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ARTICLE

# Modelling and localizing a stem taper function for *Pinus radiata* in Spain

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**Abstract:** The parsimonious taper function proposed by Riemer et al. (1995. Allg. Forst.- Jagdztg. **166**(7): 144–147) was fitted for radiata pine (*Pinus radiata* D. Don) stems in Spain by using a nonlinear mixed modelling approach. Eight candidate models (all possible expansion combinations of the three fixed parameters with random effects) were assessed, and the mixed model with three random effects performed the best according to the goodness-of-fit statistics. An evaluation data set was used to assess the performance of these models in predicting stem diameter along the bole, as well as total stem volume. Four prediction approaches were compared: one subject (tree) specific (SS) and three population specific (ordinary least squares (OLS), mean (M), and population averaged (PA)). The SS responses for a tree were estimated from a prior stem diameter measurement available for that tree, whereas OLS, M, and PA were obtained from the fixed-effects model, from the fixed parameters of mixed-effects models, and by computing mean predictions from the mixed-effects models over the distribution of random effects, respectively. Prediction errors were greater for the M and PA responses than for the OLS response, and therefore, from the prediction point of view, the use of the mixed-effects models is not recommended when an additional stem diameter measurement is not available. The mixed model with three random effects was also selected as the best model for SS estimations. Measurement of an additional stem diameter at a relative tree height of approximately 0.5 provided the best calibrations for stem diameters along the bole and total stem volume predictions. The SS approach increased the flexibility and efficiency of the selected mixed-effects model for localized predictions and thus improved the overall predictive capacity of the base model.

*Key words:* mixed-effects model, calibration, taper function, radiata pine, Spain.

**Résumé :** L'équation de défilement parcimonieuse proposée par Riemer et al. (1995. Allg. Forst.- Jagdztg. **166**(7): 144–147) a été étalonnée pour des tiges de pin de Monterey (*Pinus radiata* D. Don) en Espagne en utilisant une approche de modélisation non linéaire à effets mixtes. Huit modèles candidats (toutes les combinaisons possibles d'expansion des trois paramètres fixes avec effets aléatoires) ont été évalués. Le modèle mixte avec trois effets aléatoires s'est avéré le meilleur selon les statistiques de qualité d'ajustement. Un jeu de données de validation a été utilisé pour évaluer la performance de ces modèles à prédire les diamètres de la tige le long du tronc ainsi que le volume total de la tige. Quatre approches de prédiction ont été comparées : une approche spécifique à un sujet (l'arbre) (SS) et trois approches spécifiques à la population (moindres carrés ordinaires (MCO), moyenne (M) et moyenne par population (MP)). Les résultats de l'approche SS pour un arbre ont été estimés à partir d'une mesure préalable du diamètre de la tige disponible pour cet arbre, tandis que ceux des approches MCO, M et MP ont été obtenus respectivement avec le modèle à effets fixes, à partir des paramètres fixes des modèles à effets mixtes, et par le calcul des prédictions moyennes des modèles à effets mixtes pour la distribution des effets aléatoires. Les erreurs de prédiction étaient plus grandes pour les estimations faites avec M et MP que pour les estimations faites avec MCO. Donc, du point de vue de la prédiction, l'utilisation des modèles à effets mixtes n'est pas recommandée quand une mesure supplémentaire de diamètre de la tige n'est pas disponible. Le modèle mixte avec trois effets aléatoires a également été choisi comme le meilleur modèle pour les estimations au moyen de l'approche SS. La mesure d'un diamètre supplémentaire de la tige à une hauteur relative de l'arbre de l'ordre de 0,5 a produit les meilleurs étalonnages pour prédire les diamètres le long du tronc et le volume total de la tige. L'approche SS a augmenté la flexibilité et l'efficacité du modèle à effets mixtes sélectionné pour les prévisions locales et a ainsi amélioré la capacité prédictive globale du modèle de base. [Traduit par la Rédaction]

*Mots-clés :* modèle non linéaire à effets mixtes, étalonnage, équation de défilement, pin de Monterey, Espagne.

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## Introduction

Radiata pine (*Pinus radiata* D. Don) plantations occupy approximately 300 000 ha (Ministerio de Medio Ambiente (MMA) 2001) in Spain, especially in the Atlantic region in the north of the country. The oldest stands were planted in the 1920s, and plantations are currently being established at a rate of more than 6000 ha·year$^{-1}$ (Álvarez-Álvarez 2004). Radiata pine is, therefore, one of the most commonly used species in reforestation programmes, particularly those involving abandoned agricultural land. At present, radiata pine timber (1.55 million m$^3$·year$^{-1}$) represents 13% of the total volume of timber harvested annually in Spain (Ministerio de Agricultura, Alimentación y Medio Ambiente (MAGRAMA) 2010), which indicates the importance of this species in the country's forestry industry.

Because of the economic importance of radiata pine, several static and dynamic whole-stand growth models (Canga 2007; Castedo-Dorado et al. 2007; Sevillano-Marco et al. 2009) and individual-tree growth models (Crecente-Campo et al. 2012) have been developed for the species. Taper equations are useful, flexible tools for estimating total and merchantable stem volumes in both growth projection models and forest inventories.

Foresters have used many different models of varying complexity in attempting to describe tree taper (Burkhart and Tomé 2012, ch. 2), which indicates both the importance of the subject in forest management and the lack of a single, widely applied tree taper model. Recent research has focused on the development of new model forms and on approaches to account for observed between-tree variability in stem profile for certain species. Traditional approaches consider the inclusion of auxiliary variables (covariates) in the model structure (Burkhart and Tomé 2012, p. 24). The covariates that were used to increase the performance of stem profile models included crown dimensions (e.g., Leites and Robinson 2004), stand and site variables (e.g., Sharma and Zhang 2004; Calama and Montero 2006), and additional stem diameter measurements apart from diameter at breast height (1.3 m; e.g., Trincado and Burkhart 2006; Yang et al. 2009*a*). The latter is considered one of the best approaches because, in general, little improvement in model performance is achieved by including both crown dimensions and stand or site variables as covariates (Muhairwe et al. 1994). Moreover, recent technological advances (e.g., telescopic and laser dendrometers) enable accurate upper-stem diameter measurements at a reasonable cost (e.g., Rodríguez et al. 2014).

The mixed-effects modelling approach has become a widely used statistical technique for developing taper functions. The use of fixed-effects parameters (hereafter called fixed parameters) and random effects enables the between-tree and within-tree variability of the observed data (repeated measurements made along the stem) to be taken into account. This approach also enables prediction of a subject-specific (SS) response (obtained using both fixed parameters and random effects) for a new tree if at least one stem diameter measurement (in addition to diameter at breast height) from this tree is available (this procedure is generally referred to as calibrating or localizing the stem taper function). In the absence of any additional data for use in calibration, two obvious procedures emerge: (*i*) use the fixed part of the mixed-effects model (with the random effects assumed to be zero), which provides the mean (M) response, or (*ii*) use a fixed-effects model fitted by ordinary least squares (OLS), which provides an OLS response. Additionally, population-specific responses can be computed from mixed-effects models by taking the average prediction over the multivariate normal distribution of random effects (e.g., McCulloch and Searle 2001). These predictions provide the so-called population-averaged (PA) response (Fortin 2013).

Although SS responses appear to provide the most accurate and precise estimations, their predictive ability depends on both the number and the position of the stem diameter measurements selected. Comparison of the predictive ability of the four approaches

**Fig. 1.** Distribution of radiata pine stands in Spain.


(OLS, M, PA, and SS), the latter of which is assessed for stem diameter measurements at various relative heights, enables selection of the most appropriate approach from a practical point of view.

The parsimonious variable-form taper function developed by Riemer et al. (1995) has performed adequately in describing radiata pine stem profiles in northwestern Spain (Castedo-Dorado and Álvarez-González 2000). In the aforementioned study, model fitting was carried out by OLS; however, within-tree autocorrelation errors were not modelled, and the usefulness of additional stem diameters for improving the predictive capability of the taper function was not demonstrated. Therefore, the objectives of this study were as follows: (*i*) to fit the variable-form taper model proposed by Riemer et al. (1995) by using a nonlinear mixed-effects modelling approach with tenable statistical assumptions; (*ii*) to compare the predictive ability of OLS, M, PA, and SS approaches for predicting stem diameter along the bole and total stem volume; and (*iii*) to determine the best approach for practical purposes on the basis of the results obtained from an evaluation data set.

## Material and methods

### Data

Taper data were measured on 1395 trees from 369 sites (between two and five trees per site) located throughout the areas where most radiata pine plantations are established in Spain (Galicia, Asturias, El Bierzo, Basque Country, and Navarre) (Fig. 1). Sample trees were selected in both thinned and unthinned stands that were 5–62 years old to cover the existing range of site qualities. Diameter at breast height (*D*, cm; all diameters were measured outside the bark unless otherwise noted) was measured to the nearest 0.1 cm in each tree. The trees were later felled, and total tree height (*H*, m) was measured to the nearest 0.01 m. The trees were cut into logs at 1 to 2.5 m intervals for the first 4 to 5 m of bole length and thereafter at 1 to 2 m intervals. In each section, two perpendicular diameters (*d*, cm) were measured to the nearest 0.1 cm, and these measurements were then arithmetically averaged. The height of each section from ground level (*h*, m) was also recorded to the nearest centimetre. Log volumes were calculated as conical frustums, whereas the top section of the tree was treated as a cone. These values were used to obtain the total outside bark stem volume (*V*, m$^3$).

Abnormal data points were visually detected by plotting relative diameter (*d*/*D*) against relative height (*h*/*H*) for all taper measurements. These extreme values were corrected (if they represented mistakes in measuring the bole sections or in the transcription of field notes) or removed (if they were due to deformations caused by large knots or abiotic damage in the stem). Sections corre-

**Table 1.** Summary statistics for the fitting and evaluation data sets.

| | Data set | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | Fitting (928 trees; 10 188 sections) | | | | Evaluation (467 trees; 5733 sections) | | | |
| Variable | Mean | Minimum | Maximum | SD | Mean | Minimum | Maximum | SD |
| No. of sections per tree | 11.0 | 2 | 25 | 3.4 | 12.3 | 2 | 28 | 4.3 |
| $D$ (cm) | 29.4 | 7.2 | 63.2 | 11.9 | 30.0 | 8.2 | 69.7 | 11.8 |
| $H$ (m) | 20.3 | 6.5 | 41.5 | 6.64 | 21.3 | 5.6 | 38.7 | 6.67 |
| $V$ ($m^3$) | 0.771 | 0.0169 | 4.57 | 0.740 | 0.814 | 0.0054 | 4.12 | 0.750 |

**Note:** SD, standard deviation; $D$, diameter at breast height outside bark; $H$, total tree height; $V$, total outside-bark stem volume.



sponding to breast height and total tree height were not considered, because Riemer's model ensures that $d = D$ and $d = 0$ when $h = 1.3$ and $h = H$, respectively. Lastly, a total of 15 921 bole sections were used for analysis.

The selected data were randomly divided by sites: data from 228 sites (928 trees) were used for model fitting, and the data from the remaining 123 sites (467 trees) were used for model evaluation; this data split corresponds to approximately 70% and 30% of the whole-tree data set, respectively. The data set is large enough that the 70% to 30% split used is unlikely to reduce the precision of the parameter estimates relative to the estimates obtained with the model built from the entire data set (e.g., Trincado and Burkhart 2006). Summary statistics including mean, standard deviation, and minimum and maximum values of the main variables for both model fitting and evaluation are presented in Table 1.

**Taper function evaluated**

Although many taper functions have been developed and exhaustively evaluated, it is generally agreed that none of them can be considered "best". In this study, the variable-form model proposed by Riemer et al. (1995) was selected for evaluation for the following reasons: (*i*) a number of successful applications of this function involving different species and stem shapes have been published (Trincado et al. 1996; Hui and von Gadow 1997; Novo et al. 2003; Rojo et al. 2005; Hussein et al. 2008; Rodríguez and Lizarralde 2009); (*ii*) it performed best in a previous study based on some of the present data on radiata pine (Castedo-Dorado and Álvarez-González 2000), explaining more than 98% of the observed variability with no systematic patterns in the residuals; and (*iii*) it is a parsimonious function (it only depends on three fixed parameters), which minimizes the correlation of explanatory variables and thus avoids model-fitting problems within a mixed-effects modelling framework (Temesgen et al. 2008; Yang et al. 2009*a*).

The model proposed by Riemer et al. (1995) is a transformation of the function proposed by Brink and von Gadow (1986), known as the modified Brink function. Based on the assumption that an ideal form of a tree stem is composed by an upper and a lower part, Brink and von Gadow (1986) developed the following three-parameter function for the whole profile:

$$(1) \qquad r = \alpha + (r - \alpha)e^{\beta_1(1.3-h)} - \frac{\beta_1\alpha}{\beta_1 + \beta_2} \times [e^{\beta_2(h-H)} - e^{\beta_2(1.3-H)+\beta_1(1.3-h)}] + \varepsilon$$

where $r$ is the top radius (measured outside the bark) at height $h$ (cm), $h$ is the height above ground to $r$ (m), $\varepsilon$ is the model error, and $\alpha$, $\beta_1$, and $\beta_2$ are the model parameters to be estimated. Parameters $\beta_1$ and $\beta_2$ describe the external bending of the stem curve in the lower and upper parts of the stem, respectively (Brink and von Gadow 1986; Hui 1997).

Riemer et al. (1995) modified the model by including terms to ensure that the diameter at tree top and at breast height equaled zero and $D$, respectively, and proposed the following function:

$$(2) \qquad \frac{d}{2} = \frac{\alpha}{1 - e^{\beta_2(1.3-H)}} + \left(\frac{D}{2} - \alpha\right)\left[1 - \frac{1}{1 - e^{\beta_1(1.3-H)}}\right] + e^{-\beta_1 h}\left[\frac{\left(\frac{D}{2} - \alpha\right)e^{1.3\beta_1}}{1 - e^{\beta_1(1.3-H)}}\right] - e^{\beta_2 h}\left[\frac{\alpha e^{-\beta_2 H}}{1 - e^{\beta_2(1.3-H)}}\right] + \varepsilon$$

Hui (1997) and Hui and von Gadow (1997) subsequently expressed $\alpha$ as a linear function of the diameter at breast height ($\beta_0 D$). This resulted in the following modified version of the Riemer et al. (1995) function, which was used in the present study:

$$(3) \qquad d = 2\left\{\frac{\beta_0 D}{1 - e^{\beta_2(1.3-H)}} + \left(\frac{D}{2} - \beta_0 D\right)\left[1 - \frac{1}{1 - e^{\beta_1(1.3-H)}}\right] + e^{-\beta_1 h}\left[\frac{\left(\frac{D}{2} - \beta_0 D\right)e^{1.3\beta_1}}{1 - e^{\beta_1(1.3-H)}}\right] - e^{\beta_2 h}\left[\frac{\beta_0 D e^{-\beta_2 H}}{1 - e^{\beta_2(1.3-H)}}\right]\right\} + \varepsilon$$

Some authors (e.g., Gregoire et al. 2000; de-Miguel et al. 2012) state that taper functions should be fitted for $d^2$ rather than $d$, because the former provides less biased estimations of total or merchantable volumes. Nevertheless, we considered $d$ as the dependent variable, because estimation of height $h$ at specified top stem diameters is a key step in classifying stem portions by top diameter limits and log lengths according to the requirements of different industrial destinations (peeling, sawing, pulp wood, etc.).

**Fitting mixed-effects and fixed-effects models**

The main characteristic of mixed-effects models within a taper modelling framework is that they allow the parameter vector of the taper function to vary between subjects, i.e., regression coefficients are split into fixed parameters (common to the population) and random effects (specific to the *i*th subject). The vector of random effects ($\boldsymbol{b}_i$) is assumed to follow a multivariate normal distribution with mean zero and positive variance–covariance matrix **D**.

The empirical data used in the current study consisted of destructively sampled trees from different sites; thus, the taper function could be formulated as a multilevel mixed model, including random effects at both tree and site levels. Nevertheless, the site level was not considered because preliminary analysis (results not shown) indicated that the variance estimates for the random effects for site level were much lower than the corresponding variance estimates for tree level (between 2.4 and 3.9 times in the case of the selected mixed model with three random effects).

When developing a mixed-effects model, at least three steps are necessary: (*i*) specification of the nature of the parameters of the model, i.e., fixed parameters plus random effects or purely fixed parameters; (*ii*) determination of the structure of the between-tree variance–covariance matrix **D**; and (*iii*) determination of an appropriate within-tree variance–covariance structure for the matrix of the error term $\mathbf{R}_i$.

In this study, all possible combinations of fixed parameters and random effects were analysed. After model fitting, the best model specification was determined using Akaike's information criterion (AIC; Akaike 1974) and Schwarz's Bayesian information criterion (BIC; Schwarz 1978).

Because the data used for taper modelling consist of repeated measurements along tree stems, correlations between observations from the same tree are expected. Inclusion of random effects allows for the explanation of the variation in stem taper among trees (between-tree variability), thus accounting (at least partly) for this autocorrelation. We did not assume any special form for this matrix; hence, it was only specified to be a general symmetric positive-definite system (Pinheiro and Bates 2000, pp. 157–158).

To define the variance–covariance matrix of the error term $\mathbf{R}_i$, two components must be addressed: autocorrelation and heteroscedasticity. Although several studies have shown that within-tree correlation can be completely accounted for by including random effects alone (e.g., Yang et al. 2009*a*), this is not always the case (e.g., Trincado and Burkhart 2006; Gómez-García et al. 2013). Therefore, when necessary, we considered the inclusion of a continuous autoregressive covariance structure (CAR(1)), which is appropriate for unequally spaced measurements, within the mixed-modelling framework.

On the other hand, evaluation of heteroscedasticity was carried out by visual analysis of the residuals in regression fits. Preliminary analysis of these residuals after including random effects in the fixed-effects model did not reveal any heteroscedasticity. Therefore, no weighting factor was used to balance the error variance.

The variance–covariance parameters associated with the vector of random effects ($\boldsymbol{b}_i$) and the vector of fixed parameters ($\boldsymbol{\beta}$) were estimated with the nlme function of the nlme package (Pinheiro et al. 2013) of R statistical software (R Core Team 2013), which requires the underlying model to be linearized using a first-order Taylor series expansion around the random effects. The expansion method available in the nlme package is the first-order conditional expectation (FOCE) approximation of Lindstrom and Bates (1990), in which linearization is done by setting the random parameters to their current estimated best linear unbiased predictors (Yang and Huang 2013). For SS predictions on a new tree, the FOCE method has the advantage that it directly adds random effects to the corresponding fixed parameters in the underlying base model, thus maintaining the base model form and its biological expectations (Yang and Huang 2013).

The nlme function enables the use of two model-fitting procedures: maximum likelihood and restricted maximum likelihood. The former was used for comparison of various model forms, because it provides comparable maximum likelihood values (AIC and BIC statistics) for alternative models. The latter was used to obtain the parameter estimates in the fitting phase, because restricted maximum likelihood estimates of the variance components are not biased (Littell et al. 2006). The nls function of R (R Core Team 2013) was used for OLS fitting.

### Predictive ability of the OLS, M, PA, and SS responses

Because the main purpose of this study was to provide the best taper model for forestry practice in predicting diameter along the bole and total stem volume, we assessed the performance of the Riemer's function by means of the OLS, M, PA, and SS responses for the evaluation data set.

The OLS response was obtained from the estimates of eq. 3 fitted by OLS. The fixed parameters of the mixed-effects models were used to obtain the estimates for the M response, assuming that the expected value for the vector of random effects was zero, i.e., $\boldsymbol{b}_i = 0$.

The PA response was obtained by computing mean predictions from the mixed-effects model over the distribution of random effects (e.g., de-Miguel et al. 2012). To implement this prediction strategy, we used a Monte Carlo integration procedure simulating 10 000 realizations of random effects and considering the covariance matrix of random effects. We assumed that random effects follow a multivariate normal distribution. Each realization was added to the corresponding fixed parameters in the base model, yielding a prediction; the 10 000 predictions were subsequently averaged to obtain the PA response. We used the Cholesky decomposition to generate the random-effects vectors (e.g., Fortin and Langevin 2012).

For making SS predictions for a new tree in the evaluation data set, at least one additional stem diameter measurement from this tree is required to predict the vector of random effects $\boldsymbol{b}_i$. For the FOCE method, this is predicted using the following expression (Vonesh and Chinchilli 1997, p. 252):

$$(4)\qquad \hat{\boldsymbol{b}}_i = \hat{\mathbf{D}}\mathbf{Z}_i^{\mathrm{T}}\left(\mathbf{Z}_i\hat{\mathbf{D}}\mathbf{Z}_i^{\mathrm{T}} + \hat{\mathbf{R}}_i\right)^{-1}[\boldsymbol{d}_i - f(\mathbf{x}_i, \hat{\boldsymbol{\beta}}, \hat{\boldsymbol{b}}_i) + \mathbf{Z}_i\hat{\boldsymbol{b}}_i]$$

where $\hat{\mathbf{D}}$ is the estimated variance–covariance matrix for random effects, $\hat{\mathbf{R}}_i$ is the estimated variance–covariance matrix for the error term, $f(.)$ is a nonlinear function of the covariate matrix $\mathbf{x}_i$, and $\mathbf{Z}_i$ is the partial derivative matrix evaluated at $\hat{\boldsymbol{\beta}}$ and $\hat{\boldsymbol{b}}_i$, i.e., $\mathbf{Z}_i = \partial f(\mathbf{x}_i, \boldsymbol{\beta}, \boldsymbol{b}_i)/\partial \boldsymbol{b}_i|_{\hat{\boldsymbol{\beta}},\hat{\boldsymbol{b}}_i}$.

Because $\hat{\boldsymbol{b}}_i$ appears on both sides of eq. 4, it must be solved iteratively (Lindstrom and Bates 1990). Once $\hat{\boldsymbol{b}}_i$ is predicted, the predicted vector of diameters $\hat{\boldsymbol{d}}_i$ for the *i*th tree (the calibrated response vector) is calculated for the FOCE method as follows:

$$(5)\qquad \hat{\boldsymbol{d}}_i = f(\mathbf{x}_i, \hat{\boldsymbol{\beta}}, \hat{\boldsymbol{b}}_i)$$

Calibration was performed with only one additional stem diameter because the improvement in predictions by use of more measurements does not usually compensate the greater sampling effort (e.g., Trincado and Burkhart 2006; Yang et al. 2009*a*). For evaluation of which measure of diameter is most appropriate for subject-specific calibrations, stem diameter values measured in the field (at irregular intervals along the bole) were used to predict random effects and to obtain subsequent SS predictions.

Similarly to Yang et al. (2009*a*, 2009*b*), performance of OLS, M, PA, and SS responses was assessed by use of the following statistics:

$$(6)\qquad \mathrm{PCE} = 100\frac{\bar{e}}{\bar{y}}$$

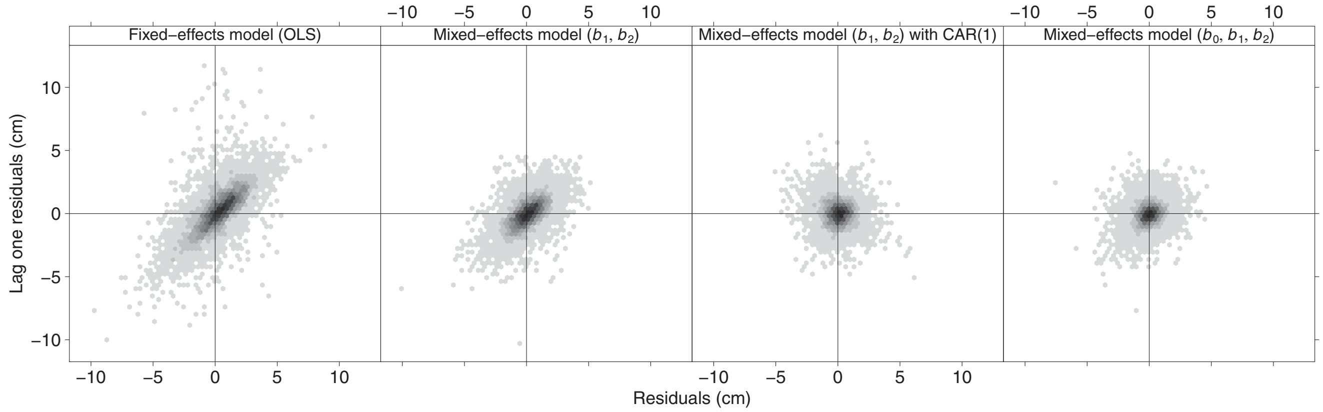
$$(7)\qquad \mathrm{RMSE} = \sqrt{\frac{\sum_{i=1}^{n}\left(\frac{\sum_{j=1}^{m_i}(y_{ij} - \hat{y}_{ij})^2}{m_i}\right)}{n}}$$

where $y_{ij}$ and $\hat{y}_{ij}$ are the observed and predicted dependent variables (diameter along the bole or total stem volume), respectively, for the *j*th observation ($j = 1, 2, \ldots, m_i$) of the *i*th tree ($i = 1, 2, \ldots, n$), $m_i$ is the number of observations within *i*th tree, $n$ is the number of trees, $\bar{y}$ is the mean of the observed values of the dependent variable, PCE is the percent mean prediction bias, $\bar{e}$ is the mean prediction bias ($\sum_{i=1}^{n}(\sum_{j=1}^{m_i}(y_{ij} - \hat{y}_{ij})/m_i)/n$), and RMSE is the root mean square error of the predictions.

PCE provides an average measure of the prediction bias in relative terms, whereas RMSE provides a better measure of model precision and was used as the primary criterion for model evaluation. For the SS approach, these statistics were calculated for 0.1 relative height classes.

Because Riemer's model cannot be directly integrated, total stem volumes were calculated by numerical integration of the taper model with the integrate function of R (R Core Team 2013).

**Fig. 2.** Correlation between raw residuals (cm) and lag-one residuals (cm) in diameter estimation for the fixed-effects model fitted by OLS and the mixed-effects model that includes random effects in $b_1$ and $b_2$, without and with a CAR(1) structure in the error term, and random effects in $b_0$, $b_1$, and $b_2$ without considering the autocorrelation structure. The grey gradient indicates the density of the data points. OLS, ordinary least squares.






## Results

### Parameter estimates and variance components

Visual comparison of residuals versus lag-one residuals of the base model fitted by OLS (the fixed-effects model) revealed strong autocorrelation (Fig. 2, first column). The same comparison with the seven combinations of fixed parameters and random effects indicated that, in all cases except in the mixed-effects model with three random effects, significant residual autocorrelation was present. In these cases, modelling the error term with the CAR(1) structure successfully captured within-tree residual autocorrelation (Fig. 2, second and third columns, exemplifies this for the mixed-effects model with random effects in parameters $b_1$ and $b_2$). For the model with random effects in $b_0$, $b_1$, and $b_2$, the mixed-effects modelling technique successfully accounted for the autocorrelation without any need to model the error term (Fig. 2, fourth column).

The estimated fixed parameters, correlation parameter, and variance components of the different models are listed in Table 2. All of the fixed-parameter estimates were significant at $p < 0.001$. As expected, the residual variance $\sigma^2$ decreased gradually as the number of random effects increased between the fixed-effects model ($\sigma^2 = 3.203$) and the most complete mixed-effects model ($\sigma^2 = 0.8861$), except in the mixed-effects model with the random effect in $b_0$ ($\sigma^2 = 3.955$).

The fitted models were compared by examination of AIC and BIC values (Table 3). For all of the models, these statistics were calculated using the raw residuals (i.e., using the maximum likelihood parameter estimates but without considering the autocorrelation parameter), because the inclusion of an autocorrelation structure artificially improves the goodness-of-fit statistics (e.g., Gregoire et al. 1995). This would invalidate direct comparison of the model with three random effects ($b_0$, $b_1$, and $b_2$) in which the error term was not modelled.

All of the mixed-effects models except for the model that included a random effect in $b_0$ performed better than the fixed-effects model. The model that included the random effect in $b_2$ was the best within the group of candidate models with one random effect, whereas the model that included the random effects in $b_1$ and $b_2$ was the best for the group of models with two random effects. For the model with random effects in $b_0$ and $b_2$, we noted that the random effects were linearly dependent. In this case, the error variance was equal to that of the mixed-effects model when only one random effect in $b_2$ was included (see Table 2), which means that adding a random effect in $b_0$ did not cause a reduction in error variance. Therefore, this model was excluded from further analyses.

As expected, the greater the number of random effects included in the model, the greater is the reduction in AIC and BIC values with respect to the PA response (approximately 19.2% for the model with three random effects). Therefore, on the basis of the reported goodness-of-fit statistics, the model that included three random effects (i.e., in $b_0$, $b_1$, and $b_2$) performed best.

Graphical analysis of the residuals revealed a more homogeneous residual variance in the mixed-effects models than in the fixed-effects model. Moderately homogeneous variance was observed over the full range of relative heights in the fixed-effects model, except for the lowest portion of the stems (Fig. 3, left). The inclusion of random effects circumvented this problem (Fig. 3, right, where $b_0$, $b_1$, and $b_2$ are random effects), thus avoiding the need to use a weighting factor to balance the error variance. No heteroscedasticity was observed on plotting residuals over $D$, $H$, or predicted values.

### Evaluation of the predictive ability of the PA, M, and SS responses

Although random effects greatly improved the model fit, this does not necessary imply that SS diameter and stem volume predictions improve accordingly when calibration is performed from only one additional diameter measurement on a standing tree. Moreover, if this measurement is not available, OLS, M, or PA responses must be used to estimate diameters along the stem and total stem volume.

Diameter ($d$) and total stem volume ($V$) were predicted for each model at the population and SS levels by using the parameter estimates, correlation parameter, and variance components included in Table 3. At the population level, the fixed-effects model was used to obtain the OLS response, whereas the fixed parameters in the mixed-effects models were used to obtain the M response. The PA response was calculated as the mean of 10 000 randomized predictions of the mixed-effects models over the distribution of random effects. To obtain SS responses, additional diameter measurements were used to calculate random effects and subsequently to predict localized $d$ and $V$. These values were used to compute RMSE (Fig. 4) and PCE (Fig. 5) by calibration relative height classes along the stem.

On the basis of the M response alone, models with one random effect in $b_0$ and $b_1$ and with two random effects in $b_0$ and $b_1$ performed best and quite similarly for $d$ and $V$ predictions: RMSE values were smaller and PCE values were close to zero. The model with three random effects yielded the poorest results for both RMSE and PCE. The same pattern was observed on the basis of the PA response. The RMSE of the M and PA responses was always greater than that of the OLS response (1.70 cm for $d$ and 0.105 $m^3$ for $V$), and the differences generally increased with the number of random effects. Regarding the PCE values, the OLS response for $d$ was closer to zero than the PA and M responses only for the model

**Table 2.** Fixed parameter estimates (standard errors), correlation parameter, and variance components of the models evaluated.

| | Random effect | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | None (OLS) | $b_0$ | $b_1$ | $b_2$ | $b_0$, $b_1$ | $b_0$, $b_2$ | $b_1$, $b_2$ | $b_0$, $b_1$, $b_2$ |
| **Fixed parameter** | | | | | | | | |
| $\beta_0$ | 0.4506 (0.0009118) | 0.4629 (0.001219) | 0.4609 (0.0009907) | 0.4526 (0.001063) | 0.4507 (0.001056) | 0.4526 (0.001063) | 0.4513 (0.0008866) | 0.4422 (0.001116) |
| $\beta_1$ | 0.8493 (0.01140) | 1.046 (0.01943) | 1.025 (0.01727) | 0.9047 (0.01264) | 0.9713 (0.01895) | 0.9052 (0.01266) | 0.9145 (0.01305) | 0.9035 (0.01588) |
| $\beta_2$ | 0.04517 (0.0005483) | 0.03987 (0.0008138) | 0.04176 (0.0007250) | 0.05273 (0.001048) | 0.04841 (0.0006253) | 0.05270 (0.001048) | 0.05666 (0.001051) | 0.06690 (0.001236) |
| **Correlation parameter** | | | | | | | | |
| $\rho$ | — | 0.7832 | 0.8460 | 0.6108 | 0.8262 | 0.6109 | 0.6879 | — |
| **Variance components for $b_i$** | | | | | | | | |
| $\sigma^2_{b_0}$ | | $7.150\times10^{-20}$ | | | 0.0004384 | $1.283\times10^{-22}$ | | 0.0007880 |
| $\sigma^2_{b_1}$ | | | 0.06489 | | 0.2233 | | 0.05791 | 0.1819 |
| $\sigma^2_{b_2}$ | | | | 0.0003177 | | $3.176\times10^{-4}$ | 0.0004352 | 0.001011 |
| $\sigma_{b_0b_1}$ | | | | | 0.009022 | | | 0.01051 |
| $\sigma_{b_0b_2}$ | | | | | | $-2.017\times10^{-13}$ | | $-6.873\times10^{-5}$ |
| $\sigma_{b_1b_2}$ | | | | | | | 0.001903 | −0.001383 |
| $\sigma^2$ | 3.203 | 3.955 | 3.018 | 2.467 | 2.455 | 2.467 | 1.598 | 0.8861 |

**Note:** $\sigma^2$: residual variance; $\sigma^2_{b_0}$, $\sigma^2_{b_1}$, and $\sigma^2_{b_2}$: variances for the random effects $b_0$, $b_1$, and $b_2$, respectively; $\sigma_{b_0b_1}$, $\sigma_{b_0b_2}$, and $\sigma_{b_1b_2}$: covariances between pairs of random effects; $\rho$: correlation parameter for the CAR(1) error structure; OLS, ordinary least squares.

**Table 3.** Goodness-of-fit statistics of the models analysed.

| Random effects | AIC | BIC | $AIC_d$ | $BIC_d$ | % AIC reduction | % BIC reduction |
|---|---|---|---|---|---|---|
| None (OLS) | 40 982 | 41 011 | 8354 | 8310 | — | — |
| $b_0$ | 41 347 | 41 390 | 8252 | 8222 | −0.9 | −0.9 |
| $b_1$ | 40 435 | 40 478 | 7340 | 7310 | 1.3 | 1.3 |
| $b_2$ | 37 682 | 37 726 | 4587 | 4558 | 8.1 | 8.0 |
| $b_0$, $b_1$ | 37 506 | 37 564 | 4411 | 4396 | 8.5 | 8.4 |
| $b_1$, $b_2$ | 34 956 | 35 014 | 1861 | 1846 | 14.7 | 14.6 |
| $b_0$, $b_1$, $b_2$ | 33 095 | 33 168 | 0 | 0 | 19.2 | 19.1 |

**Note:** Goodness-of-fit statistics obtained for all the models with the maximum likelihood procedure without considering the autocorrelation parameter in the calculations. $AIC_d$ and $BIC_d$, differences in AIC and BIC values with respect to those of the best candidate model. % AIC and BIC reduction, percentage reduction in AIC and BIC values with respect to those of the base model fitted by OLS (ordinary least squares).

with two random effects in $b_1$ and $b_2$ and the model with three random effects in $b_0$, $b_1$, and $b_2$; for $V$ predictions, the PCE of the OLS response was almost zero and always less biased than any M and PA responses.

The performance of the PA and M responses was quite similar for the six mixed-effects models that were finally evaluated. The major differences occurred in the model with three random effects, where PA response was superior for $d$ predictions and the M response was superior for $V$ predictions.

SS responses with an additional diameter measurement generally improved the $d$ and $V$ estimates, according to the RMSE and PCE values averaged by calibration relative height classes (Figs. 4 and 5, respectively). Nevertheless, for the models with one random effect in $b_0$ and $b_1$, and for additional diameters measured at relative heights lower than 0.2 or greater than 0.8, the RMSE of the SS responses was larger than that of the OLS response.

A general trend in the precision of the SS predictions for the two variables was observed: large estimation errors occurred when additional diameters for calibration were measured close to the bottom or the top of the stem, whereas errors were lowest for diameter measurements made in the middle part of the stem (relative heights of 0.4–0.7 for $d$ predictions and 0.2–0.6 for $V$ predictions). According to the PCE values, overestimation of $d$ was generally observed for most calibration relative heights in models with one and two random effects. A different pattern was observed in the model with three random effects, which had negligible bias for both $d$ and $V$ predictions for diameters measured at relative heights between 0.3 and 0.5.

Of the six mixed-effects models finally evaluated, the model with three random effects is recommended for predictions for the following reasons: (*i*) it yields the best precision of SS estimations (using calibration relative heights between 0.3 and 0.6); (*ii*) the RMSEs for the SS responses were lower than the RMSEs for the OLS responses for most calibration relative height classes; and (*iii*) the SS responses had negligible bias for most of the calibration relative height classes.

## Discussion

As expected, substantial improvements in the model fit were obtained when random effects were incorporated in the fixed-effects model (Table 3), although the latter explained almost 98% of the observed variability in stem diameter ($R^2 = 0.978$ for the OLS fit). Moreover, the degree of improvement in model fitting generally increased with the number of random effects included in the model. This was also expected because the inclusion of random effects enables the existing between-tree variation to be accounted for (e.g., Leites and Robinson 2004; Temesgen et al. 2008).

Before random effects were included in the model, there was a strong correlation between residuals of the same tree. This serial correlation was reduced (although it was still present) in the

**Fig. 3.** Plot of residuals (cm) against relative height (*h*/*H*) for the fixed-effects model fitted by OLS and for a mixed-effects model that includes $b_0$, $b_1$, and $b_2$ as random effects. OLS, ordinary least squares.

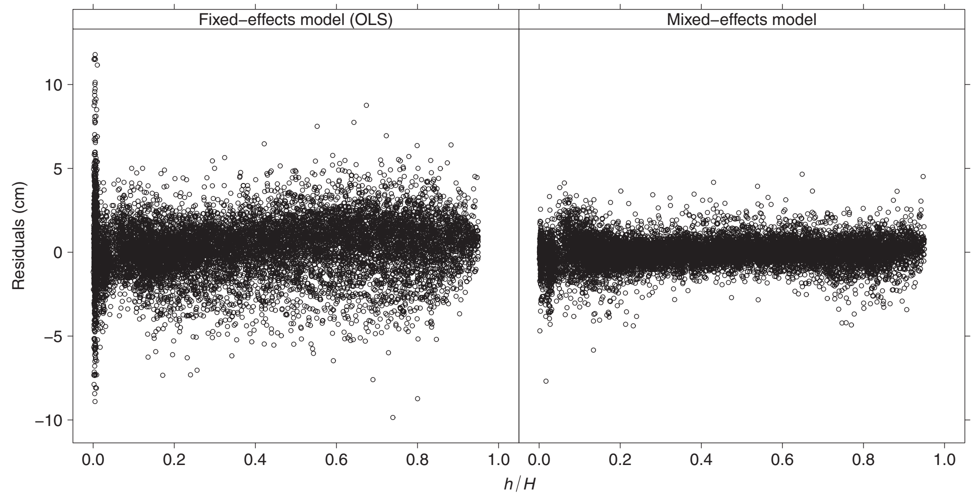
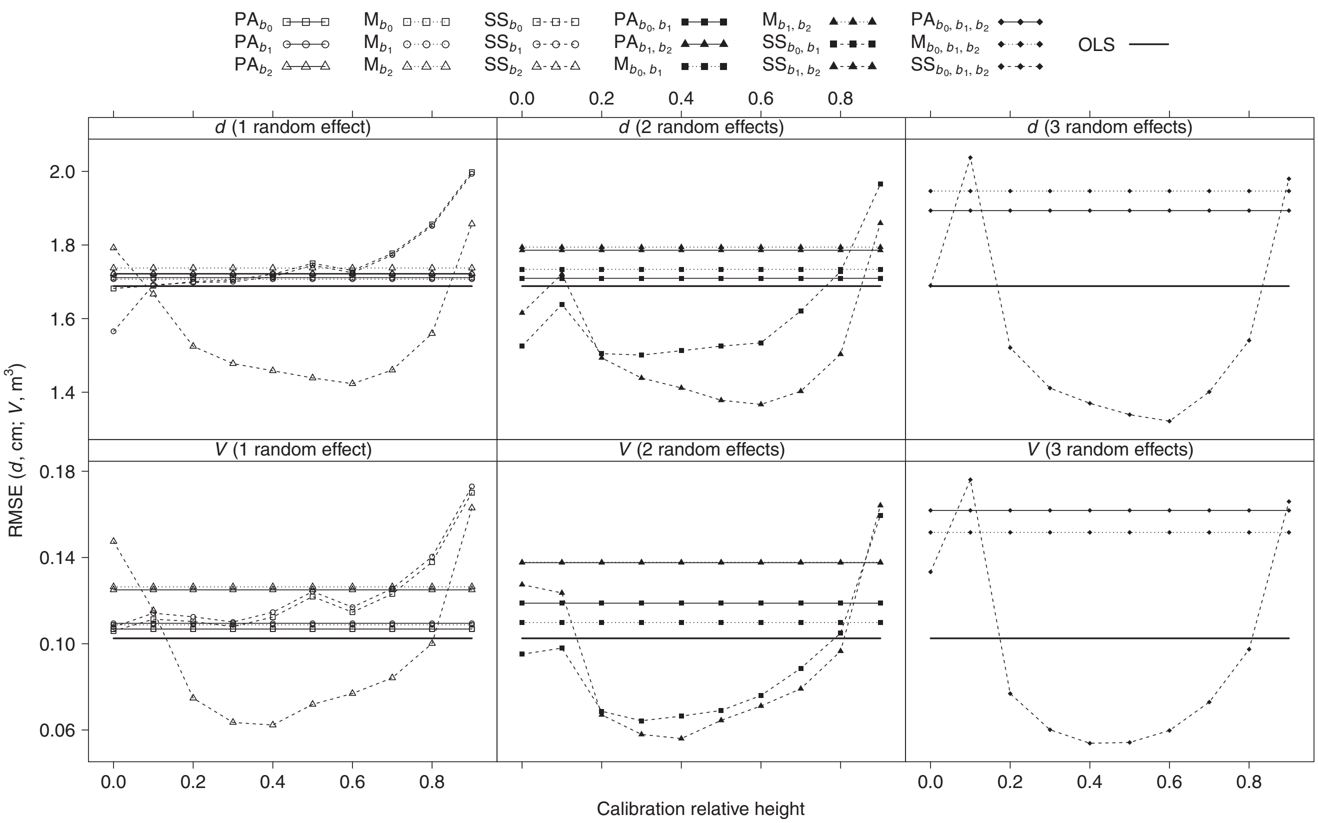



**Fig. 4.** Root mean squared error (RMSE) in predicting *d* (diameter; cm) and *V* (total stem volume; m³) from mixed-effects models that include one, two, and three random effects, calibrated with diameter measurements at 10 calibration relative heights. OLS, M, PA, and SS are the ordinary least squares, mean, population-averaged, and subject-specific responses, respectively.

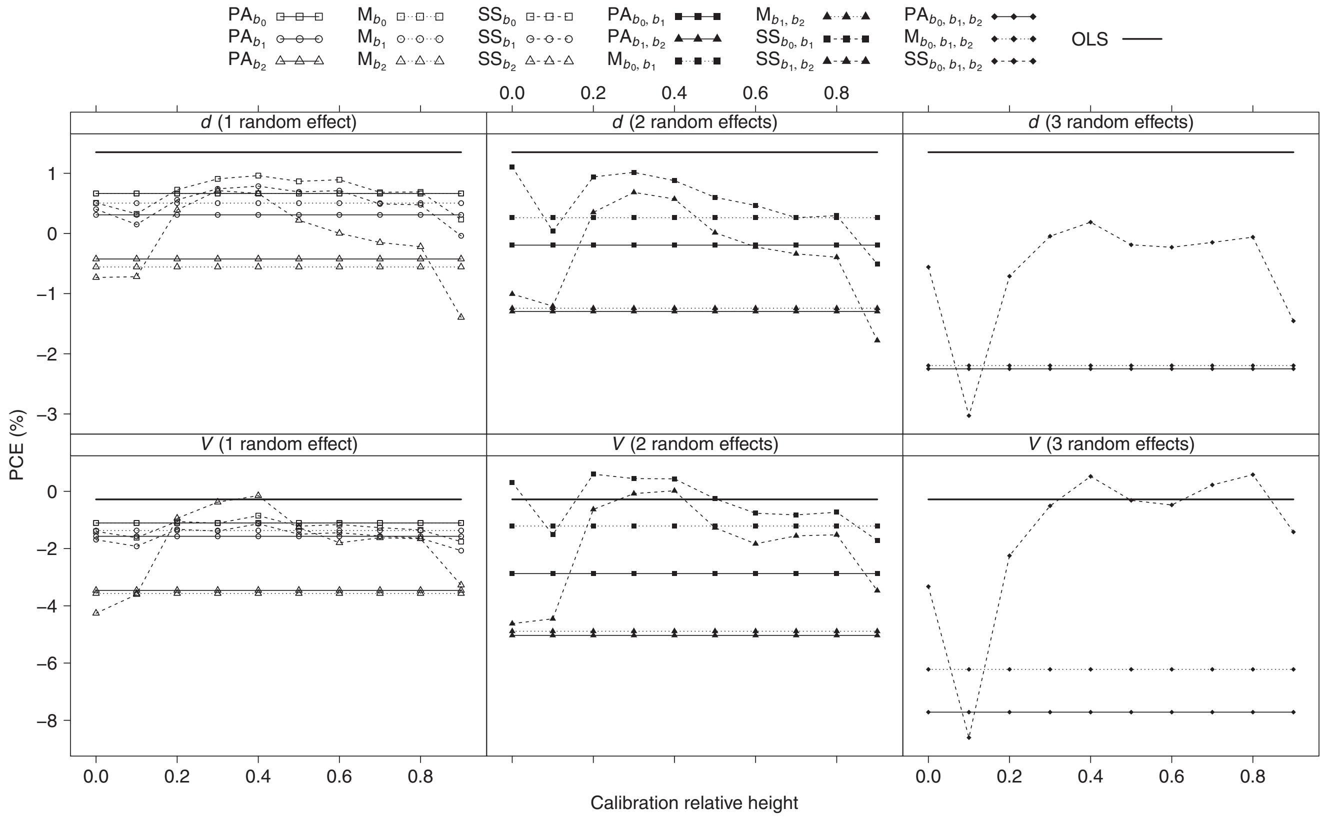



mixed-effects models that included one and two random effects, and it was completely accounted for in the model that included three random effects. Some authors have reported that mixed models account for some autocorrelation (e.g., Trincado and Burkhart 2006; Yang et al. 2009*b*; Gómez-García et al. 2013), whereas others have indicated that the correlations could be completely overcome by including random effects (e.g., Yang et al. 2009*a*). Additionally, the inclusion of three random effects was sufficient to account for the heterogeneity of the residual variance.

In summary, in the fitting phase, the mixed-effects model technique proved efficient for modelling the within-tree autocorrelation and heteroscedasticity when random effects were incorporated, at least for the best model, in which $b_0$, $b_1$, and $b_2$ are random effects.

From the point of view of the prediction approach, we found that when an additional stem diameter was available to predict the random effects, the SS response did not always yield smaller prediction errors than the OLS response. This has also been reported by other authors who tested different stem locations for calibration (e.g., Yang

**Fig. 5.** Percent mean prediction bias in predicting *d* (diameter; cm) and *V* (total stem volume; $m^3$) from mixed-effects models that include one, two, and three random effects, calibrated with diameter measurements at 10 calibration relative heights. OLS, M, PA, and SS are the ordinary least squares, mean, population-averaged, and subject-specific responses, respectively.






et al. 2009*a*). It is especially remarkable that bias was higher and precision was lower when SS calibrations were made using diameters measured at extreme relative heights (<0.2 or >0.8). This may be because Riemer's model is constrained to pass through the tip of the tree ($d = 0$ when $h = H$) and through diameter at breast height ($d = D$ when $h = 1.3$ m), a characteristic that makes it less flexible around these locations than anywhere along the bole. Because any diameter measurement used for calibration will force the taper function to pass close to its value, a small error in the diameter measurements made around these points will skew the shape of the taper function, hence yielding less accurate predictions for the rest of the bole. The 0 and 0.9 relative height classes include observations at stump level and tree tip, respectively, whereas most of the observations around diameter at breast height were in the 0.1 class. Lejeune et al. (2009) and Yang et al. (2009*a*) also found minimal prediction improvement by calibrating the model with additional diameter measurements close to both ends of the bole using the variable-exponent taper model of Kozak (1988). This may also explain why for most of the mixed-effects models, *d* and *V* predictions were enhanced when the additional diameter measurement was made in the middle part of the stem. Measurement of an additional diameter in the middle part of the stem was also identified as optimal for improving model precision in previous studies; Kozak (1998) and Cao (2009) found the diameter measured at 50% of the height above breast height to be best for western red cedar (*Thuja plicata* Donn ex D. Don) and Douglas-fir (*Pseudotsuga menziesii* (Mirb.) Franco) trees in British Columbia and planted loblolly pine (*Pinus taeda* L.) trees in Louisiana, respectively. Sabatia and Burkhart (2014) proposed a 60% value both for loblolly pine in the southern United States and radiata pine in New Zealand.

If an additional stem diameter is not available to predict the random effects, the OLS, M, or PA responses must be used. In the present study, use of the fixed part of the mixed-effects model (the M response) or use of predictions calculated from the mixed-effects models by taking the average prediction over the multivariate normal distribution of random effects (the PA response) yielded more biased and less precise predictions than the OLS response. Moreover, the predictive ability of the M and PA responses generally decreased with increasing accuracy of the SS responses in predicting *d* and *V*.

The poor performance of the M response may be expected *a priori*, because the base model is nonlinear with respect to the random effects. Our results simply confirm previous findings (e.g., Meng et al. 2009; de-Miguel et al. 2012); use of the M response as a representation of the overall mean response of the model may produce substantial prediction errors. According to Mehtätalo et al. (2015), the impact of the differences between OLS and M predictions are model and data dependent.

The poor results of the PA prediction approach with respect to the OLS approach are more surprising. The differences in the predictive performance of both approaches are notable for the selected model with three random effects: the RMSE values were 12% and 58% higher for *d* and *V* predictions from the PA response than from the OLS response, respectively. Other authors found the PA prediction approach to be more biased and less precise than using a fixed-effects model (e.g., de-Miguel et al. 2013) or only slightly better (de-Miguel et al. 2012).

According to de-Miguel et al. (2013) and Mehtätalo et al. (2015), possible reasons for the poorer performance of the PA approach are that the random effects did not follow a multivariate normal distribution or that the mixed-effects model is poorly formulated (e.g., random effects are correlated with tree dimensions). Nevertheless, in the present study, graphical analysis (results not shown) does not suggest either a lack of normality of the random effects or any correlation between these random effects and tree variables (*D* and *H*).

In summary, considering the results of the PA and M approaches, and only for prediction purposes, we do not recommend the use of mixed-effects models in the absence of calibration data. Other authors (e.g., Mehtätalo et al. 2015) have claimed that this recommendation is not adequate because the alternative approach (the use of a fixed-effects model) ignores the hierarchical structure of the data. Nevertheless, we think that the use of the fixed-effects model may be acceptable unless the aim of the model is to make inferences about the population (Fortin 2013).

Selection of the best mixed-effects model for practical purposes (including random effects in $b_0$, $b_1$, and $b_2$) was primarily based on the predictive performance of the SS responses, assuming that mixed-effects models would only be used when additional data are available for calibration (i.e., OLS estimates should be used otherwise). This model was also the first among the six mixed-effects models evaluated in the fitting phase. Other authors found differences in the best model in the fitting and calibration phases (e.g., Gómez-García et al. 2013). Nevertheless, direct comparison with the aforementioned study is not conclusive because the taper function used depends on 10 fixed parameters and not all possible combinations of fixed parameters and random effects were evaluated. One advantage of the parsimonious model of Riemer et al. (1995) is that all expansion options of fixed parameters with random effects can be easily compared.

Stable performance of the selected mixed-effects model when calibration is carried out using additional diameter measurement at relative heights between 0.3 and 0.6 allows the user to decide which point on the stem is most appropriate for making measurements without compromising the accuracy of the predictions. This is important in practice because radiata pine usually has a dense crown, and therefore, some parts of the upper stem are not always visible from the ground.

A detailed numerical example of localizing the stem taper model for SS predictions of diameter and total stem volume and the R script (R Core Team 2013) including the functions necessary for this purpose are provided in Appendix A.

## Conclusions

A nonlinear mixed-effects stem profile model was developed for radiata pine plantations in Spain on the basis of the parsimonious variable-form taper function of Riemer et al. (1995). All possible combinations of fixed parameters with random effects were evaluated to capture between-tree variation, and when necessary, a CAR(1) structure was used to capture the within-tree residual autocorrelation. Model-fitting results indicated that the inclusion of random effects for all fixed parameters provided the best fit.

An evaluation data set was used to assess the performance of the models in predicting stem diameter along the bole and total stem volume. The OLS, M, PA, and SS responses were evaluated and compared. The prediction errors were larger for the M and PA responses than for the OLS responses (although the differences depended on the combinations of random effects). Therefore, from the prediction point of view, the use of the mixed-effects models is not recommended for those cases in which an additional stem diameter measurement is not available. The SS approach did not always provide smaller prediction errors than the OLS approach: for some models and for calibrations with additional diameters measured at relative heights lower than 0.2 or greater than 0.8, the accuracy and the precision of the SS approach were even lower than those of the OLS approach.

The mixed-effects model with three random effects also performed best in the calibration phase and was, therefore, selected for prediction purposes when an additional diameter is available. Measurement of this stem diameter at relative heights of approximately 0.5 yielded the best calibrations for $d$ and $V$ predictions with the selected model and is therefore recommended for calibration. This sampling design for additional stem diameter outperformed the estimates obtained by the traditional OLS approach and will help improve $d$ and $V$ and estimates in radiata pine stands in Spain.

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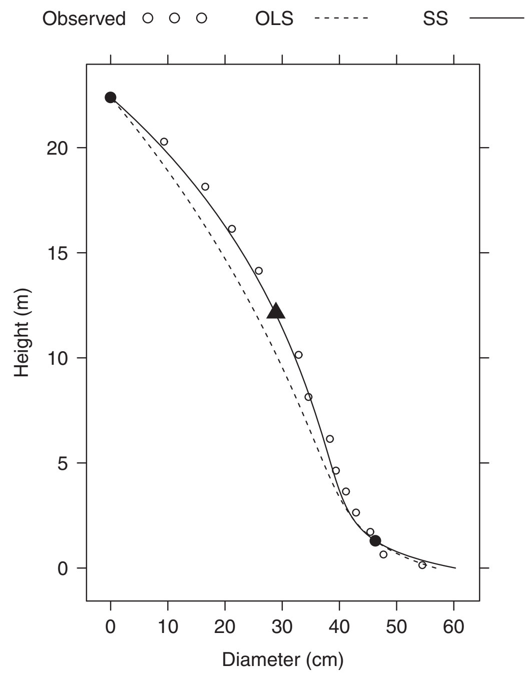
## Appendix A

To illustrate the calibration process, we used a tree with diameter at breast height $D$ = 46.3 cm and total tree height $H$ = 22.4 m. If only $D$ and $H$ are known, the PA response can be calculated by using the parameter estimates of the fixed-effects model (eq. 3) shown in Table 2.

**Fig. 6.** Observed diameters (open circles), OLS response (dashed line), and SS response obtained by calibration (solid line) for a tree of diameter at breast height $D$ = 46.3 cm and total tree height $H$ = 22.4 m (solid circles). The triangle represents the additional diameter used for calibration ($d$ = 28.9 cm, measured at $h$ = 12.14 m). OLS, ordinary least squares; SS, subject-specific.




If an additional measurement of stem diameter is available, random effects can be computed and subsequently used for SS predictions. For the mixed-effects model with three random effects (the best of the six mixed-effects models finally analysed), an additional measurement of stem diameter at a relative height of approximately 0.5 is recommended for calibration. In this example, the observation of $d$ = 28.9 cm measured at $h$ = 12.14 m above ground level was used. Additionally, the predicted variance–covariance matrix for the random effects ($\hat{\mathbf{D}}$) and predicted variance matrix for the error term ($\hat{\mathbf{R}}_i$) are known from the estimates shown in Table 2:

$$\hat{\mathbf{D}} = \begin{bmatrix} 0.0007880 & 0.01051 & -0.00006873 \\ 0.01051 & 0.1819 & 0.001383 \\ -0.00006873 & 0.001383 & 0.001011 \end{bmatrix}$$

$$\hat{\mathbf{R}}_i = \sigma^2\mathbf{\Gamma} = 0.8861$$

where gamma is a matrix describing the pattern of correlation between the measurements of the $i$th individual.

The predicted vector of random effects $\hat{\boldsymbol{b}}_i$ was calculated by solving eq. 4 iteratively. The process stops when the relative differences between random effects of the current and the previous iteration are all lower than a specified tolerance (in our case, this was fixed at $10^{-6}$). A value of zero was used as the initial random effects ($\boldsymbol{b}_i$ = [0, 0, 0]), and therefore, for this case, partial derivatives were equal to those obtained considering only the fixed parameters of the mixed-effects model, i.e., $\mathbf{Z}_i$ = [60.80 −0.003236 96.77].

The random effects vector obtained after the first iteration was $\hat{\boldsymbol{b}}_i$ = [0.006654, 0.1246, 0.01512], and the final random effects values after the whole calibration process were $\hat{\boldsymbol{b}}_i$ = [0.007280, 0.1325, 0.01484]. SS predictions were subsequently obtained by adding $\hat{\boldsymbol{b}}_i$ to the corresponding fixed parameters of the mixed-effects model, i.e., by applying eq. 5.

For purposes of demonstration, we computed the SS response for predicting the stem diameter at 20 m above ground level, the height at which stem diameter is equal to 25 cm (a merchantable diameter limit), and the cumulative volume from stump level (*v*, assuming that stump height = 0.1 m) to that height and to the tree top (i.e., the total volume, *V*)

$$d_{SS,20} = 9.02 \text{ cm}$$
$$h_{SS,25} = 14.08 \text{ m}$$
$$v_{SS(0.1,14.08)} = 1.51 \text{ m}^3$$
$$V_{SS(0.1,22.4)} = 1.67 \text{ m}^3$$

The PA and the SS responses for predicting diameter along the stem for the tree of the example are shown in Fig. 6.

The R script (R Core Team 2013) including the functions necessary for calibrating the stem taper model and obtaining SS predictions of *d*, *h*, *v*, and *V* is as follows:

```
# 1.Authorship ———————————————————
# Manuel Arias-Rodil, Ulises Diéguez-Aranda and Fernando Castedo-Dorado
# 2014
# 2.File description ———————————————————
# Compute random effects and SS predictions of the taper model by Riemer et al. (1995)
# 3.Function definitions ——————————————————
# Riemer's model
ComputeDiameterByRiemerModel <- function(D, H, h, parms){
b0 <- parms[1]
b1 <- parms[2]
b2 <- parms[3]
d <- 2 * ((b0 * D) / (1 - exp(b2 * (1.3 - H))) + (D / 2 - b0 * D) * (1 - (1 / (1 - exp(b1 * (1.3 - H))))) +
(exp(-b1 * h)) * (((D / 2 - b0 * D) * exp(1.3 * b1))/(1 - exp(b1 * (1.3 - H)))) - exp(b2 * h)*((b0 * D * exp(-b2
* H)) / (1 - exp(b2 * (1.3 - H)))))
return(d)
}
# Partial derivatives of the model with respect to fixed parameters
ComputeRiemerDerivatives <- function(D, H, h, parms){
b0 <- parms[1]
b1 <- parms[2]
b2 <- parms[3]
dvb0 <- 2 * (-(D * exp(b2 * (h - H)))/(1 - exp(b2 * (1.3 - H))) - (D * exp(b1 * (1.3 - h)))/(1 - exp(b1 *
(1.3 - H))) + D/ (1 - exp(b2 * (1.3 - H))) - D * (1 - 1/(1 - exp(b1 * (1.3 - H)))))
dvb1 <- 2 * (((D/2 - b0 * D) * (1.3 - H) * exp (b1 * (2.6 - h - H)))/(1 - exp (b1 * (1.3 - H)))2̂ + ((D/2
- b0 * D) * (1.3 - h) * exp (b1 * (1.3 - h)))/(1 - exp (b1 * (1.3 - H))) - ((D/2 - b0 * D) * (1.3 - H) * exp
(b1 * (1.3 - H)))/(1 - exp (b1 * (1.3 - H)))2̂)
dvb2 <- 2 * (- (b0 * D * (1.3 - H) * exp (b2 * (h + 1.3 - 2 * H)))/(1 - exp (b2 * (1.3 - H)))2̂ - (b0 *
D * (h - H) * exp (b2 * (h - H)))/(1 - exp (b2 * (1.3 - H))) + (b0 * D * (1.3 - H) * exp (b2 * (1.3 - H)))/(1
- exp (b2 * (1.3 - H)))2̂)
return(cbind(dvb0, dvb1, dvb2))
}
# Function for iteration calibration
DoAnIterationCalibration <- function(D, H, d, h, DMat, R, parms.f, parms.r, partial.derivatives){
parms.m <- parms.f + parms.r
Z <- partial.derivatives(D, H, h, parms.m)
fxBb <- ComputeDiameterByRiemerModel(D, H, h, parms.m)
bi <- parms.r
b <- DMat %*% t(Z) %*% solve (R + Z %*% DMat %*% t(Z)) %*% ((d - fxBb) + Z %*% bi)
return(b)
}
# Function for calibrating by expanding three fixed parameters
CalibrateRandomEffects <- function(D, H, d, h, DMat, R, fixed.parms, partial.derivatives, tolerance =
1e-06, maxit = 200){
parms.f <- fixed.parms
it <- 1
tol <- rep(1, length(fixed.parms))
parms.r.i <- rep(0, length(fixed.parms))
while (sum(tol > tolerance) > 0){
bi <- DoAnIterationCalibration(D, H, d, h, DMat, R, parms.f, parms.r.i, partial.derivatives)
it <- it + 1
parms.r.f <- parms.r.i
parms.r.i <- bi
tol <- abs((parms.r.i - parms.r.f) / parms.r.i)
if(it > maxit){
stop(“Reached maximum number of iterations without convergence”)
```

```
  }
  }
  return(c(bi[1], bi[2], bi[3]))
  }
  ComputeHeightByRiemerModel <- function(D, H, d, parms){
  h <- optimize(function(x) (d - ComputeDiameterByRiemerModel(D, H, x, parms))2̂, interval = c(0,
H))$minimum
  return(h)
  }
  ComputeVolumeByRiemerModel <- function(D, H, d, hst, parms){
  u.h <- optimize(function(x) (d - ComputeDiameterByRiemerModel(D = D, H = H, h = x, parms = parms))2̂,
interval = c(0, H))$minimum
  v <- integrate(function(x) pi/40000 * (ComputeDiameterByRiemerModel(D = D, H = H, h = x, parms =
parms))2̂, lower = hst, upper = u.h)$value
  return(v)
  }
  # 4.Execution statements ------------------
  # Estimates of variance components of random effects, error variance and fixed parameters
  D.matrix <- matrix(c(0.0007880, 0.01051, -0.00006873, 0.01051, 0.1819, 0.001383, -0.00006873, 0.001383,
0.001011), nrow = 3, ncol = 3, byrow = T)
  error.variance <- 0.8861
  fixed.parameters <- c(0.4422, 0.9035, 0.06690)
  # Partial derivatives
  ComputeRiemerDerivatives(46.3, 22.4, 12.14, fixed.parameters)
  # Calculating random effects (one iteration)
  DoAnIterationCalibration(46.3, 22.4, 28.9, 12.14, D.matrix, error.variance, fixed.parameters, parms.r =
c(0, 0, 0), ComputeRiemerDerivatives)
  # Calculating random effects (until reaching the tolerance limit)
  random.effects.calibration <- CalibrateRandomEffects(46.3, 22.4, 28.9, 12.14, D.matrix, error.variance,
fixed.parameters, ComputeRiemerDerivatives)
  # Computing mixed-effects
  mixed.effects <- fixed.parameters + random.effects.calibration
  # SS prediction of diameter at h = 20 m
  ComputeDiameterByRiemerModel(46.3, 22.4, 20, mixed.effects)
  # SS prediction of height at a top diameter limit (25 cm)
  ComputeHeightByRiemerModel(46.3, 22.4, 25, mixed.effects)
  # SS prediction of volume from 0.1 m to a top diameter limit (25 cm)
  ComputeVolumeByRiemerModel(46.3, 22.4, 25, 0.1, mixed.effects)
  # SS prediction of volume from 0.1 m to tree top
  ComputeVolumeByRiemerModel(46.3, 22.4, 0, 0.1, mixed.effects)
```