollution stock. To control this pollution in each region, the regional government implements an environmental policy by creating a market for emission permits. The government in region j determines the supply of emission permits $e_j, j = 1, 2$, and in order to produce goods, firms must purchase the required number of permits at price $\gamma_j, j = 1, 2$, determined by the market. There also exists another pollution, produced by the consumption of the homogeneous good, that does not accumulate. In this section, we look for the private solution, by assuming that households and firms take the stock of pollution and the price of the emission permits as given.

2.1 Households

A representative household of region j aims to choose his/her consumption of each variety i of the industrial good and the consumption of the homogeneous good in order to maximize his/her utility function, U_j in (1) subject to his/her budget constraint given by (2):

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[7]The assumption of a homogeneous sector without transport costs is primarily intended to preserve the simplicity and tractability of the model. While later extensions (Fujita et al. 1999) introduced product differentiation and transport costs in the homogeneous sector, to better capture core-periphery dynamics, these additions often complicate the model without substantially enhancing its explanatory power compared to the original formulation.

$$U_j = \alpha \left(\int_0^{n_1} c_{1j}(i)di + \int_0^{n_2} c_{2j}(i)di \right) - \frac{\beta - \delta}{2} \left(\int_0^{n_1} c_{1j}^2(i)di + \int_0^{n_2} c_{2j}^2(i)di \right) \quad (1)$$

$$- \frac{\delta}{2} \left(\int_0^{n_1} c_{1j}(i)di + \int_0^{n_2} c_{2j}(i)di \right)^2 + C_{H_j},$$

$$y_j = \left(\int_0^{n_1} p_{1j}(i)c_{1j}(i)di + \int_0^{n_2} p_{2j}(i)c_{2j}(i)di \right) + C_{H_j}, \quad (2)$$

where i stands for the different varieties of industrial goods, and $c_{1j}(i)$ and $c_{2j}(i)$ are consumption of a household in region j of the industrial goods produced in region 1 and 2, respectively. The same notation applies for prices ($p_{1j}(i)$ and $p_{2j}(i)$). The number of firms (equal to the number of varieties) in region 1 and 2 are n_1 and n_2, respectively, with $n_1 + n_2 = n^w$. The consumption of the homogeneous good is given by C_{H_j}, and y_j stands for per capita income of region j. Finally, as in the standard linear FC models, $\alpha > 0$ represents the intensity of preferences for industrial goods, condition $0 < \delta < \beta$ ensures the love for variety, and δ measures the substitutability between varieties.

Solving the maximization problem described by (1)-(2), the demand for the i-th variety reads:

$$c_{hj}(i) = \frac{\alpha}{\beta - \delta + \delta n^w} - \frac{1}{\beta - \delta} p_{hj}(i) + \frac{\delta}{(\beta - \delta)(\beta - \delta + \delta n^w)} P_j, \quad (3)$$

where $c_{hj}(i)$ is the demand from a representative household of the industrial variety i produced in region $h = 1, 2$ and consumed in region $j = 1, 2$; $p_{hj}(i)$ is the price of this variety; and P_j is the industrial price index in region j:

$$P_j = \int_0^{n_1} p_{1j}(i)di + \int_0^{n_2} p_{2j}(i)di. \quad (4)$$

The individual demand for the homogeneous good equals the remaining per capita income after the purchase of industrial goods:

$$C_{H_j} = y_j - \left(\int_0^{n_1} p_{1j}(i)c_{1j}(i)di + \int_0^{n_2} p_{2j}(i)c_{2j}(i)di \right). \quad (5)$$

2.2 Industrial Sector

Firms in the industrial sector produce in the region where they are settled and each unit of industrial production releases one unit of emissions to the global environment. The production marginal costs equal to $a_m + \gamma_j$ in region j, where a_m is the labor requirement and γ_j is the price of the emission permits in this region. This price is determined in the emission permit market, as will be seen in Subsection 2.4.

Firms sell their products in the markets of the two regions. Since industrial firms within each region are identical, to simplify notation we can drop the dependence on i from now on. Therefore, the profit function of a representative firm established in region j is:

$$\pi_j = \frac{1}{2}\left(p_{jj} - a_m - \gamma_j\right)x_{jj} + \frac{1}{2}\left(p_{jh} - a_m - \gamma_j - \tau\right)x_{jh}, \tag{6}$$

with $j \neq h$, x_{jj} and x_{jh} are the units produced in region j and sold in region j and h, respectively; p_{jj} and p_{jh} are the corresponding prices; and τ is the unitary transport cost. As in the linear FC model (Baldwin et al. (2011) and Ottaviano et al. (2002)), we assume that each variety can be traded at a positive cost of τ units of the homogeneous good for each unit transported from one region to the other. We assume the same for the cost of emission permits, γ_j. Since consumers (workers and capitalists) are equally distributed between the two regions and $L^w + K^w = 1$, the market size of each region is $1/2$ in (6).

Since industrial firms are monopolistic, they take into account the corresponding demand functions given by (3) in order to maximize their profits, but they neglect the effect of their decisions on the industrial price index. As a result, the optimality conditions are:

$$p_{jj} = (\beta - \delta)c_{jj} + a_m + \gamma_j, \tag{7}$$

$$p_{jh} = (\beta - \delta)c_{jh} + a_m + \gamma_j + \tau, \tag{8}$$

with $j \neq h$, and from (3) the optimal price rules are:

$$p_{jj} = \frac{1}{2}\left[\frac{\alpha(\beta - \delta)}{\beta - \delta + \delta n^w} + a_m + \gamma_j + \frac{\delta}{\beta - \delta + \delta n^w}P_j\right] \quad \text{and} \quad p_{hj} = p_{jj} + \frac{\gamma_h - \gamma_j + \tau}{2}.$$

Taking into account (4), the industrial price indexes are $P_j = n_j p_{jj} + n_h p_{hj}$. Thus, by substituting into the optimal price rules:

$$p_{jj}^* = \frac{2\alpha(\beta - \delta) + \delta\left[a_m n^w + n_j \gamma_j + n_h(\gamma_h + \tau)\right]}{2\left[2(\beta - \delta) + \delta n^w\right]} + \frac{a_m + \gamma_j}{2}, \quad p_{hj}^* = p_{jj}^* + \frac{\gamma_h - \gamma_j + \tau}{2}. \tag{9}$$

2.3 Market Equilibrium

In equilibrium demand must equal supply ($c_{jh} = x_{jh}$). Plugging the prices in (9) and the industrial price indexes into expression (3), the industrial market equilibrium yields:

$$x_{jj}^* = c_{jj}^* = \frac{2\alpha(\beta - \delta) + \delta\left[a_m n^w + n_j \gamma_j + n_h(\gamma_h + \tau)\right]}{2(\beta - \delta)\left[2(\beta - \delta) + \delta n^w\right]} - \frac{a_m + \gamma_j}{2(\beta - \delta)}, \tag{10}$$

$$x_{jh}^* = c_{jh}^* = c_{jj}^* - \frac{1}{(\beta - \delta)}\frac{\beta - \delta + \delta n_h}{2(\beta - \delta) + \delta n^w}\tau, \quad \text{with} \quad j \neq h. \tag{11}$$

As in the original linear FC model the reward to a unit of capital employed by a firm must be equal to its profits. Therefore, the profit function (6) and conditions (7) and (8) yield:

$$\pi_j^* = (\beta - \delta)\frac{(c_{jj}^*)^2 + (c_{jh}^*)^2}{2}. \tag{12}$$

Per capita income in each region has a labor component and a capital component as follows

$$y_j = \frac{\frac{L^w}{2} + \frac{K^w}{2}\bar{\pi}}{\frac{(L^w + K^w)}{2}} \quad \text{with} \quad \bar{\pi} = \frac{n_1\pi_1^* + n_2\pi_2^*}{n^w}. \tag{13}$$

Remark 1 We analyze the case when bilateral trade occurs, that is: $p_{jh}^* - a_m - \gamma_j - \tau > 0$ with $j \neq h$.[8] Additionally, to ensure that there are always workers in the homogeneous sector, the world labor supply is assumed to be sufficiently large. That is:

$$L^w > \max\left[n_1 a_m \left(c_{11}^* + c_{12}^*\right), n_2 a_m \left(c_{21}^* + c_{22}^*\right)\right].$$

2.4 Emission Permit Market

As Yanase and Kamei (2022), we assume that the government in each region distributes the emission permits through auctions. That is, the government makes emission permits available to firms at a price which is fixed by the auction. Each emission permit costs γ_j units of the homogeneous good, as assumed for trade costs in the original model by Ottaviano et al. (2002).[9]

The price of pollution permits, γ_j, is determined to ensure that the market clears. The supply of permits is determined by the regional government, and since all industrial firms within each region are equal, the supply is equal to the permits per firm times the number of firms ($n_j e_j$). The demand of emission permits depends on production. As already mentioned, we assume that each unit of industrial production in region j releases one unit of emissions. Then, the equilibrium of emission permits in region j yields:[10]

$$n_j e_j = n_j \frac{x_{j1}^* + x_{j2}^*}{2}. \tag{14}$$

We assume that the emissions in each region are equal to the amount of emission permits. That is, we eliminate the option of storing emission permits for future use. This option, known as allowance banking, is present in existing markets like the European Union Emissions Trading System. However, including the banking option complicates the analysis and

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[8] In what follows, we assume that the transport cost τ is small enough to ensure that bilateral trade always occurs. Proposition B.1 in Appendix B (online appendix) provides the explicit value of the threshold in the symmetric case. In the asymmetric case, the expression is quite complex, so we simply assume that transport costs are sufficiently small.

[9] To introduce emission permits in a tractable way, we adopt the standard methodological strategy of economic geography models (e.g.Baldwin et al. 2011), treating their cost analogously to transport costs: both are paid in units of the homogeneous good and absorbed into production costs. Since labor is the numéraire, the increased demand for the homogeneous good resulting from permit payments can be interpreted as generating higher labor income through greater employment. However, since $p_H = w$, this additional income is offset by a proportional rise in its price, leaving real income unchanged.

[10] In this expression we make use of the fact that the size of each region is: $(L^w + K^w)/2 = 1/2$.

obscures clear insights into the effects of international trade on goods markets, which is the focus of our study.

Using the industrial equilibrium expressions (10) and (11) in (14), the price of the emission permits in region j is obtained,[11]

$$\gamma_j = \left(\alpha - a_m - \frac{\tau}{2}\right) - 2(\beta - \delta)e_j - \delta E^T, \text{ where } E^T = n_1 e_1 + n_2 e_2. \qquad (15)$$

Equation (15) reveals the interconnection between the two emission permit markets, driven by the trade relationship between the regions. The price of permits in region j depends, not only on the level of emissions in the own region, but also on the other region's emissions (through total emissions). The higher the total emissions E^T, the lower the price of emission permits in each region. This type of interconnection is common in Economic Geography models and also appears in (3), where the quantity demanded depends not only on the price of the own good, but also on the price index (4).

If region 1 reduces the emission permits per firm, the price of these permits would increase. However, the trade relationship requires taking the emission permits of the other region into account. If region 2 increases its emission permits (thus lowering their price), firms in region 2 would become more competitive, enabling them to reduce their prices. This would lower the overall price index, making the prices of firms in region 1 relatively higher. Consequently, firms in region 1 would experience a decline in demand, driving down the demand for emission permits. Therefore, a reduction in the supply of emission permits has a larger effect on the reduction of actual emissions.

2.5 Pollution Dynamics

There exist two types of pollution: Z which cannot be instantaneously absorbed by nature and accumulates in the environment, and Q which is immediately absorbed by the environment.[12] The stock of pollution Z increases with industrial emissions from the two regions and is partly absorbed at rate θ. Since we assume that each unit of industrial production in region j releases one unit of emissions, using expression (14), the pollution dynamics is given by:

$$\dot{Z}(t) = n_1 e_1(t) + n_2 e_2(t) - \theta Z(t), \quad Z(0) = Z_0.$$

It is important to note that the time-dependence of the pollution stock and the emission permits is inherited by all other variables, such as the prices of permits and the consumption and production of goods.

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[11] The positivity of γ_j is ensured if the parameters satisfy the sufficient conditions stated in Corollary 3.

[12] In the New Economic Geography literature, the homogeneous sector is often called agricultural sector. With this assumption we are considering that the production of agricultural goods involves the emission of pollutants that do not persist in the environment or that are absorbed very quickly, such as wastewater, vegetable waste, some pesticides, not very large oil and grease spills, or some air pollutants, such as carbon monoxide (CO) or nitrogen dioxide (NO_2), which are all examples of non-cumulative pollutants.

On the other hand, the non-cumulative pollution can be interpreted as a negative externality of the consumption of homogenous goods, and is given by:[13]

$$Q(t) = \frac{C_{H_1}(t) + C_{H_2}(t)}{2}. \tag{16}$$

We consider a general equilibrium model in which both sectors use labor to produce goods, with a fixed labor supply. An increase in the output of one sector necessarily reduces the output of the other. Holding other factors constant, if the government reduces the industrial emission permits per firm, the price rises, raising industrial production costs. This shift releases labor, which is then reallocated to the homogeneous sector. For this reason, and to keep the model as simple as possible, we exclude a second direct environmental policy instrument on non-cumulative pollution.

3 Endogenous Determination of Emissions

We assume that each regional government knows how the private agents (households and firms) behave and how prices are formed. They incorporate this knowledge to determine the emission permits per firm that maximize social welfare, subject to the evolution of the stock of pollution.

Taking into account the market conditions and expression (1), the social welfare in region j equals the instantaneous utility of a representative household minus the environmental damages, and is given by[14]

$$SW_j(t) = \alpha \left[n_j c_{jj} + n_h c_{hj} \right] - \frac{\beta - \delta}{2} \left[n_j \left(c_{jj} \right)^2 + n_h \left(c_{hj} \right)^2 \right] - \frac{\delta}{2} \left(n_j c_{jj} + n_h c_{hj} \right)^2 + C_{H_j} - \sigma_Q Q - \frac{\sigma_z}{2} Z^2, \tag{17}$$

with $j, h = 1, 2, j \neq h$. Note that in expression (17) the different consumptions, c_{jh} and C_{H_j}, as well as the cumulative and non-cumulative pollution, Z and Q, respectively, are time-dependent, but the time argument is removed for simplicity in the presentation. Parameters σ_z and σ_Q measure the environmental damage caused by the global stock of pollution (Z) in the social welfare and by the non-cumulative pollution (Q), respectively. Moreover, from (5) taking into account (7) and (8), the individual demand for the homogeneous good reads:

$$C_{H_j} = y_j - \left\{ n_j c_{jj} \left[(\beta - \delta) c_{jj} + a_m + \gamma_j \right] + n_h c_{hj} \left[(\beta - \delta) c_{hj} + a_m + \gamma_h + \tau \right] \right\}, \tag{18}$$

with $h \neq j$, and where per capita income in each region y_j from (12) and (13) can be rewritten as:[15]

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[13] If the homogeneous good were pollution-free, the linear-quadratic structure of the welfare function (17) could produce a corner solution in the social planner's problem (presented in the next section), with the entire budget allocated to the consumption of the green homogeneous good. This is why a second type of pollution linked to the homogeneous good is introduced, which is not accumulated in Z so as not to distort the linear-quadratic structure of the differential game in the next section.

[14] Social welfare is the sum of consumer surplus, producer surplus from industry and labor income, minus damages caused by both types of pollution. Expression (17) is derived in Appendix A (online appendix).

[15] Where it is used that $n^w = K^w$ and $L^w + K^w = 1$.

$$y_j = L^w + (\beta - \delta) \left[n_1 \frac{(c_{11})^2 + (c_{12})^2}{2} + n_2 \frac{(c_{21})^2 + (c_{22})^2}{2} \right]. \tag{19}$$

Therefore, substituting (10), (11) and (15) in (17)-(19), social welfare in each region translates into a linear-quadratic function of the emission permits per firm in both regions and the pollution stock.

This section deals with the first stage of the two-stage procedure proposed to study the model. In this first stage, the problem that each regional government is facing consists in deciding the amount of emission permits per firm in the region in order to maximize the flow of discounted social welfare taking into account the dynamics of the stock of pollution, and considering a given distribution of firms. Specifically, the dynamic optimization problem that the government in region 1 is facing reads:

$$\max_{e_1} \int_0^\infty e^{-\rho t} SW_1(t)\, dt = \max_{e_1} \int_0^\infty e^{-\rho t} \left[D_1 e_1^2 + D_2 e_2^2 + D_3 e_1 e_2 + D_4 e_1 + D_5 e_2 + D_6 - \frac{\sigma_z}{2} Z^2 \right] \tag{20}$$

$$\text{s.t.:}\ \dot{Z} = n^w (s_n e_1 + (1 - s_n) e_2) - \theta Z, \quad Z(0) = Z_0, \tag{21}$$

while the problem for the government in region 2 reads:

$$\max_{e_2} \int_0^\infty e^{-\rho t} SW_2(t)\, dt = \max_{e_2} \int_0^\infty e^{-\rho t} \left[E_1 e_1^2 + E_2 e_2^2 + E_3 e_1 e_2 + E_4 e_1 + E_5 e_2 + E_6 - \frac{\sigma_z}{2} Z^2 \right] \tag{22}$$

$$\text{s.t.:}\ \dot{Z} = n^w (s_n e_1 + (1 - s_n) e_2) - \theta Z, \quad Z(0) = Z_0. \tag{23}$$

In Eqs. (21) and (23) the number of firms in each region, n_1 and n_2, has been rewritten using the following notation:

$$n_1 = n^w s_n \quad \text{and} \quad n_2 = n^w (1 - s_n),$$

where s_n is the share of industry in region 1. Moreover, constants $D_i, E_i,\ i = 1, \ldots, 6$ depend on s_n and are given by

$$D_1(s_n) = E_1(s_n) = -\frac{n^w s_n}{2} \left\{ (2\sigma_Q - 1)\delta n^w s_n + (\beta - \delta)(4\sigma_Q - 3) \right\}, \tag{24}$$

$$D_2(s_n) = E_2(s_n) = D_1(1 - s_n), \tag{25}$$

$$D_3(s_n) = E_3(s_n) = -(n^w)^2 s_n (1 - s_n)\delta(2\sigma_Q - 1) = D_3(1 - s_n), \tag{26}$$

$$D_4(s_n) = n^w s_n \left(\alpha \sigma_Q + \frac{\beta - \delta + \delta n^w s_n}{2\left[2(\beta - \delta) + \delta n^w\right]} \tau \right),$$

$$D_5(s_n) = n^w (1 - s_n) \left(\alpha \sigma_Q - \frac{\beta - \delta + n^w \delta(1 - s_n)}{2\left[2(\beta - \delta) + \delta n^w\right]} \tau \right),$$

$$D_6(s_n) = L^w(1-\sigma_Q)\text{-}n^w\tau^2\left(\frac{(4\sigma_Q\text{-}3)(\beta\text{-}\delta+\delta n^w s_n)(\beta\text{-}\delta+\delta n^w(1\text{-}s_n))}{8(\beta-\delta)\left[2(\beta-\delta)+\delta n^w\right]^2} - \frac{(\sigma_Q\text{-}1)\delta n^w(1\text{-}2s_n)^2}{4\left[2(\beta-\delta)+\delta n^w\right]^2}\right), \quad (29)$$

with $E_4(s_n) = D_5(1-s_n)$, $E_5(s_n) = D_4(1-s_n)$ and $E_6(s_n) = D_6(1-s_n)$.

The concavity of the objective function of each player with respect to its control variable for any value of s_n is granted if and only if[16]

$$\sigma_Q > \frac{3}{4}. \quad (30)$$

From now on, we assume that condition (30) is always fulfilled.

By (20)-(23) we have defined a two-player differential game, played over an infinite time horizon. Each player has one control variable, the level of pollutant emission permits per firm e_i, and there is one state variable, the accumulated pollution stock. We focus on the case where both regional governments take decisions simultaneously, and as a result, we characterize the feedback Nash equilibrium, for an exogenously given distribution of firms, $n_1 = n^w s_n$ and $n_2 = n^w(1-s_n)$.

The instantaneous welfare in each region depends on the emission permits of both regions. This fact comes as a result of the trade relations among the regions. In our model there exists two types of interactions between the regions: the interaction through the effects of emission permit levels from the other region on home welfare, and the interaction through the change in the accumulated pollution stock. As in Yanase and Kamei (2022), we consider a general equilibrium framework, and the double interaction previously described is inherited from the private agents' behavior. The richer formulation comes at the cost of more complex and tedious computations to characterize the feedback Nash equilibria. Furthermore, our analysis pays special attention to the effect of the unequal distribution of firms between the two regions and of transport costs on the environmental equilibrium strategies applied by the regions, as well as on the optimal welfare they achieve.

As commonly done in the literature when an infinite planning horizon is considered we look for stationary feedback Nash equilibria of the differential game. Thus, at any point in time, the emission permit decision of each region only depends on the level of the pollution stock at that moment (Dockner et al. (2000), Haurie et al. (2012)).

Guided by the model's linear-quadratic structure, we assume that the players' value functions are quadratic of the form

$$V_1(Z) = F_1\frac{Z^2}{2} + F_2 Z + F_3, \quad V_2(Z) = G_1\frac{Z^2}{2} + G_2 Z + G_3, \quad (31)$$

and we restrict the analysis to linear and interior strategies,[17]

$$e_1(Z) = M_1 Z + M_2, \quad e_2(Z) = N_1 Z + N_2. \quad (32)$$

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[16] See Eqs. (24) and (25).

[17] The full characterization of the value functions when emissions are allowed to be zero is very demanding, especially in the scenario where firms are unevenly distributed across regions. For this reason, we restrict the entire study to the case of strictly positive emissions in order to avoid the technical difficulty of properly pasting the value functions when emissions become zero.

Lemma 1 in Appendix A (online appendix) characterizes the feedback Nash equilibrium. The equilibrium strategies in the simplest case, when both regions are identical ($s_n = 1/2$) are presented in Appendix B (online appendix). The following proposition characterizes the equilibrium strategies in the general scenario where s_n could be different from $1/2$.

Proposition 1 *The linear feedback equilibrium strategies of the dynamic game (20)-(23) are given by*

$$e_1(Z) = e^{Sym}(Z) - \Pi(s_n), \quad e_2(Z) = e^{Sym}(Z) - \Pi(1 - s_n), \tag{33}$$

where $e^{Sym}(Z)$ is the emission permit level in the symmetric equilibrium given by[18]

$$e^{Sym}(Z) = \frac{F_1^{Sym}}{\chi} Z + \frac{\alpha \sigma_Q(\theta + \rho)}{(\theta + \rho)\chi - n^w F_1^{Sym}} + \frac{\tau}{4\chi}, \tag{34}$$

where

$$F_1^{Sym} = \frac{(2\theta + \rho)\chi - \sqrt{(2\theta + \rho)^2 \chi^2 + 4n^w \sigma_z \chi}}{2n^w} < 0, \tag{35}$$

$$\chi = (\beta - \delta)(4\sigma_Q - 3) + \delta n^w (2\sigma_Q - 1) > 0, \tag{36}$$

and

$$\Pi(s_n) = \tau(1 - 2s_n) \left[\frac{(\beta - \delta + \delta n^w)F_2^{Sym}}{\alpha \sigma_Q} + \frac{\delta n^w}{2} \right] \frac{(\beta - \delta)(4\sigma_Q - 3) + 2\delta n^w (2\sigma_Q - 1)(1 - s_n)}{2(\beta - \delta)(4\sigma_Q - 3)[2(\beta - \delta) + \delta n^w]\chi}, \tag{37}$$

with

$$F_2^{Sym} \equiv -\frac{n^w \alpha \sigma_Q F_1^{Sym}}{n^w F_1^{Sym} - (\theta + \rho)\chi}. \tag{37}$$

The global polluting emissions are:

$$
\begin{aligned}
E^T(Z) &= n^w s_n e_1(Z) + n^w(1 - s_n)e_2(Z) \\
&= n^w e^{Sym}(Z) + n^w(1 - 2s_n)^2 \frac{2F_2^{Sym}(\beta - \delta + \delta n^w) + \alpha \sigma_Q n^w \delta}{4\alpha \sigma_Q(2(\beta - \delta) + \delta n^w)\chi} \tau.
\end{aligned} \tag{38}
$$

Proof See Appendix A (online appendix).

Note that if $s_n = 1/2$ then $\Pi(1/2) = 0$ and the symmetric equilibrium strategies are obtained: $e_1(Z) = e_2(Z) = e^{Sym}(Z)$. Moreover, if trade is costless, i.e. $\tau = 0$, then $\Pi(s_n)$ and $\Pi(1 - s_n)$ vanish and emission permit equilibrium strategies[19] (and the corresponding

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[18] The superscript *Sym* stands for symmetric scenario, which is analyzed in detail in Appendix B (online appendix).

[19] From now on, we will refer to emission strategies rather than emission permit strategies, to simplify the presentation.

prices) coincide with those obtained in the symmetric case, for any value of the share of firms in each region s_n. Therefore, if trade is costless, the region with the greater number of firms is the most polluting region. Under positive trade costs, the region with the greater number of firms could be the less polluting one under certain conditions, as we prove below (see Corollary 2). The divergence in regional strategies arises solely from the existence of trade costs. An unequal distribution of firms ($s_n \neq 1/2$), in contrast, is not sufficient on its own to produce different emission strategies.

The sign of F_1^{Sym} clearly shows that the equilibrium emission strategies (33) and (34) decrease when the pollution stock increases, while the price of permits (15) increases, which is the standard result in transboundary pollution dynamic games. An increase of the pollution stock augments the marginal damage from pollution, and consequently, the players have to reduce their emissions until marginal damages and marginal revenues derived from emissions are equal at equilibrium.

It is interesting to know the effect of the trade costs τ on the equilibrium strategies. According to (34), the effect of τ in $e^{Sym}(Z)$, the emissions per firm in the symmetric case, is positive. However, its effect on $e_1(Z)$ and $e_2(Z)$ in the general case depends on the value of s_n (higher or lower than 1/2). Corollary 1 presents the effect on global emissions in the general case, whereas Corollary 2 describes the strategies adopted by the regions under an asymmetric distribution of firms.

Corollary 1 *An increase in the trade costs τ increases the global polluting emissions $E^T(Z)$.*

Proof See Appendix A (online appendix).   □

The effect of trade costs on pollution has not been investigated in depth in economic geography models with strategic behaviors, most likely because of the analytical challenges it presents. One of the models most closely related to ours, Yanase and Kamei (2022), proves, for the symmetric case, that when governments non-cooperatively determine their pollution policies and trade is frictionless, trade is better for the environment than autarky. However, with positive trade costs, the result is not maintained for all the values of the parameters. Therefore, Corollary 1 is consistent with Yanase and Kamei's message that liberalizing trade is good for the environment. We demonstrate this both for the case of an equal and of a different number of firms in the two countries. The empirical evidence on this matter is mixed. Whereas some studies suggest that tariff liberalization leads to emissions growth (Cristea et al. 2013), other studies confirm that rising energy prices and transportation costs encourages the use of less sustainable modes of transportation (Steenhof et al. 2006), resulting in increased emissions. Our contribution highlights a complementary channel: an increase in trade costs reduces the demand for industrial goods produced abroad (c_{hj} reduces if $h \neq j$), and the strategic welfare-maximizing behavior of governments leads to an increase in global emission permits.

Higher transport costs diminish the marginal expenditure devoted to the homogeneous goods, by reducing the marginal income, and increasing the marginal expenditures on industrial goods. As a result, the cost associated with the marginal non-cumulative emission ($\partial Q / \partial e_j$) falls, thus, regions can allow to further increase $E^T(Z)$.[20] This feature explains

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[20] The non-cumulative pollution, Q, depends indirectly on industrial emissions since the consumption of the homogeneous good depends on the income level and the expenditure on industrial goods. This relation can

why the steady state of pollution also increases with transport costs (see Proposition 3 below).

According to (33), the emission permits per firm in a region depend on the share of firms in that region, regardless if it is region 1 or region 2. That is, if the share of firms in region 1 is s_n, then the share of firms in region 2 is $1 - s_n$ (and vice versa) and the emission permits per firm depend on $\Pi(s_n)$ and $\Pi(1 - s_n)$, respectively (and vice versa). Therefore, to ease the presentation of the results, we name e_{s_n} as the emission permits per firm in a region with a share of firms s_n.

The following corollary compares the emission permits in the two regions in the general case with those in the case of symmetric regions.

Corollary 2 *If $\tau > 0$ and $s_n \neq 1/2$, for low values of pollution damage on welfare, $\sigma_z < \hat{\sigma}_z$, then,*

$$e_{1-s_n}(Z) < e^{Sym}(Z) < e_{s_n}(Z) \quad if \quad s_n > 1/2. \tag{39}$$

On the contrary, for high values of pollution damage on welfare, $\sigma_z > \hat{\sigma}_z$, then,

$$e_{s_n}(Z) < e^{Sym}(Z) < e_{1-s_n}(Z) \quad if \quad s_n > 1/2. \tag{40}$$

Moreover, global emissions can be higher or lower than in the symmetric scenario. Specifically,

$$E^T = n^w s_n e_{s_n}(Z) + n^w(1 - s_n) e_{1-s_n}(Z) > n^w e^{Sym}(Z) if and only if \sigma_z < \hat{\sigma}_z,$$

where

$$\hat{\sigma}_z \equiv \frac{\delta(\theta + \rho)\chi[2(\beta - \delta)(2\theta + \rho) + \delta n^w(3\theta + 2\rho)]}{[2(\beta - \delta) + \delta n^w]^2} > 0. \tag{41}$$

Proof See Appendix A (online appendix).   □

The following proposition shows that regardless of the value of the damage of cumulative pollution on welfare, σ_z, the more similar the number of firms in the two regions, the more similar will be the emission permits per firm in the two regions.

Proposition 2 *The gap between emission permits per firm across the two regions decreases as the difference in the number of firms between them narrows, that is*

$$\frac{\partial |e_{s_n}(Z) - e_{1-s_n}(Z)|}{\partial |s_n - (1 - s_n)|} > 0. \tag{42}$$

Proof See Appendix A (online appendix).

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be seen through expressions (16)-(19).

The following corollary collects the results corresponding to the price of emission permits. Denote by γ_{s_n} the price of emission permits in a region with a share of firms s_n.

Corollary 3 *If $\tau > 0$ and $s_n \neq 1/2$, for low values of pollution damage on welfare, $\sigma_z < \hat{\sigma}_z$, then,*

$$\gamma_{s_n}(Z) < \gamma^{Sym}(Z) < \gamma_{1-s_n}(Z) \quad if \quad s_n > 1/2.$$

On the contrary, for high values of pollution damage on welfare, $\sigma_z > \hat{\sigma}_z$, then,

$$\gamma_{1-s_n}(Z) < \gamma^{Sym}(Z) < \gamma_{s_n}(Z) \quad if \quad s_n > 1/2,$$

with

$$\gamma^{Sym}(Z) = \left(\alpha - a_m - \frac{\tau}{2}\right) - \left[2(\beta - \delta) + \delta n^w\right] e^{Sym}(Z). \tag{43}$$

A sufficient condition that guarantees the positivity of these prices is condition (75) in Appendix A (online appendix).

Proof See Appendix A (online appendix).

The threshold $\hat{\sigma}_z$ for the damage of the pollution stock Z in (41) is an increasing function of σ_Q (the damage from non-cumulative pollution, Q), meaning that governments weigh the damage caused by both cumulative and non-cumulative pollution, Z and Q.

The following corollary classifies the strategic behaviors that may arise.

Corollary *The strategic interactions between the regions (large-small) give rise to two different behavioral patterns, depending on the severity of the damage of the pollution on welfare, σ_z:*

- A *responsible-freerider* (R-F) behavior emerges when $\sigma_z > \hat{\sigma}_z$.
- An *irresponsible-hostage* (I-H) behavior arises when $\sigma_z < \hat{\sigma}_z$.

For a given σ_Q, if $\sigma_z > \hat{\sigma}_z$ the damage of cumulative pollution is relatively more significant than that associated with non-cumulative pollution giving rise to a *R-F behavior*. The large region acts *responsibly*, lowering its emission permits in comparison to the smaller region (see (40)).[21] This happens because, as the number of firms increases, the region realizes that it has more responsibility in total emissions. On the other hand, the less industrialized region can afford to emit more because its impact on the environment is lower, and because the more industrialized region pollutes less, thus behaving like a *freerider*.

A low pollution damage ($\sigma_z < \hat{\sigma}_z$) gives rise to an *I-H behavior*. The more industrialized region benefits more from increasing its emissions (and production) than from reducing pollution. As a result, it will adopt an environmentally *irresponsible* behavior by increasing its emissions compared to the other region (see (39)). On the other hand, the less industrial-

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[21] For brevity in the language we will use large (small) region to refer to the region with the largest (smallest) number of firms, that is, the more (less) industrialized region.

ized region is negatively affected by the increase in emissions from the more industrialized region. Then, it will be forced to reduce its emissions or increase them by a smaller amount, behaving as a *hostage* of the decisions of the large region. This behavior requires that the price gap narrows.

Note also that the threshold $\hat{\sigma}_z$ increases with the discount rate (ρ) and the natural regeneration of the environment (θ). That is, when the decision-makers are less patient (high ρ) or when the environment regenerates faster (high θ), regional governments give less importance to long-term impact of emissions. Then, in order to adopt an environmentally responsible behavior the damage of pollution, σ_z, must be even higher.

Proposition 3 *The steady state of the pollution stock, given by*

$$Z^* = n^w \frac{\alpha \sigma_Q (\theta + \rho) \chi + \frac{\left\{\beta - \delta + \delta n^w \left[s_n^2 + (1-s_n)^2 \right]\right\}(\theta + \rho)\chi - 2 F_1^{Sym} n^w [2(\beta - \delta) + \delta n^w] s_n (1 - s_n)}{2 [2(\beta - \delta) + \delta n^w]} \tau}{\left(\theta \chi - F_1^{Sym} n^w \right) \left[(\theta + \rho) \chi - F_1^{Sym} n^w \right]}, \quad (44)$$

is globally asymptotically stable. Furthermore, Z^* achieves its global minimum (maximum) value when $s_n = 1/2$ if $\sigma_z < \hat{\sigma}_z$ ($\sigma_z > \hat{\sigma}_z$), with $\hat{\sigma}_z$ defined in (41). Moreover if $s_n = 1/2$, then

$$Z^* = Z^{*Sym} = \frac{n^w}{\theta \chi - n^w F_1^{Sym}} \left(\frac{\alpha \sigma_Q (\theta + \rho) \chi}{(\theta + \rho) \chi - n^w F_1^{Sym}} + \frac{\tau}{4} \right). \quad (45)$$

Proof See Appendix A (online appendix).   □

Corollary *The steady-state pollution stock (Z^*) depends on the behavioral pattern adopted by the regions (R-F or I-H) as well as on the industrial concentration (s_n):*

- Under a R-F behavior ($\sigma_z > \hat{\sigma}_z$), greater industrial concentration results in lower Z^*,
- while under an I-H behavior ($\sigma_z < \hat{\sigma}_z$), greater industrial concentration leads to higher Z^*.

When the cumulative pollution stock is more harmful ($\sigma_z > \hat{\sigma}_z$), the large region behaves responsibly polluting less than in the symmetric case, while the small region will pollute more than in the symmetric case. As s_n further increases, more firms leave the less industrialized region and settle in the region with an increasing share of firms, going from emitting $e_{1-s_n}(Z)$ to emitting $e_{s_n}(Z)$, being greater and lower than $e^{Sym}(Z)$, respectively. As a consequence of having an increasing number of firms that pollute less, Z^* will decrease as the industry concentrates. According to Corollary 3, being a responsible more industrialized region implies having higher emission permit price. The opposite occurs when pollution is less harmful ($\sigma_z < \hat{\sigma}_z$). As the number of firms increases, more firms go from emitting $e_{1-s_n}(Z)$ to emitting $e_{s_n}(Z)$, being lower and greater than $e^{Sym}(Z)$, respectively; thus, Z^* will increase as the industry concentrates.

Additionally, the steady state of the pollution stock (44) always increases with transport costs.

Proposition 4 *Social welfare of the two regions at the Nash equilibrium depends on the initial level of pollution Z_0 and on the distribution of firms in the following way:*

$$V_1(Z_0) = \frac{F_1^{Sym}}{2}Z_0^2 + F_2(s_n)Z_0 + F_3(s_n),$$

$$V_2(Z_0) = \frac{F_1^{Sym}}{2}Z_0^2 + F_2(1-s_n)Z_0 + F_3(1-s_n),$$

where s_n is the share of firms in Region 1 and, hence, $1-s_n$ the share in region 2. Constant F_1^{Sym}, given in (35), is independent of s_n, and

$$F_2(s_n) = F_2^{Sym}\left(1 - \frac{\beta - \delta + \delta n^w}{2\alpha\sigma_Q(2(\beta-\delta)+\delta n^w)}(1-2s_n)\tau\right), \tag{4}$$

with F_2^{Sym} given in (37). The expression of $F_3(s_n)$ is obtained directly from (71) and (72) (in the online appendix), although its explicit form is rather cumbersome.

Proof It is straightforward from the proof of Proposition 1. □

Consequently, even when the regions have a different share of firms, $s_n \neq 1/2$, if there were no trade costs ($\tau=0$), both regions would achieve the same welfare, equal to that attained in the symmetric case. If there are trade costs, $\tau>0$, the welfare in each region depends on the share of firms settled in that region. In the following proposition, to ease the presentation of the results, we denote by V_{s_n} social welfare at the Nash equilibrium in a region with a share of firms s_n.

Proposition 5 *If the industry is dispersed, $s_n \in (0,1)$, then there is a threshold for the pollution damage, $\tilde{\sigma}_z(s_n)$ (with $\hat{\sigma}_z < \tilde{\sigma}_z(s_n)$, $d\tilde{\sigma}_z/ds_n > 0$ and $\lim_{s_n \to 0} \tilde{\sigma}_z(s_n) = \lim_{s_n \to 1} \tilde{\sigma}_z(s_n) = \infty$), such that:*

- *If $\sigma_z < \tilde{\sigma}_z(s_n)$, then the region with the larger share of firms attains the highest level of optimal welfare, that is:*

$$V_{1-s_n}(Z^*) < V_{s_n}(Z^*) \quad \text{if and only if} \quad s_n > 1/2.$$

- *If $\sigma_z > \tilde{\sigma}_z(s_n)$, then the region with the larger share of firms attains the lowest level of optimal welfare, that is:*

$$V_{s_n}(Z^*) < V_{1-s_n}(Z^*) \quad \text{if and only if} \quad s_n > 1/2.$$

However, if the industry is completely agglomerated in one region ($s_n = 0$ or $s_n = 1$) regardless of the pollution damage σ_z,

$$V_0(Z^*) < V_1(Z^*).$$

Proof See Appendix A (online appendix). □

Once the steady state of the pollution stock is reached, Z^*, if the industry is fully agglomerated, the core region will unambiguously enjoy greater welfare than the peripheral region. On the other hand, when the industrial activity is equally distributed, both regions attain the same welfare. Between these two extreme scenarios, when firms are operating in both regions, but the distribution is not identical, the more industrialized region would achieve greater welfare if the pollution damage is not too high ($\sigma_z < \tilde{\sigma}_z(s_n)$). Otherwise, the more industrialized region will attain lower welfare.

As a first conclusion derived from our analysis, we have that when a I-H behavior arises ($\sigma_z < \hat{\sigma}_z < \tilde{\sigma}_z(s_n)$), the irresponsible large region always attains higher social welfare. The small hostage region always loses. However, when a R-F behavior arises, there are two possible outcomes regarding welfare. On the one hand, if the pollution damage is not too high ($\hat{\sigma}_z < \sigma_z < \tilde{\sigma}_z(s_n)$), the responsible region maintains a larger welfare than the freerider. On the other hand, if the pollution damage is sufficiently high ($\hat{\sigma}_z < \tilde{\sigma}_z(s_n) < \sigma_z$), the small freerider region attains a larger welfare. Finally, when the industry is fully agglomerated in one region ($\tilde{\sigma}_z(s_n) \to \infty$), the large region (responsible or irresponsible) always reaches the largest welfare.

A second conclusion is that for the small freerider region to win, the pollution damage has to be high enough. This implies that the large region must behave in a sufficiently responsible manner. The more responsible the large region is, the more benefit the small freerider region can obtain, and therefore, the greater the possibilities of attaining greater welfare. Additionally, as the number of firms increases in the large region, for the freerider to maintain the higher welfare, a higher pollution damage is needed. That is, for the freerider to keep winning in terms of welfare, the large growing region must behave even more responsible, such that the shrinking freerider region can compensate the lower number of industrial firms with a sufficiently high increase in its emissions.

However, when the industry is fully agglomerated, there is no pollution damage that can ensure the freerider wins in terms of welfare, simply because there are no industrial firms in the small region that can benefit from the higher emissions (or the lower price of the emission permits). Therefore, when the industry is fully agglomerated, the large region (responsible or irresponsible) always attains greater welfare.

4 Spatial Distribution of Firms: Equilibria and Stability

This section deals with the second stage of the two-stage procedure proposed to study the model. In the previous section, in the first stage, we assumed that the distribution of firms, s_n, was exogenously given. However, in the economic geography literature, firm distributions are typically treated as endogenous. In this section we focus on the long-run equilibria for the distribution of firms, and the pollution stock, and their stability properties. We wonder how the emission strategies obtained in the last section affect the long-run allocation of firms.

For the purpose of this section, we assume that the equilibrium emission strategies are functions of the pollution stock (Z), and the distribution of firms (s_n), as given by (33). Therefore, pollution dynamics (21), is considered here as a two-variable function:

$$\dot{Z} = n^w [s_n e_1(Z, s_n) + (1 - s_n) e_2(Z, s_n)] - \theta Z. \tag{46}$$

In the long run, capital moves from one region to the other according to nominal profits, and hence, the following differential equation describes how the share of firms, s_n, evolves over time:[22]

$$\dot{s}_n = s_n \left(1 - s_n\right) \left[\pi_1(Z, s_n) - \pi_2(Z, s_n)\right], \tag{47}$$

where $\pi_j(Z, s_n)$ can be obtained from expressions (10)-(12), and (33).

In the long-run equilibria, firms will no longer have any incentive to change their location, $\dot{s}_n = 0$, and the stock of pollution will remain constant at its steady state, $\dot{Z} = 0$. We denote the steady state values by s_n^* and Z^*, respectively.

Proposition 6 *There always exist at least three equilibria for all $\tau < \bar{\tau}$:*

- A symmetric equilibrium, characterized by an identical emission permit policy in the two regions and an identical level of welfare, with:

$$s_n^* = 1/2, \qquad Z^* = Z^{*^{Sym}} \quad \text{and} \quad V_{1/2}(Z^*) = V^{Sym}(Z^*). \tag{48}$$

- And two agglomeration equilibria with greater welfare in the industrialized region:

$$s_n^* = 0, 1, \qquad Z^* = Z^{*^{Agg}} = \frac{n^w \left(\theta + \rho\right) \chi \left[\alpha \sigma_Q + \frac{1}{2} \frac{\beta - \delta + \delta n^w}{2(\beta - \delta) + \delta n^w} \tau\right]}{\left(-n^w F_1 + \theta \chi\right) \left[-n^w F_1 + (\theta + \rho) \chi\right]} \quad \text{and} \quad V_0(Z^*) < V_1(Z^*),$$

where $Z^{*^{Sym}}$ is given by expression (45), and $\bar{\tau}$ is the maximum transport cost that allows trade.[23] Furthermore, from Proposition 3: $Z^{*^{Agg}} \gtrless Z^{*^{Sym}}$ when $\hat{\sigma}_z \gtrless \sigma_z$.

Proof See Appendix C (online appendix).   □

Note that $Z^{*^{Agg}}$ is equal to the steady state of the pollution stock in (44) when $s_n = 0, 1$. At the symmetric equilibrium, the industrial activity is equally distributed between the regions and firms emit according to (34). While at the agglomeration equilibria, one of the regions becomes an industrial core, with all the industrial firms, and the other region (periphery) produces only the homogeneous good, with their equilibrium emission strategies given by (33) with $s_n = 1$ ($s_n = 0$). Moreover, since the pollution at the steady state depends on the value of the pollution damage, σ_z, if pollution damage is low ($\sigma_z < \hat{\sigma}_z$), an I-H behavior arises, industrial concentration generates higher levels of emissions and the steady state of pollution will be larger at the agglomeration equilibria. But, if the damage from pollution is high ($\sigma_z > \hat{\sigma}_z$), R-F behavior appears and the steady state of pollution will be higher at the symmetric equilibrium (Proposition 3).

As the following propositions show the stability of the equilibria of the dynamic system (46)-(47) depends on pollution damage (σ_z) and transport costs (τ). Figure 3 illustrates the
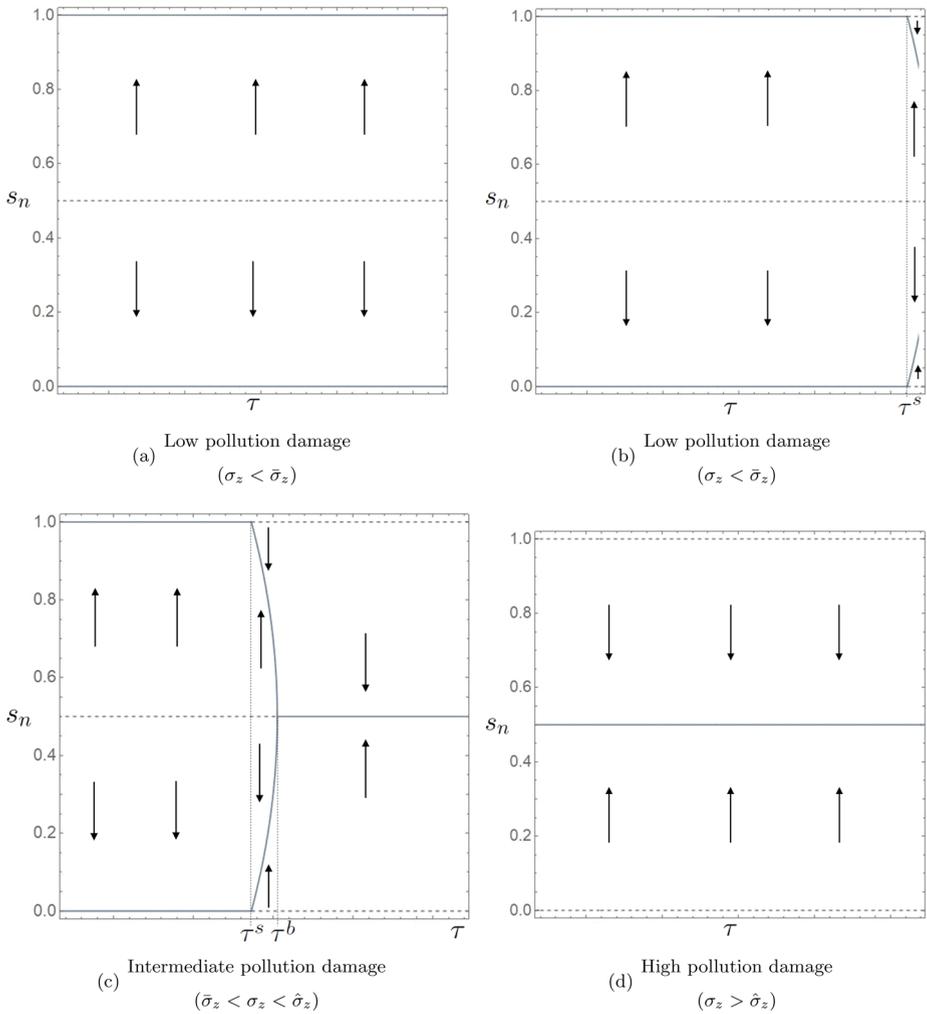

Fig. 3 These diagrams represent the long-run distribution of firms, s_n, for each value of transport costs, τ. Solid and dashed lines represent stable and unstable equilibria, respectively. Arrows depict the basins of attraction of the stable equilibria

equilibrium values of s_n as trade costs τ vary, over the different ranges of pollution damage. Solid (dashed) lines correspond to stable (unstable) equilibria, and arrows depict the basins of attraction of the stable equilibria.

Proposition 7 *When $\sigma_z < \hat{\sigma}_z$, the symmetric equilibrium is stable for $\tau > \tau^b$, and unstable for $\tau < \tau^b$; while the agglomeration equilibria are unstable for $\tau > \tau^s$ and stable for $\tau < \tau^s$, with:*

$$\tau^b = -\frac{\bar{U}_0}{\bar{U}_1(1/2)}, \quad \tau^s = -\frac{\bar{U}_0}{\bar{U}_1(1)} = -\frac{\bar{U}_0}{\bar{U}_1(0)}, \quad and \quad \tau^s < \tau^b, \tag{49}$$

where $\bar{U}_0$ and $\bar{U}_1(s_n)$, are defined in Appendix C, and $\hat{\sigma}_z$ is given by expression (41).

Proof See Appendix C (online appendix).   □

If $\tau \in \left(\tau^s, \tau^b\right)$ the following proposition proves the existence of a supercritical pitchfork bifurcation.

Proposition 8 *When $\sigma_z < \hat{\sigma}_z$, and $\tau \in \left(\tau^s, \tau^b\right)$ there exist two intermediate interior equilibria, equidistant from the symmetric one ($s_n^* = 1/2$), which are locally stable. Moreover, if τ approaches τ^b, these equilibria converge to the symmetric equilibrium, and if τ approaches τ^s, they converge to the agglomeration equilibria. Additionally,*

$$Z^{*^{Sym}} < Z^* < Z^{*^{Agg}} \quad and \quad V_{1-s_n^*}(Z^*) < V_{s_n^*}(Z^*) \quad if \quad s_n^* > 1/2.$$

Proof See Appendix C (online appendix).   □

The results of Propositions 7 and 8 are illustrated in Fig. 3(c). When $\sigma_z < \hat{\sigma}_z$, the symmetric equilibrium is stable for high values of transport costs, while agglomeration equilibria are stable for low values of transport costs. Moreover, Proposition 8 states that when the symmetric equilibrium becomes unstable ($\tau < \tau^b$), two new equidistant interior equilibria appear. That is, as transport costs diminish, the "transition" between the symmetric equilibrium towards the agglomeration equilibria is smooth, since these branches of new equilibria are stable.

However, a particular case arises when the pollution damage parameter, σ_z, is much lower, as the following proposition shows.

Proposition 9 *When $\sigma_z < \bar{\sigma}_z$, the symmetric equilibrium is unstable and the agglomeration equilibria are stable for $\tau < \tau^s$, where τ^s is given by (49), and $\bar{\sigma}_z$ ($\bar{\sigma}_z < \hat{\sigma}_z$) is defined in (96) in Appendix C.*

Proof See Appendix C (online appendix).   □

Figure 3(a) and 3(b) illustrate the results in Proposition 9. These figures emerge as special cases of Proposition 7, with τ^b larger than the maximum transport cost that allows interregional trade in the symmetric equilibrium (i.e. $\bar{\tau}^{Sym} < \tau^b$). The low pollution damage raises the steady-state pollution stock ($Z^{*^{Sym}}$), which reduces the production for export. Ultimately this implies that to maintain trade (and positive levels of production) the maximum transport cost must be lower with a lower pollution damage, such that, with $\sigma_z < \bar{\sigma}_z$, τ^b is unreachable. In Fig. 3(a), τ^s is larger than the transport cost that allows trade at the agglomeration equilibria, while in Fig. 3(b) is smaller.

Interestingly, the results from Propositions 7-9 are not obtained in the standard FC model (Ottaviano et al. 2002). The standard FC model lacks what are called agglomeration forces in the new economic geography literature. One of these agglomerative forces is the market size effect, which measures the willingness of firms to settle where the market is larger, and by doing so attract workers which tend to further increase the size of the market. However, in the FC models workers do not move between regions. Capital is the only mobile factor, and its rewards are spent

in the region of origin of the capitalists. Thus, the allocation of firms does not change either the allocation of expenditure or the size of the market. As a result, the standard FC models, with symmetric regions, always exhibit a stable symmetric equilibrium. In our setting, the strategic interaction between the two regional governments gives rise to an agglomeration force that is absent in the economic geography models.

Proposition 10 *When $\sigma_z > \hat{\sigma}_z$, the symmetric equilibrium is the only stable interior equilibrium and the agglomeration equilibria are unstable for all $\tau < \bar{\tau}$.*

Proof See Appendix C (online appendix).   □

This last case is depicted in Fig. 3(d) and it is the one we would obtain in the standard FC model. In our model, depending on the value of parameter σ_z, three cases can be distinguished: low pollution damage ($\sigma_z < \bar{\sigma}_z$), intermediate pollution damage ($\bar{\sigma}_z < \sigma_z < \hat{\sigma}_z$), and high pollution damage ($\sigma_z > \hat{\sigma}_z$), as Figure 3 represents.

The stability of the equilibria can be explained by differentiating expression (47) with respect to s_n, and evaluating at the symmetric equilibrium. That is, the change of the profit differential when a firm located in region 1 decides to move towards region 2:[24]

$$\left.\frac{d\dot{s}_n}{ds_n}\right|_{s_n=1/2} = \frac{1}{4}\left(\frac{d\pi_1}{ds_n} - \frac{d\pi_2}{ds_n}\right) = \frac{1}{4}\left.\frac{\partial(\pi_1 - \pi_2)}{\partial s_n}\right|_{\bar{\gamma}} + \frac{1}{4}\left[\frac{\partial(\pi_1 - \pi_2)}{\partial\gamma_1}\frac{\partial\gamma_1}{\partial s_n} + \frac{\partial(\pi_1 - \pi_2)}{\partial\gamma_2}\frac{\partial\gamma_2}{\partial s_n}\right], \quad (50)$$

where the first term can be obtained using the optimal demands (10)-(11), and taking the emission permit prices as given ($\bar{\gamma}$),

$$\left.\frac{\partial(\pi_1 - \pi_2)}{\partial s_n}\right|_{\bar{\gamma}} = -\frac{2\delta n^w \tau^2}{(\beta - \delta)\left[2(\beta - \delta) + \delta n^w\right]} < 0, \quad (51)$$

which is a standard dispersion force in the economic geography models: the competition effect.

Starting from the symmetric equilibrium, a negative sign of expression (50) implies that when a firm changes its location, the profit differential reduces, becoming negative, and the defecting firm will have incentives to return to region 2; therefore, the symmetric equilibrium is stable. However, if the sign is positive, by moving from region 2 towards region 1, the defecting firm is increasing its profit differential. Therefore, more firms will find it profitable to change their location, and the symmetric equilibrium would be unstable.

The following Remark summarizes the forces at work.

Remark The following three forces shape the long-run distribution of firms:

- Competition effect: a larger number of firms increases competition and reduces profits, driving firms away (dispersion force).
- R-F behavior: a reduction in the large region's emissions raises the price of permits, thereby decreasing profits and driving firms away (dispersion force).

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[24] The proof of Proposition 7 in Appendix C (online appendix) shows why it is sufficient to look at this derivative.

- I-H behavior: higher emissions in the large region lead to a lower price of permits, thereby increasing profits and attracting more firms (agglomeration force).

Considering the three forces in play, let us analyze Propositions 7 to 10 through Fig. 3. When $\sigma_z > \hat{\sigma}_z$, the pollution damage is high (Fig. 3(d)), only the symmetric equilibrium is stable, and the economic activity is evenly dispersed across the regions. On the one hand, the competition effect discourages agglomeration. On the other hand, a high pollution damage results in a R-F behavior, which reinforces the competition effect, making the symmetric equilibrium stable. Therefore, welfare is equalized between the regions, and the steady state of pollution will be the highest possible.[25]

When $\sigma_z < \bar{\sigma}_z$, pollution damage is low (Figs. 3(a) and 3(b)), the agglomeration equilibria are stable for $\tau < \tau^s$, and the economic activity is agglomerated in one of the regions. Given that $\bar{\sigma}_z < \hat{\sigma}_z$, regions adopt an I-H behavior, that encourages industrial concentration. Since the pollution damage is very low, the effect of the I-H behavior completely overcomes the competition effect, giving rise to stable agglomeration equilibria. The region that becomes the industrial core will enjoy greater welfare compared to its neighbor. The steady state of pollution will be the highest possible.

When $\bar{\sigma}_z < \sigma_z < \hat{\sigma}_z$, pollution damage is intermediate (Fig. 3(c)), a mix situation appears, where the symmetric equilibrium is stable for high transport costs ($\tau > \tau^b$) and the agglomeration equilibria are stable for low transport costs ($\tau < \tau^s$). In this scenario regions adopt an I-H behavior that favors the agglomeration of the industry. However, due to the intermediate pollution damage, the agglomeration force is weaker than in the previous case (when $\sigma_z < \bar{\sigma}_z$). On the other hand, the competition effect favors the dispersion of the industry, and, according to expression (51), its strength is proportional to the transport costs, τ. The interaction of these two forces determines the final industrial distribution. When transport costs are high, the competition effect is stronger and overcomes the agglomeration force, leading to a stable symmetric equilibrium, identical welfare in both regions, and to a minimum at $Z^{*^{Sym}}$. However, when transport costs are low, the I-H behavior is stronger, causing the agglomeration of the firms. Then, the irresponsible region ends up attaining the greatest welfare, and $Z^{*^{Agg}}$ corresponds to a maximum.

Furthermore, the agglomeration forces arising from I-H behavior are stronger near the symmetric equilibrium, and weaker near the agglomeration equilibria. On the other hand, as already pointed out, the higher the transport costs, the stronger the competition effect. These two features explain why the symmetric equilibrium will lose its stability before the agglomeration equilibria become stable. Ultimately, this gives rise to the emergence of two additional stable interior equilibria, which in the economic geography literature is known as a smooth transition from the symmetric equilibrium to the agglomeration equilibria. In these interior equilibria, the large industrial region enjoys greater welfare compared to the small region, while the steady state of pollution will be larger than in the symmetric case, but lower than in the full agglomeration scenario.

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[25] When we say the highest or the lowest we are exclusively referring to a given value of τ and σ_z. It should not be seen as a comparison between scenarios with different pollution damages or transport costs.

5 Concluding Remarks

With the aim of studying the environmental policy of two governments that interact strategically and dynamically, we developed a simple linear FC model which incorporates non-cumulative pollution linked to the homogeneous good, and cumulative transboundary pollution which is a by-product of the industrial sector. This last feature is a novelty in the economic geography models. The love for variety inherited from the NEG framework gives rise to trade patterns which vary with transport costs. As a result of this, social welfare in each region depends on their own production (emissions) and on the production coming from the foreign region. Additionally, incomes also exhibit this interaction due to the possibility of exporting industrial goods. In the dynamic game literature, this interdependence and the role of transport costs are much less explored. In this setup, regional governments have the authority to impose an environmental policy by creating a market of emission permits in order to maximize the welfare of their population, taking into account the environmental damages from both types of pollution, non-cumulative and cumulative, as well as the evolution over time of the cumulative pollution stock.

The model is formulated as a dynamic game, and the game is solved following a two-stage procedure. In the first stage, the governments determine the equilibrium emission permits with the number of firms taken as given. In the second stage, the long-run distribution of firms is determined with the equilibrium decisions of the governments from the first stage taken as inputs. We asked two main research questions: 1. How do the regional governments face the environmental challenge?; that is, which are their environmental policies? 2. How do firms react to these policies and how do they affect the distribution of the industry?

With respect to our first question, we found that the equilibrium emission permit strategies present two distinct behaviors depending on the pollution damage parameter. When pollution damage is high, a responsible-freerider (R-F) behavior emerges. The more industrialized region, the region with the largest share of the industry, acts *responsibly* by reducing its emission permits per firm, while the less industrialized region behaves as a *freerider*, increasing its emission permits per firm. This behavior implies that the stock of pollution at the steady state reaches a minimum if firms are completely agglomerated in an industrial core. Likewise, when pollution damage is low, an irresponsible-hostage (I-H) behavior emerges. The more industrialized region *irresponsibly* increases its emissions, while the less industrialized region is forced to reduce theirs (*hostage*). In this case, the long-run pollution stock is at the maximum when the industry is agglomerated in one of the regions.

Regarding our second question about the firms' reaction, we found that the industrial allocation is highly influenced by transport costs and by the behavior of the regional governments. The underlying economic geography model gives rise to the standard competition effect. Firms have incentives to stay disperse in order to avoid high levels of competition. Meanwhile, the behavior arising from strategic interplay can act as a dispersion or an agglomeration force. If the large region behaves *responsibly* (R-F), the price of the emission permits increases which cuts the benefits. In this case there is no incentive for firms to agglomerate, and the competition effect is reinforced, giving rise to a stable symmetric equilibrium. However, if the government in the more industrialized region acts *irresponsibly* (I-H), the price of its emission permits reduces, allowing for greater benefits. As a consequence, more firms will be willing to move to the large region, triggering an agglomeration process. This leads to scenarios where industrial activity becomes fully or partially agglomerated.

The existence of stable agglomeration equilibria and an unstable symmetric equilibrium is new in FC models. In the standard model, the absence of expenditure reallocation prevents agglomeration forces from emerging. As a result, to avoid high competition, firms always disperse between locations. In our setting, the strategic interaction between the two regional governments gives rise to a new agglomeration force, that differs from that of the economic geography models. This force emerges due to the irresponsible (I-H) behavior of the government of the large industrialized region.

For the case of the I-H behavior our results show that the large irresponsible region always obtains more welfare than the small hostage region. However, when regions follow a R-F behavior, the small freerider region can, in same cases, exploit the responsible behavior of its neighbor to gain a welfare advantage.

Our analysis complements previous papers that analyze emission quotas in a dynamic setting. Our model has been enriched by integrating equilibrium market interactions into the transboundary pollution dynamic game. A first possible extension of our model could be the inclusion of pollution abatement activities, for example, of an end-of-pipe type. Another extension could be to assess the environmental damage of the transport sector, which is absent in our model. These extensions are part of our research agenda.

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Data availability We do not analyse or generate any datasets, because our work proceeds within a theoretical and mathematical approach.

Declarations

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