Monochromators: versatile design procedure for specific purposes

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A design procedure is described to obtain monochromators for specific applications, which are made from optical devices that can be easily acquired or built. This procedure makes use of a constant angular deviation system consisting of a transmission grating, operating in a symmetrical arrangement, and a flat mirror which makes a fixed angle with it. An analysis of the characteristic parameters of the design and of the constraints, which have an effect on them, is presented as well as two examples of applications of the procedure.

I. Introduction

A new computer aided design procedure is described in this paper. This procedure can be regarded as very useful in producing a monochromator with one property emphasized above the others, so that it will be better suited for a specific application than are commercial monochromators. At the same time, the design can be easily performed, since all the elements from which such a monochromator is made are optical devices that can be easily obtained or made in a laboratory.

The basis of the design consists of the combination of a transmission grating operating in a symmetrical arrangement and a flat mirror which makes a fixed angle with the grating so that the angular deviation of the light that passes through the system is constant. To make things easier and improve efficiency, we consider only a volume holographic grating produced by recording the interference of two infinite plane wavefronts in a thick plate.[1] In addition, the design has other standard devices such as systems for collimation and collection, mirrors, and lenses.[2]

Figure 1 shows some of the elements of the optical device. The grating diffracts the wavelengths of the input beam into different directions so that they are focused at different points. Let ϵ be the angle between the input collimated beam and the normal direction to the grating and Λ the spatial period of the grating. When ϵ has such a value that for a given wavelength λ it satisfies

$$2\Lambda \sin\epsilon = \lambda, \tag{1}$$

the grating is operating symmetrically, or, what amounts to the same thing, it satisfies the Bragg condition.[3] The angular deviation of a beam of wavelength λ passing through the system is equal to $\pi - 2\alpha$, where α is the angle between the grating and the mirror.[4] This fact allows us to obtain a system of constant deviation throughout the entire range of wavelengths of interest simply by rotating the grating–mirror group around an axis perpendicular to the planes which contain both of them. Thus all the wavelengths which satisfy Eq. (1) are focused at the same point of the focal plane of the collector system.

From Fig. 1 and previous considerations, we establish that the characteristic parameters of the design are:

(1) the angle between the grating and mirror,
(2) the coordinates of the axis of rotation,
(3) the sense of rotation,
(4) the height of incidence of the collimated beam from the vertex of the system (the point of intersection between the axis made by the plane containing the mirror and the one containing the grating and the normal plane to both of them),
(5) the width of the collimated beam,
(6) the spatial period of the diffraction grating,
(7) the focal length and diameter of the collector system,
(8) the vertex–mirror and vertex–grating distances, and
(9) the sizes of the mirror and grating.

Obviously, the design will be under several constraints, some caused by the usual demands of a monochromator and others from the consistency of the de-

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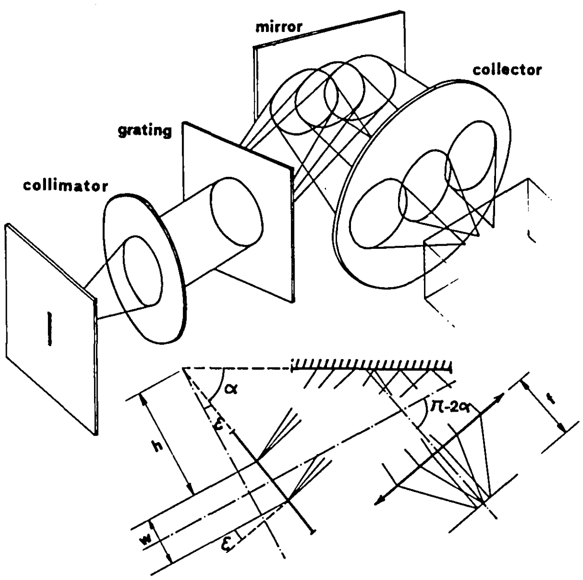

Fig. 1. Diagram of the optical device.

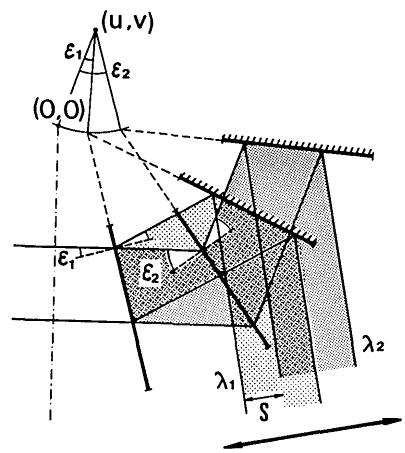

Fig. 2. Two overlapping system positions showing the displacement S that the output beam experiences for two different wavelengths, which satisfy Eq. (1).

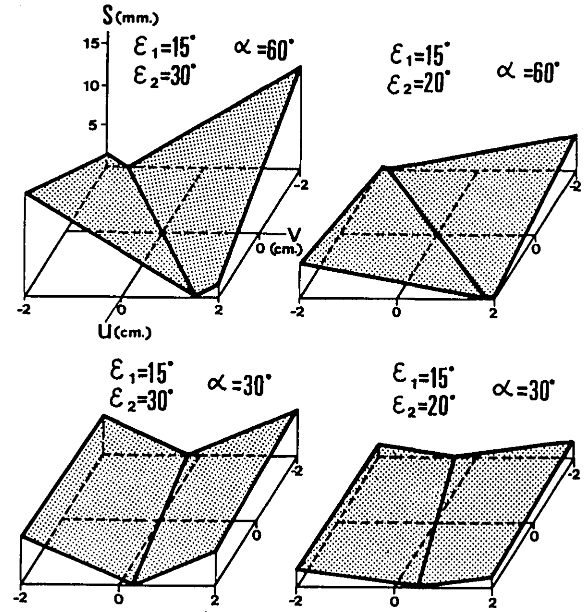

Fig. 3. Displacement values as a function of the coordinates of the rotation axis (u,v) for different values of α, ϵ_1, and ϵ_2.

sign procedure itself. In the next section, we discuss how some of the former parameters can be affected by these constraints and the organization of them to create a concise algorithm describing our method. Finally, in Sec. III, we show two examples that illustrates the field of application of the method.

II. Design Procedure

In the following we assume that, for a wavelength equal to zero, the grating is perpendicular to the incident collimated beam, and the origin of coordinates is found on the vertex of the system. The grating works in the first order. We also assume that any rotation of the grating–mirror system will be in the positive sense (counter-clockwise). We call ϵ the angle that satisfies Eq. (1) and θ that which corresponds to the light diffracted by the grating. By $\lambda_{\min}$ and $\lambda_{\min}$ we designate the minimum and maximum wavelengths of the range where the monochromator will work (spectral domain).[2]

A. Study of the Restrictions on the Parameters

1. Coordinates of the Rotation Axis

Let us consider as arbitrary the coordinates of the rotation axis (u,v). Let λ_1 and λ_2 be two wavelengths for which Eq. (1) is satisfied. The positions of the system corresponding to these wavelengths are represented in Fig. 2. This figure shows that the output beam of wavelength λ_1 appears displaced in relation to the output beam of wavelength λ_2, so that it is incident at different locations on the collector lens (or mirror) with the dangerous consequences that this implies: the requirement for a large lens, the different influence of aberrations, and so on. It is, therefore, essential to minimize this displacement and even make it zero.

If (x_1,y_1) and (x_2,y_2) are the coordinates of the vertex of the system after rotating through angles ϵ_1 and ϵ_2, respectively, around (u,v) we find that the absolute value of the displacement is given by

$$S = |\sin 2\alpha \cdot [(x_2 - x_1) + \tan\alpha \cdot (y_2 - y_1)]|. \qquad (2)$$

Figure 3 shows the values of S computed from Eq. (2) as a function of the coordinates of the rotation axis. Note that there exists a straight line passing through the origin of the coordinate system so that all its points make the displacement equal to zero. Since the slope of the line depends on α and on ϵ_1, ϵ_2 or λ_1, λ_2, the only zero of S which is independent of these magnitudes is $(u,v) = (0,0)$. Thus, the necessary and sufficient condition such that the displacement becomes zero, for every wavelength and every angle between grating and mirror, is that the rotation axis contain the vertex of the system. In the following this axis is assumed.

It follows from Eq. (2) that S has small values when the rotation axis slightly deviates from the vertex of the system. This fact becomes important during the performance of the monochromator, because mechanical faults that produce such a situation on the axis do not seriously increase the displacement value.

2. *Focal Length of the Collector System*

Certainly during the design period it must be possible to specify the angular dispersion of the monochromator, $\Delta\theta/\Delta\lambda$, where $\Delta\lambda$ is the spectral interval that will be observed around each wavelength and $\Delta\theta$ is the angular interval between the directions corresponding to the extreme wavelengths of $\Delta\lambda$. The collector system will make in its focal plane an image with a width Δx of the spectral interval. This width is related to the angular interval by

$$\frac{\Delta x}{2} = f \cdot \tan\left(\frac{\Delta\theta}{2}\right), \tag{3}$$

where we assume that the collector system, with focal length f, is perfect. It is possible to consider Δx as the width of a detector (slit, vidicon, etc.) placed in the focal plane.

Let us now suppose that the spatial period of the grating is fixed. Since the grating law is not linear, it is not exactly equivalent to take $\Delta\lambda$ around a wavelength as it is to take it around a different one. Thus, to define the angular dispersion, we choose a particular wavelength as a reference λ_r inside the spectral domain and the spectral interval that we want to inspect around this wavelength $\Delta\lambda_r$. Let ϵ_r be the value given by Eq. (1). The lack of linearity also implies that if we require the angular semi-interval $\Delta\theta/2$ taken from ϵ_r to the increasing angles (also increasing wavelengths) to contain the spectral semi-interval $\Delta\lambda_r/2$ we are sure that in the total angular interval $\Delta\theta$ we will observe a larger spectral interval, although by a much smaller quantity, than $\Delta\lambda_r$. Thus we have to require

$$\Lambda \cdot \left[\sin\epsilon_r + \sin\left(\epsilon_r + \frac{\Delta\theta}{2}\right)\right] = \lambda_r + \frac{\Delta\lambda_r}{2} \cdot \tag{4}$$

Using Eqs. (1), (3), and (4) we find

$$f = \frac{\Delta x/2}{\tan\left[\arcsin\left(\frac{\lambda_r + \Delta\lambda_r}{2\Lambda}\right) - \arcsin\left(\frac{\lambda_r}{2\Lambda}\right)\right]} \cdot \tag{5}$$

Accordingly, with fixed $\Delta\lambda_r$ and Δ_x, that is, fixed angular dispersion, we obtain the right focal length for the desired monochromator.

The angles corresponding to the extreme wavelengths of the observable spectral interval taken around each wavelength λ, generally different from λ_r, are

$$\theta_+ = \epsilon + \arctan\left(\frac{\Delta x/2}{f}\right), \tag{6a}$$

$$\theta_- = \epsilon - \arctan\left(\frac{\Delta x/2}{f}\right), \tag{6b}$$

and the fraction of the observable interval that contains wavelengths bigger than λ is

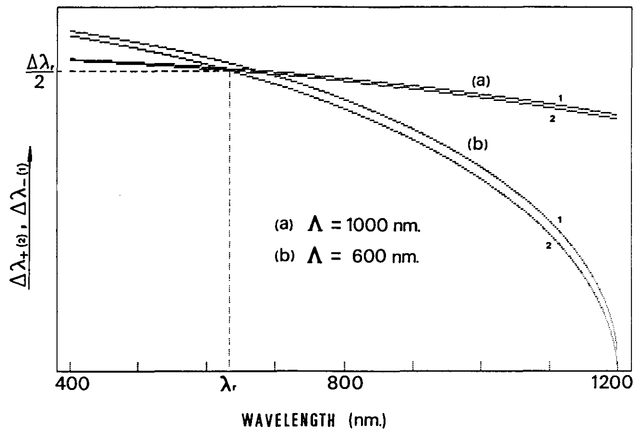

Fig. 4. Variation with wavelength of the fractions of the observable interval ($\Delta\lambda_-$,ΔL_+) for two different spatial periods of the grating.

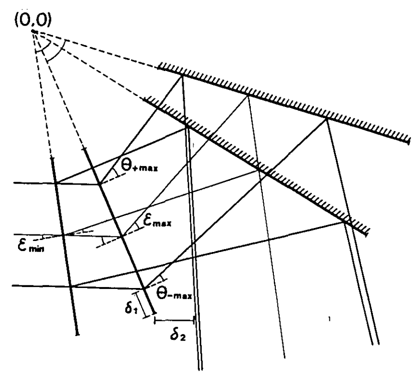

Fig. 5. Ray path for the extreme positions of the system.

$$\Delta\lambda_+ = \tfrac{1}{2} \cdot [2\Lambda \sin\theta_+ - \lambda],$$

while the fraction that contains wavelengths smaller than λ is

$$\Delta\lambda_- = -\tfrac{1}{2} \cdot [2\Lambda \sin\theta_- - \lambda].$$

Figure 4 shows the dependence between both of them with the wavelength and the period of the grating. Note that when the grating period takes on smaller values, the length of the observable interval, $\Delta\lambda = \Delta\lambda_- + \Delta\lambda_+$, may vary notably with wavelength.

3. *Height of Incidence of the Collimated Beam*

Let us again suppose that the spatial period of the grating is fixed. We assume that the collimated beam is incident at height H taken from the vertex. It could happen that the beam diffracted by the grating, after being reflected in the mirror, intercepts the first one again, and in that way the output beam suffers from vignetting. As the wavelength increases, a greater height of incidence is needed to avoid vignetting, so that λ_{max} is what fixes the condition for avoiding vignetting. The positions of the diffracted beam for the two extreme wavelengths of the spectral domain are those shown in Fig. 5. Let θ_{+max} be the angle of diffraction given by Eq. (6a) with $\epsilon = \epsilon_{max}$. Taking δ_1 as the

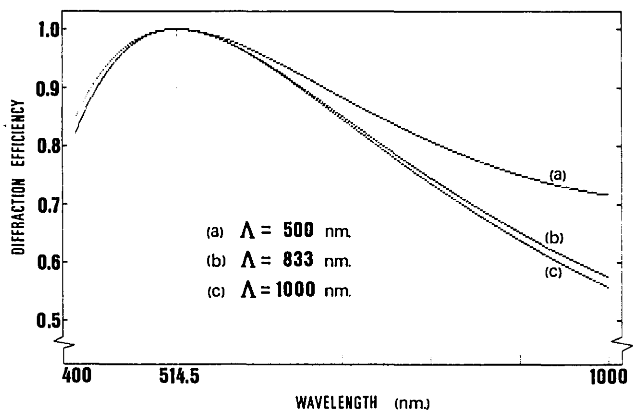

Fig. 6. Diffraction efficiency curves for three values of Λ when the Bragg condition is satisfied for each wavelength in the spectral domain. Refractive-index plate has been taken as 1.52, construction wavelength at 514.5 nm, and plate thickness is 6.5 μm.

distance between the lower extreme of the incident collimated beam and the lower border of the grating, it is possible to show that the minimum value of the height of incidence which avoids vignetting is

$$H = \tfrac{1}{2} \cdot (W + \delta_1 \cdot \cos\epsilon_{max}) \cdot [\cot\alpha \cdot \cot(\alpha - \theta_{+max}) - 1], \quad (7)$$

when $\theta_{+max} < \alpha < \pi/4 + \theta_{+max}/2$. If $\theta_{+max} < \alpha$ and $\alpha > \pi/4 + \theta_{+max}/2$, there is no vignetting for any height of incidence. If $\theta_{+max} > \alpha$, it is not possible to avoid vignetting by means of choosing H, because some rays of the diffracted beam are incident in the mirror at a negative angle with respect to the normal direction (as the rays reach the normal by the shortest route, they follow clockwise); thus, part of the reflected beam is incident again on the grating. This case can be solved by suitable values of Λ, as we see in the following section.

If, in addition to avoiding vignetting, we wish to make the distance between the output beam and the lower border of the grating δ_2 (see Fig. 5), we have to derive a relation similar to Eq. (7).

4. *Spatial Period of the Diffraction Grating*

In the previous sections we assumed that the spatial period of the grating Λ was fixed. The results that we obtain in this section will allow us to fix this parameter.

Let us assume that $\alpha\epsilon(0,\pi)$. Of the constraints that restrict the values of Λ, two are limiting conditions to the design:

(1) all diffracted rays of interest have to reach the mirror;

(2) no ray may be reflected by the mirror at a negative angle with respect to the normal direction.

There is an additional condition that is directed to improving the uses of the monochromator:

(3) the diffraction efficiency of the grating must be as high and uniform as possible in the spectral range of interest.

It is possible to demonstrate that the first condition is fulfilled only if

$$\alpha < \frac{\pi}{2} + \theta_{-min}, \quad (8a)$$

while the second condition is fulfilled only if

$$\theta_{+max} < \alpha. \quad (8b)$$

In addition, if $\alpha > \pi/2$ can be, then it could have $\theta_{+max} > \pi/2$ even when conditions (8) are fulfilled; but this cannot be allowed because in such a case some rays of the input collimated beam would be reflected by the grating instead of transmitted by it. We must, therefore, add another condition:

$$\theta_{+max} < \frac{\pi}{2}\,. \quad (8c)$$

According to Eqs. (5) and (6), relations (8) become

$$\arcsin\left(\frac{\lambda_{min}}{2\Lambda}\right) - \left[\arcsin\left(\frac{\lambda_r + \Delta\lambda_r}{2\Lambda}\right) - \arcsin\left(\frac{\lambda_r}{2\Lambda}\right)\right] > \alpha - \frac{\pi}{2}\,, \quad (9a)$$

$$\arcsin\left(\frac{\lambda_{max}}{2\Lambda}\right) + \left[\arcsin\left(\frac{\lambda_r + \Delta\lambda_r}{2\Lambda}\right) - \arcsin\left(\frac{\lambda_r}{2\Lambda}\right)\right] < \min\left\{\alpha,\frac{\pi}{2}\right\}, \quad (9b)$$

When we solve for Λ in these inequalities, we find that the values of Λ are restricted to belonging to an interval $(\Lambda_{min},\Lambda_{max})$. A discussion of the solution of (9) is in the Appendix. In addition, we find in the Appendix that α also presents a limitation in its values; in fact, it must satisfy

$$\alpha < \frac{\pi}{2} + \arcsin[\lambda_{min}/(\lambda_r + \Delta\lambda_r)]; \quad (10)$$

otherwise there is no value of Λ that satisfies (9a).

Let us now consider the diffraction efficiency of the grating. On the basis of the Kogelnik coupled wave theory[5,6] the diffraction efficiency of a volume transmission hologram has the form

$$\eta = \frac{\sin^2(\sqrt{\phi^2 + \chi^2})}{1 + \chi^2/\phi^2}\,, \quad (11)$$

where ϕ is the modulation parameter which depends on the wavelength and the spatial period of the grating (assuming that the laser wavelength used for the hologram construction, refractive index, and thickness of the hologram are fixed), and the parameter χ is a measure of the deviation from the Bragg condition. When this condition [i.e., Eq. (1)] is satisfied, $\chi = 0$, so that Eq. (11) becomes $\eta = \sin^2\phi$. This magnitude shows us how the diffraction efficiency changes along the spectral domain (see Fig. 6). Note that the efficiency decreases as the period increases. On the other hand, for the wavelengths of the observable spectral interval, taken around one of them which satisfies Eq. (1), a deviation is produced from the Bragg condition. In this case Eq. (11) shows us how the diffraction efficiency changes in the neighborhood of such a wavelength. In Fig. 7 we see that as Λ increases, the efficiency in general becomes uniform, while its value at the Bragg angle decreases, as we have already seen in Fig. 6, and the maximum shifts. Thus it will be convenient to take for Λ sufficiently large values that the

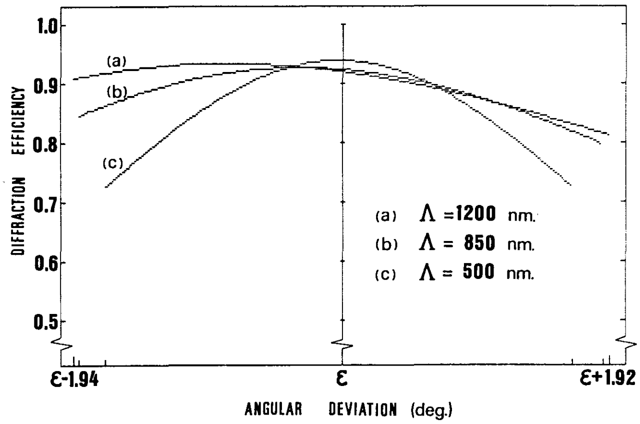

Fig. 7. Diffraction efficiency in the neighborhood of $\lambda = 632.8$ nm as a function of the angular deviation from the Bragg condition measured into the grating for three values of Λ (when outside the grating the semiangular deviation is 3°).

uniformity of the diffraction efficiency allows us to observe all the points belonging to a spectral profile in similar conditions.

5. *Width of the Collimated Beam*

The width of the incident collimated beam w is fixed as soon as the monochromator aperture, or similarly, the f/No., is chosen, because

$$f/\text{No.} = f/w,$$

where f is the focal length of the collector system, previously fixed by the choice of the angular dispersion as shown in Sec. II.A.2.

B. Development of the Procedure

This study of the constraints allows us to establish the following classification of the parameters:

(1) free parameters: the rotation sense;

(2) limited parameters: the coordinates of the rotation axis, the height of the incident beam, the spatial period of the grating, and the angle between the grating and mirror;

(3) dependent parameters: the width of the beam, the focal length, diameter, and center of the collector system, the vertex–mirror and vertex–grating distances, and the mirror and grating sizes.

Broadly speaking, the procedure is developed as follows. When the demands of the desired monochromator and all the free parameters are fixed, one determines the existance intervals of the limited parameters. After choosing the values of each of them, in their corresponding interval, one calculates the values of the dependent parameters. If the final system is not the very best, one may repeat the whole process with other values of the eligible parameters (free and limited). The steps of this calculation are as follows:

(1) Fix the spectral domain ($\lambda_{\min},\lambda_{\max}$), the reference wavelength λ_r, and the angular dispersion (given by $\Delta\lambda_r$ and Δx) of the monochromator.

(2) Calculate the biggest angle between the grating and mirror and choose a smaller value than that one.

(3) Test for Λ following the study developed in the Appendix to calculate the interval ($\Lambda_{\min},\Lambda_{\max}$) and choose a value.

(4) Calculate the diffraction efficiency curves of the grating and study their uniformity; if it is not adequate, choose another value of Λ.

(5) Fix the aperture of the monochromator as well as the mechanical limitations of the optical device (the distance between the output beam and the low border of the grating etc.).

(6) Calculate the height of incidence of the collimated beam that avoids vignetting and is also compatible with the mechanical limitations.

(7) Compute the dependent parameter values by using a ray tracing.

All the previous steps, except those in which one has to select values, are made by computer.

An alternate way is to follow steps (1)–(5) and then repeat (6) and (7) for different allowed values of Λ and different apertures. In this way, it is possible to obtain tables with values of important utility in order to compare the characteristics of different designs. Afterwards the efficiency curves of every design are studied.

C. Remarks

There is no constraint on the rotation sense. Thus, although until now we have assumed that the grating–mirror group rotates in the positive sense, it may also rotate in the opposite sense. But, in this case, the condition for avoiding vignetting for the output beam is less restrictive, because the rays that are incident on the mirror are farther from the normal direction than in the first case. On the contrary, the mirror sizes that are needed for a monochromator with the same characteristics are notably bigger. The study of the system when it rotates in the negative sense is completely analogous to the preceding analysis.

Even though the only limitation on α is the one given by Eq. (10), the choice of the value of this parameter must be done with the idea that, in practice, it is easier to determine the values of some angles than others. This fact affects alignment of the collector system, because its optical axis must be situated making an angle equal to π-2α with the direction of the input beam.

Finally, we note that handling the parameters and even their classifications is in some way arbitrary. One of them, chosen as a dependent parameter, could have been chosen as free, and then another parameter would become dependent and take its values as a function of the former. The same discussion is applicable to the limited parameters. We have made a choice that we think gives more versatility to the design.

III. Applications: Examples

In this section, we use the above design procedure in two examples.

The first is the realization of a monochromator to be used in plasma spectroscopy. The main use of this

Table I. Summary of Design Parameters for the Monochromator to be Used in Plasma Spectroscopy[a]

(Deg)	(nm)	f(cm)	f/No.	h(cm)	w(cm)	MS(cm)	GS(cm)	VMD(cm)	VGD(cm)	DC(cm)
45	1000	29.02	8	3.97	3.63	5.58	5.24	3.94	3.55	4.63

[a] MS and GS are the mirror and grating sizes, respectively; VMD and VGD are the vertex–mirror and vertex–grating distances, respectively; DC is the diameter of the collector system.

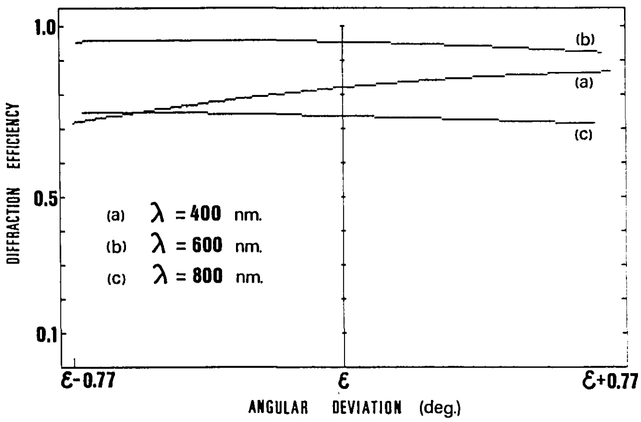

Fig. 8. Diffraction efficiency curves of the first design for three different wavelengths. (The angular deviation is measured into the grating.)

monochromator is to project the spectrum of the H_β lines in an optical multichannel analyzer, although it will be also used to observe spectrums centered in wavelengths belonging to the 400–800-nm interval.

The requirements of such a monochromator are the following. It must have high luminosity, but it does not need a large aperture because the input beam was previously collimated by the experiment. It does not require high resolution because the width at half-height of the H_β profile[7] is 4 nm. Thus, to observe the whole profile, we need an interval of 20 nm centered at 486.3 nm. The resolution will be determined by the fact that we project 40 nm on a vidicon which has a width of 12 mm. It is also necessary that the diffraction efficiency around each wavelength be as uniform as possible. This requirement obliges us to compromise in our choice of the spatial period of the grating (inside its interval of existence), because a smaller period gives a small focal length, mirror, and grating sizes. However, the diffraction efficiency curves are far from uniformity. Table I shows the choice of the spatial period of the grating as well as the values of the other parameters that we have determined. Figures 6(c) and 8 shows the efficiency curves corresponding to this design. We are currently making a prototype that will be the subject of another paper.

The second example is the construction of another monochromator applicable to the spectroscopy of atmospheric absorption.[8] Unlike the former monochromator, this one does not need spatial resolution because it will be used to observe the luminous flux of 1 nm of the spectrum through a slit of 60 μm. In addition, it does not need high luminosity because the source is the sun. However, it is necessary that this monochromator work in a wide spectral range: 400–1000 nm in view of the type of detector that will be used. However, these demands do not impose serious restrictions on the method of design, so that it is possible to obtain several models, all with very good characteristics. In view of the freedom in the choice of the parameters, these have taken with standard values: focal lengths of the photographic objectives etc. Their values are in Table II, and the corresponding efficiency curve is in Fig. 6(a).

IV. Conclusions

In this paper we have described a procedure for the design of monochromators for specific applications which are easy to construct. As a result, this procedure reduces the user's task to choosing the values of some parameters (in which there are same possibilities of choice). The automation of the subsequent calculations provides an easy way to perform the trial and error technique: by repetition of the process for different choices of these parameters until obtaining the best system.

The two examples show the versatility of this procedure, which is especially well suited for obtaining low resolution monochromators, although it is equally useful when high resolution is demanded.

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Appendix

Let us consider inequality (9a). Depending on the values of α there are different groups of values of Λ that satisfy (9a). [The minimum value for Λ is $(\lambda_r + \Delta\lambda_r)/2$ in order to ensure that all the functions that appear in

Table II. Summary of Design Parameters for the Monochromator to be Used in Atmosphere Spectroscopy[a]

(Deg)	(nm)	f(cm)	f/No.	h(cm)	w(cm)	MS(cm)	GS(cm)	VMD(cm)	VGD(cm)	DC(cm)
45	833	4.71	4	5.77	1.18	2.79	3.74	5.57	5.44	1.70

[a] See footnote of Table I.

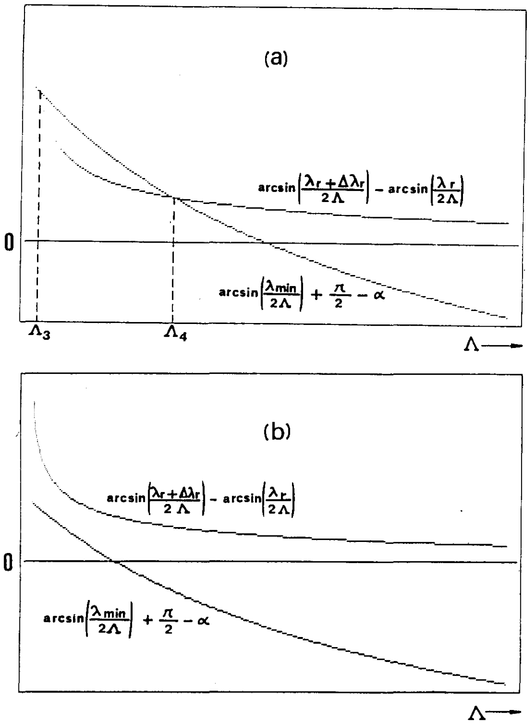

Fig. 9. Two possible cases when condition (C)(2) of the Appendix is satisfied.

this inequality are defined.] All the different possibilities are explained:

(A) $\alpha < \arcsin[\lambda_r/(\lambda_r + \Delta\lambda_r)]$.

Inequality (9a) is fulfilled for all $\Lambda > (\lambda_r + \Delta\lambda_r)/2$.

(B) $\arcsin[\lambda_r/(\lambda_r + \Delta\lambda_r)] < \alpha < \pi/2$.

Now there are two cases:

(1) $\arcsin[\lambda_{min}/(\lambda_r + \Delta\lambda_r)] > \alpha - \arcsin[\lambda_r/(\lambda_r + \Delta\lambda_r)]$.

And again (9a) is fulfilled for all $\Lambda > (\lambda_r + \Delta\lambda_r)/2$.

(2) $\arcsin[\lambda_{min}/(\lambda_r + \Delta\lambda_r)] < \alpha - \arcsin[\lambda_r/(\lambda_r + \Delta\lambda_r)]$.

Now there is Λ_1 for which (9a) is fulfilled if $\Lambda > \Lambda_1$.

(C) $\pi/2 < \alpha < \pi/2 + \arcsin[\lambda_{min}/(\lambda_r + \Delta\lambda_r)]$.

There are the following cases:

(1) $\arcsin[\lambda_{min}/(\lambda_r + \Delta\lambda_r)] > \alpha - \arcsin[\lambda_r/(\lambda_r + \Delta\lambda_r)]$.

There is a Λ_2 for which (9a) is fulfilled if $\Lambda < \Lambda_2$.

(2) $\arcsin[\lambda_{min}/(\lambda_r + \Delta\lambda_r)] < \alpha - \arcsin[\lambda_r/(\lambda_r + \Delta\lambda_r)]$.

In this second case there are two possibilities, which are shown in Fig. 9. (It is not possible to find in an explicit way the condition that allows us to distinguish between the two cases, but it is possible to find it numerically.) When it is the case shown in Fig. 9(a) there are Λ_3 and Λ_4 so that condition (9a) is fulfilled if $\Lambda_3 < \Lambda < \Lambda_4$. In the case of Fig. 9(b) there is no value of Λ that satisfies the condition.

(D) $\alpha > \pi/2 + \arcsin[\lambda_{min}/(\lambda_r + \Delta\lambda_r)]$.

There is no value of Λ that fulfills condition (9a).

Let us now consider inequality (9b). Let $\varphi = \min(\alpha,\pi/2)$. There are the following possibilities:

(A) $\lambda_r < \lambda_{max} < \lambda_r + \Delta\lambda_r$.

To ensure that all the functions in (9b) are defined, the minimum value of Λ must be, as it was before, $(\lambda_r + \Delta\lambda_r)/2$, and there are two cases:

(1) $\arcsin[\lambda_{max}/(\lambda_r + \Delta\lambda_r)] < \varphi - \pi/2 + \arcsin[\lambda_r/(\lambda_r + \Delta\lambda_r)]$.

Now (9b) is fulfilled for all $\Lambda > (\lambda_r + \Delta\lambda_r)/2$.

(2) $\arcsin[\lambda_{max}/(\lambda_r + \Delta\lambda_r)] > \varphi - \pi/2 + \arcsin[\lambda_r/(\lambda_r + \Delta\lambda_r)]$.

There is a Λ_1 so that (9b) is fulfilled if $\Lambda > \Lambda_1$.

(B) $\lambda_{max} > \lambda_r + \Delta\lambda_r$.

At first sight, the minimum value of Λ must now be $\lambda_{max}/2$. It is possible to demonstrate that there is a Λ_2 so that (9b) is fulfilled if $\Lambda > \Lambda_2$.

Note that in general the simultaneous verification of the studied inequalities demands that the choice of Λ be restricted to an interval $(\Lambda_{min},\Lambda_{max})$.

The Λ with subscripts that appear in the previous paragraphs will be found with the help of the numerical calculus skills used to solve trascendental equations. In some cases we use methods of absolute convergency, and in other cases it is enough with the iterative methods of conditional convergency, which converge to a solution more rapidly.

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