Assessing the profitability of cooperative advertising programs in competing channels[1]

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Highlights

- Profitability of cooperative advertising programs for competing channels.

- Results for decentralized vs integrated competing channels.

- Cooperative advertising benefits competing manufacturers in most cases.

- Cooperative advertising does not benefit competing retailers in most cases.

- Total channel profit can be significantly improved with cooperative advertising.

Assessing the profitability of cooperative advertising programs in competing channels

Abstract

A large literature studied the profitability (effectiveness) of cooperative advertising programs (CAPs) in distribution channels, but very few studies modeled pricing decisions in competitive markets *under different channel structures. This paper fills this gap.* We propose a game-theoretic model where two competing channels make pricing and promotional decisions. *The efectiveness of CAPs is studied under different channel structures to examine how vertical and horizontal externalities can impact the effectiveness of CAPs.* Each channel structure can be integrated or decentralized to account for different vertical interaction effects, resulting in three cases: (i) both channels are decentralized ($\mathcal{DD}$), (ii) both are integrated ($\mathcal{II}$), and (iii) a hybrid structure where one channel is decentralized and is competing with an integrated channel ($\mathcal{DI}$). *We solve six non-cooperative games: (1) both manufacturers offer CAPs under $\mathcal{DD}$, (2) only one manufacturer offers a CAP under $\mathcal{DD}$, (3) both manufacturers do not offer CAPs under $\mathcal{DD}$, (4) the decentralized manufacturer offers a CAP under $\mathcal{DI}$, (5) the decentralized manufacturer does not offer a CAP under $\mathcal{DI}$, and (6) the channel problem under $\mathcal{II}$.* Then, we obtain and compare equilibrium profits and strategies across these games. The main results indicate that the profitability of CAPs depends on the levels of price competition and of the advertising effects. Also,

while manufacturers benefit from CAPs, retailers may not find such programs profitable. Finally, the decentralized or integrated structure of the competing channel significantly impacts the effects of cooperative advertising. For example, CAPs can effectively coordinate the $\mathcal{DD}$ channel and even help it exceed profits earned by a vertically integrated channel. However, in the $\mathcal{DI}$ case, although CAPs can improve total channel profits, they do not fully coordinate the channel.

1   Introduction

Cooperative advertising programs represent a significant investment for distribution channels. Recent reports estimate that about \$36 billion are being paid by manufacturers to retailers in cooperative advertising funds, which represents about 12% of total advertising costs (Borrell Associates Report 2015). Such large investments are offered as incentives to the retailers to promote manufacturers' products.

There is an extensive literature that studies the effectiveness of cooperative advertising programs for firms in the distribution channel. Most of this literature is focused on the profitability of cooperative advertising programs in channels where the manufacturer and the retailer are not facing competition from similar companies (bilateral monopolies). Recent reviews by Aust and Buscher (2014) and Jørgensen and Zaccour (2014) of this literature discuss the varying modeling assumptions and set-ups. A consistent result in the bilateral monopoly literature is that cooperative advertising acts as an incentive that

increases retailer's advertising, thereby expanding demand, and in most cases increasing the profits for each channel member (Dant and Berger 1996; Jørgensen et al. 2000; Huang and Li 2001; Yue et al. 2006; Karray and Zaccour 2007; Xie and Ai 2006; Yan 2010; Yang et al. 2013; Karray 2013; Jørgensen et al. 2003; Karray and Zaccour 2005; He et al. 2009; Tsao 2015).

A few recent studies have looked at the effectiveness of cooperative advertising programs in channels where some form of competition exists at the retailing, manufacturing or both levels of the channel. Bergen and John (1997) showed that cooperative advertising programs can benefit a single manufacturer selling through multiple retailers. Karray and Zaccour (2007) found that cooperative advertising programs could lead to prisoner's dilemma situations for competing manufacturers and retailers. Assuming prices to be exogenous, they find that cooperative advertising can ultimately decrease every firm's profit. Liu et al. (2014) consider a two-manufacturers, two-retailers channel and evaluate the effectiveness of cooperative advertising assuming exogenous cooperative advertising rates. They find that these programs do not benefit the channel if they lead to a significant decrease in the channel members' unit margins. Focusing on retail competition only, Karray and Amin (2015) showed that a monopolistic manufacturer selling through competing retailers should not offer cooperative advertising programs when price competition is low and these programs are not effective enough to reach the level of profits earned by an integrated channel. *Finally, also considering only retail competition Aust and Buscher (2014) study the effects of (horizontal) collusion between retailers on the strategies of all channel mem-*

bers. Modeling the dynamic effects of advertising in a channel with competing retailers and assuming constant prices, He et al. (2011) and Chutani and Sethi (2012) find that the retailers might not benefit from cooperative advertising. Finally, Chutani and Sethi (2014) also consider the dynamic effects of advertising and assuming a fixed total market demand, show (numerically) that cooperative advertising does not benefit competing retailers.

The main insights from the literature are as follows: (1) While cooperative advertising is shown to be beneficial for firms in bilateral monopoly channels, this result does not always hold in markets where at least one channel member faces competition. (2) In addition to the retailers' marketing efforts, pricing decisions matter when assessing the effectiveness of cooperative advertising. This is because these programs affect the channel members' unit margins, which in turn affect the profitability of cooperative advertising. (3) The results are sensitive to the type of horizontal and vertical externalities that are retained, both in terms of number of players at each level of the channel and of their decision variables.

Building on the above lessons, the objective of this paper is to investigate the effectiveness of cooperative advertising *for competing channels under different channel structures. Specifically, we consider two competing channels and study how cooperative advertising strategies and outcomes vary with the structure of the marketing channel. First, we determine the impact of cooperative advertising programs for two competing decentralized channels (DD structure). This contributes to the existing literature which typically either ignored pricing in competitive channels or considered exogenous advertising decisions. Next, we assess the profitability of cooperative advertising for competing channels with one*

4

centralized while the other is decentralized (DI structure). Comparison of results across the DD and DI structures helps identify whether the decentralized or integrated structure of the competing channel impacts the effects of the cooperative advertising program for a decentralized channel. Finally, as vertical integration is usually considered to be the ideal channel coordination mechanism, it is important to compare the value of cooperative advertising programs for competing channels in both the DI and the DD channels to the integrated channel set-up (II structure). The effects of cooperative advertising programs are compared against the II channel structure which provides a benchmark against which the effectiveness of the cooperative advertising program is assessed. We address these issues with a game-theoretic model that takes into account different channel structures, with pricing and advertising decisions. More specifically, we aim at answering the following research questions:

1. Under what conditions manufacturers are better off implementing a cooperative advertising program (CAP)?

2. Is it in the best interest of retailers to accept the CAP designed by the manufacturers?

3. Does a CAP lead to higher total channel's profit?

4. How do the effects generated by cooperative advertising vary with the channel structure of the competition (decentralized vs. integrated)?

5. How does the total channel's profit with a CAP compare to the profit of an integrated channel?

5

Our main results can be summarized as follows. (i) Although the results vary to some extent with the parameter values, a first general message is that a CAP is beneficial to manufacturers in a very large portion of the parameter space. (ii) The reverse is observed for retailers; indeed, it is rarely the case (in terms of parameter values) that a CAP improves their profits. (iii) It is not always true that a CAP leads to higher total channel's profit. Combining the previous points, the implication is that a CAP may not be feasible altogether. (iv) A CAP may lead to higher total profits in the decentralized channels than to the ones earned by integrated ones.

The rest of the paper is organized as follows. In Section 2, we introduce the model, and present the different variants that are relevant to answer our research questions. We briefly discuss the different equilibrium solutions in Section 3. In Section 4, we deal with the effectiveness of CAPs in the decentralized channels and in Section 5, we do the same for the case where a decentralized channel faces an integrated firm. Finally, we conclude and discuss future research avenues in Section 6.

2   Model

We consider a market with two manufacturers (M_1 and M_2) selling their partially substitutable products through two exclusive retailers (R_1 and R_2). The products are indexed by $i = 1, 2$. *All notations are summarized in the Appendix (Table A.1).*

Denote by p_i the retail price of product i and by w_i its wholesale price. The retail price

6

is decided by retailer R_i and the wholesale price by manufacturer M_i.[1] Retailer R_i can undertake activities a_i to promote its product, e.g., flyers, displays, local advertising, etc. The demand D_i for product i depends on the retail prices and promotion efforts of the two competing products, that is,

$$D_i = f\left(p_i, p_{3-i}, a_i, a_{3-i}\right), \quad i = 1, 2,$$

with

$$\frac{\partial D_i}{\partial p_i} < 0, \quad \frac{\partial D_i}{\partial p_{3-i}} \geq 0, \quad \frac{\partial D_i}{\partial a_i} \geq 0, \quad \frac{\partial D_i}{\partial a_{3-i}} \leq 0.$$

The above inequalities state that each demand is increasing in its own promotional activities and in the competitor's price and is decreasing in own price and in the competitor's promotional activities. We retain the following linear form for demand:

$$D_i = v - p_i + \beta p_{3-i} + \rho a_i - \delta a_{3-i}, \quad i = 1, 2. \tag{1}$$

The positive parameter v represents the baseline market potential, that is, the market size before accounting for promotional activities. For simplicity, we assume that v is the same for both products, which means that they have the same brand-equity value. *Also, the effect of a product's own price on demand is normalized to one for simplicity and without*

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[1]We assume that the manufacturer does not set the retail price, i.e., there is no price maintenance contract in effect. This assumption has been widely used in the literature and represents common practice in many exclusive channels. We thank an anonymous reviewer for this comment.

loss of generality. The parameter $\beta \in (0,1)$ represents the intensity of price competition between the two products. Low levels of β denote a marketplace where products are highly differentiated, while high levels of β imply that there is an intense price competition between the two brands. *Since $\beta \in (0,1)$, this ensure that the marginal effect of own price on demand is higher than the marginal effect of competing product price on demand. This is aligned with common economic pricing properties.* The positive parameters ρ and δ measure the effects of own and competing product's promotions on demand, respectively. The values of ρ and δ vary with consumer characteristics, the kind of promotional campaign undertaken by the retailers, and the characteristics of the product category (Alba et al. 1994; Blattberg et al. 1995). In any event, we suppose that $\rho \geq \delta$, which means that own promotions have at least the same effect (in absolute value) on demand than the competing brand's promotions. Further, we suppose that the impact of own advertising effort on demand is lower than the price impact, that is, we assume $\rho < 1$. These assumptions can then be summarized as follows:

$$0 < \beta, \delta, \rho < 1. \tag{2}$$

The above inequalities make sense economically and, as we will see later on, ensure strict concavity of the profit functions. Finally, we note that the linear demand function has been commonly used in the marketing and economics literature (e.g., Ingene and Parry 2007; Cai et al. 2012; Karray 2015).

Promotional costs for the manufacturers and for the retailer are assumed convex, mean-

8

ing that marginal costs of advertising are increasing *(Karray 2013 and 2015; Liu et al. 2014)*. For simplicity, we retain a quadratic functional form, that is, $c_i(a_i) = a_i^2$. When manufacturer M_i offers a cooperative advertising program with a participation or support rate t_i to retailer R_i, the manufacturer i's portion of the retailer's promotional expenses for its product is $t_i a_i^2$, $t_i \in (0, 1]$ while the rest is paid for by retailer i, i.e., $(1 - t_i)a_i^2$. We assume that production cost is constant and set it equal to zero, without any loss of generality.

2.1 Channel Structure and Strategic Interactions

In order to address our research questions, we shall characterize equilibrium strategies and outcomes in the following three channel's structures:

Decentralized-Decentralized ($\mathcal{DD}$): Manufacturer M_i sells its product through its exclusive retailer $R_i, i = 1, 2$ and each firm acts independently. The optimization problems are given by

$$\begin{aligned} \max_{w_i, t_i} \pi_{M_i} &= w_i D_i - t_i a_i^2, \\ \max_{p_i, a_i} \pi_{R_i} &= (p_i - w_i) D_i - (1 - t_i) a_i^2. \end{aligned}$$

Decentralized-Integrated ($\mathcal{DI}$): Manufacturer M_1 sells its product through its exclusive retailer R_1 and both players act independently. Manufacturer M_2 and retailer R_2

9

form a vertically integrated firm, denoted by I_2. The optimization problems are as follows:

$$\max_{w_1,t_1} \pi_{M_1} = w_1 D_1 - t_1 a_1^2,$$

$$\max_{p_1,a_1} \pi_{R_1} = (p_1 - w_1) D_1 - (1 - t_1) a_1^2,$$

$$\max_{p_2,a_2} \pi_{I_2} = p_2 D_2 - a_2^2.$$

As the model is symmetric, that is, players at any layer of the channel face the same parameter values, there is no need to consider the mirror Integrated-Decentralized case.

Integrated-Integrated ($\mathcal{II}$): The market is served by two fully integrated competitors $I_i, i = 1, 2$. In this duopoly competition, the optimization problems are as follows:

$$\max_{p_i,a_i} \pi_{I_i} = p_i D_i - a_i^2.$$

In the $\mathcal{DD}$ structure, we have both vertical and horizontal interactions (and externalities). In the $\mathcal{DI}$ structure, one vertical interaction vanishes. Finally, in the $\mathcal{II}$ channel, we only have horizontal interaction, meaning that the classical double-marginalization problem totally disappears.

2.2   Cooperative Advertising

In the $\mathcal{DD}$ structure, a cooperative advertising can be offered by none, one or the two manufacturer(s). In the $\mathcal{DI}$ structure, manufacturer M_1 can support or not its retailer.

Using the subscript $\mathcal{C}$ to refer to a situation where a cooperative advertising program is available (positive support rate) and by $\emptyset$ to the scenario where it is zero (no cooperative advertising program is offered), we end up with six cases, namely:

$$
\mathcal{DD} \text{ channel} : \begin{cases} \mathcal{D}_\emptyset \mathcal{D}_\emptyset & M_1 \text{ and } M_2 \text{ do not offer support,} \\[2mm] \mathcal{D}_\mathcal{C} \mathcal{D}_\emptyset, & \text{Only } M_1 \text{ supports its retailer,} \\[2mm] \mathcal{D}_\mathcal{C} \mathcal{D}_\mathcal{C} & M_1 \text{ and } M_2 \text{ support their retailers.} \end{cases}
$$

$$
\mathcal{DI} \text{ channel} : \begin{cases} \mathcal{D}_\emptyset \mathcal{I} & M_1 \text{ does not support its retailer,} \\[2mm] \mathcal{D}_\mathcal{C} \mathcal{I}, & M_1 \text{ supports its retailer.} \end{cases}
$$

$$
\mathcal{II} \text{ channel} : \begin{cases} \text{Cooperative advertising is not an issue.} \end{cases}
$$

As alluded to it before, the case $\mathcal{D}_\emptyset \mathcal{D}_\mathcal{C}$ does not need to be considered because by symmetry it is strategically equivalent to $\mathcal{D}_\mathcal{C} \mathcal{D}_\emptyset$. Table 1 gives the list of variables and profit functions in the different games.

(Insert Table 1)

3   Equilibrium Solutions

In the cooperative advertising and marketing channel literature (see the surveys in Ingene et al. 2012; Aust and Buscher 2014; Jørgensen and Zaccour 2014), the typical assumption in a decentralized dyad channel is that the manufacturer acts first as leader and the retailer next as follower. The decentralized manufacturer(s) is (are) a first-mover in our setup based

11

on the empirical evidence that cooperative advertising programs are usually initiated by manufacturers. The survey by the National Register Publishing shows that manufacturers announce their coop advertising rates before the retailers decide of their advertising.[2] For pricing, this sequence of play is also based on the empirical evidence that a Stackelberg game where manufacturers are leaders is often appropriate for pricing behaviors in channels (Sudhir 2001). When there is competition at a layer of the channel, the assumption is that the agents at that layer compete à la Nash. Consequently, we have here a Stackelberg interaction between the two levels of the marketing channel and Nash competition at each level.

As usual in a Stackelberg game, the determination of the equilibrium proceeds in the reverse order of the information flow (by backwards induction). That is, we first solve for the second-stage equilibrium to obtain the retailers' reaction functions and next solve the first-stage equilibrium problem (when there are two manufacturers, as in the $\mathcal{DD}$ channel) or the first-stage optimization problem (when there is only one manufacturer, as in the $\mathcal{DI}$ channel). In the $\mathcal{II}$ channel, there is no vertical interaction and we solve for Nash equilibrium. More details follow.

- $\mathcal{D_C D_C}$: First, we consider the retailers' problems and solve the following first-order equilibrium conditions:

$$\frac{\partial \pi_{R_i}}{\partial p_i} = \frac{\partial \pi_{R_i}}{\partial a_i} = 0, \ i = 1, 2,$$

[2]See Co-opadvertisingprograms.com, the on-line database of NRP for coop advertising programs (http://www.co-opsourcebook.com/coop_sample.htm).

which yields the reaction functions to the manufacturers' decision variables, that is,

$$a_i = A_i(w_1, w_2, t_1, t_2), \quad i = 1, 2,$$

$$p_i = P_i(w_1, w_2, t_1, t_2), \quad i = 1, 2.$$

We insert them in the manufacturers' optimization problems to get

$$\max_{w_i, t_i} \pi_{M_i} = w_i D_i - t_i \left(A_i(w_1, w_2, t_1, t_2) \right)^2,$$

where

$$
\begin{aligned}
D_i &= v - P_i(w_1, w_2, t_1, t_2) + \beta P_{3-i}(w_1, w_2, t_1, t_2) + \rho A_i(w_1, w_2, t_1, t_2) \\
&\quad - \delta A_{3-i}(w_1, w_2, t_1, t_2).
\end{aligned}
$$

We solve for a Nash equilibrium, i.e., obtain the values of w_1, w_2, t_1 and t_2 that solve the following equilibrium conditions:

$$\frac{\partial \pi_{M_i}}{\partial w_i} = \frac{\partial \pi_{M_i}}{\partial t_i} = 0, \quad i = 1, 2.$$

The solution to the above system is next inserted in the retailers' reaction functions to obtain their retail prices and promotion efforts as functions of the model's parameter values.

13

- $\mathcal{D_C D_\emptyset}$ and $\mathcal{D_\emptyset D_\emptyset}$: A similar approach as above is followed to solve these games. However, in $\mathcal{D_C D_\emptyset}$, there is no variable t_2 and in $\mathcal{D_\emptyset D_\emptyset}$ both t_1 and t_2 are immaterial.

- $\mathcal{D_C I}$: In this game, we have one manufacturer M_1 and two retailers: R_1 and I_2 (the integrated firm). As above, we start by considering the first-order equilibrium conditions of the downstream players, that is,

$$\frac{\partial \pi_{R_1}}{\partial p_1} = \frac{\partial \pi_{R_1}}{\partial a_1} = \frac{\partial \pi_{I_2}}{\partial p_2} = \frac{\partial \pi_{I_2}}{\partial a_2} = 0,$$

which provides their reaction functions to manufacturer M_1 decision variables, namely:

$$a_i = A_i^* (w_1, t_1), \quad i = 1, 2,$$

$$p_i = P_i^* (w_1, t_1), \quad i = 1, 2.$$

We insert these reaction functions in manufacturer M_1 optimization problem to get

$$\max_{w_1, t_1} \pi_{M_1} = w_1 \left(v - P_1^* (w_1, t_1) + \beta P_2^* (w_1, t_1) + \rho A_1^* (w_1, t_1) - \delta A_2^* (w_1, t_1) \right)$$

$$- t_1 \left(A_1^* (w_1, t_1) \right)^2 .$$

Next, we solve the following first-order optimality conditions:

$$\frac{\partial \pi_{M_1}}{\partial w_1} = \frac{\partial \pi_{M_1}}{\partial t_1} = 0.$$

14

The solution is inserted in the retailers' reaction functions to obtain their decisions as functions of the parameter values.

- $\mathcal{D}_\emptyset \mathcal{I}$: We follow a similar approach as for solving $D_\mathcal{C} I$, with however no support rate variable t_1.

- $\mathcal{II}$: This is a standard duopoly game, with the optimization problems given by

$$\max_{p_i, a_i} \pi_{I_i} = p_i \left(v - p_i + \beta p_{3-i} + \rho a_i - \delta a_{3-i} \right) - a_i^2.$$

The first-order Nash equilibrium conditions are as follows:

$$\frac{\partial \pi_{I_i}}{\partial p_i} = \frac{\partial \pi_{I_i}}{\partial a_i} = 0, \quad i = 1, 2.$$

The detailed equilibrium results for the six cases are provided in the Appendix. We make the following observations: (i) Under the assumption in (2), and some other mild conditions that are stated in the Appendix, we obtain a unique equilibrium in each of the six games. (ii) The equilibrium strategies and payoffs are given in closed forms, in all but one case. In any event, the expressions, in particular those of players' outcomes, are very long and highly non-linear in the parameter values and do not offer any qualitative insight. Consequently, we must resort to numerical simulations to proceed with the necessary comparisons.

The results are presented in figures where we let vary the three model's key parameters,

15

namely, the degree of price competition β, and the coefficients of promotional activities, i.e., δ and ρ. We retain three values for β $(0.25, 0.50, 0.75)$ to represent a low, medium and high degree of price competition.[3] For δ and ρ, we show the results for the whole interval of admissible values, that is, $(0, 1)$.[4]

To avoid repeating ourselves, we mention the following result that holds true in all simulations: in the space (ρ, δ), we have feasible solutions only in the region below the bisector. Feasibility refers to equilibrium solutions for which decision variables (prices and promotion efforts), margins, profits and demands assume nonnegative values. Note that in this feasible region, the constraint $\rho \geq \delta$ is always satisfied. In all figures, we clearly denote the region where only the game $\mathcal{D}_\emptyset \mathcal{D}_\emptyset$ is feasible by R_0.

4   Effectiveness of CAP in Decentralized Channels

To assess the effectiveness of a CAP in the $\mathcal{D}\mathcal{D}$ channel, we need to compute and compare the equilibrium outcomes in the scenarios $\mathcal{D}_\emptyset \mathcal{D}_\emptyset$ and $\mathcal{D}_\mathcal{C} \mathcal{D}_\mathcal{C}$. The results are used to answer our research questions. Namely; under what conditions manufacturers are better off implementing a cooperative advertising program (CAP)? When these conditions are verified, is it in the best interest of retailers to accept the CAP designed by the manufacturers? And, does a CAP lead to higher total channel's profit?

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[3]We performed all analyses for other values of β lower and higher than 0.5 and found that the main qualitative results hold.

[4]The numerical results were generated considering a grid of $(0, 1)$ for each parameters δ and ρ with a mesh of 0.005. That is, for each scenario (a fixed value of β) we computed optimal strategies and profits at $40,000$ different points.

These three questions are addressed by comparing equilibrium profits obtained in the $\mathcal{D}_\emptyset \mathcal{D}_\emptyset$ and $\mathcal{D}_\mathcal{C} \mathcal{D}_\mathcal{C}$ for the manufacturers, retailers and the total channel. Finally, we also compare the total channel's profit with a CAP to the profit of an integrated channel to assess whether CAP can coordinate the competitive $\mathcal{D}\mathcal{D}$ channel.

4.1 Profitability of CAP

Figures 1a, 1b and 1c show the results for the different retained values of β. Given our assumption that the manufacturers are leaders in their channels, the first step is to check under what conditions these agents are willing to offer a CAP. Next, in the region of the parameter space where a CAP is offered, we check if the retailer is better off (profit-wise) accepting the CAP. We have the following regions of interest in Figures 1a, 1b and 1c:

R1: A CAP is Pareto-profit improving with respect to its absence, that is, both the manufacturer and the retailer realize a higher payoff under a CAP.

R2: A CAP leads to a higher profit for the manufacturer only.

R3: A CAP is not implemented because it would deteriorate the manufacturer's profit.

These regions reveal some interesting insights. Whereas the portion of the parameter space where each manufacturer is interested in offering a CAP is large, that is, in regions R1 and R2, the region of acceptability by the retail partner is very small. In fact, for medium and high degree of price competition, the union of R1 and R2 occupies the whole space where a solution is feasible. For a low value of β, this region is smaller, but still

of considerable size (Figure 1.a). Here, the optimality of a CAP for the manufacturers requires that the marginal impact on demand of adverse promotion, measured by δ, to be sufficiently low. The striking result, and probably the most important takeaway, is that the retailers do not benefit from the manufacturers' CAP in most cases. To be more precise, the retailers would be happy to be supported by their manufacturers, only when (i) the degree of price competition is relatively high, and (ii) the impact of adverse promotion is near zero, that is, δ is very small.

This result can be explained by looking at the effects of CAPs on strategies and demands. In the DD channel structure, the cooperative advertising programs lead to higher manufacturers' wholesale prices, which aggravates the double marginalization problem. Further, higher pricing and advertising strategies of the retailers boost demand for each product when advertising competitive effects are low enough. The additional revenues generated by CAPs for the manufacturers (even when demands decrease with CAPs) exceed the cost of their promotional support. Ultimately, this results in higher profits for the decentralized manufacturers.

For the retailers, the higher manufacturers' prices and the increased costs of promotions (even with the manufacturer's support) result in limited retail margins and inflated promotional costs. The additional demand units generated by CAPs for low levels of adverse promotional effects are not large enough to compensate for the retailers' increased costs. The only exception is when these adverse promotional effects are very small (lower than 0.04), in which case the increase in demand can boost revenues enough to result in higher

18

retail profits.

This raises an important methodological (with practical ramifications) question, namely: can a retailer reject a CAP based on the argument that it deteriorates its profits? From a purely game-theoretic point of view, a follower has no choice but to act as such. Indeed, the leader makes an announcement and the follower reacts by best replying to this announcement, and this is the end of story. More on this below.

(Insert Figure 1)

As mentioned in the introduction, Karray and Zaccour (2007) found that CAPs are part of a Nash equilibrium of a prisoner's dilemma game when prices are exogenous. Do we have the same situation here? The answer is unambiguous, it is never the case here. To check for this, we need to compute and compare the equilibrium payoffs for the two manufacturers in four games, namely: $\mathcal{D}_\emptyset \mathcal{D}_\emptyset, \mathcal{D}_\mathcal{C} \mathcal{D}_\emptyset, \mathcal{D}_\emptyset \mathcal{D}_\mathcal{C}$ and $\mathcal{D}_\mathcal{C} \mathcal{D}_\mathcal{C}$.

In the usual matrix representation, we have the following payoffs, where in each cell, the first entry is for manufacturer M_1 and the second is for manufacturer M_2.

$$M_2$$

$$
\begin{array}{cccc}
 & & C & \emptyset \\
 & C & \left(\pi_{M_1}^{CC}, \pi_{M_2}^{CC}\right) & \left(\pi_{M_1}^{C\emptyset}, \pi_{M_2}^{C\emptyset}\right) \\
M_1 & & & \\
 & \emptyset & \left(\pi_{M_1}^{\emptyset C}, \pi_{M_2}^{\emptyset C}\right) & \left(\pi_{M_1}^{\emptyset\emptyset}, \pi_{M_2}^{\emptyset\emptyset}\right)
\end{array}
$$

To have $(\mathcal{C}, \mathcal{C})$ as Nash equilibrium of a prisoner's dilemma game, the following inequalities must hold true:

$$\pi_{M_1}^{\mathcal{C}\mathcal{C}} < \pi_{M_1}^{\emptyset\emptyset} \text{ and } \pi_{M_2}^{\mathcal{C}\mathcal{C}} < \pi_{M_2}^{\emptyset\emptyset},$$

19

that is, both players would be better off if *both* do not implement a CAP (this means that $(\emptyset, \emptyset)$ is Pareto improving with respect to $(\mathcal{C}, \mathcal{C})$), and

$$\text{for } M_1 \quad : \quad \pi_{M_1}^{\emptyset\mathcal{C}} < \pi_{M_1}^{\mathcal{C}\mathcal{C}}, \quad \pi_{M_1}^{\emptyset\emptyset} < \pi_{M_1}^{\mathcal{C}\emptyset},$$

$$\text{for } M_2 \quad : \quad \pi_{M_2}^{\mathcal{C}\emptyset} < \pi_{M_2}^{\mathcal{C}\mathcal{C}}, \quad \pi_{M_2}^{\emptyset\emptyset} < \pi_{M_2}^{\emptyset\mathcal{C}},$$

that is, the best reply to $\emptyset$ (do not implement a CAP) is $\mathcal{C}$ (implement a CAP), and the best reply to $\mathcal{C}$ (implement a CAP) is $\mathcal{C}$ (implement a CAP).

We checked for these inequalities for all retained values of β, δ and ρ and concluded the following:[5]

Claim 1 *For all parameter values, it is never the case that cooperative advertising programs are part of a Nash equilibrium of a prisoner's dilemma game.*

The above result contradicts the finding in Karray and Zaccour (2007). The methodological and practical implication here is that it is not neutral to assume, as these authors did, that prices are not strategic variables.

In Figures 2a, 2b and 2c, we provide the results for the total channel's profit. Total channel's profit corresponds to the sum of profit of a manufacturer and its exclusive retailer. In these Figures, S1 is the region where a CAP leads to higher total channel profit, S2 denotes the region where the manufacturers benefit from CAP but the total channel does

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[5] That is, $\beta \in \{0.25, 0.50, 0.75\}$ and δ and ρ in a grid $(0, 1)$ with a mesh of 0.005.

not, and S3 is the region where neither the manufacturer nor the total channel benefit from CAP.

(Insert Figure 2)

The main finding here is that in a sizeable part of the region where a CAP is implemented by the manufacturers, the total channel's profit is higher than without a cooperative advertising program. Also, the size of this region increases with the value of β. Putting together this result and the previous one, we get that when a CAP yields a higher total channel's profit, it is essentially to the advantage of the manufacturers. Back to the point of acceptability of CAPs by retailers, in many papers in the literature, see the surveys in Aust and Buscher (2014) and Jørgensen and Zaccour (2014), an additional step is considered in the analysis of effectiveness of CAPs. Indeed, it is assumed that (i) the players will implement a CAP whenever it leads to a higher total channel's profit, and (ii) the players next split the dividend of a CAP using, e.g., an egalitarian principle. This dividend is measured by

$$\Delta = \text{total channel's profit with CAP } - \text{ total channel's profit without CAP},$$

and each agent will get its profit without CAP plus half of Δ. The argument in following this approach is twofold. First, CAPs are voluntary programs, where manufacturers announce their coop rates, then retailers have to formally reimbursement (usually by filling up forms and completing other administrative requirements). Hence, while the coop rate

21

is announced, the retailer can still completely opt out of it. Second, CAPs are Pareto-improving, and this is clearly an incentive to engage in such programs. Although we have all the data required to perform this exercise, we refrain from going in this direction because our focus is on cooperative advertising in a noncooperative game context.

4.2   Can CAP coordinate the competitive $\mathcal{DD}$ channel?

In bilateral monopoly channels, a recurrent result is that a CAP is a partial coordinating mechanism. This means that while Δ is strictly positive, it is lower than the gap between the fully integrated channel's profit and the total channel's profit with a CAP. A natural question is how this result is affected by channel's competition? To answer this question, we computed total channel's profits under vertical integration, that is, the equilibrium profits of the $\mathcal{II}$ channel. The relevant region for this comparison is S1 in Figures 2a, 2b and 2c. Recall that in S1, we have two properties that are satisfied, namely: (i) the manufacturers benefit from a CAP and will hence implement it, and (ii) the total channel's profit is higher with a CAP. Consequently, what remains to be done is to compare the profits in this region (for all parameter values) to the profits that would be realized under vertical integration. Let us define by $\pi_{M_i}^{\mathcal{D}_C\mathcal{D}_C} + \pi_{R_i}^{\mathcal{D}_C\mathcal{D}_C}$ the total channel's profit in the game $\mathcal{D}_C\mathcal{D}_C$, and recall that the profit in the $\mathcal{II}$ channel is denoted π_{I_i}. In whole region of interest S1, our findings are as follows: (i) For a low value of β (0.25 more precisely), we have

$$\pi_{M_i}^{\mathcal{D}_C\mathcal{D}_C} + \pi_{R_i}^{\mathcal{D}_C\mathcal{D}_C} < \pi_{I_i}, \quad i = 1, 2,$$

22

and (ii) for $\beta \in \{0.50, 0.75\}$, we have

$$\pi_{M_i}^{\mathcal{D_C}\mathcal{D_C}} + \pi_{R_i}^{\mathcal{D_C}\mathcal{D_C}} > \pi_{I_i}, \quad i = 1, 2.$$

Recalling our results that for a low degree of price competition a CAP is never attractive for the retailer, the first inequality is not that surprising. The second inequality carries an important message, that is, while both vertical and horizontal externalities are still present in the $\mathcal{D_C}\mathcal{D_C}$ case, the channel performs better than under (Bertrand) duopoly competition between two integrated firms. This result is in sharp contrast with the usual one in a dyad mentioned above. So, in a competitive channel with a sufficiently high degree of price competition, cooperative advertising outperforms integrated competitive firms.

To shed an additional light on these results, we compare prices and advertising efforts under the two scenarios (see Appendix for details). Prices are lower in the $\mathcal{II}$ channel for all retained values of β, δ and ρ, which is expected because of the elimination of double marginalization in the $\mathcal{II}$ channel. For advertising, the ordering depends on the parameter values. For $\beta = 0.25$, we have higher advertising efforts under $\mathcal{II}$ structure than under $\mathcal{D_C}\mathcal{D_C}$, for all values of δ and ρ. Here, cooperative advertising is not succeeding in boosting retailer's advertising spending. (Recall that in this case, retailers never want a CAP.) For $\beta = 0.5$, the ordering depends essentially on own advertising impact; when ρ is high, advertising effort is higher under $\mathcal{D_C}\mathcal{D_C}$ than $\mathcal{II}$. For $\beta = 0.75$, it always holds in region S1 that advertising levels are higher under $\mathcal{D_C}\mathcal{D_C}$ than $\mathcal{II}$. In conclusion, it is hard to make

23

a general statement as the ordering of advertising efforts highly depends on parameter values. Still, the important message here is that in a competitive channel, profits with a CAP can be higher than profits under vertical integration.

We recapitulate our findings in the following claims, and by the same token answer our research questions:

Claim 2 *When the degree of price competition is moderate or high ($\beta = 0.50$ or 0.75), the manufacturers are always better off implementing CAPs in a $\mathcal{DD}$ channel structure. When β is low, a CAP is optimal for the manufacturer only when the marginal impact on demand of adverse promotion, measured by δ, is sufficiently low.*

Claim 3 *The region where the total channel's profit is higher under a CAP is a subset of the region in which a CAP leads to higher profits to manufacturers.*

Claim 4 *In a $\mathcal{DD}$ channel, the retailer is (almost) never interested by a CAP.*

Claim 5 *For medium and high degree of price competition (β), total channel's profit with a CAP is higher than the profit of an integrated firm competing against another integrated firm. For low β, the result is the other way around.*

5   Effectiveness of CAP in $\mathcal{DI}$ channels

In this case, a decentralized channel is facing competition from a vertically integrated firm. Our purpose is to investigate the effectiveness of CAPs for the decentralized channel. Recall

24

that we aim at answering the following research questions: (1) Under what conditions the manufacturer in the decentralized channel is better off implementing a CAP? (2) Is it in the best interest of the retailer in the decentralized channel to accept the CAP designed by the manufacturer? (3) Does a CAP lead to higher total decentralized channel's profit? (4) Finally, how does the total channel's profit with a CAP compare to the profit of an integrated channel?

To avoid repetitions, we mention from the outset three straightforward elements: (i) cooperative advertising is an issue only in the decentralized channel; (ii) the integrated channel has solved the double marginalization problem and hence it is, everything else being equal, more price competitive in the market; (iii) the higher the value of β, the tougher is the price competition for the decentralized channel. A direct consequence of the last two items is that the decentralized channel is in more need of advertising to compensate for the price handicap. Of course, the lower the value of δ (marginal impact of competitive advertising), the higher is the efficiency of own advertising in raising demand. The implication of all this is that comparatively to the $\mathcal{DD}$ case, it is intuitive to expect that in the $\mathcal{DI}$ channel, the decentralized manufacturer would be interested in incentivizing, through a CAP, its retailer to invest more in advertising.

In all our simulations in this scenario, that is, for all values of β, δ and ρ, we obtain the following result. When a CAP is implemented, we observe that the retailer advertises at a higher level, and the wholesale and retail prices are higher in the decentralized channel. In the meantime, the integrated firm also invests more in advertising and sells its product

25

at a higher price than when no CAP is offered in the decentralized channel. Interestingly, demand ends up being higher for both channels, which means that the (net) positive advertising effect seems to dominate the (net) negative pricing effect for both retailers (the one in the decentralized channel and the integrated firm). A comparison of the profits of the players in the decentralized channel is provided in Figures 3a, 3b and 3c. In these figures U2 denotes the region where the manufacturer benefits from CAP but the retailer does not. Based on these figures, we state the following results.

(Insert Figure 3)

Claim 6 *In a $\mathcal{DI}$ channel structure and for all parameter values, the manufacturer achieves a higher profit with a CAP than without this program.*

Claim 7 *In a $\mathcal{DI}$ channel structure and for all parameter values, the decentralized retailer achieves a lower profit when a CAP is implemented by the manufacturer than without this program.*

The intuition behind this result is that in this channel structure, a CAP is not helping at all in mitigating the double marginalization problem, but in fact is exacerbating it, with the manufacturer benefiting from a higher wholesale price. This is because with higher pricing and advertising strategies, the demand of the decentralized channel increases. The boosted revenues for the manufacturer are sufficient to cover for the added cost of promotion. However, for the retailer, the increase in the wholesale price along with the increase in promotional efforts (even with the manufacturer's support) squeezes both costs

26

and margins. These effects ultimately lead to lower retailer's profits.

(Insert Figure 4)

Figures 4a, 4b and 4c exhibit the regions where a CAP also leads to a higher total channel's profit. The total channel profit represents the sum of the profits of the retailer and the manufacturer in the decentralized channel. In Figure 4, the region V1 represents parameter values for which both the manufacturer's and the total decentralized channel's profits increase with CAP. The region V2 corresponds to the area where the manufacturer's profit increases but the total channel's profit decreases. Our findings can be summarized as follows.

Claim 8 *When the price competition is low, the total channel's profit is higher with a CAP than without it when δ is very low for all values of ρ, and when the effects of both own promotion (ρ) and competitor's promotion (δ) are very high. For higher values of β, a sufficient condition for obtaining a larger total channel's profit is for δ to be sufficiently low.*

These findings indicate that, depending on the levels of price competition (β) and promotion effects (ρ and δ), CAP can be an effective coordination mechanism for a decentralized channel facing an integrated competitor. This is the case when the market is characterized by high levels of price competition and limited effects of promotional competition, or for low levels of price competition and very high or very low promotional competition. Under these conditions, the manufacturer's increased wholesale price and

27

demand, thereby revenues result in gains that exceed the retailer's losses. However, CAPs can also lower total channel profits due to the aggravation of both vertical (higher retail prices) and horizontal externalities (higher retail and promotional efforts by the integrated competitor).

Finally, as we did in the previous scenario, we compare the total decentralized channel's profit with a CAP (that is, $\pi_{M_1}^{\mathcal{D_C I}} + \pi_{R_1}^{\mathcal{D_C I}}$) to the profit under full integration (π_{I_1}).

Claim 9 *For all parameter values, we obtain*

$$\pi_{M_1}^{\mathcal{D_C I}} + \pi_{R_1}^{\mathcal{D_C I}} < \pi_{I_1}.$$

In a $\mathcal{DI}$ channel structure, although CAPs can boost total profits in the decentralized channel, they cannot help achieve the level of profits earned by a fully integrated channel. This result is similar to the one that was repeatedly shown in a bilateral monopoly setting, namely, that cooperative advertising does not lead to the fully integrated channel's profit.

6 Conclusions

A large literature studied the effectiveness of cooperative advertising programs (CAPs) in distribution channels. However, very few studies have considered pricing decisions in competitive channels *and the effects of channel structure has been largely ignored in this literature.* This paper fills to some extent this gap and investigates how vertical and

28

horizontal externalities can impact the effectiveness of cooperative advertising programs. In particular, we evaluate the strategic implications of CAPs by studying whether the implementation of such programs can improve profits, and when they do, whether they can partially or fully coordinate the channel. *We do so considering different channel structures to study how can the decentralized or integrated structure of the competing channel impact the effectiveness of CAPs. The effects of cooperative advertising programs in each channel structure are then evaluated against the benchmark effects of vertical integration.*

We propose a game-theoretic model where two competing channels make pricing and promotional decisions. Each channel structure can be decentralized or integrated to account for different vertical interaction effects, resulting in three cases: (i) both channels are decentralized ($\mathcal{DD}$), (ii) both are integrated ($\mathcal{II}$), and (iii) a hybrid structure where one channel is decentralized and is competing with an integrated channel ($\mathcal{DI}$). We solve six non-cooperative games, then obtain and compare equilibrium profits and strategies across these games.

The findings indicate the following three main results. First, the profitability of cooperative advertising programs depends on the levels of price competition and of the advertising effects: both the direct and competitive effects. Second, cooperative advertising seems to be beneficial to manufacturers but rarely benefits retailers. Third, the decentralized or integrated structure of the competing product's channel significantly impacts the effects of cooperative advertising.

In particular, while cooperative advertising benefits the decentralized manufacturer

29

when facing an integrated competitor, it may not be beneficial when the competing channel is decentralized, specifically when products are highly differentiated and promotions lead to strong competitive effects. In these conditions, only manufacturers competing with an integrated channel should offer cooperative advertising programs. Further, in most cases, the decentralized retailer should opt out of cooperative advertising programs if the competing product is sold by an integrated channel. However, it should accept such an offer when the competing retailer is decentralized and products are characterized by intense price competition with low competitive promotional effects.

In both channel structures ($\mathcal{DD}$ and $\mathcal{DI}$), cooperative advertising can be effective in boosting the total profit of the decentralized channel, especially when price competition is high, making it possible for the manufacturers to implement such programs by sharing their additional profits with their retailers. Finally, CAPs can be effective channel coordination mechanisms. In the $\mathcal{DD}$ structure, they can even lead to higher profits than those achieved from vertical integration, mainly because of horizontal externalities that boost advertising spending and raise prices to consumers. However, in the $\mathcal{DI}$ channel, although they can improve total channel profits, CAPs do not fully coordinate the channel.

The different effects of CAPs across channel structures are due to the combination of vertical and horizontal externalities. Contrary to the DD structure, in the $\mathcal{DI}$ case, the competitor does not implement cooperative advertising and does not bear the cost of double marginalization. This leads to different strategic choices by the competing channel when it is integrated than when it is decentralized, and ultimately impacts the strategies

30

of the manufacturer and retailer evaluating the cooperative advertising agreement. For example, we find that in the $\mathcal{DD}$ structure, coop advertising rates are higher than in the $\mathcal{DI}$ case, and so are advertising and pricing strategies in most cases.

In conclusion, this paper shows that competitive interactions in the channel significantly impact the assessment of cooperative advertising effectiveness. Future research can build on these findings to study the case where the competing channels are asymmetric, or when channel members' strategies have long term effects (e.g., carryover effects of promotions). Finally, other considerations related to *asymmetrical (or incomplete) information*[6] *or* to inventory build-up and holding costs as a result of promotions can also be of interest.

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31

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Channel structure	Decision makers	Games	Decision variables	Profit functions
DD	M_i R_i	$D_\emptyset D_\emptyset$	w_i p_i, a_i	$\pi_{M_i} = w_i D_i$ $\pi_{R_i} = (p_i - w_i)D_i - a_i^2$
		$D_C D_C$	w_i, t_i p_i, a_i	$\pi_{M_i} = w_i D_i - t_i a_i^2$ $\pi_{R_i} = (p_i - w_i)D_i - (1 - t_i)a_i^2$
		$D_C D_\emptyset$	w_i, t_1 p_i, a_i	$\pi_{M_1} = w_1 D_1 - t_1 a_1^2$ $\pi_{M_2} = w_2 D_2$ $\pi_{R_1} = (p_1 - w_1)\,D_1 - (1 - t_1)a_1^2$ $\pi_{R_2} = (p_2 - w_2)\,D_2 - a_2^2$
DI	M_1 R_1 I_2	$D_\emptyset I$	w_1 p_i, a_i	$\pi_{M_1} = w_1 D_1$ $\pi_{R_1} = (p_1 - w_1)D_1 - a_1^2$ $\pi_{I_2} = p_2 D_2 - a_2^2$
		$D_C I$	w_1, t_1 p_i, a_i	$\pi_{M_1} = w_1 D_1 - t_1 a_1^2$ $\pi_{R_1} = (p_1 - w_1)D_1 - (1 - t_1)a_1^2$ $\pi_{I_2} = p_2 D_2 - a_2^2$
II	I_i	II	p_i, a_i	$\pi_{Ii} = p_i D_i - a_i^2$

Table 1: Variables and outcomes in the different games $(i = 1, 2)$

7 Appendix

Table A.1: List of notations

w_i	Wholesale price of manufacturer i, $w_i > 0$
t_i	Cooperative advertising participation or support rate of manufacturer i, $t_i \in (0,1)$
p_i	Wholesale price of retailer i, $p_i > w_i$
a_i	Advertising effeort of retailer i, $a_i > 0$
D_i	Demand for product i, $D_i > 0$
π_{M_i}	Profit of manufacturer i, $\pi_{M_i} > 0$
π_{R_i}	Profit of retailer i, $\pi_{R_i} > 0$
π_{I_i}	Profit of integrated manufacturer-retailer dyad i, $\pi_{I_i} > 0$
v	Baseline demand parameter, $v > 0$
β	Effect of competing product's price on demand, $\beta \in (0,1)$
ρ	Effect of own product's promotions on demand, $\rho \in (0,1)$
δ	Effect of competing product's promotions on demand, $\delta \in (0, \rho)$
R_i	Retailer i
M_i	Manufacturer i
$\mathcal{DD}$	Both manufacturer-retailer dyads are decentralized
$\mathcal{DI}$	One manufacturer-retailer dyads is decentralized while the other is integrated
$\mathcal{II}$	Both manufacturer-retailer dyads are integrated

7.1 Equilibrium Solutions for $\mathcal{DD}$ Channel

We obtain feedback equilibrium solutions using backward induction for three games. The first game is $\mathcal{D}_\emptyset\mathcal{D}_\emptyset$, where manufacturers withhold any cooperative advertising support to the retailer ($t_i = 0$, $i = 1, 2$). In the second game, that is, $\mathcal{D}_\mathcal{C}\mathcal{D}_\emptyset$, manufacturer 1 offers cooperative advertising at a positive rate t_1, while manufacturer 2 withholds any cooperative advertising support to the retailer, $t_2 = 0$. In the third game, that is, $\mathcal{D}_\mathcal{C}\mathcal{D}_\mathcal{C}$, the manufacturers offer cooperative advertising at a rate $t_i \neq 0$ ($i = 1, 2$).

7.1.1 $\mathcal{D}_\emptyset\mathcal{D}_\emptyset$ case

Proposition 1 *The equilibrium strategies in the game $\mathcal{D}_\emptyset\mathcal{D}_\emptyset$ where no cooperative advertising is implemented are given by*

$$w_1 = w_2 = \frac{v(\rho(\delta + \rho) - 2(\beta + 2))}{2(2\beta^2 + \beta - 4) + \rho(\delta + 2\rho - \beta(2\delta + \rho))}, \tag{3}$$

$$p_1 = p_2 = \frac{v(24 - 8\beta^2 + 6\beta\delta\rho - (10 + \delta^2)\rho^2 + \rho^4)}{[2(\beta - 2) + \rho(\rho - \delta)]\left[2(2\beta^2 + \beta - 4) + \rho(\delta + 2\rho - \beta(2\delta + \rho))\right]}, \tag{4}$$

$$a_1 = a_2 = \frac{v\rho(4 - 2\beta^2 + \beta\delta\rho - \rho^2)}{[2(\beta - 2) + \rho(\rho - \delta)]\left[2(2\beta^2 + \beta - 4) + \rho(\delta + 2\rho - \beta(2\delta + \rho))\right]}. \tag{5}$$

The manufacturers' and retailers' optimal profits are given by:

$$\pi_{M_1} = \pi_{M_2} = \frac{2v^2(4 + 2\beta - \delta\rho - \rho^2)(-4 + 2\beta^2 - \beta\delta\rho + \rho^2)}{[2(\beta - 2) + \rho(\rho - \delta)]\left[2(2\beta^2 + \beta - 4) + \rho(\delta + 2\rho - \beta(2\delta + \rho))\right]^2}, \tag{6}$$

$$\pi_{R_1} = \pi_{R_2} = \frac{v^2(4 - \rho^2)(-4 + 2\beta^2 - \beta\delta\rho + \rho^2)^2}{[2(\beta - 2) + \rho(\rho - \delta)]\left[2(2\beta^2 + \beta - 4) + \rho(\delta + 2\rho - \beta(2\delta + \rho))\right]^2}. \tag{7}$$

38

Proof. Second stage: In this stage of the game, the retailers compete à la Nash, and each retailer chooses its retail price, p_i, and the promotional effort, a_i, $i = 1, 2$, in order to maximize its profit π_{R_i}, that is,

$$\max_{p_i, a_i} \pi_{R_i} = (p_i - w_i)D_i - a_i^2, \tag{8}$$

where the demand D_i for product i given in (1).

The solution to problem (8) gives the retailers' reaction functions, that is, p_i and a_i, $i = 1, 2$, as functions of the wholesale prices, w_i, $i = 1, 2$.

The retailers' profits are strictly concave functions in the decision variables, p_i, a_i, if

$$4 - \rho^2 > 0,$$

which is satisfied by (2).

From the first-order optimality conditions for the problem in (8), the following expressions can be derived:

$$p_i = w_i + \frac{-2v + (w_1 + w_2)(1 - \beta)}{-4 + 2\beta - \rho(\delta - \rho)} + \frac{(w_1 - w_2)(1 + \beta)}{4 + 2\beta - \rho(\delta + \rho)}, \quad i = 1, 2, \tag{9}$$

$$a_i = \frac{1}{2}\rho \left(\frac{-2v + (w_1 + w_2)(1 - \beta)}{-4 + 2\beta - \rho(\delta - \rho)} + \frac{(w_1 - w_2)(1 + \beta)}{4 + 2\beta - \rho(\delta + \rho)} \right), \quad i = 1, 2. \tag{10}$$

First stage: In this stage, the manufacturers play a Nash game and choose their wholesale prices, w_i, $i = 1, 2$ in order to maximize their profits. Manufacturer i's optimization

problem is given by

$$\max_{w_i} \pi_{M_i} = w_i D_i, \tag{11}$$

where D_i is defined in (1). Using the retailers' reaction functions in (9)-(10), the manufac-turers' profit functions become

$$\pi_{M_i} = \frac{2w_i \left[4w_i - 2(v(2+\beta) + \beta(w_j + \beta w_i)) + (v - w_j + \beta w_i)\delta\rho + (v - w_i + \beta w_j)\rho^2 \right]}{[-4 + 2\beta - \rho(\delta - \rho)] [4 + 2\beta - \rho(\delta + \rho)]},$$

where $i, j = 1, 2$, $i \neq j$.

The solution to the manufacturers' problems gives us the wholesale prices, w_i for $i = 1, 2$. The manufacturers' profits are strictly concave functions in their decision variables w_i, if

$$\frac{4 - 2\beta^2 + \beta\delta\rho - \rho^2}{[-4 + 2\beta - \rho(\delta - \rho)] [4 + 2\beta - \rho(\delta + \rho)]} < 0,$$

which is clearly satisfied by (2).

>From the first-order conditions for each manufacturer i's problem in (11), we get the manufacturers' optimal strategies in (3).

Replacing these expressions into the retailers' reaction functions in (9)-(10), we obtain the optimal retail prices and promotional strategies in (4) and (5).

Once the manufacturers' and retailers' optimal strategies are known, their optimal profits in (6) and (7) immediately follow. ∎

40

7.1.2 $\mathcal{D}_{\mathcal{C}}\mathcal{D}_{\emptyset}$ case

The equilibrium strategies in the game $\mathcal{D}_{\mathcal{C}}\mathcal{D}_{\emptyset}$ where manufacturer 1 offers cooperative advertising, while manufacturer 2 does not offer cooperative advertising cannot be completely characterized in an analytical way. To get the solution for this case, we follow the same steps as in the proof of Proposition 1. Firstly, in the second stage of the game, we characterize the retailers' reaction functions.

Second stage: The retailers choose their prices, p_i, and their promotional efforts, a_i, in order to maximize their profits, which read as follows:

$$\max_{p_1,a_1} \pi_{R_1} = (p_1 - w_1)D_1 - (1 - t_1)a_1^2, \quad \max_{p_2,a_2} \pi_{R_2} = (p_2 - w_2)D_2 - a_2^2 \qquad (12)$$

where D_i is the demand function for product i given in (1).

The retailers' profits are strictly concave functions in their decision variables, p_i and a_i, if the following conditions are satisfied:

$$4(1 - t_1) - \rho^2 > 0, \quad 4 - \rho^2 > 0.$$

Taking into account the above conditions, and maximizing π_{R_i} with respect to retailer i's decisions, p_i, and a_i, we get the optimal retail prices and promotional efforts as functions of the wholesale prices, $w_i, i = 1, 2$ and of manufacturer 1's cooperative advertising rate,

41

t_1, as follows:

$$p_1 = \frac{\Lambda_1(t_1, w_1, w_2)}{\Lambda_2(t_1)}, \tag{13}$$

$$a_1 = \frac{\Lambda_3(t_1, w_1, w_2)}{\Lambda_2(t_1)}, \tag{14}$$

$$p_2 = \frac{\Lambda_4(t_1, w_1, w_2)}{\Lambda_2(t_1)}, \tag{15}$$

$$a_2 = \frac{\Lambda_5(t_1, w_1, w_2)}{\Lambda_2(t_1)}, \tag{16}$$

where

$$\Lambda_1(t_1, w_1, w_2) = (8(1 - t_1) + 2\beta\delta\rho - (6 - 2t_1 + \delta^2)\rho^2 + \rho^4)w_1$$

$$+ 2(1 - t_1)v(4 + 2\beta - \rho(\delta + \rho)) + 2(1 - t_1)(\delta\rho + \beta(2 - \rho^2))w_2,$$

$$\Lambda_2(t_1) = 4(1 - t_1)(4 - \beta^2) + 2(2 - t_1)\beta\delta\rho - (8 - 4t_1 + \delta^2)\rho^2 + \rho^4,$$

$$\Lambda_3(t_1, w_1, w_2) = \rho\left[-4w_1 + 2\beta(w_2 + \beta w_1) + (w_2 - \beta w_1)\delta\rho + (w_1 - \beta w_2)\rho^2\right]$$

$$+ v(4 + 2\beta - \rho(\delta + \rho))\rho,$$

$$\Lambda_4(t_1, w_1, w_2) = 4(1-t_1)(2(v+w_2)+(v+w_1)\beta)+2(-v+w_1+(1-t_1)\beta w_2)\delta\rho$$

$$-(2v+2\beta w_1+(6-4t_1+\delta^2)w_2)\rho^2+\rho^4 w_2,$$

$$\Lambda_5(t_1, w_1, w_2) = \rho\left[2(1-t_1)(\beta w_1+v(2+\beta)-(2-\beta^2)w_2)-(v-w_1+\beta w_2)\delta\rho\right]$$

$$-(v-w_2+\beta w_1)\rho^3.$$

First stage: The manufacturers take into account the retailers' reaction functions obtained in the second stage and play Nash to choose the wholesale prices, w_i for $i = 1, 2$ and the cooperative advertising rate, t_1. Both manufacturers maximize their profits such as

$$\max_{w_1, t_1} \pi_{M_1} = w_1 D_1 - t_1 a_1^2, \quad \max_{w_2} \pi_{M_2} = w_2 D_2. \tag{17}$$

With the help of Mathematica 10.1 we can deduce the conditions ensuring that the manufacturer 1's profit, π_{M_1}, is a concave function in w_1 and t_1 and manufacturer 2's profit, π_{M_2}, is a concave function in w_2. The resulting expressions are very long and we refrain from writing them as they do not offer any qualitative insight.

Replacing the retailers' reaction functions (13)-(16) obtained in Stage 2 in π_{M_i}, and taking into account the conditions that ensure the concavity of π_{M_i}, the optimality conditions

43

for the manufacturers' problem are obtained as follows

$$\frac{\partial \pi_{M_1}}{\partial t_1} = 0, \; \frac{\partial \pi_{M_i}}{\partial w_i} = 0 \quad i = 1, 2.$$

From equation $\frac{\partial \pi_{M_2}}{\partial w_2} = 0$, one can get one expression for the cooperative advertising support rate t_1 as a function of the wholesale prices, w_1 and w_2:

$$t_1 = \frac{v\left[2(2+\beta) - \rho(\delta+\rho)\right] - 2(4 - 2\beta^2 + \beta\delta\rho - \rho^2)w_2 + (2\beta + \delta\rho - \beta\rho^2)w_1}{2\left[(2+\beta)v + \beta w_1 - 2(2 - \beta^2)w_2\right]}. \quad (18)$$

Substituting this expression of t_1 into the other optimality conditions, a system of two non-linear equations in variables w_1 and w_2 is obtained. With the help of Mathematica 10.1, the equation obtained from $\frac{\partial \pi_{M_1}}{\partial w_1} = 0$ can be factorized in two factors. If the first factor is zero, three possible pairs (w_1, w_2) are obtained. We remove the three possibilities because they imply null promotional investments. The second factor can be viewed as a quadratic polynomial for w_2 where the first-order coefficient and the independent term depend on the parameters β, ρ and δ as well as on w_1. Solving the following quadratic polynomial equation

$$Aw_2^2 + B(w_1)w_2 + C(w_1) = 0,$$

under condition $(B(w_1))^2 - 4AC(w_1) \geq 0$, one gets two possible expressions for w_2:

$$w_2 = \frac{-B(w_1) \pm \sqrt{(B(w_1))^2 - 4AC(w_1)}}{2A}. \quad (19)$$

The coefficients $A, B(w_1)$ and $C(w_1)$ are very long expressions and we refrain from writing them as they do not offer any qualitative insight.

Substituting the two possible expressions of w_2 as a function of w_1 into the optimality condition $\frac{\partial \pi_{M_1}}{\partial t_1} = 0$, one gets a highly non-linear equation in variable w_1 which is not analytically solvable. The solutions of this equation are numerically approximated using Matlab 7.6 for fixed values of parameters β, ρ and δ. Given each triplet (β, ρ, δ) for each solution w_1 we compute the corresponding w_2 and t_1 in (18) and (19) and choose the optimal solution in such a way that the objective functions in (17) are maximized.

7.1.3 $\mathcal{D}_\mathcal{C}\mathcal{D}_\mathcal{C}$ case

Proposition 2 *The equilibrium strategies for the game where a cooperative advertising is implemented in the channel is given by the following solution:*

$$p_1 = p_2 = \frac{2(1-t)(v+w) + \rho(\delta - \rho)w}{2(1-t)(2-\beta) + \rho(\delta - \rho)}, \tag{20}$$

$$a_1 = a_2 = \frac{(v-(1-\beta)w)\rho}{2(1-t)(2-\beta) + \rho(\delta - \rho)}, \tag{21}$$

$$t_1 = t_2 = -\frac{\left[\rho(16 + 2\delta^2 - 3\rho^2) + \beta\delta(-8 + \rho^2) - \rho\sqrt{\Delta}\right]}{8\beta\delta + 4(-6 + \beta^2)\rho}, \tag{22}$$

$$w_1 = w_2 = -v\frac{Numw - (2\delta(2\delta - \rho) + \beta^2(-4 + \rho^2) - \beta(-8 + 2\delta\rho + \rho^2))\sqrt{\Delta}}{Denw}, \tag{23}$$

where

$$\Delta = 16\beta^4 + 4\delta^4 - 8\beta^3\delta\rho + (8 - 3\rho^2)^2 + 2\beta\delta\rho(56 + 2\delta^2 - 3\rho^2) - 4\delta^2(8 + 3\rho^2)$$

$$+ \quad \beta^2(\delta^2(\rho^2 - 16) - 8(8 + \rho^2)),$$

$$Numw = 8(2(-2 + \beta)\beta(-6 + \beta(2 + 3\beta)) + (-4 + (10 - 7\beta)\beta)\delta^2 + \delta^4)$$

$$+ \quad 4\delta(-12 + 28\beta - 10\beta^2 + 3\beta^3 + (-5 + 2\beta)\delta^2)\rho$$

$$+ \quad 4\beta(-18 + \beta(7 - 4(-2 + \beta)\beta) + \delta^2)\rho^2$$

$$+ \quad (12 + \beta(-18 + \beta + \beta^2))\delta\rho^3 + \beta(6 + (-3 + \beta)\beta^2)\rho^4,$$

$$Denw = 16(2(-2+\beta)(-1+\beta)\beta(-4+\beta+2\beta^2) - (-1+\beta)(8+5(-2+\beta)\beta)\delta^2 + (-2+\beta)\delta^4)$$

$$+ \quad 8\delta(8 + 9\delta^2 + \beta(-44 - 3\delta^2 + \beta(51 + \beta(-17 + 2\beta) + \delta^2)))\rho$$

$$- \quad 4((-1 + \beta)\beta(1 + \beta)(26 + \beta(-22 + 5\beta)) - 3(-2 + (-5 + \beta)\beta)\delta^2)\rho^2$$

$$+ \quad 2(-9 + \beta(42 + \beta(-12 + (-2 + \beta)\beta)))\delta\rho^3$$

$$+ \quad (-3 + \beta)\beta(3 + \beta(3 + (-3 + \beta)\beta))\rho^4.$$

This equilibrium requires $\Delta \geq 0$.

Proof. We follow the same steps as in the proof of Proposition 1.

Second stage: The retailers choose the prices, p_i, and their promotional efforts, a_i, in order to maximize their profits, which read as follows:

$$\max_{p_i,a_i} \pi_{R_i} = (p_i - w_i)D_i - (1 - t_i)a_i^2, \tag{24}$$

46

where D_i is the demand function for product i given in (1).

The retailers' profits are strictly concave functions in their decision variables, p_i and a_i, if the following conditions are satisfied:

$$4(1 - t_i) - \rho^2 > 0, \quad i = 1, 2.$$

Taking into account the above conditions, and maximizing π_{R_i} with respect to retailer i's decisions p_i, and a_i, we get the optimal retail prices and promotions as functions of the wholesale prices, w_i, and the cooperative advertising rates, t_i, for $i = 1, 2$:

$$p_i = \frac{\Omega_1(t_i, t_j, w_i, w_j)}{\Omega_2(t_i, t_j)}, \tag{25}$$

$$a_{i1} = \frac{\Omega_3(t_i, t_j, w_i, w_j)}{\Omega_2(t_i, t_j)}, \tag{26}$$

where

$$
\begin{aligned}
\Omega_1(t_i, t_j, w_i, w_j) &= (2(1 - t_i)(v + \beta w_j) + w_i(6 - 2t_i - 4t_j + \delta^2))\rho^2 - \rho^4 w_i \\
&+ 2((1 - t_i)(v - w_j) - (1 - t_j)\beta w_i)\delta\rho \\
&- 4(1 - t_i)(1 - t_j)(2(v + w_i) + (v + w_j)\beta), \\
\Omega_2(t_i, t_j) &= 4(\beta^2 - 4)(1 - t_i)(1 - t_j) + 2\beta\delta\rho(t_i + t_j - 2) \\
&+ (\delta^2 - 4(t_i + t_j - 2))\rho^2 - \rho^4,
\end{aligned}
$$

47

$$\Omega_3(t_i, t_j, w_i, w_j) \quad = \quad \rho\left[2(1-t_j)(w_j\beta+v(2+\beta)+w_i(\beta^2-2))-(v-w_j+w_i\beta)\delta\rho\right]$$

$$- \quad (v-w_i+w_j\beta)\rho^3.$$

First stage: The manufacturers take into account the retailers' reaction functions obtained in the second stage and play Nash to choose the wholesale prices, w_i, and the cooperative advertising rate, t_i for $i = 1, 2$. Each manufacturer M_i maximizes its profit given by

$$\max_{w_i, t_i} \pi_{M_i} = w_i D_i - t_i a_i^2. \tag{27}$$

With the help of Mathematica 10.1, we can deduce the conditions ensuring that each manufacturer's profit function, π_{M_i} $(i = 1, 2)$, is a concave function in w_i and t_i. The resulting expressions are very long and we refrain from writing them as they do not offer any qualitative insight.

Replacing the retailers' reaction functions obtained in Stage 2 in π_{M_i}, and taking into account the conditions that ensure the concavity of π_{M_i} in variables w_i and t_i, the optimality conditions for the manufacturers' problem are obtained as follows

$$\frac{\partial \pi_{M_i}}{\partial t_i} = 0, \; \frac{\partial \pi_{M_i}}{\partial w_i} = 0 \quad i = 1, 2.$$

By symmetry, the above system of four equations simplifies to two equations. >From

the first equation one can get two expressions for the wholesale price, that is,

$$w_1 = w_2 = w = \frac{v}{1 - \beta},$$

and

$$w_1 = w_2 = w = f(t),$$

where $f(t)$ denotes a known function of t.

Substituting the first expression of w into the second optimality condition, a non-linear equation in variable t is obtained, which admits three different solutions. However, these three solutions are removed because they imply null advertising investments.

Substituting the second expression of w into the second optimality condition, we obtain an equation in t that admits four solutions. Two of these solutions are removed again because they lead to null advertising investments. Therefore, the two feasible solutions are given by the pairs: (w^I, t^I) and (w^{II}, t^{II}), where w^I, t^I are w and t, in (23) and (22), respectively, and

$$
\begin{aligned}
w^{II} &= -\frac{v(2 + \beta)(4 + 2\beta - \rho(\delta + \rho))}{2\beta^3 + \beta\rho(\delta + \rho) - 2(4 + (2\delta - \rho)(\delta + \rho)) - \beta^2((\delta + \rho)\rho - 6)}, \\
t^{II} &= \frac{4 + 2\beta - \rho(\delta + \rho)}{2(2 + \beta)}.
\end{aligned}
$$

This last solution is removed because it does not satisfy the concavity conditions of π_{M_i} in variables w_i and t_i. Therefore, only one possibility for (w, t) remains feasible and its

49

expressions is reported in (23) and (22). The remainder optimal expressions (20) and (21) can be easily obtained taking into account (25) and (26) and the symmetric manufacturers' optimal strategies. ∎

7.2 Equilibrium solutions in $\mathcal{DI}$ Channel

We obtain feedback equilibrium solutions using backward induction for two games. In the first game, the decentralized manufacturer (M_1) does not offer any cooperative advertising support to its retailer ($t_1 = 0$). In the second game, M_1 offers a cooperative advertising at a rate $t_1 \neq 0$ ($i = 1, 2$).

7.2.1 $\mathcal{D}_\emptyset \mathcal{I}$ case

Proposition 3 *The equilibrium strategies for the game where no cooperative advertising is implemented in the $\mathcal{DI}$ channel are given by*

$$w_1 = \frac{v(-4 - 2\beta + \rho(\delta + \rho))}{2(-4 + 2\beta^2 - \rho(\delta\beta - \rho))}, \tag{28}$$

$$p_1 = \frac{v(24 - 8\beta^2 + 6\beta\delta\rho - (10 + \delta^2)\rho^2 + \rho^4)}{2(-4 + 2\beta^2 - \rho(\delta\beta - \rho))(-4 + 2\beta - \rho(\delta - \rho))}, \tag{29}$$

$$p_2 = \frac{v(8 - 4\beta^2 + \delta\rho - 2\rho^2 + \beta(2 + 2\delta\rho - \rho^2))}{(-4 + 2\beta^2 - \rho(\delta\beta - \rho))(-4 + 2\beta - \rho(\delta - \rho))}, \tag{30}$$

$$a_1 = \frac{v\rho}{8 - 4\beta + 2\delta\rho - 2\rho^2}, \tag{31}$$

$$a_2 = \frac{v\rho(8 - 4\beta^2 + \delta\rho - 2\rho^2 + \beta(2 + 2\delta\rho - \rho^2))}{2(-4 + 2\beta^2 - \rho(\delta\beta - \rho))(-4 + 2\beta - \rho(\delta - \rho))}. \tag{32}$$

50

The manufacturer's and retailers' optimal profits are given by:

$$\pi_{M_1} = \frac{v^2(-4 - 2\beta + \rho(\delta + \rho))}{2(-4 + 2\beta^2 - \rho(\delta\beta - \rho))(-4 + 2\beta - \rho(\delta - \rho))}, \tag{33}$$

$$\pi_{R_1} = \frac{v(4 - \rho^2)}{4(-4 + 2\beta - \rho(\delta - \rho))^2}, \tag{34}$$

$$\pi_{I_2} = \frac{v^2(4 - \rho^2)(8 - 4\beta^2 + \delta\rho - 2\rho^2 + \beta(2 + 2\delta\rho - \rho^2))^2}{4(-4 + 2\beta^2 - \rho(\delta\beta - \rho))^2(-4 + 2\beta - \rho(\delta - \rho))^2}. \tag{35}$$

Proof. Second stage: In this stage, the retailers play a Nash game between them. Each retailer chooses its retail price, p_i, and promotions, a_i $(i = 1, 2)$, to maximize their profits. Retailer 1's problem is given by

$$\max_{p_1, a_1} \pi_{R_1} = (p_1 - w_1)D_1 - a_1^2, \tag{36}$$

where D_1 is the demand function for product 1 given in (1).

The integrated firm problem is

$$\max_{p_2, a_2} \pi_{I_2} = p_2 D_2 - a_2^2, \tag{37}$$

where D_2 is the demand function for product 2 given in (1).

The solution to problems (36) and (37) gives the retailers' reaction functions, that is, p_i and a_i $(i = 1, 2)$, as functions of the wholesale price, w_1.

The retailers' profits are strictly concave functions of their decision variables, p_i and a_i, if

$$4 - \rho^2 > 0,$$

which is satisfied by (2).

Taking into account the condition above to ensure the concavity of π_{R_1} and π_{I_2}, and from the first-order optimality conditions for the problems in (36) and (37), the following expressions can be derived:

$$p_1 = w_1 + \frac{-2v + w_1(1-\beta)}{-4 + 2\beta - \rho(\delta - \rho)} - \frac{w_1(1+\beta)}{4 + 2\beta - \rho(\delta + \rho)}, \tag{38}$$

$$p_2 = \frac{-2v + w_1(1-\beta)}{-4 + 2\beta - \rho(\delta - \rho)} + \frac{w_1(1+\beta)}{4 + 2\beta - \rho(\delta + \rho)}, \tag{39}$$

$$a_1 = \frac{1}{2}\rho \left(\frac{-2v + w_1(1-\beta)}{-4 + 2\beta - \rho(\delta - \rho)} - \frac{w_1(1+\beta)}{4 + 2\beta - \rho(\delta + \rho)} \right), \tag{40}$$

$$a_2 = \frac{1}{2}\rho \left(\frac{-2v + w_1(1-\beta)}{-4 + 2\beta - \rho(\delta - \rho)} + \frac{w_1(1+\beta)}{4 + 2\beta - \rho(\delta + \rho)} \right). \tag{41}$$

First stage: In this stage, M_1 chooses its wholesale price, w_1 to maximize its profit such as

$$\max_{w_1} \pi_{M_1} = w_1 D_1, \tag{42}$$

where the demand D_1 defined in (1). The manufacturer knows the retailers' pricing and advertising reaction functions derived in Stage 2, and incorporates this information when deciding its optimal wholesale pricing. We then replace the retail prices and promotions

by those obtained reaction functions in (38) - (41) in the manufacturer's objective function in (55). This function then reads:

$$\pi_{M_1} = w_1 \left(\frac{-2v + w_1(1-\beta)}{-4 + 2\beta - \rho(\delta - \rho)} + \frac{w_1(1+\beta)}{-4 - 2\beta + \rho(\delta + \rho)} \right).$$

The solution to the manufacturer's problem gives us the wholesale price, w_1. The profit function, π_{M_1}, is strictly concave function in its decision variable, w_1, if

$$\frac{4 - 2\beta^2 + \beta\delta\rho - \rho^2}{[-4 + 2\beta - \rho(\delta - \rho)] [4 + 2\beta - \rho(\delta + \rho)]} < 0,$$

which is satisfied by (2).

Taking into account the condition above, from the first-order condition for manufacturer M_1's problem in (55), we get its optimal strategy in (28).

Replacing this expression into the retailers' reaction functions in (38) to (41) we obtain the optimal retail prices and advertising strategies in (29) and (32).

Once the manufacturer's and retailers' optimal strategies are known, their optimal profits in (33) to (35) immediately follow. ∎

7.2.2 $\mathcal{D}_\mathcal{C}\mathcal{I}$ case

Proposition 4 *The equilibrium strategies for the game where a cooperative advertising is implemented in the channel is given by the following solution:*

$$p_1 = -\frac{v(4+2\beta-\rho(\delta+\rho))(96-2(18+\delta^2)\rho^2+3\rho^4-\beta\delta\rho(-20+\rho^2)+2\beta^2(-16+\rho^2))}{Den}, \quad (43)$$

$$p_2 = \frac{2vNum}{Den}, \quad (44)$$

$$a_1 = \frac{2v(4+2\beta-\rho(\delta+\rho))(2\beta^2\rho-2(6+\delta^2)\rho+3\rho^3-\beta\delta(-4+\rho^2))}{Den}, \quad (45)$$

$$a_2 = \frac{v\rho Num}{Den}, \quad (46)$$

$$t_1 = \frac{-2\beta^2\rho+\beta\delta(4+\rho^2)+\rho(-4-2\delta^2+\rho^2)}{2\beta^2\rho-\beta\delta(-4+\rho^2)+\rho(-2\delta^2+3(-4+\rho^2))}, \quad (47)$$

$$w_1 = \frac{v(4+2\beta-\rho(\delta+\rho))(64-2(14+\delta^2)\rho^2+3\rho^4-\beta\delta\rho(-12+\rho^2)+2\beta^2(-8+\rho^2))}{Den}, \quad (48)$$

where

$$
\begin{aligned}
Den \;=\; & -512+4(100+12\delta^2+\delta^4)\rho^2-4(26+3\delta^2)\rho^4+9\rho^6-4\beta^3\delta\rho(-20+\rho^2) \\[4pt]
& +\; \beta^4(-16+\rho^2)+2\beta\delta\rho(36+2\delta^2-3\rho^2)(-4+\rho^2)+\beta^2(\delta^2(16-32\rho^2+\rho^4) \\[4pt]
& +\; 12(32-12\rho^2+\rho^4)),
\end{aligned}
$$

$$
\begin{aligned}
Num \;=\; & -128+8\beta(-12+6\beta+4\beta^2-\delta^2)+4\delta(6-3\beta(3+2\beta)+\delta^2)\rho \\[4pt]
& +\; 2(34+3\delta^2-\beta(\beta(3+\beta)-2(9+\delta^2)))\rho^2+(-6+\beta(3+\beta))\delta\rho^3-3(3+\beta)\rho^4.
\end{aligned}
$$

Proof. We follow the same steps as in the proof of Proposition 3 .

Second stage: In this stage, the retailers play a Nash game between them. Each retailer chooses its retail price, p_i, and promotions, a_i $(i = 1, 2)$ to maximize its profit. Therefore, retailer 1's problem can be written as

$$\max_{p_1, a_1} \pi_{R_1} = (p_1 - w_1)D_1 - (1 - t_1)a_1^2, \tag{49}$$

where D_1 given in (1).

The integrated firm's problem can be written as

$$\max_{p_2, a_2} \pi_{I_2} = p_2 D_2 - a_2^2, \tag{50}$$

where D_2 is the demand function for product 2 given in (1).

The solution to problems (49) and (50) gives the retailers' reaction functions, that is, p_i and a_i, $i = 1, 2$, as functions of the wholesale price, w_1, and the cooperative advertising rate, t_1.

The retailers' profits are strictly concave functions of their decision variables, p_i and a_i, if the following conditions are satisfied:

$$4(1 - t_1) - \rho^2 > 0, \quad 4 - \rho^2 > 0.$$

Taking into account the above conditions, and maximizing the retailers' profits, we get the optimal retail prices and promotions as functions of the wholesale price, w_1, and the cooperative advertising rate, t_1:

$$p_1 = \frac{\Gamma_1(t_1, w_1)}{\Gamma_2(t_1)}, \tag{51}$$

$$a_1 = \frac{\Gamma_3(t_1, w_1)}{\Gamma_2(t_1)}, \tag{52}$$

$$p_2 = \frac{\Gamma_4(t_1, w_1)}{\Gamma_2(t_1)}, \tag{53}$$

$$a_2 = \frac{\rho \Gamma_4(t_1, w_1)}{2\Gamma_2(t_1)}, \tag{54}$$

where

$$\Gamma_1(t_1, w_1) = 2v(1-t_1)(4 + 2\beta - \rho(\delta + \rho)) + w_1(8 + 2\beta\delta\rho - (6 + \delta^2)\rho^2 + \rho^4$$
$$+ 2t_1(-4 + \rho^2)),$$

$$\Gamma_2(t_1) = 4(4 - \beta^2)(1 - t_1) - 2\beta\delta\rho(t_1 - 2) + (-8 + 4t_1 - \delta^2)\rho^2 + \rho^4,$$

$$\Gamma_3(t_1, w_1) = \rho \left[w_1(-4 + 2\beta^2 - \beta\delta\rho + \rho^2) + v(4 + 2\beta - \rho(\delta + \rho)) \right],$$

$$\Gamma_4(t_1, w_1) = 4(1 - t_1)(w_1\beta + v(2 + \beta)) - 2(v - w_1)\delta\rho - 2(v + w_1\beta)\rho^2.$$

First stage: In this stage, M_1 chooses its wholesale price, w_1, and its cooperative advertising rate, t_1, in order to maximize its profit as follows:

$$\max_{w_1, t_1} \pi_{M_1} = w_1 D_1 - t_1 a_1^2, \tag{55}$$

where D_1 is the demand function defined in (1). At this stage, the manufacturer knows the retailers' pricing and promotion reaction functions derived in Stage 2, it incorporates this information when deciding its strategy. We then replace the retail prices and promotions by the obtained reaction functions in (51) - (54) in the manufacturer's objective function in (55). This function reads:

$$\pi_{M_1} = w_1 \left(\frac{-2v + w_1(1 - \beta)}{-4 + 2\beta - \rho(\delta - \rho)} + \frac{w_1(1 + \beta)}{-4 - 2\beta + \rho(\delta + \rho)} \right).$$

The solution to the manufacturer's problem gives us the wholesale price, w_1, and the cooperative advertising rate, t_1.

With the help of Mathematica 10.1, we can deduce the conditions ensuring that the manufacturer 1's profit, π_{M_1}, is a concave function in the manufacturer 1's decision variables, w_1 and t_1. The resulting expressions are very long and we refrain from writing them as they do not offer any qualitative insight.

Replacing the retailers' reaction functions obtained in Stage 2 in π_{M_1}, and taking into account the conditions that ensure the concavity of π_{M_1} in its choice variables w_1 and t_1, the optimality conditions for the manufacturer's problem are obtained as follows:

$$\frac{\partial \pi_{M_1}}{\partial t_1} = 0, \ \frac{\partial \pi_{M_1}}{\partial w_1} = 0.$$

>From the first equation one can get two expressions of the wholesale price, namely:

$$w_1 = \frac{v(-4 - 2\beta + \delta\rho + \rho^2)}{-4 + 2\beta^2 - \beta\delta\rho + \rho^2},$$

$$w_1 = g(t_1),$$

where $g(t_1)$ denotes a known function of t_1.

Substituting the first expression of w_1 into the second optimality condition, a non-linear equation in variable t_1 is obtained and admits two different solutions. However, these two solutions are removed; one because it implies null promotions, and the other because it does not satisfy the conditions that ensure the concavity of π_{M_1} in the choice variables w_1 and t_1.

Substituting the second expression of w_1 into the second optimality condition, one can obtain an equation in t_1 that admits two solutions. One of these solutions is removed again because it does not satisfy the conditions that ensure the concavity of π_{M_1} in the choice variables w_1 and t_1. Therefore, there is a unique feasible solution and its expressions is reported in (48) and (47). The remainder optimal expressions (44), (46) and (45) can easily be obtained taking into account (51) - (54) and the manufacturer 1's optimal strategies. ∎

7.3 Equilibrium Solution in the II Channel

In this channel structure each manufacturer and retailer acts as one vertically integrated firm.

Proposition 5 *The equilibrium strategies in the II channel are given by*

$$p_1 \;=\; p_2 = \frac{2v}{4 - 2\beta + \rho(\delta - \rho)}, \tag{56}$$

$$a_1 \;=\; a_2 = \frac{\rho v}{4 - 2\beta + \rho(\delta - \rho)}. \tag{57}$$

The channel's optimal profits are:

$$\pi_{I_1} = \pi_{I_2} = \frac{v^2(4 - \rho^2)}{[4 - 2\beta + \rho(\delta - \rho)]^2}.$$

Proof. The optimization problem of firm $i = 1, 2$ is given by

$$\max_{p_i, a_i} \pi_{I_i} = p_i D_i - a_i^2, \tag{58}$$

where D_i is the demand function for product i given in (1).

The profit function π_{I_i} is strictly concave in p_i and a_i if

$$4 - \rho^2 > 0, \quad i = 1, 2,$$

which is satisfied by (2).

Taking into account the above condition, from the first-order optimality conditions, the optimal retail prices and advertising investments in (56) and (57) immediately follow. ∎

59
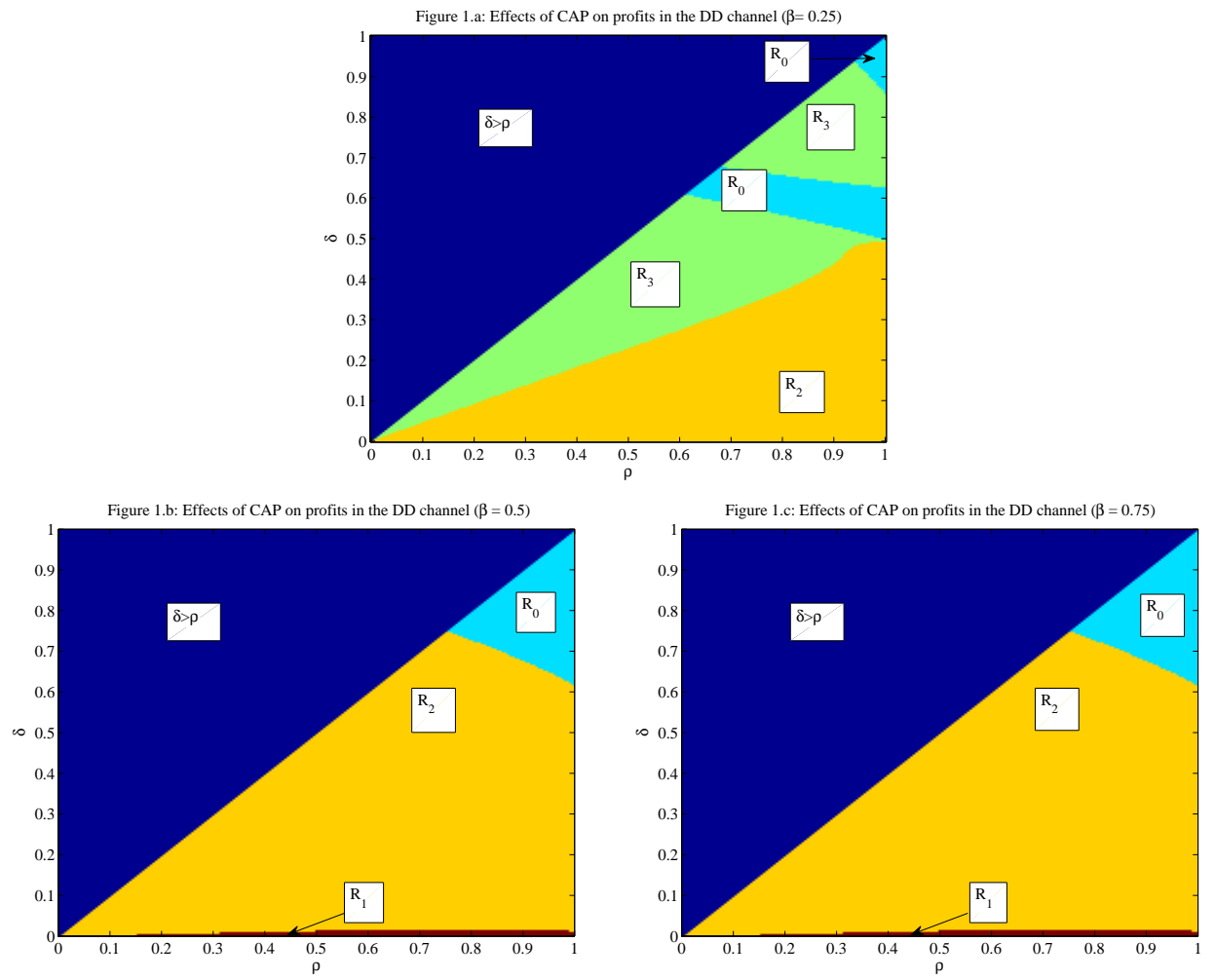
Figure(s)

Figure 1: Effects of CAP on profits in the $\mathcal{DD}$ channel.

1
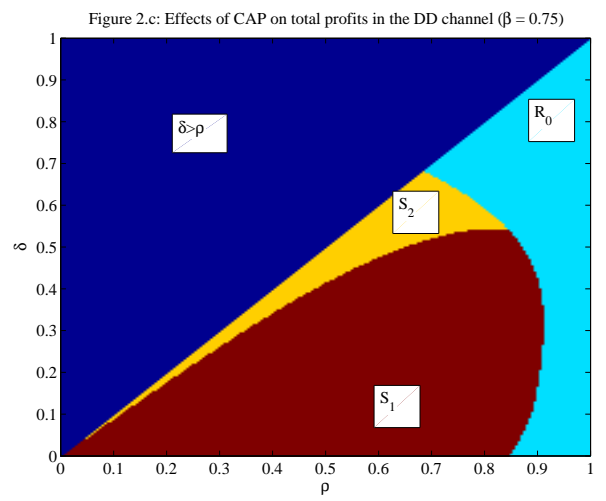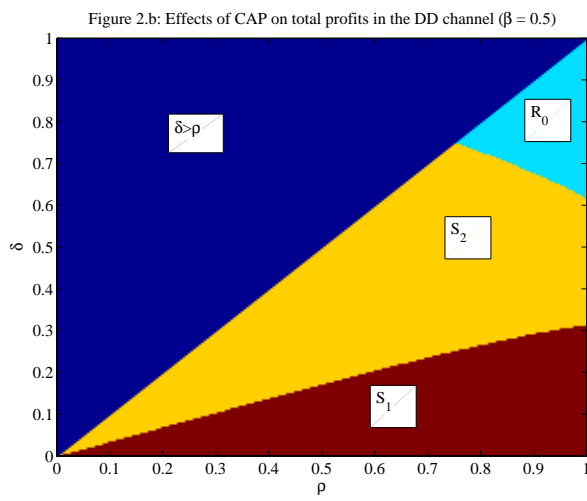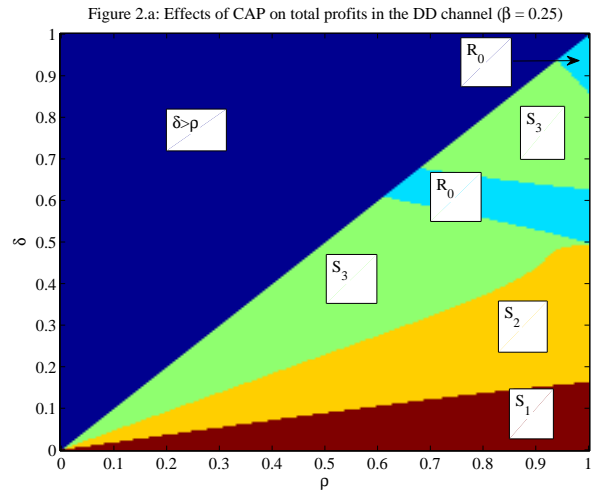
Figure 2: Effects of CAP on total profits in the $\mathcal{DD}$ channel.

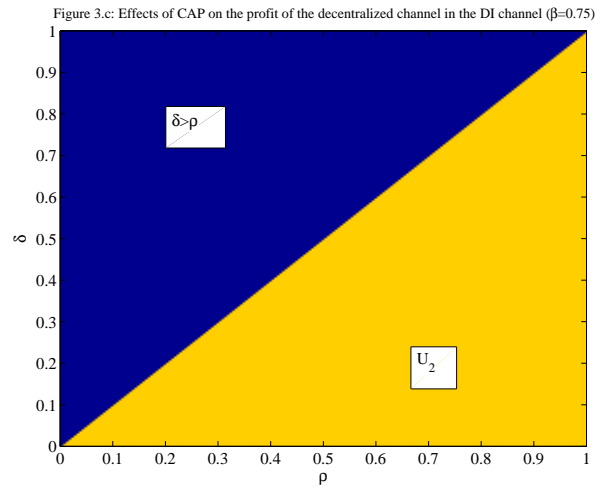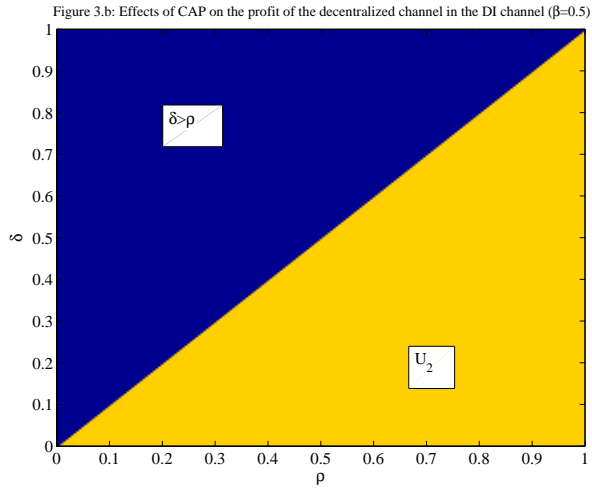Figure 3.a: Effects of CAP on the profit of the decentralized channel in the DI channel (β=0.25)

Figure 3.b: Effects of CAP on the profit of the decentralized channel in the DI channel (β=0.5)

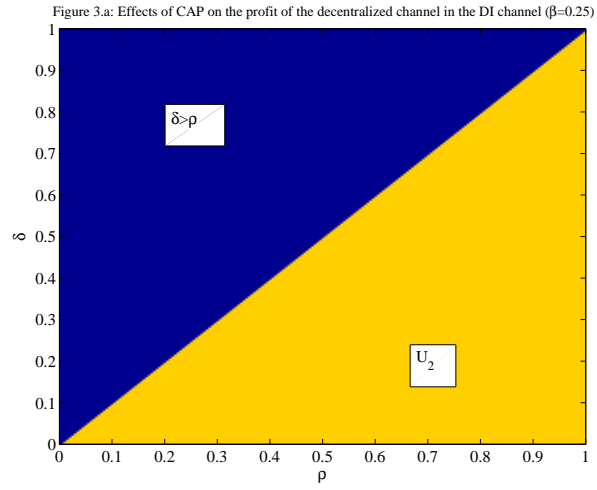Figure 3.c: Effects of CAP on the profit of the decentralized channel in the DI channel (β=0.75)

Figure 3: Effects of CAP on the profit of the decentralized channel in the $\mathcal{DI}$ channel.
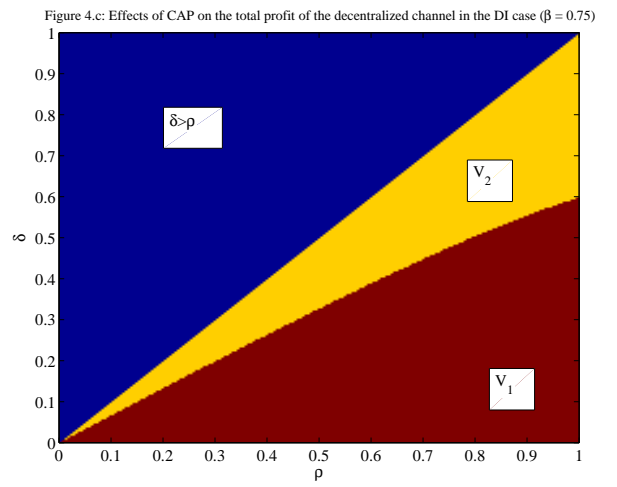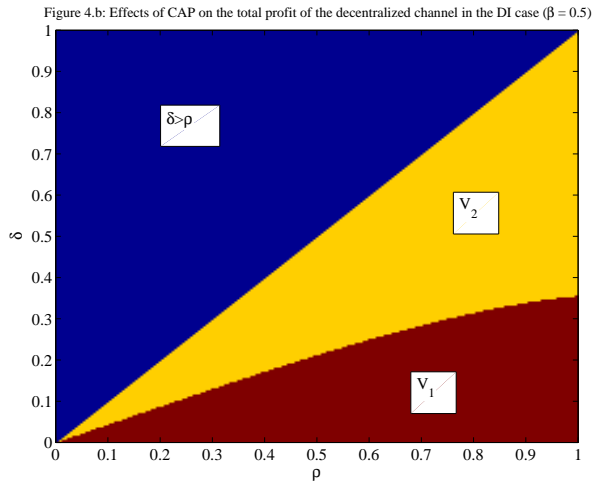
Figure 4.a: Effects of CAP on the total profit of the decentralized channel in the DI case (β = 0.25)

Figure 4.b: Effects of CAP on the total profit of the decentralized channel in the DI case (β = 0.5)
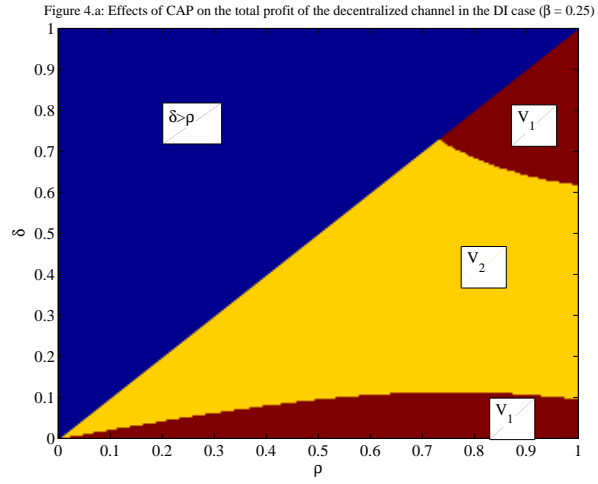
Figure 4.c: Effects of CAP on the total profit of the decentralized channel in the DI case (β = 0.75)

Figure 4: Effects of CAP on the total profit of the decentralized channel in the $\mathcal{DI}$ case