Loans Amortization with Payments Constant in Real Terms

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Abstract In this paper we present the so-called debts amortization with payments constant in real terms. In this kind of loan, the payments are increasing according to the Consumer Price Index (CPI). Thus, the borrower assigns the same proportion of his/her salary to the discharge of the loan, without reducing his/her purchasing power due to this increase, but with the advantage of a minor first payment for the total amortization of the principal over the agreed loan period. We will use capitalization or discount functions with the inflation implicit and we will study the inflation rate dependence on the interest rate; taking into account that there exists a strong correlation between both magnitudes, it is expected that the final loan duration is not widely modified with respect to the initially agreed one. This work presents the theoretical treatment and some practical applications of this new proposed financial product, especially, for mortgage amortization.

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JEL Classification G21, G24.

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1. Introduction

Debts with payments adjusted to the rates of inflation corresponding to the loan periods were introduced by De Pablo (1991;1998). In these works there was appearing the calculation of the first payment using either the French method[1] or payments increasing in geometric progression, by correcting later the following ones according to the rate of inflation of the corresponding loan period. As an immediate consequence, this correction of payments gave rise, in the case of a rising inflationary situation, to a reduction in the loan term, whereby they were labelled "loans of variable duration". Likewise, all the financial magnitudes involved in the amortization schedule of the loan (outstanding principal at beginning of period, interest due at end of period, and principal repaid at the end of period) were re-calculated.

Later, Cruz *et al.* (1996) adjusted the loan payments to the income expected by companies belonging to the Agricultural Sector and, after, García *et al.* (2001) and Cruz and Muñoz (1998) generalized the previous result to the cash-flows generated by the transaction investment to which the loan amount was assigned, either in certainty, risk or uncertainty context, giving rise to a method of amortization "fitted" to the financed company (López and Cazorla (1998)).

However, a constraint of the previous works is that the interest rate was constant during the complete loan term, whereby, in this paper, we will follow the same approach but introducing variable instead of constant interest rates which is the case of most loans borrowed at present, especially mortgage loans. In this way, we expect that the loan duration is not affected too much, since a very strong correlation is estimated between inflation and interest rates (Cruz and González (2007)).

This paper is organized as follows: In Section 2 we present the problem of fitting the loan payments according to the rate of inflation in the previous period

[1] Or methods of constant payments.

and the interest rate corresponding to the current period. This will be solved following two procedures: first, re-calculating at every instant the new payment, and, second, calculating only the first payment to fit, later, the following ones according to the rates of inflation, and the interests due at the end of each period according to the current interest rates. This way, some examples will be proposed in bull, bassist and plane situations for the rates of inflation. In Section 3 the fit between inflation and real interest rates is studied, concluding that a strong dependence exists which allows us to present some empirical applications of loans with inflation and interest rates already known, and other debts where these magnitudes are projected in future. In all considered cases it is observed that there exists no significant variation in the loan term, as foreseen. Thus, in Section 4 an empirical application is presented and, finally, Section 5 summarizes and concludes.

2. Adjustment of the Loan Payments According to Inflation and Interest Rate

In this paper we will consider loans whose payments are variable in geometric progression, being the common ratio of the progression one plus the average inflation rate expected for the loan term. This is because we have to take into account that the rates of inflation calculated or foreseen for different periods are accumulative over time.

It is well-known that the present value of an ordinary annuity variable in geometric progression for n years with money worth i is denoted by $A_{(a,q)n\rceil i}$, where a is the first payment and q the common ratio of the progression (Cruz and Valls (2008)):

$$A_{(a,q)_{n}\rceil i} = a\frac{1-\left(\frac{q}{1+i}\right)^{n}}{(1+i)-q}.$$

In this paper we will use $q = 1 + g$ as common ratio of the progression, being g the rate of inflation for the year previous to the beginning of the term which is provisionally taken as the average inflation rate expected for the interval $[0, n]$. Moreover, we will use the interest rate i_1 (EURIBOR plus a margin or differential) in force for this first year. Therefore, in this case, we write:

$$C_0 = a_1 \frac{1 - \left(\frac{1+g}{1+i_1}\right)^n}{i_1 - g}, \tag{1}$$

being C_0 the loan amount and a1 the first payment. However, when finishing the first loan period, the real inflation rate for this year will have been g_1 instead of g, whereby we would have to re-calculate the payment for the second year according to g_1 and not to g:

$$C_1 = a_2 \frac{1 - \left(\frac{1+g_1}{1+i_2}\right)^{n-1}}{i_2 - g_1}, \tag{2}$$

being C_1 the outstanding principal at the end of the first period, a_2 the second payment, and i_2 the interest rate in force for the second year. Repeating this reasoning for one more year, we would have:

$$C_2 = a_3 \frac{1 - \left(\frac{1+g_2}{1+i_3}\right)^{n-2}}{i_3 - g_2}, \tag{3}$$

and so on. In general, we would write:

$$C_s = a_{s+1} \frac{1 - \left(\frac{1+g_s}{1+i_{s+1}}\right)^{n-s}}{i_{s+1} - g_s}, s = 0, 1, \ldots, n-1, \tag{4}$$

being:

- C_s the outstanding principal at the end of period s,
- a_{s+1} the payment corresponding to period $s+1$,
- g_s the rate of inflation of period s, and
- i_{s+1} the interest rate in force for period $s+1$.

Let us suppose that the initial rate of inflation g has a well known absolute average increase or decrease k in the interval $[0, n]$. Next, we are going to denote by variable x the average increment or decrement of the initial interest rate i_1 with respect to k. Thus, $k+x$ represents the absolute average increase or decrease of the initial interest rate i_1. If we had to calculate the first new payment (which obviously is a function of x), we would write:

$$C_0 = a_1'(x) \frac{1 - \left(\frac{1+g+k}{1+i_1+k+x}\right)^n}{i_1 - g + x}. \tag{5}$$

Taking into account that the quotient in expression (5) depends, among others, on the variable x, we are going to denote it by $f(x)$, that is to say:

$$f(x) = \frac{1 - \left(\frac{1+g+k}{1+i_1+k+x}\right)^n}{i_1 - g + x}. \tag{6}$$

First, in order to study its increase or decrease, we are going to calculate the derivative of $f(x)$; later, we will study the relationship between $f(0)$ and the quotient in expression (1) to deduce, finally, on the basis of the increase or decrease of $f(x)$, the increases or decreases of the new payment. In effect, it is possible to easily verify that:

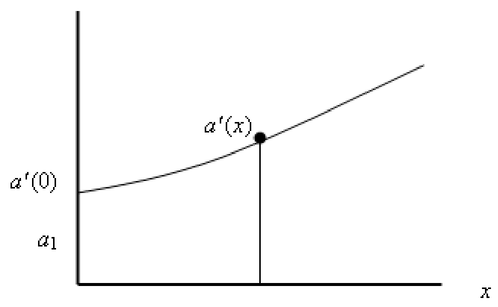
$$f'(x) = \frac{(n\frac{i_1-g+x}{1+i_1+k+x} + 1)\left(\frac{1+g+k}{1+i_1+k+x}\right)^n - 1}{(i_1 - g + x)^2}. \tag{7}$$

Next, we are going to study the relationship between $f(0)$ (when the absolute average increase or decline of i_1 and g over $[0, n]$ coincide) and the quotient of equation (1). However, it is possible to show that, when i_1 is less than g (which is the most likely situation in our study):

- If $k > 0$, $\frac{1+g+k}{1+i_1+k} < \frac{1+g}{1+i_1}$, then $\frac{1-\left(\frac{1+g+k}{1+i_1+k}\right)^n}{i_1-g} > \frac{1-\left(\frac{1+g}{1+i_1}\right)^n}{i_1-g}$, and so $a_1'(x) < a_1$,

- If $k < 0$, $\frac{1+g+k}{1+i_1+k} > \frac{1+g}{1+i_1}$, then $\frac{1-\left(\frac{1+g+k}{1+i_1+k}\right)^n}{i_1-g} < \frac{1-\left(\frac{1+g}{1+i_1}\right)^n}{i_1-g}$, and so $a_1'(x) > a_1$.

If there was taking place a decrease of interest and inflation rates, but this decrease was minor for interest rates, then $k < 0$ and $x > 0$. However, taking into account that, for $x = 0$, $a'(0) > a_1$ and that, in the interval $[0, +\infty)$, $f'(x) > 0$, it will follow that $a'(x) > a_1$.



Therefore, because of this result, we expect some higher payments for the first years of the loan period which, eventually, might be compensated by a lower value of the progression common ratio (take into account that we are assuming a situation in which the rate of inflation is decreasing). Nevertheless, this procedure is very laborious, whereby we are going to simplify the calculations in the following way.

Once calculated the first payment, the same as in the previous paragraph, we are going to keep this first payment up to the end of the loan period, but with the following difference: because of the payments are increasing in geometric progression, the payment increase rate to be used every year will be the rate of inflation of the previous period, obtaining the so-called *loans with payments constant in real terms*. This is because, during the complete loan period, we will be paying for the same payment as calculated for the first period but in real terms; therefore, this is a loan of variable duration.

This kind of loan can be amortized either at constant or at variable interest rate; nevertheless, as we will see later on, in order to minimize the variability of the loan term, we will have to choose the case of variable interest rate. Therefore, in every period we will have to calculate the discharged interest with the interest rate in force for this period. Thus, we have now a loan operation whose payments are increasing in geometric progression, with common ratio equal to one plus the inflation rate of the previous period, which makes the payments constant in real terms. Moreover, the interest rate is also variable, whereby the duration will be also variable.

As a consequence of the last statement, the proposed method provides an amortization schedule with its own characteristics in which the payments only can be known once the inflation rate for the previous period is known, whereas, for the calculation of the interest due and the principal repaid at the end of period, it will be necessary, moreover, the knowledge of the reference interest rate for the period.

In order to construct the debt amortization schedule, we are going to fill the table in line by line as follows:

1. Once known the interest rate in force for the first period and the rate of inflation of the previous period (which will be taken as a reference for the calculation of the first payment), we will calculate the first payment, which will be taken as a reference for the calculation of the remainder payments:

$$a_1 = \frac{C_0(i_1 - g)}{1 - \left(\frac{1+g}{1+i_1}\right)^n}$$

and

$$a_s = min\left\{a_1 \prod_{k=1}^{s}(1 + g_{k+1}), C_{s-1}(1 + i_s)\right\}.$$

2. We will calculate the interest amount discharged at the end of the first period and, by difference with its corresponding payment, we will obtain the principal amount repaid at the end of period which, in the first periods, might be negative; starting from this amount, we will calculate the interest due, the

principal repaid and the outstanding principal at the end of the first period:

$$I_1 = C_0 i_1,$$
$$A_1 = a_1 - I_1,$$
$$C_1 = C_0 - A_1.$$

3. At the end of the second period, the inflation rate of the first period is already known, so we will be able to calculate the second payment amount and, analogously to the previous paragraph, we will find the interest due, the principal repaid and the outstanding principal at the end of the second period, and so on:

$$I_s = C_{s-1} i_s,$$
$$A_s = a_s - I_s,$$
$$C_s = C_{s-1} - A_s.$$

This iterative procedure will be repeated up to an instant s such that:

$$\left| a_1 \prod_{k=1}^{s} (1 + g_{k-1}) - C_{s-1} i_s \right| > C_{s-1},$$

in whose case

$$A_s = C_{s-1}.$$

3. Correlation between Interest and Inflation Rates

It is well-known that the inflation rate (g) is directly correlated with interest rates (i) and, therefore, taking into account that:

$$A_s = min \left\{ \left| a_1 \prod_{k=1}^{s} (1 + g_{k-1}) - C_{s-1} i_s \right|, C_{s-1} \right\}$$

we can state the following:

1. In the case of loans with amortization periods in which the rate of inflation is constant, it is expected that the interest rates are also constant and, therefore, the amortization term will be equal to the foreseen one.
2. In the case of raising rates of inflation, the interest rates will also increase, whereby the increase of the principal repaid, due to the increase of the inflation, will be corrected by the opposite effect because of the raise of interest rates.
3. If the rate of inflation decreases, the interest rate will also diminish, whereby the decrease in the principal repaid due to the decrease of the rate of inflation, will be corrected because of the decrease of interest rates.

Though it is true that in the first payments of the loan the effect of the interest is higher than that of the inflation, in the last payments the opposite effect takes place, whereby it is expected that the final loan amortization term can oscillate in a small margin of time.

4. Empirical Application

Now let us see some practical examples of how the same loan could be amortized in different situations with respect to the expected evolution of the inflation and interest rates, as described in Section 3. In order to do this, we are going to suppose three situations. The first one represents a period in which the interest rates, as well as the inflation, remain constant. The second situation describes the case in which both magnitudes increase, and, finally, the third one of the situations studies the case in which both magnitudes diminish. For this first application, we are going to suppose, a loan to be amortized by annual payments over the next 10 years at a 3.10% as initial value of interest rate and 3.20% as inflation rate.

As it can be observed, in the second situation the last payment would have to be lower than the corresponding to the initial rate of inflation, since, in opposite

FIRST SITUATION: *The magnitudes remain constant.*

Year	i_s	g_s	a_s	I_s	A_s	C_s
0		3.20%				100,000.00 €
1	3.10%	3.20%	10,265.08 €	3,100.00 €	7,165.08 €	92,834.92 €
2	3.10%	3.20%	10,593.56 €	2,877.88 €	7,715.68 €	85,119.24 €
3	3.10%	3.20%	10,932.56 €	2,638.70 €	8,293.86 €	76,825.38 €
4	3.10%	3.20%	11,282.40 €	2,381.59 €	8,900.81 €	67,924.57 €
5	3.10%	3.20%	11,643.44 €	2,105.66 €	9,537.77 €	58,386.79 €
6	3.10%	3.20%	12,016.03 €	1,809.99 €	10,206.03 €	48,180.76 €
7	3.10%	3.20%	12,400.54 €	1,493.60 €	10,906.93 €	37,273.83 €
8	3.10%	3.20%	12,797.36 €	1,155.49 €	11,641.87 €	25,631.96 €
9	3.10%	3.20%	13,206.87 €	794.59 €	12,412.28 €	13,219.68 €
10	3.10%		13,629.49 €	409.81 €	13,219.68 €	0.00 €

SECOND SITUATION: *The interest rates annually increase 0.15% and the inflation rates 0.20%*

Year	i_s	g_s	a_s	I_s	A_s	C_s
0						100,000.00€
1	3.10%	3.20%	10,265.08€	3,100.00€	7,165.08€	92,834.92€
2	3.25%	3.40%	10,593.56€	3,017.13€	7,576.43€	85,258.49€
3	3.40%	3.60%	10,974.93€	2,898.79€	8,076.14€	77,182.35€
4	3.55%	3.80%	11,391.98€	2,739.97€	8,652.00€	68,530.35€
5	3.70%	4.00%	11,847.66€	2,535.62€	9,312.03€	59,218.31€
6	3.85%	4.20%	12,345.26€	2,279.90€	10,065.35€	49,152.96€
7	4.00%	4.40%	12,888.45€	1,966.12€	10,922.33€	38,230.63€
8	4.15%	4.60%	13,481.32€	1,586.57€	11,894.75€	26,335.88€
9	4.30%	4.80%	14,128.42€	1,132.44€	12,995.98€	13,339.90€
10	4.45%		14,834.84€	593.63€	14,241.22€	-901.32€

case, the total principal repaid would be greater than the loan amount. But the adjustment would be carried out in a unique period.

In the third case, we can observe that one more period will be necessary with a small payment, since, due to a decreasing evolution of inflation, we need more time to completely amortize the principal.

Now let us see how the interest rates are correlated with the rates of inflation. It is well-known that "During the inflationary periods, lenders demand a compensation for the depreciation of the money purchasing power. Therefore, the nominal interest rate (Ayres (1963))or market rate tends to include a premium

THIRD SITUATION: *The interest rates annually decrease 0.15%, whereas the rate* of inflation annually diminishes 0.20%.

Year	i_s	g_s	a_s	I_s	A_s	C_s
0						100,000.00€
1	3.10%	3.20%	10,265.08€	3,100.00€	7,165.08€	92,834.92€
2	3.00%	3.05%	10,578.16€	2,785.05€	7,793.12€	85,041.80€
3	2.90%	2.90%	10,884.93€	2,466.21€	8,418.72€	76,623.08€
4	2.80%	2.75%	11,184.27€	2,145.45€	9,038.82€	67,584.26€
5	2.70%	2.60%	11,475.06€	1,824.78€	9,650.28€	57,933.98€
6	2.60%	2.45%	11,756.20€	1,506.28€	10,249.91€	47,684.07€
7	2.50%	2.30%	12,026.59€	1,192.10€	10,834.49€	36,849.58€
8	2.40%	2.15%	12,285.16€	884.39€	11,400.77€	25,448.81€
9	2.30%	2.00%	12,530.86€	585.32€	11,945.54€	13,503.26€
10	2.20%		12,762.69€	297.07€	12,465.61€	1,037.65€

equal to the rate of the expected inflation (Fisher's hypothesis)" (Mochón (2005)), and moreover, the fluctuation of the reference interest rates also constitutes a measure of macroeconomic politics, either to foment the consumption (decrease of interest rates) and therefore to increase the inflation, or to stop it, in whose case the rates are increased to achieve that the rate of inflation diminishes. It is for this reason whereby, beside both magnitudes being very correlated, they present a big delay in their correlation, or, in words of Blanchard (2003) about the existing relationship between the nominal interest rates and the inflation in different countries: " Since this relationship is only verified in the medium term, it would not be expected that the inflation and the nominal interest rates were evolving in the same way in all countries and at any time". In our case, the combination with a greater correlation is that one in which the interest rates are regressed on the rates of inflation retarded a period and with a delay of six periods, in whose case the regression, with 89.47% correlation, remains of the following form:

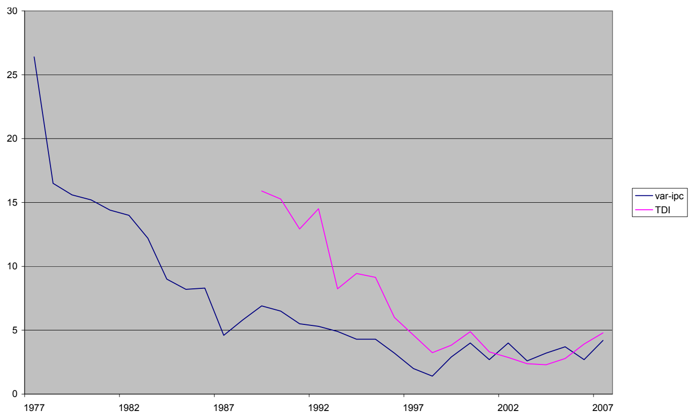
$$i_s = 1.3795 g_{s-1} + 1.0177 g_{s-6} - 4.1148.$$

Dependent Variable: MIBOR - EURIBOR
Method: Least Squares
Sample(adjusted): 1990 2007
Included observations: 19 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-4.114765	0,9781	-4,207008	0,00067
VARIPC(-1)	1.379539	0,3185	4,331256	0,00052
VARIPC6	1.017662	0,1798	5,66044	3,5E-05
	R-squared			0,911336929
	Adjusted R-squared			0,900254045
	S.E. of regression			1,474447889

Source: Instituto Nacional de Estadística and www.euribor.com/es.

This estimation has been carried out with the data of last 19 annual interest rates, and of the last 25 annual rates of inflation. In the following graph we can observe the high correlation between both magnitudes.



Taking into account the last regression, we are going to determine, in the first table below, the foreseen interest rates for an initial inflation rate of 3% and an arithmetic increasing of 0.20%. Analogously, in the second table, we are going to determine the foreseen interest rates for an initial inflation rate of 4.25% and an arithmetic decreasing of 0.15%.

Year	g	i (estimated)	Year	g	i (estimated)
-6	2.00		6	4.20	5.46
-5	2.20		7	4.40	5.94
-4	2.40		8	4.60	6.42
-3	2.60		9	4.80	6.89
-2	2.80		10	5.00	7.37
-1	3.00		11	5.20	7.85
1	3.20	3.06	12	5.40	8.33
2	3.40	3.54	13	5.60	8.81
3	3.60	4.02	14	5.80	9.29
4	3.80	4.50	15	6.00	9.77
5	4.00	4.98			

Year	g	i (estimated)	Year	g	i (estimated)
-6	5.00		6	3.35	6.04
-5	4.85		7	3.20	5.68
-4	4.70		8	3.05	5.32
-3	4.55		9	2.90	4.96
-2	4.40		10	2.75	4.60
-1	4.25		11	2.60	4.24
1	4.10	7.84	12	2.45	3.88
2	3.95	7.48	13	2.30	3.52
3	3.80	7.12	14	2.15	3.16
4	3.65	6.76	15	2.00	2.80
5	3.50	6.40			

4.1. Retrospective Application

In this paragraph we are going to present an example of how a loan had been amortized using this new method of amortization, compared to the traditional method of constant payments.

As observed in the next amortization schedules, in case of having requested a loan to be amortized by the method of constant payments over 15 years at a variable interest rate, the first payment had been 18,161.42€, and the total amount of the due interest, 92,703.65€.

On the other hand, if the same loan had been requested to be amortized by the method proposed here, the first payment had been 13,346.58€, whereas the total of the due interest had been 92,687.63€. Moreover, the payback period had been three years less than in the previous case.

AMORTIZATION SCHEDULE. CONSTANT PAYMENTS						
Year	i_s	A_s	I_s	a_s	C_s	*MIBOR*
					100,000.00€	
1990	16.27%	1,893.42€	16,268.00€	18,161.42€	98,106.58€	15.27%
1991	13.93%	2,625.20€	13,661.34€	16,286.54€	95,481.38€	12.93%
1992	15.51%	2,684.78€	14,806.30€	17,491.08€	92,796.60€	14.51%
1993	9.24%	4,542.34€	8,572.55€	13,114.89€	88,254.26€	8.24%
1994	10.45%	4,649.34€	9,218.16€	13,867.50€	83,604.92€	9.45%
1995	10.14%	5,211.72€	8,474.19€	13,685.91€	78,393.20€	9.14%
1996	6.99%	6,546.40€	5,482.82€	12,029.23€	71,846.80€	5.99%
1997	5.62%	7,357.94€	4,039.95€	11,397.89€	64,488.86€	4.62%
1998	4.24%	8,105.63€	2,734.33€	10,839.96€	56,383.23€	3.24%
1999	4.81%	8,329.88€	2,709.78€	11,039.66€	48,053.35€	3.81%
2000	5.89%	8,543.40€	2,829.86€	11,373.27€	39,509.94€	4.89%
2001	4.29%	9,264.66€	1,693.40€	10,958.06€	30,245.28€	3.29%
2002	3.88%	9,700.70€	1,172.91€	10,873.62€	20,544.58€	2.88%
2003	3.38%	10,101.42€	695.02€	10,796.45€	10,443.15€	2.38%
2004	3.30%	10,443.15€	345.04€	10,788.20€	0.00 €	2.30%
TOTAL		**100,000.00€**	**92,703.65€**	**192,703.65€**		

AMORTIZATION SCHEDULE. CONSTANT PAYMENTS							
Year	i_s	A_s	I_s	a_s	C_s	*MIBOR*	*Var-CPI*
					100,000.00€		
1990	16.27%	**-2,921.42€**	16,268.00€	13,346.58€	102,921.42€	15.27%	6.50%
1991	13.93%	**-251.16€**	14,331.81€	14,080.65€	103,172.58€	12.93%	5.50%
1992	15.51%	**-1,172.05€**	15,998.97€	14,826.92€	104,344.63€	14.51%	5.30%
1993	9.24%	5,914.08€	9,639.36€	15,553.44€	98,430.55€	8.24%	4.90%
1994	10.45%	5,941.17€	10,281.07€	16,222.24€	92,489.38€	9.45%	4.30%
1995	10.14%	7,545.07€	9,374.72€	16,919.79€	84,944.31€	9.14%	4.30%
1996	6.99%	11,520.22€	5,941.00€	17,461.23€	73,424.09€	5.99%	3.20%
1997	5.62%	13,681.82€	4,128.64€	17,810.45€	59,742.27€	4.62%	2.00%
1998	4.24%	15,526.73€	2,533.07€	18,059.80€	44,215.55€	3.24%	1.40%
1999	4.81%	16,458.53€	2,125.00€	18,583.53€	27,757.01€	3.81%	2.90%
2000	5.89%	17,692.26€	1,634.61€	19,326.87€	10,064.75€	4.89%	4.00%
2001	4.29%	10,064.75€	431.38€	10,496.13€	0.00€	3.29%	2.70%
2002	3.88%					2.88%	4.00%
2003	3.38%					2.38%	2.60%
2004	3.30%					2.30%	3.20%
TOTAL		**100,000.00€**	**92,687.63€**	**192,687.63€**			

4.2. Practical Application with Projected Interests

Finally, we will see how the examples of Section 4 would be amortized, but using the projected interests.

SECOND SITUATION: INCREASING INFLATION						
Year	i_s	g_s	a_s	I_s	A_s	C_s
0		3.00%				100,000.00€
1	3.06%	3.20%	10,243.16€	3,059.10€	7,184.06€	92,815.94€
2	3.54%	3.40%	10,591.42€	3,284.33€	7,307.09€	85,508.85€
3	4.02%	3.60%	10,972.72€	3,435.73€	7,536.99€	77,971.86€
4	4.50%	3.80%	11,389.68€	3,506.72€	7,882.96€	70,088.90€
5	4.98%	4.00%	11,845.27€	3,488.23€	8,357.04€	61,731.87€
6	5.46%	4.20%	12,342.77€	3,368.28€	8,974.49€	52,757.37€
7	5.94%	4.40%	12,885.85€	3,131.54€	9,754.31€	43,003.07€
8	6.42%	4.60%	13,478.60€	2,758.72€	10,719.87€	32,283.19€
9	6.89%	4.80%	14,125.57€	2,225.80€	11,899.77€	20,383.43€
10	7.37%	5.00%	14,831.85€	1,503.09€	13,328.76€	7,054.66€
11	7.85%	5.20%	15,603.11€	553.79€	15,049.31€	-7,994.65€

THIRD SITUATION: DECREASING INFLATION						
Year	i_s	g_s	a_s	I_s	A_s	C_s
0		4.25%				100,000.00€
1	7.84%	4.10%	12,499.58€	7,836.58€	4,663.01€	95,336.99€
2	7.48%	3.95%	12,993.32€	7,128.34€	5,864.97€	89,472.02€
3	7.12%	3.80%	13,487.06€	6,368.09€	7,118.97€	82,353.05€
4	6.76%	3.65%	13,979.34€	5,565.28€	8,414.06€	73,939.00€
5	6.40%	3.50%	14,468.62€	4,730.81€	9,737.81€	64,201.18€
6	6.04%	3.35%	14,953.32€	3,876.90€	11,076.41€	53,124.77€
7	5.68%	3.20%	15,431.82€	3,017.01€	12,414.82€	40,709.96€
8	5.32%	3.05%	15,902.49€	2,165.57€	13,736.92€	26,973.04€
9	4.96%	2.90%	16,363.66€	1,337.85€	15,025.82€	11,947.22€
10	4.60%	2.75%	16,813.66€	549.61€	16,264.05€	-4,316.83€

Observe that, in the first table, with estimated interest rates, the payback is kept constant, regularizing, only, the last payment.

In the second case, when the rate of inflation is decreasing, we will need one more period for a last very small payment.

5. Conclusions

The aim of this paper has been the reduction of the first payments of a loan in order to make easier the access to the credit of those sectors of the population with minor income, since it is an usual practice the increase of the salaries according to the rate of inflation and, in consequence, the debtor would keep, during the whole life of the loan, the same purchasing power, because the payment will always represent the same proportion of his/her salary.

In Section 4.1 we have seen how the reduction of the first payment is very high, because, with the proposed amortization method, it decreases 26%. On the other hand, and in comparison with the references on this topic, the introduction of variable interest rates in this method of amortization allows us to keep, in most times, the duration of the loan, since, as explained in Section 4, the effects of the fluctuations of interest rates are, in some sense, compensated by the fluctuations of the rate of inflation that it is expected take place in the same way.

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