Fighting store brands through the strategic timing of pricing and advertising decisions[1]

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Abstract

This paper investigates whether manufacturers can use the timing (sequence) of their pricing and advertising decisions to benefit from or to deter store brand (SB) introductions. We develop and solve six sequential game theoretic models for a bilateral channel where different timing of these decisions are considered before and after the retailer introduces a store brand. Comparisons of equilibrium solutions across games show that the sequence of pricing and advertising decisions in the channel significantly impacts the profitability of a store brand entry by the retailer. Such impact depends on: (1) whether each channel member decides on pricing and advertising simultaneously or sequentially prior to the SB entry, (2) whether the timing chosen for these decisions changes following the SB introduction, and (3) the intensity of competition between the store and national brands. In particular, the SB entry leads to losses for the manufacturer when the sequence of advertising and pricing decisions is kept unchanged after the SB entry even when it is much differentiated from the NB. These results offer new perspectives on the effects of store brand entry in distribution channels, and suggest that for low levels of competition intensity between the NB and the SB, the manufacturer can either

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prevent or benefit from the retailer's brand given an adjustment in the sequence of the manufacturer's decisions.

Keywords: OR in Marketing; Advertising and pricing; Decision timing; Store and national brands; Game theory.

1 Introduction

Store brands (SB), also known as private labels, have been increasing in popularity for a few decades. A recent Nielsen Homescan study reports that more than two-thirds of total U.S. households (70%) buy store brands as alternatives to national brand (NB) products (The Nielsen Company, 2016). In 2015, the sales of SB totaled \$118.4 billion in the US, with an increase of \$2.2 billion over the previous year and a growth rate of 5% over a two year period (Private Label Manufacturers Association (PLMA), 2016). These sales span a variety of retail sectors including supermarkets, drug stores, specialty retailers, and others. The dominance of store brands is even more prevalent in Europe as evident in large market shares that vary between 40% and 53% (PLMA, 2016).

The success of store brands has been a source of concern for national brand manufacturers who have seen increased competition from these brands and shrinking market shares. To compete against store brands, manufacturers of national brands use different marketing strategies. While the existing game-theoretic literature about SB focuses on pricing strategies (Sethuraman, 2009), we focus on two strategic tools related to advertising (promotion) and pricing. These two marketing mix elements have a large influence on purchases of consumer products (Hwang and Thomadsen, 2016).

In addition to studying both local advertising and pricing decisions when assessing the effects of SB introductions, we consider the timing (sequence) in which these decisions are made in the channel. Since the SB literature has been focused on pricing strategies, the issue of the timing for pricing and local advertising agreement has not been discussed in the literature. In particular, there have been few game-theoretic analyses about SB-modeled pricing and advertising strategies, and most of these studies consider that such decisions are simultaneously determined by each channel member (Karray and Zaccour, 2006; Amrouche and Yan, 2015). In reality, channel members can have long-standing agreements on advertising, information they then use when deciding on their pricing strategies. For example, Soberman and Parker (2006) make this assumption when investigating the effects of SB introduction using a Hotelling model of demand. Conversely, in some channels, the retailer and the manufacturer can have a long standing agreement on prices, for example, to implement an everyday low pricing strategy, while non-price promotional and advertising campaigns (features, displays, local advertising) can be decided given the pricing strategy in place. Finally, as assumed in the literature, channel members can decide on their advertising and pricing strategies simultaneously.

In this paper, we take into consideration the timing (sequence) in which pricing and advertising decisions are made in the channel when assessing the effects of a store brand introduction. In particular, we consider a channel that is led by the NB manufacturer, and study whether a price agreement that precedes or succeeds advertising (as opposed to a simultaneous choice of these decisions) can impact the success of the store brand introduction. We aim at addressing the following research questions: (1) What are the effects of a store brand introduction on channel members' profits given different decision-making timing (sequences) for pricing and advertising decisions? (2) In a channel led by the national brand manufacturer, can the latter strategically use the timing of advertising and pricing decisions to deter store brand introductions? (3) If yes, then under what conditions should pricing precede advertising decisions or vice-versa, or should they be chosen simultaneously?

2 Literature review

Two literature streams are relevant to this paper. The first relates to the effects of store brand introductions on the strategies and profitability of channel members. The second is about the timing (sequence) of decision-making. The analytical literature about store brand introductions points to the importance of advertising decisions in the channel. A few papers have examined advertising and pricing decisions when studying the effects of store brand introductions. This literature has mostly assumed that whenever these decisions are made by a channel member, they are made simultaneously. In this paper, we study whether the timing of pricing and advertising decisions can influence the strategic implications of store brand introductions.

2.1 Store brand literature

The literature about store brand introductions showed that retailers can benefit from introducing a store brand mainly by expanding the product category, gaining better margins, and differentiating the retail offering (Chintagunta et al., 2002). In particular, store brands can expand the product category at the retail store by either attracting new customers or by better serving NB consumers (Gruca et al., 2001; Pauwels and

Srinivasan, 2004). Not only can retailers gain extra revenues from selling the store brand, they can also expand sales for the NB by attracting new consumers. Further, store brands can allow retailers to obtain better price terms from NB manufacturers (e.g., Narasimhan and Wilcox, 1998). Retailers also usually gain higher margins on SBs than on NBs because they procure the SB from unbranded manufacturers operating at lower costs than NB manufacturers who have to pay the extra costs to brand and market their products. Despite these benefits, store brand introductions are not always successful. Their survival depends on factors related to consumer price sensitivity and preference for the NBs, and to the SB positioning strategy by the retailer against the existing NBs (Sayman et al., 2002; Scott-Morton and Zettelmeyer, 2004; Choi and Coughlan, 2006; Nasser et al., 2013; see reviews by Sethuraman, 2009 and Pauwels and Srinivasan, 2009).

Extant research finds that the increased popularity of store brands has harmed NB manufacturers by squeezing their shares and shrinking their margins as retailers get better price deals on national brands, and competition from store brands lowers the latter's retail prices (Scott-Morton and Zettelmeyer, 2004; Pauwels and Srinivasan, 2004; Meza and Sudhir, 2010). However, some studies find that NB manufacturers can also benefit from SB entry through demand expansion or price competition (Ailawadi and Harlam, 2004; Bonfrer and Chintagunta, 2004; Chintagunta et al., 2002; Pauwels and Srinivasan, 2004). The SB entry can also improve the NB manufacturer's profitability when the channel is led by the retailer, due to the higher NB demand and wholesale price following SB introduction by the retailer (Ru et al., 2015).

Given the harmful effects of store brand introductions for national manufacturers, a few studies have examined strategies that manufacturers can implement to either preempt or deter SB entry. Notably, Nasser et al. (2013) propose that NB manufacturers can use their pricing and product line strategies to deter SB entry. They identify conditions under which the NB manufacturer should either reposition his product, choose to supply the SB, or extend his product line by introducing a flanker NB. Jin et al. (2017) propose that pricing and dual channel strategies can be effective in deterring SB threats. Other researchers look at channel coordination mechanisms as tools to be used against SB entry. For example, Karray and Zaccour (2006) propose the use of cooperative advertising programs while Fang et al. (2013) propose revisiting the NB manufacturer's pricing contract. In his extensive literature review about store brands, Sethuraman (2009) notes the importance of national brand advertising in fighting against store brands. This is because advertising can help differentiate the national brand through increased brand equity, which provides NB manufacturers with more bargaining power to negotiate pricing deals with retailers (Abe, 1995; Connor and Peterson, 1992; Amrouche and Yan, 2015). However, when NB manufacturers increase their advertising spending to fight against a SB introduction, the retailer can react by increasing the price of both national and store brands to better discriminate between NB and SB consumers. This result may be due to the fact that the retailer uses pricing in reaction to the manufacturers' pre-set advertising levels. Alternatively, when the pricing contract is negotiated between channel members prior to manufacturer's advertising announcement, this result may not hold. Finally, empirical evidence points to the fact that SB shares are negatively related to NB advertising and promotions (Sethuraman and Gielens, 2014).

2.2 Strategic decision timing (sequence) literature

In game theoretic analyses, the strategic timing of certain decisions has received some attention in the microeconomics literature mainly in studies related to competitive strategies. In this literature, the focus has been on studying the effects of different sequences of decision-making for a specific strategy (e.g., price or quantity) chosen by competing firms. The decision sequence is usually described as simultaneous (a la Bertrand) or sequential (a la Stackelberg) (Bertrand, 1883; von Stackelberg, 1934). In a channel context, the issue of decision-making timing has been tightly linked to channel leadership (Lee and Staelin, 1997; Jorgensen et al., 2001; Shi et al., 2013). Again, the focus in this literature has been on studying how the sequence in which firms make their decisions impacts the equilibrium solutions. When each firm makes multiple decisions, the usual assumption is that these decisions are made simultaneously. In this paper, we do not look at the effect of channel leadership but rather assume a manufacturer's leadership in the channel, which has been predominantly assumed in the analytical literature about store brands (see Sethuraman, 2009). While we focus on manufacturer leadership, we consider different sequences of decision-making for each of the manufacturer's strategies, namely price and advertising.

The issue of whether the NB manufacturer should lead the channel in advertising first, in pricing first or in both decisions simultaneously is significant because the retailer observes each of the manufacturer's announced decisions before making his own. Therefore, depending on which information is announced to the retailer (advertising, pricing or both), the latter will react by choosing different levels of advertising and pricing which will then impact the demands, revenues and profits for each of the store and the national brands. This issue has received very little attention in the distribution channel literature. A notable exception is the work by Karray (2013) who showed that the order in which pricing and advertising decisions are announced by the channel leader can significantly impact the strategic outcomes of each channel member. Indeed, in reality, the retailer and the manufacturer do not necessarily make their advertising and pricing decisions simultaneously. In some cases, pricing agreements between manufacturers and retailers are established first. This is the case of channels that adopt an everyday low pricing (EDLP) strategy such as the one famously established between Proctor & Gamble and Walmart. This will mean that advertising efforts (e.g., store flyers and manufacturer promotions) are decided given the pricing established in the channel. In other cases, the retailer and the manufacturer do not have such long term agreements and might make these decision simultaneously (e.g., to allow for flexibility in an unstable economic environment), or decide of advertising first (e.g., to get contractual agreement with media agencies) before making their pricing decisions. We study in this paper whether such differences in decision-making sequences has an impact on the store brand's introduction success.

Considering both the manufacturer's and the retailer's pricing and advertising decisions, we allow the manufacturer to decide on the sequence in which these decisions are chosen. In particular, the manufacturer can: (1) decide on his advertising and pricing simultaneously, followed by simultaneous decision-making by the retailer, (2) decide on his advertising, followed by the retailer's advertising, then decide on his pricing followed by the retailer's prices, or (3) choose his price, followed by the retailer's price, then decide on his advertising followed by the retailer's advertising. In each of these different setups, the manufacturer leads the channel and the retailer is the follower. We investigate whether the sequence of decision-making for pricing and advertising can impact the effects of a store brand introduction by the retailer.

In particular, we characterize the optimal pricing and advertising strategies of a manufacturer facing a strategic retailer and a SB introduction. We identify conditions under which the NB manufacturer should first announce to the retailer his advertising, his pricing or both simultaneously, following a retailer's decision to introduce a store brand. Contrary to the existing literature, we assess the manufacturer's announcement sequence of different strategies as a potential mechanism to counter the threat of SB introduction.

The remainder of the paper is organized as follows. Section 2 describes the model and assumptions. Section 3 derives the equilibrium solutions for six games. Namely, for each of the three sequences of decision-making described above, we solve the game without a store brand and with the SB. Section 4 assesses the impact of a store brand introduction on channel members' optimal strategies and profits. Section 5 concludes and discusses the managerial and theoretical implications of this research.

3 The Model

We consider a distribution channel formed by one manufacturer (M) and one retailer (R). The manufacturer sells the national brand (NB) to the retailer who offers it to consumers. The manufacturer decides on the NB wholesale price (w) and also of the level of his NB local advertising (a_m). The retailer sets the NB retail price (p_n) and the local advertising effort for the NB (a_n).

w	Wholesale price of NB manufacturer, $w > 0$
p_s	Retail price for the SB, $p_s > 0$
p_n	Retail price for the NB, $p_n > w$
a_s	Advertising effort of the retailer for the SB, $a_s > 0$
a_n	Advertising effort of the retailer for the NB, $a_n > 0$
a_m	Advertising effort of the manufacturer, $a_m > 0$
d_n	Demand for the NB product, $d_n > 0$
d_s	Demand for the SB product, $d_s > 0$
Π_m	Profit of the NB manufacturer, $\Pi_m > 0$
Π_r	Profit of the retailer, $\Pi_r > 0$
Π_{ch}	Profit of the total channel, $\Pi_{ch} = \Pi_r + \Pi_m$
g_n	Baseline demand parameter for the NB, $g_n > 0$
g_s	Baseline demand parameter for the SB, $0 < g_s < g_n$
θ	Effect of competing product's price on demand, $\theta \in (0, 1)$
b	Effect of manufacturer's advertising effort on demand, $b > 0$
c	Effect of retailer's advertising effort on demand, $c > 0$
R	Retailer
M	Manufacturer
SB	Store brand
NB	National brand
$N1$	The manufacturer decides simultaneously of pricing and advertising, no SB is offered
$N2$	The manufacturer decides sequentially (advertising then pricing), no SB is offered
$N3$	The manufacturer decides sequentially (pricing then advertising), no SB is offered
$SN1$	The manufacturer decides simultaneously of pricing and advertising, a SB is offered
$SN2$	The manufacturer decides sequentially (advertising then pricing), a SB is offered
$SN3$	The manufacturer decides sequentially (pricing then advertising), a SB is offered

Table 1: List of Notations

In this paper, both the manufacturer's and the retailer's advertising decisions consist in local advertising activities or non-price promotions aimed at stimulating sales of their products **in the short term**. **Examples of such activities include features and**

displays for retailers and contests, sweepstakes, product samples and local media ads for manufacturers (Reid et al., 2005; Kalra and Shi, 2010). For clarity, a list of notations is included in Table 1.

Besides offering the NB, the retailer has also the option to introduce his store brand (SB). We assume that the latter is supplied by an unbranded manufacturer who sells the SB at cost to the retailer.[4] When the SB is introduced by the retailer, the latter sets the price to consumers (p_s) as well as set the local advertising level to promote the SB (a_s). The scenario where only the national brand is sold by the retailer is the benchmark scenario, whereas the scenario where both the NB and the SB are offered in the store is the alternative. We start by presenting the model for the alternative scenario (both SB and NB are offered), then discuss the simpler benchmark set-up (only the NB is offered).

3.1 Both the national and store brands are offered

The demand function for the SB (s) and for the NB (n) are given by d_i such as[5]

$$d_i = \frac{\alpha_i - \theta\alpha_j - p_i + \theta p_j}{1-\theta^2}, \quad i,j = n,s, \quad i \neq j, \tag{1}$$

where

$$\alpha_n = g_n + ba_m + ca_n, \tag{2}$$

$$\alpha_s = g_s + ca_s. \tag{3}$$

The expressions α_i ($i = n, s$) represent the expanded base demand of brand i, g_i is the baseline demand of product i when no advertising is being done for it. The baseline demand of each brand is increased by the advertising efforts of the manufacturer and the retailer. The effect of manufacturer's advertising on his baseline demand is denoted by the positive parameter b, whereas retailer advertising effect on demand is represented by the positive parameter c. Finally, product substitution is represented by the positive

[4] **The supplier selection decision is assumed exogenous to our model.**

[5] **The demand function in (1) has been obtained through maximization of the representative consumer surplus ($U - p_n d_n - p_s d_s$), where U is the utility function of a representative consumer $U = \sum_{i=n,s} \left(\alpha_i d_i - d_i^2/2\right) - \theta d_n d_s$. This linear quadratic formulation of U has been commonly used in the marketing and economics literature (e.g., Spence, 1976; Ingene and Parry, 2007; Cai et al., 2012; Liu et al., 2014; Karray et al., 2017). It exhibits the classical economic properties that: 1. The representative consumer's utility of owning a product decreases as the consumption of the substitute product increases, 2. The marginal utility for a product diminishes as the consumption of the product increases, and 3. The value of using multiple substitutable products is less than the sum of the separate values of using each product on his own (Samuelson, 1974).**

parameter $\theta \in (0, 1)$. When $\theta = 0$, the products are purely monopolistic; as θ goes to 1, the products converge to purely substitutable.

Incorporating the expressions of α_i in the demand functions (d_i) for $i = n, s$, we get

$$d_n = \frac{1}{1-\theta^2}\left[(g_n - \theta g_s) + ba_m + ca_n - \theta ca_s - p_n + \theta p_s\right], \tag{4}$$

$$d_s = \frac{1}{1-\theta^2}\left[(g_s - \theta g_n) + ca_s - \theta\left(ba_m + ca_n\right) - p_s + \theta p_n\right]. \tag{5}$$

When the retailer offers both store and national brands, the profit maximization problems of the manufacturer (Π_m) and the retailer (Π_r) are then given by

$$\begin{aligned} \max_{w,a_m} \Pi_m &= wd_n - a_m^2, \\ \max_{p_n,p_s,a_s,a_n} \Pi_r &= (p_n - w)d_n + p_s d_s - a_n^2 - a_s^2. \end{aligned}$$

This formulation implies the following three main assumptions. First, for simplicity, the production costs of both the national and the store brands are assumed the same and equal to zero for simplicity. Second, the advertising costs of both the manufacturer and the retailers are quadratic to represent increasing marginal costs of advertising. Third, the inventory and other procurement decisions are assumed exogenous to the problem at hand. These assumptions are very commonly used in the analytical literature about store brand introductions (see Sethuraman (2009) for a review showing the commonality of these assumptions) and the marketing channels literature (e.g., Ingene and Parry, 2007; Cai et al., 2012).

Note, however, that although this model is used to derive the results in the paper, some further analyses have been conducted with a more complicated model to include alternative formulations. First, a model with quadratic (square rooted) effect of advertising on demand and a linear advertising cost has been used. We found that all results presented in the paper hold under this alternative formulation. Second, we derived some of the results considering advertising costs parameters that are different for the retailer and for the manufacturer. Although these additional parameters complicated the results considerably, we found that the main results in the paper hold. We then retained the simpler parsimonious model presented here.

3.2 Benchmark model: only the national brand is offered

We now present the model for the benchmark scenario where only a national brand is offered by the retailer. In this case, the consumer can only choose the national brand (there is no alternative competing product available in the marketplace). Therefore, the demand for the national brand product is given by

$$d_n = g_n + ba_m + ca_n - p_n. \tag{6}$$

The manufacturer and the retailer problems consist of maximizing their profits in this scenario and are as follows

$$\begin{aligned} \max_{w,a_m} \Pi_m &= wd_n - a_m^2, \\ \max_{p_n,a_n} \Pi_r &= (p_n - w)d_n - a_n^2. \end{aligned}$$

3.3 Sequence of decision-making scenarios (games)

We assume that the channel is led by the manufacturer. This is a common assumption for decentralized dyadic channels in the marketing channel literature (Ingene et al., 2012). This sequence of play is also supported by empirical evidence showing that a Stackelberg game where manufacturers are leaders is often appropriate for pricing decisions in channels (Sudhir, 2001). As well, this is also aligned with the literature about national and store brands and is supported by empirical evidence (see the survey by Sethuraman, 2009). Being the channel leader, the manufacturer always announces his decision(s) first, then the retailer can react to the manufacturer's decision(s) and choose his own. Therefore, the retailer will react by making the same kind of decision(s) that is (are) announced by the manufacturer. In other words, the manufacturer is the leader for both decisions considered in this paper (advertising and pricing) and the retailer follows by making the same kind of decision(s) that the manufacturer announces.

Since this paper aims to assess whether the manufacturer can use the sequence in which advertising and pricing decisions are made to alter the strategic effects of a store brand introduction by the retailer, six scenarios need to be considered (Table 2). In the first three scenarios, the benchmark model is considered, (i.e., the retailer carries only the national brand and does not offer the store brand), and one of the following three can occur.

- ($N1$): The manufacturer makes his advertising and pricing decisions simultaneously, then the retailer reacts by also simultaneously setting both his advertising and pricing strategies, knowing the manufacturer's advertising and pricing decisions.
- ($N2$): The manufacturer first decides on his NB advertising, and the retailer then

sets his advertising decision, knowing the manufacturer's advertising. Afterwards, the manufacturer announces his wholesale price, knowing his own and the retailer's advertising. Finally, the retailer sets his retail price for the NB, knowing the manufacturer's advertising and wholesale price and the retailer's advertising.

- ($N3$): The manufacturer first announces his wholesale price, the retailer then sets his retail price for the NB, knowing the manufacturer's price. Subsequently, the manufacturer announces his NB advertising, knowing his own and the retailer's prices. Finally, the retailer sets his advertising level for the NB, knowing all previously announced decisions.

Each of these three situations ($N1$, $N2$ and $N3$) serve as benchmark scenarios. Following the store brand introduction by the retailer, similar scenarios are encountered. The manufacturer can set both advertising and pricing decisions simultaneously, or sequentially (with advertising first or pricing first). In any event, the retailer follows by reacting to the manufacturer's announcement by making decisions similar to the ones made by the manufacturer. However, contrary to the benchmark scenarios, the retailer is also choosing his decisions for the store brand, consisting of local advertising efforts for the SB (a_s) and in the SB retail price (p_s). Note that the retailer makes the same type of decision(s) (advertising, pricing or both) for the NB and the SB simultaneously. Therefore, there are three scenarios ($SN1$, $SN2$ and $SN3$) as described in Table 2.

	Simultaneous (1)	Advertising then pricing (2)	Pricing then advertising (3)
	$SN1$	$SN2$	$SN3$
SB & NB	Stage 1: M chooses a_m & w	Stage 1: M chooses a_m	Stage 1: M chooses w
	Stage 2: R chooses p_n, p_s, a_n & a_s	Stage 2: R chooses a_n & a_s	Stage 2: R chooses p_n & p_s
		Stage 3: M chooses w	Stage 3: M chooses a_m
		Stage 4: R chooses p_n & p_s	Stage 4: R chooses a_n & a_s
	$N1$	$N2$	$N3$
NB alone	Stage 1: M chooses a_m & w	Stage 1: M chooses a_m	Stage 1: M chooses w
(Benchmark)	Stage 2: R chooses p_n & a_n	Stage 2: R chooses a_n	Stage 2: R chooses p_n
		Stage 3: M chooses w	Stage 3: M chooses a_m
		Stage 4: R chooses p_n	Stage 4: R chooses a_n

Table 2: Games and decision sequence

Next, we derive equilibrium solutions in each of the six scenarios and obtain the equilibrium profits of the manufacturer and the retailer. We then compare equilibrium solutions in each of the scenarios in the SB case ($SN1$, $SN2$ and $SN3$) with each of the benchmark scenarios ($N1$, $N2$ and $N3$). The results will indicate the strategic effects of a SB introduction by the retailer for each channel member, given different initial decision-making sequences by the manufacturer.

4 Equilibrium solutions

4.1 Equilibrium in the benchmark scenarios

In each scenario of the benchmark scenarios ($N1$, $N2$ and $N3$), we solve the game by backward induction. In $N1$, the game is played in two stages. We start by solving the retailer's problem, then use the obtained reaction functions to write the manufacturer's profit function and solve his problem to get the equilibrium manufacturer's advertising and wholesale price. In $N2$, the game is played in four stages. We start with stage 4 by solving the retailer's pricing problem to get the NB price. Then, we use the NB price expression in the manufacturer's problem and solve the latter to obtain the wholesale price. The obtained expressions of both prices are then used to write the retailer's problem and to solve it in the retail advertising strategy. The solution, along with all other pricing variables, are then injected in the manufacturer's problem and used to obtain the equilibrium advertising strategy for the NB. A similar approach is used in $N3$, but instead of starting by solving the retailer's problem in price we start by solving the retailer's problem in advertising.

Proposition 1 *The equilibrium solution obtained in each of the three benchmark scenarios is included in Table 3.*

A detailed description of the solution methodology and expressions of the reaction functions and second-order conditions are included in Appendix A. The obtained solutions verify the positivity conditions for each channel members' prices, advertising, demands, margins and profits. We also characterize the concavity conditions ensuring that the extremum are interior maximum. We denote by the feasible region the parameters space in b and c where both the positivity and concavity conditions are satisfied in each scenario $N1$, $N2$ and $N3$.

	N1	N2	N3
w	$\frac{4-c^2}{8-b^2-2c^2}$	$\frac{8\left(16-c^2\right)}{256-32b^2-32c^2+c^4}$	$\frac{4}{8-b^2c^2}$
a_m	$\frac{b}{8-b^2-2c^2}$	$\frac{32b}{256-32b^2-32c^2+c^4}$	$\frac{2b}{8-b^2c^2}$
p_n	$\frac{6-c^2}{8-b^2-2c^2}$	$\frac{12\left(16-c^2\right)}{256-32b^2-32c^2+c^4}$	$\frac{2\left(12-b^2c^2+2b^2-2c^2\right)}{(8-b^2c^2)(4-c^2)}$
a_{rn}	$\frac{c}{8-b^2-2c^2}$	$\frac{c\left(16-c^2\right)}{256-32b^2-32c^2+c^4}$	$\frac{c\left(4-b^2c^2+2b^2\right)}{(8-b^2c^2)(4-c^2)}$
d_n	$\frac{2}{8-b^2-2c^2}$	$\frac{4\left(16-c^2\right)}{256-32b^2-32c^2+c^4}$	$\frac{2\left(4-b^2c^2+2b^2\right)}{(8-b^2c^2)(4-c^2)}$
Π_m	$\frac{1}{8-b^2-2c^2}$	$\frac{32}{256-32b^2-32c^2+c^4}$	$\frac{4}{(8-b^2c^2)(4-c^2)}$
Π_r	$\frac{4-c^2}{(b^2+2c^2-8)^2}$	$\frac{\left(16-c^2\right)^3}{(256-32b^2-32c^2+c^4)^2}$	$\frac{\left(4-b^2c^2+2b^2\right)^2}{(8-b^2c^2)^2(4-c^2)}$

Table 3: Equilibrium solutions in the benchmark scenarios (N1, N2 and N3)

4.2 Equilibrium in the SB games

When the retailer offers both store and national brands, we solve each of the scenarios ($SN1$, $SN2$ and $SN3$) described in Table 2 by backward induction. In $SN1$, a two-stage game is played by the manufacturer and the retailer. We start by solving the retailer's problem in both the NB and SB prices and advertising efforts simultaneously, then use the obtained reaction functions to write the manufacturer's profit function and solve his problem to get the equilibrium NB manufacturer's advertising and wholesale price simultaneously.

In $SN2$, a four-stage game is played. We start by solving the retailer's pricing problem in stage 4 and get the NB and SB prices, then use these price expressions in the manufacturer's problem and obtain the wholesale price. The obtained expressions of retail and wholesale prices are then used to write the retailer's problem and to solve it in the retailer's advertising strategies for the NB and the SB. The obtained solution, along with all other pricing variables, are then injected in the manufacturer's problem and used to obtain the equilibrium advertising strategy for the NB. A similar approach is used in $SN3$, but instead of starting by solving the retailer's problem in prices we start by solving the retailer's problem in advertising strategies.

Proposition 2 *The equilibrium solution obtained in each of the three SB games is included in Table 4.*

Appendix B provides a detailed description of the solution methodology and expressions of the reaction functions and second-order conditions for each of the games described here ($SN1$, $SN2$ and $SN3$). We identify the feasible region, namely the parameter space in g_s, g_n, θ, b and c where both the positivity and concavity conditions are satisfied in each scenario $SN1$, $SN2$ and $SN3$. Since the expressions of the retailer's profits are lengthy, we have omitted them here for ease of presentation. All other abbreviated expressions are included in Appendix C.

	$SN1$	$SN2$	$SN3$
w	$\frac{\left(\left(4-c^2\right)^2-16\theta^2\right)\left(4-c^2-4\theta g_s\right)}{(4-c^2)(32(1-\theta^2)-4b^2+2c^4+(b^2-16)c^2)}$	$\frac{8\left(1-\theta^2\right)\left(4-c^2-4\theta g_s\right)Y}{den(SN2)}$	$\frac{4\left(1-\theta^2\right)^2\left(4-c^2-4\theta g_s\right)}{den(SN3)}$
a_m	$\frac{b\left(4-c^2-4\theta g_s\right)}{32(1-\theta^2)-4b^2+2c^4+(b^2-16)c^2}$	$\frac{32b\left(1-\theta^2\right)\left(4-c^2-4\theta g_s\right)\left(c^4-4\right)}{den(SN2)}$	$\frac{2b\left(1-\theta^2\right)\left(4-c^2-4\theta g_s\right)}{den(SN3)}$
p_n	$\frac{2\theta g_s\left[\left(c^2-4\right)\left(b^2+4\right)+16\theta^2\right]+\left(c^2-4\right)[24\theta^2-\left(c^2-4\right)^2]}{(4-c^2)(32(1-\theta^2)-4b^2+2c^4+(b^2-16)c^2)}$	$\frac{num(p_n^{SN2})}{den(SN2)}$	$\frac{num(p_n^{SN3})}{den(SN3)(c^2-4-4\theta)(c^2-4+4\theta)}$
p_s	$\frac{2\left(c^2\theta\left(c^2-4\right)+g_s\left(2c^4+\left(b^2+4\theta^2-16\right)c^2-4b^2-32\theta^2+32\right)\right)}{(4-c^2)(32(1-\theta^2)-4b^2+2c^4+(b^2-16)c^2)}$	$\frac{num(ps^{SN2})}{den(SN2)}$	$\frac{num(p_s^{SN3})}{den(SN3)(c^2-4-4\theta)(c^2-4+4\theta)}$
a_n	$\frac{c\left(4-c^2-4\theta g_s\right)}{32(1-\theta^2)-4b^2+2c^4+(b^2-16)c^2}$	$\frac{c\left(4-c^2-4\theta g_s\right)Y}{den(SN2)}$	$\frac{cX\left(4-c^2-4\theta g_s\right)}{den(SN3)(c^2-4-4\theta)(c^2-4+4\theta)}$
a_s	$\frac{c\left(4\theta\left(c^2-4\right)+g_s\left(2c^4+\left(b^2-16\right)c^2-4b^2-16\theta^2+32\right)\right)}{(4-c^2)(32(1-\theta^2)-4b^2+2c^4+(b^2-16)c^2)}$	$\frac{num(ars^{SN2})}{den(SN2)}$	$\frac{num(a_{rs}^{SN3})}{den(SN3)(c^2-4-4\theta)(c^2-4+4\theta)}$
d_n	$\frac{2\left(4-c^2-4\theta g_s\right)}{32(1-\theta^2)-4b^2+2c^4+(b^2-16)c^2}$	$\frac{4\left(4-c^2-4\theta g_s\right)\left(c^4+64\left(1-\theta^2\right)-4\left(5-3\theta^2\right)\right)}{den(SN2)}$	$\frac{2X\left(4-c^2-4\theta g_s\right)}{den(SN3)(c^2-4-4\theta)(c^2-4+4\theta)}$
d_s	$\frac{2\left(\left(2c^4+\left(b^2-16\right)c^2-4b^2-16\theta^2+32\right)g_s+4\theta\left(c^2-4\right)\right)}{(4-c^2)(32(1-\theta^2)-4b^2+2c^4+(b^2-16)c^2)}$	$\frac{num(ds^{SN2})}{den(SN2)}$	$\frac{num(d_s^{SN3})}{den(SN3)(c^2-4-4\theta)(c^2-4+4\theta)}$
Π_m	$\frac{\left(4-c^2-4\theta g_s\right)^2}{(4-c^2)(32(1-\theta^2)-4b^2+2c^4+(b^2-16)c^2)}$	$\frac{32\left(1-\theta^2\right)\left(4-c^2-4\theta g_s\right)^2}{den(SN2)}$	$\frac{4\left(1-\theta^2\right)^2\left(4-c^2-4\theta g_s\right)^2}{den(SN3)(c^2-4-4\theta)(c^2-4+4\theta)}$

Table 4: Equilibrium solutions in the store brand scenarios (SN1, SN2 and SN3)

5 Effects of the store brand introduction

We now discuss the effects of the store brand entry on the profits of the manufacturer, retailer and the total channel. We compare equilibrium profits obtained in the three benchmark scenarios ($N1$, $N2$ and $N3$) to those in scenarios where a SB is offered by the retailer ($SN1$, $SN2$ and $SN3$). Comparisons across scenarios Ni and SNj for i, $j = 1, 2, 3$ yields in total 9 comparisons.

The methodology for obtaining the results is as follows. Given that some equilibrium expressions are very long and highly non-linear in the parameters' values, in particular those obtained in the three scenarios where a SB is offered ($SN1$, $SN2$ and $SN3$), the comparisons of closed-form solutions offer qualitative insights only in some cases and we have to resort to numerical simulations to obtain meaningful results. For the numerical analyses, we fix g_n to 1 and retain five values for g_s $(0.1, 0.25, 0.5, 0.75, 0.9)$ to consider different scenarios for the SB's baseline sales. This is to represent the most common scenario where the baseline sales for the SB are smaller than for the NB. With the help of Mathematica 11.0 and specifically using the Reduce command[6], we get the results in terms of the three model's key parameters, namely, the degree of price competition (θ), and the coefficients of promotional activities (b and c). In most cases, we show the results for the whole interval of admissible values for θ, b and c. When this is not possible due to the complexity of the analytical expressions, we retain five values for θ $(0.1, 0.25, 0.5, 0.75, 0.9)$ to represent very low, low, medium, high and very high degrees of price competition.[7]

Some of the results are presented in figures, where we fix θ to a specific value and show the results for the whole interval of admissible values for the advertising effects (b and c). In all numerical simulations, feasibility refers to equilibrium solutions for which decision variables (prices and advertising), margins, profits and demands are strictly positive and the concavity conditions are satisfied. In each figure, we compare profits obtained from two different scenarios (games) and the region where these conditions are not satisfied for either one or both games is indicated by "UF". All proof as well as feasibility conditions are included in Appendix D. The results of all comparisons are summarized in the following claim.

Claim 1: *Comparisons of equilibrium profits of the retailer, manufacturer and the entire channel across scenarios Ni and SNj for i, $j = 1, 2, 3$ are summarized in Table*

[6] "Reduce" solves equations or inequalities for variables and eliminates quantifiers.

[7] The numerical results were generated considering a grid of $(0, 3)$ for each of the parameters b and c with a mesh of 0.005. That is, for each scenario (a fixed value of θ), we computed optimal strategies and profits.

5.

		$SN1$	$SN2$	$SN3$
Manufacturer				
	$N1$	$-$	$-$	$-$
	$N2$	$\pm$	$-$	$\pm$
	$N3$	$\pm$	$\pm$	$-$
Retailer				
	$N1$	$\pm$	$\pm$	$\pm$
	$N2$	$+$	$\pm$	$\pm$
	$N3$	$\pm$	$\pm$	$\pm$
Channel				
	$N1$	$\pm$	$\pm$	$\pm$
	$N2$	$+$	$\pm$	$\pm$
	$N3$	$\pm$	$\pm$	$\pm$

Table 5: Effects of SB introduction on profits

In Table 5, the signs "$+$", "$-$" and "$\pm$" corresponding to row Ni and column SNj, mean that the effect of the SB introduction is positive, negative or can be either positive or negative depending on the model's parameters, respectively. To study how the manufacturer's decision sequence can impact the effects generated by a store brand entry, the results in Table 5 are analyzed in two stages. First, we assess the effects of store brand entry, assuming that the manufacturer does not change the sequence of his decision-making for pricing and advertising. Then, we expand the analysis to include the effects of SB introduction when the manufacturer changes his decision sequence after the SB entry.

5.1 The manufacturer does not change his decision sequence

We start by assessing the effects of SB entry when the manufacturer adopts the same sequence of decision-making before and after store brand entry. This analysis will allow us to address our first research question. Namely; what are the effects of a store brand introduction on channel members' profits given different decision-making (announcement) sequences by the NB manufacturer?

We study the case where the manufacturer simultaneously announces his price and advertising before as well as after the store brand introduction by comparing equilibrium solutions and profits obtained in scenarios $SN1$ and $N1$. This is the sequence of decision-making extensively assumed in the literature. We also study the effects of the SB entry when the manufacturer adopts sequential decision-making for pricing and advertising (such as when advertising is decided prior to pricing) by comparing equilibrium solutions and profits obtained in scenarios $SN2$ and $N2$. Finally, we discuss the results related to the effects of SB entry when the manufacturer also adopts sequential decision-making but chooses pricing prior to advertising by comparing equilibrium solutions and profits obtained in scenarios $SN3$ and $N3$. The effects of the SB entry on the profits of the retailer, the manufacturer and the channel for each decision-making sequence are summarized in the following result.

Result 1: When the manufacturer keeps the same decision-making sequence before and after the SB entry, the manufacturer incurs losses. The retailer and the total channel can gain or lose from the SB entry.

Result 1 shows that the SB introduction harms the manufacturer's profit no matter the sequence of decision-making that he adopts as long as the same sequence is kept unchanged after the SB entry. In this regard, the sequence of decision-making for pricing and advertising does not qualitatively change the outcome for the manufacturer whose profit will diminish in all cases. Therefore, we can conclude that whenever the manufacturer adopts the same decision-making sequence before and after the SB entry, the SB is considered a threat for his profitability no matter the strength of the SB, the level of price competition with the NB or the advertising effects. Note that this result is consistent with the extent marketing literature that has studied the implications of a SB entry in the distribution channel and has noted the threatening impact of such introductions for the manufacturer. Most previous studies considered a simultaneous decision-making sequence in a channel led by the manufacturer (Scott-Morton and Zettelmeyer, 2004; Pauwels and Srinivasan, 2004; Meza and Sudhir, 2010). We extend this result to the other two decision sequences. The manufacturer's losses can mainly be explained by the lower margins earned on the NB. Further, as the retailer gets a better price deal on the national brand, he lowers the retail price and advertising effort, resulting overall in lower sales and shrinking revenues for the manufacturer.

Looking at the retailer, Result 1 shows that, qualitatively, the impact of SB introduction on profits is also similar across different scenarios of decision-making sequences. Whether the latter is simultaneous or corresponds to any of the two sequential decision-making scenarios, the retailer might lose or gain with the SB entry as long as the sequence of decision-making is consistent. Further numerical explorations show that $\Pi_r^{SNi} > \Pi_r^{Ni}$ $if \ \ \theta \geq \Phi_i$, $(i = 1, 2, 3)$. These thresholds are different when advertising is set simultaneously with price (N1 and SN1), prior to price (N2 and SN2) or following price (N3 and

SN3) and can depend on the levels of the SB baseline demand (g_s) (see Tables A1 and A2 in the Appendix).

This means that given high enough levels of price competition, the retailer will always benefit from the SB. For low levels of price competition ($\theta < \Phi_i$), the impact of the SB entry on the retailer's profits can be positive or negative depending on the values of advertising effects (b and c). Figure 1 shows an example of such mixed effects for the three scenarios. As we can see, when advertising is set prior to pricing (N2 and SN2), the retailer will not benefit from the SB in a very small region of the parameter domain characterized by very high levels of both advertising effects (b and c). When advertising is announced after pricing (N3 and SN3) or when these decisions are announced simultaneously (N1 and SN1), the retailer will not benefit from the SB entry when the manufacturer's advertising effect (b) is large but the retailer's advertising is not highly effective (low c). Under these conditions, the store brand entry leads to an increased wholesale price, which in turn squeezes the retail margin on the NB. The retailer also boosts his advertising spending for the NB. Although these changes lead to higher demand for the NB, the retailer's revenues are not large enough to pay for the additional advertising expenses and the retailer ends up losing with the SB entry.

While the qualitative result is the same across the various decision scenarios, two differences can be noted. First, the thresholds on theta (Φ_i) that guarantee positive retailer profits due to SB entry differ across scenarios. Therefore, everything else being the same, a SB entry might benefit the retailer under a specific decision sequence but might be harmful to the retailer's profit given a different decision sequence set-up. Second, for specific values of g_s and θ, the conditions on the advertising effects that would lead to a negative profit impact for the retailer are also different. For example, in SN2, a negative effect on the retailer's profit occurs when both advertising effects are large, while high manufacturer's advertising effects with low retail advertising effectiveness are required for the SB to result in losses for the retailer in SN3. This suggests that the sequence of decision-making influences the role of advertising effects when assessing the profitability of SB introductions by the retailer.

The results for the total channel mimic those obtained for the retailer except that larger threshold values on θ are found for the total channel. This suggests that even when the retailer finds it profitable to introduce the store brand, his gain might not be enough to compensate for the manufacturer's losses, which leads to a decline in the total channel profit. However, as the price competition level increases, the retailer's gains exceed the manufacturer's losses, which results in an overall positive effect for the channel.

The implications of these results are as follows. From a theoretic (modeling) perspective, they reveal the influence of the commonly assumed simultaneous decision-making sequence for pricing and advertising in the SB literature. In particular, while this assumption does not qualitatively change the result for the manufacturer, it does impact the retailer and the total channel. In fact, SB entry constitutes a definite threat to the NB manufacturer in all scenarios. However, for the retailer, the viability of the SB entry greatly depends on the chosen decision sequence by the manufacturer and the model's parameters. While low price competition (θ) is required for negative effects to emerge for the retailer, the values of θ required for such effects as well as the ranges of advertising effects (b and c) that are conducive for a harmful entry for the retailer change from one scenario to another. A managerial implication of this result is that, given everything else the same (competition levels, advertising effects, etc.), different decision-making sequences can lead the retailer to introduce the SB in some cases while not finding such a strategy profitable in others.

Insert Figure 1 about here

5.2 The manufacturer can change his decision sequence

In the previous section, we studied scenarios where the manufacturer does not change his decision-making sequence following the SB introduction by the retailer. The results from the previous section shed some light into the implications of such commitments for the NB manufacturer and the retailer. From a theoretic (modeling) perspective, they also question the usual assumption of simultaneous decision-making for pricing and advertising in the SB literature and reveal the implications of relaxing such assumption.

In this section, we further expand our analysis to consider situations where the NB manufacturer is not committed to any sequence of decision-making so he can choose to change it following the retailer's decision to introduce the SB. For a rational manufacturer, a change in the sequence of decision-making should be considered since the output of the game can depend on such a sequence and since the NB manufacturer and the retailer play a different game when the SB is introduced. Therefore, the manufacturer can decide to also lead the channel differently by modifying the sequence in which pricing and advertising decisions are announced. This analysis allows us to address our second and third research questions. Namely, in a channel led by the manufacturer, can the NB manufacturer strategically use their decision-making sequence for advertising and pricing to deter store brand introductions? And, if yes, then under what conditions should the NB manufacturer announce pricing first, advertising first or both simultaneously?

We focus in this section on the results of comparisons across scenarios Ni and SNj for i, $j = 1, 2, 3$ and $i \neq j$, which yields six comparisons.

Result 2: When the manufacturer changes the sequence of decision-making following SB introduction, he will lose if the benchmark is a simultaneous decision-making sequence ($N1$), and could lose or gain if the benchmark is a sequential decision-making (either N2 or N3).

Result 2 shows that, for manufacturers who simultaneously choose their pricing and advertising prior to SB entry, a change in the decision-making sequence cannot prevent the harmful effects of the SB. This means that, in this case, the manufacturer cannot use a different decision-making sequence to benefit from the SB entry. Alternatively, when the benchmark is a sequential decision-making sequence (either N2 or N3), SB entry can lead to a drop or an increase in the manufacturer's profit. In this case, depending on the model's parameters, the manufacturer can benefit from the store brand introduction by switching from a sequential decision scenario to a different sequential decision arrangement or to simultaneous one. Specifically, our numerical analysis shows that the manufacturer's profit could increase only when the price competition level is low enough ($\theta < \Delta_i$)[8].

To illustrate these cases, we show in Figure 2 the effects of SB entry on the manufacturer's profits in the different scenarios. As we can see, the manufacturer benefits from the SB when **the retailer's** advertising is highly effective (high c) if he switches from N2 to SN1 or to SN3. Alternatively, when the benchmark is N3, the manufacturer benefits from the SB when he changes his decision-making sequence to SN1 or SN2 for high enough levels of his advertising effect (b) combined with low levels the retailer's advertising effect (c).

Insert Figure 2 about here

These results provide useful insights to manufacturers about the implications of their decision-making process. They show that the negative impact of SB introduction can be overturned by simply changing the sequence of decision-making for price and advertising. They also shed light on how the simultaneous decision-making assumption in the literature can affect the analysis of the SB introduction effects.

Note however that the manufacturer would only benefit from the SB entry if the retailer actually introduces the SB, which would occur if the retailer also benefits from the

[8] Δ_i ($i = 1, 2, 3$) are reported in Tables A1 and A2 in the Appendix).

SB introduction under the same market conditions that are beneficial to the manufacturer. Therefore, it is necessary to examine the effects of the changes in decision-making sequences on the retailer's profit. Such effects can also be useful to the manufacturer in order to preempt a harmful SB entry. In other words, when a change in decision-making sequence cannot lead to beneficial impact of SB entry for the manufacturer, the latter should consider whether such change could alter the result for the retailer. This occurs when the retailer's profit decreases while it would have increased had the sequence of decision-making been kept unchanged. Given the findings in Table 5 and combining the comparisons of the manufacturer's profit in Result 2 to those for the retailer, we can derive the following result.

Result 3: When the manufacturer changes the sequence of decision-making following SB introduction, and for low levels of price competition:

1. Both the manufacturer and the retailer can benefit from the SB introduction if the benchmark is N2 (N3) and the manufacturer's (retailer's) advertising effect, c (b), is high.

2. The manufacturer can preempt SB entry by the retailer for high (intermediate) levels of advertising effects if the benchmark is N1 (N3).

3. In all other market conditions, the manufacturer cannot avoid the negative effects of SB entry on his profits.

These results provide interesting insights to manufacturers about whether changing their decision sequence can be an effective strategy to fight store brand introductions. They show that, given a low level of competition between the NB and the SB and for certain levels of advertising effects, the manufacturer can benefit from the SB or preempt its entry simply by changing his decision-making sequence. In other cases, the decision sequence does not qualitatively change the harmful outcome of the SB introduction for the manufacturer.

In particular, the first finding indicates that when the benchmark is a sequential decision (N2 or N3) and advertising effects are high, both the retailer and the manufacturer will benefit from the SB entry. Under these market conditions, the SB is not a threat but rather an opportunity to the manufacturer. Looking at the total demand, it is clear that the retailer benefits from the SB entry in this case mainly because the total demand expands with the SB entry (Appendix E). Therefore, while the manufacturer always loses following the SB entry when he keeps his decision-making unchanged (Result 1), a switch to a different decision sequence can lead to a gain for both the manufacturer and the retailer if the benchmark is N2 (N3) and the manufacturer's (retailer's) advertising effect, c (b), is high.

The second finding in Result 3 shows that the manufacturer can preempt SB entry by the retailer for low levels of price competition. For this to occur, two conditions need to be satisfied; 1- both the retailer's and the manufacturer's profits should decrease following the SB entry, and 2- the effect on the retailer's profit should be positive while the effect on the manufacturer's profit should be negative given the same sequence of decisions before and after SB entry.[9] These market conditions are illustrated in Figure 3 in those regions colored in yellow where both the retailer's and the manufacturer's profits decrease following the SB entry. We can see that these regions are characterized by high (intermediate) levels of advertising effects if the benchmark is N1 (N3). Further analyses of the total demand show that this result is driven by the effect on the NB demand. In fact, for such advertising effects, the retailer's total demand decreases following the SB introduction because of the increase in the NB retail price, which ultimately leads to shrinking revenues and a loss for the retailer.

Finally, the last finding in Result 3 indicates that in all other market conditions, the manufacturer cannot avoid the negative effects of the SB entry on his profit since the retailer's profit is higher with the SB no matter the manufacturer's sequence of decisions and the manufacturer's profit is shrunk as a consequence. These market conditions are indicated in Figure 3 by those areas where the retailer's profit increases while the manufacturer's profit decreases (green colored areas). They are characterized by low enough retailer's and manufacturer's advertising effects when the benchmark is N1, low retailer's advertising effect when the benchmark is N2 and low manufacturer's advertising when the benchmark is N3. Note the differences in the market conditions necessary for a harmful impact of the SB on the manufacturer's profit across the different scenarios. Again, this indicates the importance of the manufacturer's decision-making sequence and their effects on the implications of SB entry.

Insert Figure 3 about here

6 Conclusions

This paper investigates the effects of store brand entry on channel members' profits. We focus on two strategic marketing decisions related to pricing and advertising and consider a manufacturer-led channel. The existing literature predominantly assumes simultaneous pricing and advertising decisions in such a context. In this paper, we

[9] Given that the feasible domain in the six games can be different, we consider the most restrictive feasible domain in these comparisons. This means that all compared equilibria are feasible.

investigate whether different decision sequences can impact the effects of store brand entry. When the manufacturer announces a decision first (advertising or pricing), this can impact the retailer's reactions in setting their pricing and advertising strategies and thereby the success of the store brand. We also study whether the manufacturer can use his decision sequence as a strategic tool to benefit from or to fight a store brand entry. We develop a game theoretic model based on consumer utility and solve six games (scenarios) where different decision-making sequence choices are considered before and after the store brand introduction by the retailer.

The main findings indicate that the sequence in which the manufacturer announces his advertising and pricing decisions significantly impacts the effects of a store brand's entry by the retailer. This impact can be categorized in two ways. First, the effects of store brand entry depend on whether the manufacturer adopts simultaneous or sequential decision-making prior to the SB entry. Second, this impact depends on whether the manufacturer changes the sequence of his decision-making following the SB introduction. These results offer an interesting new perspective on the impact of store brand entry for manufacturers of NB.

In situations where the NB manufacturer does not change his decision-making sequence following the SB introduction by the retailer, our results shed some light into the implications of such commitments for the NB manufacturer and for the retailer. In particular, we find that SB entry constitutes a definite threat to NB manufacturers who do not change their decision-making sequence following a SB entry. Whether the manufacturer announces his pricing and advertising simultaneously or sequentially (in any order), the retailer's decision to introduce a store brand will reduce the NB demand and consequently result in losses for the manufacturer.

The implications of these results are as follows. From a theoretic (modeling) perspective, they reveal the influence of the commonly assumed simultaneous decision-making sequence for pricing and advertising in the SB literature. In particular, while this assumption does not qualitatively change the result for the manufacturer, it does impact the retailer and the total channel. In fact, SB entry constitutes a definite threat to the NB manufacturer in all scenarios. However, for the retailer, the viability of the SB entry greatly depends on the chosen decision sequence by the manufacturer and the model's parameters. While low price competition is required for negative effects to emerge for the retailer, the required competition intensity for such effects as well as the ranges of advertising effects that are conducive for a harmful entry for the retailer change from one scenario to another. A managerial implication of this result is that, given everything else the same (competition levels, advertising effects, etc.), different decision-making sequences can lead the retailer to introduce SB in some cases while not finding such a strategy profitable in others.

The effect of a SB introduction can change significantly when the manufacturer alters the sequence of his decision-making following the SB entry. In fact, the NB manufacturer may not be committed to any sequence of decision-making so he can choose to change it following the retailer's decision to introduce the SB. In this case, our results suggest that the manufacturer can either benefit from the SB entry along with the retailer or can preempt the threat of a harmful entry by changing his decision sequence, which indirectly benefits him by lowering the retailer's profit, thereby deterring the retailer from introducing the SB in the first place.

Given a simultaneous decision sequence prior to SB entry, the NB manufacturer will lose when the retailer introduces a store brand, whether the manufacturer keeps the same simultaneous decision sequence or switches to a sequential one. In this situation, the manufacturer cannot use the order of his decision-making as a tool to directly benefit from store brand entry. However, the retailer can incur losses if the manufacturer switches to sequential decision-making after SB entry when the market is characterized by low price competition levels and high levels of advertising effects. Therefore, although he cannot benefit from a store brand entry, the manufacturer who simultaneously decides of his price and advertising can deter SB entry by simply switching to sequential decision-making, no matter which decision he announces first. Hence, a change in the decision sequence can indirectly benefit the manufacturer by preempting SB entry and avoiding its harmful effects.

Given a sequential decision sequence prior to SB entry, a change in the decision-making sequence can be effective in two ways; either by making the SB entry also beneficial to the manufacturer or by making it unprofitable for the retailer, hence preempting entry. When the manufacturer changes his decision sequence following SB entry, the retailer would still benefit from such an entry when the price competition between the two brands is high, but his profit may decrease for low levels of price competition. For example, when the manufacturer announces advertising prior to (after) pricing, then reverses this sequence or uses a simultaneous one, the retailer does not benefit from introducing the SB when the manufacturer's (retailer's) advertising is very effective.

Future research can relax some of our assumptions. For example, we assumed deterministic utility functions; future work can consider uncertainty in consumer preferences. Also, we focused our analysis on local advertising activities (e.g., flyers and displays). This set-up can be extended to include national advertising efforts and model the long-term effects of advertising decisions in a dynamic model. Competition at the manufacturing or retailing levels or both may also change the results obtained with our model.[10] Finally, other marketing mix decisions can be considered such as the positioning of the store brand in comparison to the national brand **as well as other operational considerations (e.g., fulfillment and contractual agreements)**. While these considerations could significantly complicate the model and the results, they can further advance our knowledge about the effects of store brand introductions and provide strategies for national brand manufacturers when dealing with such competitive effects.

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Appendix

1 Appendix A: Equilibrium in the benchmark scenarios

We obtain feedback equilibrium solutions using backward induction for the three benchmark games where the retailer does not offer a store brand. The first game is $N1$, where the manufacturer decides simultaneously of advertising and pricing. In the second game, that is, $N2$, the manufacturer decides of advertising prior to pricing. In the third game ($N3$), the manufacturer decides of pricing prior to advertising. In each of these three games, the retailer is the follower; he makes the same decision(s) than the manufacturer in the previous stage of the game.

1.1 The $N1$ game

In $N1$, the game is played in two stages. First, we consider the retailer's problem in the second stage given by

$$\max_{p_n,a_n} \Pi_r = (p_n - w)\, d_n - a_n^2, \tag{1}$$

where d_n is given by (1) and solve the following first-order equilibrium conditions:

$$\frac{\partial \Pi_r}{\partial p_n} = \frac{\partial \Pi_r}{\partial a_n} = 0,$$

which yields the reaction functions to the manufacturer's decision variables, that is,

$$a_n = A_n^{N1}(w, a_m), \quad p_n = P_n^{N1}(w, a_m). \tag{2}$$

The retailer's profit is a strictly concave function in the decision variables p_n and a_n iff

$$4 - c^2 > 0.$$

We then insert the retailer's reaction functions into the manufacturer's optimization problem given by

$$\max_{w,a_m} \Pi_m = w d_n - a_m^2, \tag{3}$$

where

$$d_n = g_n + b a_m + c A_n^{N1}(w, a_m) - P_n^{N1}(w, a_m),$$

and solve the manufacturer's first-order optimality conditions given by

$$\frac{\partial \Pi_m}{\partial w} = \frac{\partial \Pi_m}{\partial a_m} = 0.$$

to get the equilibrium wholesale price and advertising effort given in Table 3. The manufacturer's profit is a strictly concave function in the decision variables w and a_m iff

$$\begin{aligned} 4 - c^2 &> 0, & (4) \\ b^2 + 2c^2 - 8 &< 0. & (5) \end{aligned}$$

The equilibrium manufacturer's expressions are next inserted into the retailers' reaction functions to obtain the equilibrium retail price and advertising effort of the retailer as functions of the model's parameter values. This equilibrium solution exists iff both conditions in (4) and (5) are satisfied. These two conditions are satisfied if and only if the following inequalities apply:

$$0 < c < 2, \quad 0 < b < \sqrt{2(4 - c^2)}. \tag{6}$$

Note that the upper bound for b is a strictly decreasing and strictly concave function of c. It takes the maximum value of $2\sqrt{2} \simeq 2.82843$ for $c = 0$ and zero for $c = 2$. Finally (6) also ensure positive equilibrium output in N1 (in Table 3) as well as a positive retailer's margin at equilibrium ($p_n - w > 0$).

1.2 The $N2$ game

In $N2$, the game is played in four-stages. To solve the game backwards, we start by stage 4 and solve the retailer's problem in

$$\max_{p_n} \Pi_r = (p_n - w)\, d_n - a_n^2, \tag{7}$$

where d_n is given by (**??**) and solve the following first-order condition: $\frac{\partial \Pi_r}{\partial p_n} = 0$, which provides the retailer's price reaction function to his advertising and to the manufacturer's decision variables, namely:

$$p_n = P_n^{N2}(w, a_m, a_n). \tag{8}$$

The retailer's profit is a strictly concave function in the decision variables p_n for any values of the model's parameter ($\frac{\partial^2 \Pi_r}{\partial p_n^2} = -2 < 0$). In stage 3, we insert this reaction function into the manufacturer's pricing problem to get

$$\max_{w} \Pi_m = w\left(g_n + ba_m + ca_n - P_n^{N2}(w)\right) - a_m^2, \tag{9}$$

and solve the manufacturer's first-order optimality condition $\frac{\partial \Pi_m}{\partial w} = 0$ to get the manufacturer's wholesale price as a function of advertising efforts a_m and a_n such as:

$$w = W^{N2}(a_m, a_n). \tag{10}$$

The manufacturer's profit is a strictly concave function in the decision variable w for any values of the model's parameter ($\frac{\partial^2 \Pi_m}{\partial w^2} = -1 < 0$). The manufacturer's price reaction function is then inserted into the retailer's price reaction function in (8) and into the retailer's profit function.

In stage 2, we solve the retailer's advertising problem given by

$$\max_{a_n} \Pi_r = \left(P_n^{N2}(a_m, a_n) - W^{N2}(a_m, a_n)\right)\left(g_n + ba_m + ca_n - P_n^{N2}(a_m, a_n)\right) - a_n^2. \tag{11}$$

The retailer's profit is a strictly concave function in the decision variable a_n iff

$$16 - c^2 > 0. \tag{12}$$

We solve the retailer's first-order condition given by $\frac{\partial \Pi_r}{\partial a_n} = 0$. The solution gives the retailer's advertising strategy as function of the manufacturer's advertising decision variable

$$a_n = A_n^{N2}(a_m), \tag{13}$$

which is then inserted in the pricing reaction functions of the manufacturer and of the retailer. In stage1, these functions are placed in the manufacturer's advertising problem in

$$\max_{a_m} \Pi_m = W^{N2}(a_m)\left(g_n + ba_m + cA_n^{N2}(a_m) - P_n^{N2}(a_m)\right) - a_m^2. \tag{14}$$

Solving the manufacturer's first-order optimality condition ($\frac{\partial \Pi_m}{\partial a_m} = 0$) gives the equilibrium solution for a_m. The manufacturer's profit is a strictly concave function in the decision variable a_m iff

$$32\left(b^2 + c^2 - 8\right) - c^4 < 0. \tag{15}$$

The equilibrium solution for a_m is then next inserted in the manufacturer's and retailer's reaction functions to obtain the equilibrium retail and wholesale prices, and the advertising effort of the retailer as functions of the model's parameters.

This equilibrium solution exists iff both conditions in (12) and (15) are satisfied. These two conditions are satisfied if and only if the following inequalities apply:

$$0 < c < 4, \quad 0 < b < \frac{16 - c^2}{4\sqrt{2}}. \tag{16}$$

Note that the upper bound for b is a strictly decreasing and strictly concave function of c. It takes the maximum value of $2\sqrt{2}$ for $c = 0$ and zero for $c = 4$. Finally (16) also ensure positive equilibrium output in N2 (in Table 3) as well as a positive retailer's margin at equilibrium ($p_n - w > 0$).

1.3 The $N3$ game

In $N3$, the equilibrium solution is played by solving a four-stage game . We follow a similar approach as for $N2$. To solve the game backwards, we start by stage 4 and solve the retailer's advertising problem given by

$$\max_{a_n} \Pi_r = (p_n - w)\, d_n - a_n^2, \tag{17}$$

where d_n is given by (**??**) and solve the following first-order condition: $\frac{\partial \Pi_r}{\partial a_n} = 0$, which provides the retailer's advertising reaction function to his pricing and to the manufacturer's decision variables, namely:

$$a_n = A_n^{N3}(w, a_m, p_n). \tag{18}$$

The retailer's profit is a strictly concave function in the decision variable a_n for any values of the model's parameter ($\frac{\partial^2 \Pi_r}{\partial a_n^2} = -2 < 0$). In stage 3, we insert this reaction function into the manufacturer's advertising problem to get

$$\max_{a_m} \Pi_m = w\left(g_n + b a_m + c A_n^{N3}(w, a_m, p_n) - p_n\right) - a_m^2, \tag{19}$$

and solve the manufacturer's first-order optimality condition $\frac{\partial \Pi_m}{\partial a_m} = 0$ to get the manufacturer's advertising as a function of pricing strategies p_n and w such as:

$$a_m = A_m^{N3}(p_n, w). \tag{20}$$

The manufacturer's profit is a strictly concave function in the decision variable a_m for any values of the model's parameter ($\frac{\partial^2 \Pi_m}{\partial a_m^2} = -2 < 0$). The expression in (20) is then inserted into the retailer's advertising reaction function in (18) and into the retailer's profit function. Now we solve the retailer's pricing problem in stage 2 of the game. The retailer's problem is given by

$$\max_{p_n} \Pi_r = (p_n - w)\left(g_n + bA_m^{N3}(p_n, w) + cA_n^{N3}(w, p_n) - p_n\right) - \left[A_n^{N3}(w, p_n)\right]^2, \quad (21)$$

and solve the retailer's first-order condition given by $\frac{\partial \Pi_r}{\partial p_n} = 0$. The solution gives the retailer's NB price as function of the manufacturer's wholesale price

$$p_n = P_n^{N3}(w), \quad (22)$$

The retailer's profit is a strictly concave function in the decision variable p_n iff

$$4 - c^2 > 0. \quad (23)$$

The expression in (21) is then inserted in the advertising reaction functions of the manufacturer in (20) and of the retailer in (18). Next, these functions are placed in the manufacturer's pricing problem in stage 1 which is

$$\begin{aligned} \max_{w} \Pi_m &= w\left(g_n + bA_n^{N3}(w) + cA_n^{N3}(w) - P_n^{N3}(w)\right) \\ &\quad - \left[A_m^{N3}(w)\right]^2. \end{aligned} \quad (24)$$

Solving the manufacturer's first-order optimality condition ($\frac{\partial \Pi_m}{\partial w} = 0$) gives the equilibrium solution for w in the $N3$ scenario. The manufacturer's profit is a strictly concave function in the decision variable w iff

$$\frac{b^2c^2 - 8}{2(4 - c^2)} < 0. \quad (25)$$

Finally, the equilibrium wholesale price is inserted in the manufacturer's and retailer's reaction functions to obtain the equilibrium retail price, and the advertising effort of the retailer as functions of the model's parameters.

This equilibrium solution exists iff both conditions in (23) and (25) are satisfied. It can be proved that if these two conditions are satisfied, then the conditions ensuring positive retailer's margin, $p_n - w$, and positive a_n, d_n and p_n, given by $4 + b^2(2 - c^2) > 0$ and $2(6 - c^2) + b^2(2 - c^2) > 0$, respectively, are satisfied too. Conditions (23) and (25) are satisfied if and only if the following inequalities apply:

$$0 < c < 2, \quad 0 < b < \frac{2\sqrt{2}}{c}. \quad (26)$$

Note that the upper bound for b is a strictly decreasing and strictly convex function of c. It tends to infinity as c approaches zero, and attains the value of $\sqrt{2} \simeq 1.41421$ for $c = 2$.

2 Appendix B: Equilibrium in the SB games

We obtain feedback equilibrium solutions using backward induction for the three games where the retailer does offer a store brand. The first game is $SN1$, where the manufacturer decides simultaneously of advertising and pricing. In the second game, that is, $SN2$, the manufacturer decides of advertising prior to pricing. In the third game ($SN3$), the manufacturer decides of pricing prior to advertising. In each of these three games, the retailer is the follower; he makes the same decision(s) than the manufacturer in the previous stage of the game.

2.1 The $SN1$ game

In this case, the game is played in two stages. First, we consider the retailer's problem in the second stage given by

$$\max_{p_n, p_s, a_s, a_n} \Pi_r = (p_n - w)\, d_n + p_s d_s - a_n^2 - a_s^2, \tag{27}$$

where d_n is given by (**??**) and d_s in (**??**) and solve the following first-order equilibrium conditions:

$$\frac{\partial \Pi_r}{\partial p_n} = \frac{\partial \Pi_r}{\partial a_n} = \frac{\partial \Pi_r}{\partial p_s} = \frac{\partial \Pi_r}{\partial a_s} = 0,$$

which yields the retailer's reaction functions to the manufacturer's decision variables, that is,

$$a_n = A_n^{SN1}(w, a_m), \quad p_n = P_n^{SN1}(w, a_m), \quad a_s = A_s^{SN1}(w, a_m), \quad p_s = P_s^{SN1}(w, a_m). \tag{28}$$

The retailer's Hessian matrix is given by

$$\begin{pmatrix} \frac{-2}{1-\theta^2} & \frac{2\theta}{1-\theta^2} & \frac{c}{1-\theta^2} & \frac{-\theta c}{1-\theta^2} \\ \frac{2\theta}{1-\theta^2} & \frac{-2}{1-\theta^2} & \frac{-\theta c}{1-\theta^2} & \frac{c}{1-\theta^2} \\ \frac{c}{1-\theta^2} & \frac{-\theta c}{1-\theta^2} & -2 & 0 \\ \frac{-\theta c}{1-\theta^2} & \frac{c}{1-\theta^2} & 0 & -2 \end{pmatrix}.$$

Therefore, for any $\theta \in (0, 1)$, as per our model specification, the retailer's profit is a strictly concave function in the decision variables p_n, p_s, a_n, a_s iff

$$\begin{aligned} c^2 + 4\theta^2 - 4 &< 0, && (29) \\ \left(c^2 + 4\theta^2 - 4\right)\left(c^2 - 4\theta^2 - 4\right) - 16\theta^2(1-\theta^2) &> 0. && (30) \end{aligned}$$

We then insert the expressions in (28) into the manufacturer's optimization problem given by

$$\max_{a_m, w} \Pi_m = w d_n (w, a_m) - a_m^2, \tag{31}$$

where

$$\begin{aligned} d_n (w, a_m) &= \frac{1}{1-\theta^2}[g_n - \theta g_s + b a_m + c A_n^{SN1}(w, a_m) \\ &\quad -\theta c A_s^{SN1}(w, a_m) - P_n^{SN1}(w, a_m) + \theta P_s^{SN1}(w, a_m)], \end{aligned}$$

and solve the manufacturer's first-order conditions given by

$$\frac{\partial \Pi_m}{\partial w} = \frac{\partial \Pi_m}{\partial a_m} = 0.$$

The solution to the above system gives the equilibrium solution for w and a_m in the $SN1$ scenario. The manufacturer's Hessian matrix is given by

$$\begin{pmatrix} \frac{4c^2-16}{(c^2+4\theta^2-4)(c^2-4\theta^2-4)} & \frac{8b-2bc^2}{(c^2+4\theta^2-4)(c^2-4\theta^2-4)} \\ \frac{8b-2bc^2}{(c^2+4\theta^2-4)(c^2-4\theta^2-4)} & \frac{-2\left(16-16\theta^2-8c^2+c^4\right)}{(c^2+4\theta^2-4)(c^2-4\theta^2-4)} \end{pmatrix}.$$

Therefore, for any $\theta \in (0, 1)$, the manufacturer's profit is a strictly concave function in the decision variables w, a_{ms} iff

$$\begin{aligned} \frac{-4\left(4-c^2\right)}{(c^2+4\theta^2-4)(c^2-4\theta^2-4) - 16\theta^2(1-\theta^2)} &< 0, && (32) \\ 4\left(4-c^2\right)\left(2\left(16-16\theta^2-8c^2+c^4-2b^2\right) - b^2c^2\right) &> 0. && (33) \end{aligned}$$

The manufacturer's equilibrium expressions are next inserted in the retailer's reaction functions to obtain the equilibrium retail prices and advertising effort of the retailer as functions of the model's parameter values.

Conditions (29) and (30) imply that condition (32) is satisfied too. Furthermore, under condition (29), condition (33) simplifies as

$$2\left(16 - 16\theta^2 - 8c^2 + c^4 - 2b^2\right) + b^2c^2 > 0. \tag{34}$$

With the help of Mathematica 11.0 and specifically using the "Reduce" command[1], one gets that the three concavity conditions (29), (30) and (34) are satisfied if and only if the following inequalities apply:

$$0 < c < 2, \quad 0 < b < \sqrt{2(4-c^2)}, \quad 0 < \theta < \frac{(4-c^2)(8-b^2-2c^2)}{4\sqrt{2}}. \tag{35}$$

Note that the upper bound for b is a strictly decreasing and strictly concave function of c. It takes the maximum value of $2\sqrt{2}$ for $c = 0$ and zero for $c = 2$.

Manipulating analytically the positivity conditions using Mathematica 11.0, we can prove, first, that the conditions in (35) imply that w, a_m, d_n, a_n, p_n, the retailer's optimal profits, Π_r^{SN1}, and margin, $p_n - w$, are positive. Second, that if the condition ensuring that a_s is positive is satisfied, then p_s is positive too. Third, that all the concavity and positivity conditions are satisfied if and only if the following inequalities apply:

$$\begin{gathered} 0 < c < 2, \quad 0 < b < \sqrt{2(4-c^2)}, \quad 0 < \theta < \frac{1}{8}\left(-4+c^2+\sqrt{(4-c^2)(36-4b^2-9c^2)}\right), \\ \frac{4(4-c^2)\theta}{2((4-c^2)^2-8\theta^2)-b^2(4-c^2)} < g_s < 1. \end{gathered} \tag{36}$$

2.2 The $SN2$ game

In $SN2$, the game is played in four stages. To solve the game backwards, we start by stage 4 and solve the retailer's pricing problem given by

$$\max_{p_n, p_s} \Pi_r = (p_n - w)\, d_n + p_s d_s - a_n^2 - a_s^2, \tag{37}$$

where d_n is given by (**??**) and d_s in (**??**) and solve the following first-order conditions:

$$\frac{\partial \Pi_r}{\partial p_n} = \frac{\partial \Pi_r}{\partial p_s} = 0,$$

which provides the retailer's price reaction functions to his advertising and to the manufacturer's decision variables, namely:

$$p_n = P_n^{SN2}(w, a_m, a_n, a_s), \quad p_s = P_s^{SN2}(w, a_m, a_n, a_s). \tag{38}$$

The retailer's concavity conditions are given by $\frac{-2}{1-\theta^2} < 0$ and $\frac{4}{1-\theta^2} > 0$. Both of these conditions are satisfied for any $\theta \in (0, 1)$. In stage 3, we insert this reaction function

[1]Reduce solves equations or inequalities for variables and eliminates quantifiers.

into the manufacturer's pricing problem to get

$$\max_{w} \Pi_m = w d_n \left(w, a_m, a_n, a_s\right) - a_m^2. \tag{39}$$

where $d_n \left(w, a_m, a_n, a_s\right)$ is the demand of the NB after replacing the retail prices by the reaction functions in (38). Then, we solve the manufacturer's first-order optimality condition $\frac{\partial \Pi_m}{\partial w} = 0$ to get the manufacturer's wholesale price as a function of advertising efforts a_m, a_s and a_n such as:

$$w = W^{SN2} \left(a_m, a_n, a_s\right). \tag{40}$$

The manufacturer's concavity condition is given by $\frac{-1}{1-\theta^2} < 0$, which is satisfied for any $\theta \in (0, 1)$. The expression in (40) is then inserted into the retailer's price reaction functions in (38) and into the retailer's profit function. Now we solve the retailer's advertising problem in stage 2 of the game. The retailer's problem is given by

$$\begin{aligned} \max_{a_n, a_s} \Pi_r &= \left(P_n^{SN2} \left(a_m, a_n, a_s\right) - W^{SN2} \left(a_m, a_n, a_s\right)\right) d_n \left(a_m, a_n, a_s\right) \\ &\quad + P_s^{SN2} \left(a_m, a_n, a_s\right) d_s \left(a_m, a_n, a_s\right) - a_n^2 - a_s^2, \end{aligned} \tag{41}$$

and solve the retailer's first-order conditions given by

$$\frac{\partial \Pi_r}{\partial a_n} = \frac{\partial \Pi_r}{\partial a_s} = 0.$$

The solution gives the retailer's advertising strategy as function of the manufacturer's advertising variable

$$a_n = A_n^{SN2} \left(a_m\right), \quad a_s = A_s^{SN2} \left(a_m\right), \tag{42}$$

The retailer's concavity conditions are given by

$$\frac{c^2 - 16 \left(1 - \theta^2\right)}{8 \left(1 - \theta^2\right)} < 0, \tag{43}$$

$$\frac{c^4 - 20c^2 + 64 - 12 \left(4 - c^2\right) \theta^2}{16 \left(1 - \theta^2\right)} > 0. \tag{44}$$

The expressions in (42) are then inserted in the pricing reaction functions of the manufacturer in (40) and of the retailer in (38). Next, these functions are placed in the manufacturer's advertising problem in stage 1 which is

$$\max_{a_m} \Pi_m = W^{SN2}\left(a_m\right) d_n\left(a_m\right) - a_m^2. \tag{45}$$

Solving the manufacturer's first-order optimality condition ($\frac{\partial \Pi_m}{\partial a_m} = 0$) gives the equilibrium solution for a_m in $SN2$. The manufacturer's concavity condition is as follows:

$$32b^2(4-c^2)^2(1-\theta^2) - (c^4 + 64(1-\theta^2) + 4c^2(3\theta^2 - 5))^2 < 0. \tag{46}$$

Finally, the obtained equilibrium is inserted in the manufacturer's and retailers' reaction functions to obtain the equilibrium retail and wholesale prices, and the advertising efforts of the retailer as functions of the model's parameters.

Using the "Reduce" command in Mathematica 11.0, one gets that the three concavity conditions (43), (44) and (46) are satisfied if and only if the following inequalities apply:

$$0 < c < 2, \quad 0 < b < \frac{16 - c^2}{4\sqrt{2}}, \quad 0 < \theta < \bar{\theta}(b, c). \tag{47}$$

where $\bar{\theta}(b, c)$ is given by:

$$\bar{\theta}(b,c) = \frac{1}{2(16-3c^2)}\sqrt{(4-c^2)\left[4(16-b^2)(4-c^2) + 3c^4 - 2b\sqrt{2\left(2b^2(4-c^2)^2 + c^2(128 - 40c^2 + 3c^4)\right)}\right]}.$$

Note that the upper bound for b is a strictly decreasing and strictly concave function of c. It takes the maximum value of $2\sqrt{2}$ for $c = 0$ and zero for $c = 2$.

In this scenario, there are eight positivity conditions that have to be added in order to ensure that the decisions (prices and advertising), margins, profits and demands are strictly positive.

Manipulating analytically the positivity and concavity conditions in (47) using Mathematica 11.0, we can prove first, that conditions (47) and the conditions guaranteeing a_m, a_n, w, d_n and d_s positive imply that p_n, p_s, Π_r and $(p_n - w)$ are positive. Second, all the concavity and positivity conditions are satisfied if and only if the following inequalities apply:

$$0<c<2,\quad \begin{cases} 0<b<3\frac{c\sqrt{8-c^2}}{4\sqrt{2}}, & \begin{cases} 0<\theta<\frac{1}{4}(4-c^2), & \underline{g}_s(c,b,\theta)<g_s<1 \\ \text{or} & \\ \frac{1}{4}(4-c^2)<\theta<\bar{\theta}(b,c), & \underline{g}_s(c,b,\theta)<g_s<\frac{4-c^2}{4\theta} \end{cases} \\ b=3\frac{c\sqrt{8-c^2}}{4\sqrt{2}}, & \quad 0<\theta<\frac{1}{4}(4-c^2), \quad \underline{g}_s(c,b,\theta)<g_s<1 \\ 3\frac{c\sqrt{8-c^2}}{4\sqrt{2}}<b<\frac{16-c^2}{4\sqrt{2}}, & \quad 0<\theta<\bar{\theta}(b,c), \quad \underline{g}_s(c,b,\theta)<g_s<1, \end{cases} \tag{48}$$

where

$$\underline{g}_s(c,b,\theta)=\frac{\theta(c^2-8)\left(c^4+64(1-\theta^2)+4c^2(-5+3\theta^2)\right)}{c^6+32b^2(4-c^2)(1-\theta^2)+4c^4(5\theta^2-9)-512(2-3\theta^2+\theta^4)+32c^2(12-13\theta^2+3\theta^4)}.$$

2.3 The $SN3$ game

In this case, the equilibrium solution is played by solving the four stage game described in Table 2. We follow a similar approach as for $SN2$. To solve the game backwards, we start by stage 4 and solve the retailer's advertising problem given by

$$\max_{a_n,a_s}\Pi_r=(p_n-w)\,d_n+p_s d_s-a_n^2-a_s^2, \tag{49}$$

where d_n is given by (**??**) and d_s by (**??**) and solve the following first-order conditions:

$$\frac{\partial\Pi_r}{\partial a_n}=\frac{\partial\Pi_r}{\partial a_s}=0,$$

which provides the retailer's advertising reaction functions to his pricing and to the manufacturer's decision variables, namely:

$$a_n=A_n^{SN3}\left(w,a_m,p_n,p_s\right),\quad a_s=A_s^{SN3}\left(w,a_m,p_n,p_s\right). \tag{50}$$

The retailer's Hessian matrix is given by

$$\begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix},$$

and the concavity conditions are then satisfied for any values of the model parameters.

In stage 3, we insert the obtained retailer's reaction functions in (50) into the manufacturer's advertising problem to get

$$\max_{a_m} \Pi_m = w d_n \left(w, a_m, p_n, p_s\right) - a_m^2. \tag{51}$$

and solve the manufacturer's first-order optimality condition $\frac{\partial \Pi_m}{\partial a_m} = 0$ to get the manufacturer's advertising as a function of pricing strategies p_n, p_s and w such as:

$$a_m = A_m^{SN3} \left(p_n, p_s, w\right). \tag{52}$$

The manufacturer's concavity condition is satisfied for any values of the model parameters ($\frac{\partial^2 \Pi_m}{\partial a_m^2} = -2 < 0$).

In stage 2, the manufacturer's pricing reaction functions are then inserted into the retailer's advertising reaction functions in (52) and into the retailer's profit function. Next, we solve the retailer's pricing problem given by

$$\max_{p_n, p_s} \Pi_r = \left(p_n - w\right) d_n \left(w, p_n, p_s\right) + p_s d_s \left(w, p_n, p_s\right) - \left[A_n^{SN3} \left(p_n, p_s, w\right)\right]^2 - \left[A_s^{SN3} \left(p_n, p_s, w\right)\right]^2, \tag{53}$$

and solve the retailer's first-order conditions

$$\frac{\partial \Pi_r}{\partial p_n} = \frac{\partial \Pi_r}{\partial p_s} = 0.$$

The solution yields the retailer's prices as function of the manufacturer's wholesale price

$$p_n = P_n^{SN3} \left(w\right), \quad p_s = P_s^{SN3} \left(w\right), \tag{54}$$

The retailer's concavity conditions are satisfied iff

$$\left(c^2 + 4\right) \theta^2 + c^2 - 4 \; < \; 0, \tag{55}$$

$$\left(c^2 + 4\theta - 4\right) \left(c^2 - 4\theta - 4\right) \; > \; 0. \tag{56}$$

In stage 1, we insert the retailer's obtained pricing functions in the advertising reaction functions of the manufacturer in (52) and of the retailer in (50). Next, these functions are placed in the manufacturer's pricing problem which is

$$\max_{w} \Pi_m = w d_n \left(w\right) - \left[A_m^{SN3} \left(w\right)\right]^2. \tag{57}$$

Solving the manufacturer's first-order optimality condition ($\frac{\partial \Pi_m}{\partial w} = 0$) gives the equilibrium solution for w in $SN3$. The manufacturer's concavity condition is satisfied iff

$$-\frac{b^2c^4+32(1-\theta^2)^2-4c^2(2(1-\theta^2)^2+b^2(1+\theta^2))}{(c^2+4\theta-4)(c^2-4\theta-4)}<0. \tag{58}$$

Finally, the equilibrium wholesale price is inserted in the manufacturer's and retailer's reaction functions to obtain the equilibrium retail price, and the advertising efforts of the retailer as functions of the model's parameters.

Using the "Reduce" command in Mathematica 11.0, one gets that the three concavity conditions (55), (56) and (58) are satisfied if and only if the following inequalities apply:

$$0<c<2, \quad \begin{cases} 0<b<\frac{c\sqrt{8-c^2}}{4\sqrt{2}}, & 0<\theta<\frac{1}{4}(4-c^2), \\ \frac{c\sqrt{8-c^2}}{4\sqrt{2}}<b<\frac{2\sqrt{2}}{c}, & 0<\theta<\tilde{\theta}(b,c), \end{cases} \tag{59}$$

where $\tilde{\theta}(b,c)$ is given by:

$$\tilde{\theta}(b,c)=\frac{1}{2}\sqrt{\frac{16-(4-b^2)c^2+bc\sqrt{2c^4+64-(24-b^2)c^2}}{4-c^2}}.$$

Conditions (59) imply that w, a_m, a_n, d_n and p_n are positive and that $p_n-w>0$.

Additional conditions ensuring p_s, a_s, d_s and π_r positive must be imposed. Finally, Mathematica is unable to analytically obtained the conditions on parameters c, b, θ and g_s that satisfied all these conditions.

3 Appendix C: Abbreviated expressions

In Table 4, the abbreviated expressions are as follows:

$num(p_n^{SN2})=-2[32\theta^5 g_s(16-3c^2)+4\theta^4(152c^2-15c^4-384)$
$+g_s\theta^3(4c^4+32c^2(b^2+6)-128(b^2+8))+(3072-1472c^2+196c^4-5c^6)\theta^2$
$-g_s\theta(32+32b^2+8c^2-c^4+128)(c^2-4)+6c^6-144c^4+864c^2-1536]$,

$num(a_s^{SN2})=(32((1/32)c^6g_s+((3/4)g_s\theta^2-(9/8)g_s+(1/8)\theta)c^4+((9/2)g_s\theta^4+(3/2)\theta^3+g_s(b^2-31/2)\theta^2-(5/2)\theta+(-b^2+12)g_s)c^2-4(\theta-1)(\theta+1)(6g_s\theta^2+2\theta+g_s(b^2-8))))c$,

$num(p_s^{SN2})=-2(g_s+\theta)c^6+(-56g_s\theta^2-24\theta^3+72g_s+40\theta)c^4-64(\theta-1)(6g_s\theta^2-2\theta+g_s(b^2-12))(\theta+1)c^2+256g_s(\theta-1)(\theta+1)(b^2+8\theta^2-8)$,

$num(d_s^{SN2})=-2(g_s-\theta)c^6+(-40g_s\theta^2+24\theta^3+72g_s-56\theta)c^4+(-192g_s\theta^4-320\theta^3+(-64b^2+832)g_s\theta^2+448\theta+(64b^2-768)g_s)c^2+256(4g_s\theta^2+4\theta+g_s(b^2-8))(\theta-1)(\theta+1)$,

$den\,(SN2)=Y^2-32b^2(4-c^2)^2(1-\theta^2)$,

$Y=c^4+64(1-\theta^2)-4c^2(5-3\theta^2)-16\theta^2$,

$num(p_n^{SN3}) = -2(b^2c^6g_s\theta - 4b^2c^4g_s\theta^3 + 2c^6\theta^4 + b^2c^6 - 4b^2c^4g_s\theta + 2b^2c^4\theta^2 + 8b^2c^2g_s\theta^3 - 8b^2c^2\theta^4 + 32b^2g_s\theta^5 - 4c^6\theta^2 - 28c^4\theta^4 - 16c^2g_s\theta^5 - 48c^2\theta^6 - 64g_s\theta^7 - 10b^2c^4 - 8b^2c^2g_s\theta - 24b^2c^2\theta^2 - 64b^2g_s\theta^3 - 32b^2\theta^4 + 2c^6 + 56c^4\theta^2 + 32c^2g_s\theta^3 + 224c^2\theta^4 + 192g_s\theta^5 + 192\theta^6 + 32b^2c^2 + 32b^2g_s\theta + 64b^2\theta^2 - 28c^4 - 16c^2g_s\theta - 304c^2\theta^2 - 192g_s\theta^3 - 5)76\theta^4 - 32b^2 + 128c^2 + 64g_s\theta + 576\theta^2 - 192)$,

$num(a_n^{SN3}) = -c(c^2 + 4g_s\theta - 4)[b^2c^4 - 2(2\theta^4 + (b^2-4)\theta^2 + 3b^2 + 2)c^2 - 16\theta^4 + 8(b^2 + 4)\theta^2 - 8b^2 - 16]$,

$num(p_s^{SN3}) = -2b^2(g_s + \theta)c^6 + (4b^2\theta^3 + 16g_s\theta^4 + 8\theta^5 + 16b^2g_s + 12b^2\theta - 32g_s\theta^2 - 16\theta^3 + 16g_s + 8\theta)c^4 + (16(\theta+1))(2\theta^4g_s - 2\theta^3 + g_s(b^2-10)\theta^2 + (b^2+2)\theta + (2b^2+8)g_s)(\theta - 1)c^2 - 256g_s(\theta-1)^3(\theta+1)^3$,

$num(a_s^{SN3}) = -c(b^2c^6g_s - 4b^2c^4g_s\theta^2 - 8c^4g_s\theta^4 - 8b^2c^4g_s + 4b^2c^4\theta + 16b^2c^2g_s\theta^2 - 8b^2c^2\theta^3 + 32b^2g_s\theta^4 + 16c^4g_s\theta^2 + 64c^2g_s\theta^4 - 16c^2\theta^5 + 64g_s\theta^6 + 16b^2c^2g_s - 24b^2c^2\theta - 32b^2g_s\theta^2 - 32b^2\theta^3 - 8c^4g_s - 128c^2g_s\theta^2 + 32c^2\theta^3 - 256g_s\theta^4 + 64\theta^5 + 32b^2\theta + 64c^2g_s - 16c^2\theta + 320g_s\theta^2 - 128\theta^3 - 128g_s + 64\theta)$,

$num(d_n^{SN3}) = (2(b^2c^4 + (-4\theta^4 + (8-2b^2)\theta^2 - 6b^2 - 4)c^2 + 16\theta^4 - (8b^2+32)\theta^2 + 8b^2 + 16))(c^2 + 4g_s\theta - 4)$,

$num(d_s^{SN3}) = -2(b^2c^6g_s - 4b^2c^4g_s\theta^2 - 8c^4g_s\theta^4 - 8b^2c^4g_s + 4b^2c^4\theta + 16b^2c^2g_s\theta^2 - 8b^2c^2\theta^3 + 32b^2g_s\theta^4 + 16c^4g_s\theta^2 + 64c^2g_s\theta^4 - 16c^2\theta^5 + 64g_s\theta^6 + 16b^2c^2g_s - 24b^2c^2\theta - 32b^2g_s\theta^2 - 32b^2\theta^3 - 8c^4g_s - 128c^2g_s\theta^2 + 32c^2\theta^3 - 256g_s\theta^4 + 64\theta^5 + 32b^2\theta + 64c^2g_s - 16c^2\theta + 320g_s\theta^2 - 128\theta^3 - 128g_s + 64\theta)$,

$X = b^2c^4 - 2(2\theta^4 + (b^2-4)\theta^2 + 3b^2 + 2)c^2 + 16\theta^4 - 8(b^2+4)\theta^2 + 8b^2 + 16$,

$den\,(SN3) = b^2c^4 + 32(1-\theta^2)^2 - 4c^2\left[2(1-\theta^2)^2 + b^2(1+\theta^2)\right].$

The expressions of the retailer's profits in all scenarios ($SN1$, $SN2$ and $SN3$) were too long so we omit them here for ease of presentation.

4 Appendix D: Comparison of equilibrium solutions across games

We present here the results obtained from comparing equilibrium profits in each pair of scenarios N_i and SN_j $(i, j = 1, 2)$. All results are summarized in Table A.1 and the accompanying Table A.2.

4.1 N1 vs. SN1

We compare the manufacturer's, retailer's and total channel's profits obtained at equilibrium in the N1 and SN1 games under conditions in (36) that ensure that the concavity and positivity conditions for the scenarios N1 and SN1 are satisfied.

Using Mathematica 11.0 we analytically prove that conditions in (36) imply $\Pi_m^{SN1} < \Pi_m^{N1}$. However, conditions in (36) do not always imply that $\Pi_r^{SN1} > \Pi_r^{N1}$ and $\Pi_{ch}^{SN1} > \Pi_{ch}^{N1}$.

Using the command "Reduce" of Mathematica, for any values of g_s, we analytically show that under conditions in (36):

$$\begin{aligned} &\Pi_r^{SN1} > \Pi_r^{N1} \quad \text{if} \quad \theta \geq 0.09, \quad \pm \quad otherwise \\ &\Pi_{ch}^{SN1} > \Pi_{ch}^{N1} \quad \text{if} \quad \theta \geq 0.2, \quad \pm \quad otherwise. \end{aligned}$$

4.2 N1 vs. SN2

We compare the manufacturer's profits for the N1 and SN2 games. These two games can be compared given the conditions in (6) and (48) that ensure that the concavity and positivity conditions for the scenarios N1 and SN2 are satisfied. Using Mathematica, we are able to derive the conditions on these parameters where both the N1 and SN2 games are feasible (i.e., (6) and (48) are satisfied). These conditions are complex so we omit them here for the sake of clarity. We compare the manufacturer's profits in the N1 and SN2 games under these conditions. We prove analytically with Mathematica that

$$\Pi_m^{SN2} < \Pi_m^{N1}.$$

For the retailer and total channel profits, using Mathematica, for any value of the parameters satisfying conditions (6) and (48), we cannot prove analytically the signs of the profits comparisons. Therefore, for fixed values of g_s, we find that

$$\begin{array}{c} \Pi_r^{SN2} < \Pi_r^{N1} \quad \text{if} \quad \theta \geq \Phi_4 \\ \Pi_r^{SN2} > \Pi_r^{N1} \quad \text{if} \quad \theta \geq \Phi_5 \\ \pm \quad otherwise \end{array}$$

$$\begin{array}{c} \Pi_{ch}^{SN2} < \Pi_{ch}^{N1} \quad \text{if} \quad \theta \geq \Gamma_1 \\ \Pi_{ch}^{SN2} > \Pi_{ch}^{N1} \quad \text{if} \quad \theta \geq 0.91 \\ \pm \quad otherwise \end{array}$$

where Φ_4, Φ_4 and Γ_1 take different values depending on the value of g_s (see Table A.2).

4.3 N1 vs. SN3

We compare the manufacturer's profits for the N1 and SN3 games under the concavity and positivity conditions ensuring that both games are feasible (6) and (59). Mathematica is unable to analytically describe the conditions on the parameters characterizing all these conditions. We have numerically checked that $\Pi_m^{SN3} < \Pi_m^{N1}$ for $g_s \in \{0.01, 0.05, 0.1, 0.25, 0.6, 0.75, 0.9\}$ and $\theta \in \{0.01, 0.05, 0.1, 0.25, 0.5, 0.75, 0.9\}$ when conditions are (6) and (59).

We carry out the comparison of the retailer's and total channel profits in the N1 and SN3 games for some fixed values of parameter g_s for any value of the parameters satisfying conditions (6) and (59). The results summarize as follows:

$$\begin{array}{c} \Pi_m^{SN3} > \Pi_m^{N1} \quad \text{if} \quad \theta \geq \Phi_6, \quad \pm \quad otherwise \\ \Pi_{ch}^{SN3} > \Pi_{ch}^{N1} \quad \text{if} \quad \theta \geq \Gamma_2, \quad \pm \quad otherwise \end{array}$$

where Φ_6 and Γ_2 take different values depending on the value of g_s (see Table A.2).

4.4 N2 vs. SN1

We compare the manufacturer's, the retailer's and the total channel equilibrium profits obtained in the N2 and SN1 games. These two games can be compared when the conditions in (16) and (36) that ensure that the concavity and positivity conditions for the scenarios N2 and SN1 are satisfied. We have proved that if conditions (36) are satisfied, then (16) are satisfied too.

For any value of the parameters satisfying (36) and using Mathematica we can derive the following results:

$$\Pi_m^{SN1} < \Pi_m^{N2} \quad \text{if} \quad \theta \geq 0.57, \quad \pm \quad \textit{otherwise}$$
$$\Pi_r^{SN1} > \Pi_r^{N2}$$
$$\Pi_{ch}^{SN1} > \Pi_{ch}^{N2}.$$

4.5 N2 vs. SN2

We compare the manufacturer's, retailer's and total equilibrium profits in the N2 and SN2 games. These two games can be compared when the conditions in (16) and (48) that ensure that the concavity and positivity conditions for the scenarios N2 and SN2 are satisfied. We have proved that if conditions (48) are satisfied, then (16) are satisfied too.

In general, we cannot compare using Mathematica the profits in the N2 and SN2 games for any value of the parameters satisfying (48). Therefore, we compare these profits for some fixed values of parameter g_s. The results summarize as follows:

$$\Pi_m^{SN2} < \Pi_m^{N2}$$
$$\Pi_r^{SN2} > \Pi_r^{N2} \quad \text{if} \quad \theta \geq \Phi_2, \quad \pm \quad \textit{otherwise}$$
$$\Pi_{ch}^{SN2} > \Pi_{ch}^{N2} \quad \text{if} \quad \theta \geq \Gamma_3, \quad \pm \quad \textit{otherwise}$$

where Φ_2 and Γ_3 take different values depending on the value of g_s (see Table A.2).

4.6 N2 vs. SN3

We compare the manufacturer's, the retailer's and total channel profits for the N2 and SN3 games under the concavity and positivity conditions ensuring that both games are feasible, conditions (16) and (59), respectively.

In general, we cannot compare using Mathematica the profits in the N2 and SN2 games for any value of the parameters satisfying conditions (16) and (59). Therefore, we compare these profits for some fixed values of parameter g_s. The results summarize as follows:

$$\Pi_m^{SN3} < \Pi_m^{N2} \quad \text{if} \quad \theta \geq \Delta_1, \quad \pm \quad \textit{otherwise}$$
$$\Pi_r^{SN3} > \Pi_r^{N2} \quad \text{if} \quad \theta \geq \Phi_7, \quad \pm \quad \textit{otherwise}$$
$$\Pi_{ch}^{SN3} > \Pi_{ch}^{N2} \quad \text{if} \quad \theta \geq \Gamma_4, \quad \pm \quad \textit{otherwise}$$

where Δ_1, Φ_7 and Γ_4 take different values depending on the value of g_s (see Table A.2).

4.7 N3 vs. SN1

We compare the manufacturer's, the retailer's and the total channel equilibrium profits obtained in the N3 and SN1 games. These two games can be compared when the condition in (26) and (36) that ensure that the concavity and positivity conditions for the scenarios N3 and SN1 are satisfied. We have proved that if conditions (36) are satisfied, then (26) are satisfied too.

We compare the profits for some fixed values of parameter g_s. The results summarize as follows:

$$\begin{array}{lll} \Pi_m^{SN1} < \Pi_m^{N3} & \text{if} \quad \theta \geq \Delta_2, & \pm \quad otherwise \\ \Pi_r^{SN1} > \Pi_r^{N3} & \text{if} \quad \theta \geq \Phi_8, & \pm \quad otherwise \\ \Pi_{ch}^{SN1} > \Pi_{ch}^{N3} & \text{if} \quad \theta \geq \Gamma_5, & \pm \quad otherwise \end{array}$$

where Δ_2, Φ_8 and Γ_5 take different values depending on the value of g_s (see Table A.2).

4.8 N3 vs. SN2

We compare the manufacturer's, the retailer's and the total channel equilibrium profits obtained in the N3 and SN2 games. These two games can be compared when the condition in (26) and (48) that ensure that the concavity and positivity conditions for the scenarios N3 and SN2 are satisfied.

We compare the profits for some fixed values of parameter g_s. The results summarize as follows:

$$\begin{array}{lll} \Pi_m^{SN2} < \Pi_m^{N3} & \text{if} \quad \theta \geq \Delta_3, & \pm \quad otherwise \\ \Pi_r^{SN2} > \Pi_r^{N3} & \text{if} \quad \theta \geq \Phi_9, & \pm \quad otherwise \\ \Pi_{ch}^{SN2} > \Pi_{ch}^{N3} & \text{if} \quad \theta \geq \Gamma_6, & \pm \quad otherwise \end{array}$$

where Δ_3, Φ_9 and Γ_6 take different values depending on the value of g_s (see Table A.2).

4.9 N3 vs. SN3

We compare the manufacturer's profits for the N3 and SN3 games under the concavity and positivity conditions ensuring that both games are feasible (conditions (26) and (59), respectively). Mathematica is unable to analytically describe the conditions on the parameters characterizing all these conditions. We compare the profits for some fixed values of parameters g_s and θ. The results summarize as follows:

$$\Pi_m^{SN3} < \Pi_m^{N3}$$

$$\begin{array}{lll} \Pi_r^{SN3} > \Pi_r^{N3} & \text{if} \quad \theta \geq \Phi_3, & \pm \quad otherwise \\ \Pi_{ch}^{SN3} > \Pi_{ch}^{N3} & \text{if} \quad \theta \geq \Gamma_7, & \pm \quad otherwise \end{array}$$

where Φ_3 and Γ_7 take different values depending on the value of g_s (see Table A.2).

Table A.1: Summary of results: Effects of SB introduction on profits

		$SN1$	$SN2$	$SN3$
Manufacturer				
	$N1$	$-$	$-$	$-$
	$N2$	$-$ if $\theta \geq 0.57$ $\pm$ *otherwise*	$-$	$-$ if $\theta \geq \Delta_1$ $\pm$ *otherwise*
	$N3$	$-$ if $\theta \geq \Delta_2$ $\pm$ *otherwise*	$-$ if $\theta \geq \Delta_3$ $\pm$ *otherwise*	$-$
Retailer				
	$N1$	$+$ if $\theta \geq \Phi_1$, $\pm$ *otherwise* $\Phi_1 = 0.09$	$-$ if $\theta \geq \Phi_4$ $+$ if $\theta \geq \Phi_5$ $\pm$ *otherwise*	$+$ if $\theta \geq \Phi_6$, $\pm$ *otherwise*
	$N2$	$+$	$+$ if $\theta \geq \Phi_2$, $\pm$ *otherwise*	$+$ if $\theta \geq \Phi_7$, $\pm$ *otherwise*
	$N3$	$+$ if $\theta \geq \Phi_8$, $\pm$ *otherwise*	$+$ if $\theta \geq \Phi_9$, $\pm$ *otherwise*	$+$ if $\theta \geq \Phi_3$, $\pm$ *otherwise*
Channel				
	$N1$	$+$ if $\theta \geq 0.2$, $\pm$ *otherwise*	$-$ if $\theta \geq \Gamma_1$ $+$ if $\theta \geq 0.91$ $\pm$ *otherwise*	$+$ if $\theta \geq \Gamma_2$, $\pm$ *otherwise*
	$N2$	$+$	$+$ if $\theta \geq \Gamma_3$, $\pm$ *otherwise*	$+$ if $\theta \geq \Gamma_4$, $\pm$ *otherwise*
	$N3$	$+$ if $\theta \geq \Gamma_5$, $\pm$ *otherwise*	$-$ if $\theta \geq \Gamma_6$, $\pm$ *otherwise*	$+$ if $\theta \geq \Gamma_7$, $\pm$ *otherwise*

Table A.2: values of Δ_i, Φ_i and Γ_i[2]

g_s	0.1	0.25	0.5	0.75	0.9
Δ_1			0.57	0.57	0.61
Δ_2			0.67	0.71	0.69
Δ_3	0.2	0.45	0.67	0.71	0.69
Φ_2	0.11	0.23	0.34	0.37	0.39
Φ_3	0.06	0.15	0.31	0.48	0.62
Φ_4	0.20	0.48			
Φ_5				0.69	0.67
Φ_6	0.09	0.18	0.28	0.41	0.59
Φ_7	0.06	0.13	0.26	0.41	0.51
Φ_8			0.66	0.71	0.71
Φ_9	0.2	0.48		0.71	0.73
Γ_1	0.2	0.48			
Γ_2	0.09	0.18	0.29	0.44	0.56
Γ_3					0.77
Γ_4	0.06	0.14	0.28	0.44	0.56
Γ_5		0.43	0.65	0.75	0.8
Γ_6	0.2	0.46	0.76		0.93
Γ_7		0.18	0.37	0.57	0.72

5 Appendix E: Effect of SB on total demand

Insert Figure A.1

[2] empty cells means that there is no determinate value of Δ_i, Φ_i and Γ_i for the corresponding values of g_s.