Pricing of demand-related products: Can ignoring cross-category effect be a smart choice?*

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July 9, 2020

Abstract

This paper studies pricing strategies of competing retailers offering substitutable products in multiple product categories. For such retailers, in addition to accounting for within-category pricing effects, cross-category effects can also influence consumers' purchase decisions and thereby impact the retailers' optimal pricing strategies. We model consumers' utility for purchasing substitutable products from the same category and from other categories as well. We then solve a game-theoretic model to identify the retailers' optimal prices and profits. Our results show that accounting for cross-category effects largely influences the retailers' pricing and profitability. In particular, cross category effects influence the sensitivity of prices to within-category substitution levels. While cross-category effects have an impact on retailers' equilibrium strategies, this effect is only relevant when within-category substitution effects are present. We also find that intentionally ignoring cross-category effects, leads to lower prices for categories that are either substitutable or highly complementary, and to higher prices otherwise. When not accounting for the fact that the two categories are demand-related

*Research of the first and third authors is supported by the National Sciences and Engineering Research Council of Canada (NSERC). The second author's research is partially supported by MINECO under projects ECO2014-52343-P, and ECO2017-8227-P (AEI) and by Junta de Castilla y León VA024P17 and VA105G18 co-financed by FEDER funds.

implies lower prices when this information is accounted for, then at least one retailer chooses to disregard the cross-category effect at equilibrium. Finally, we find evidence for prisoner's dilemma situations where both retailers ignore cross-category effect at equilibrium while accounting for it would yield them higher profits.

Keywords: Pricing; Retailing; Game theory; Product substitution and complementarity; Information value.

1 Introduction

Competing retailers offering multiple product categories must make complex pricing decisions. The price for one product may impact and be impacted by the retailer's price of competing products in the same category, by prices of other items in different categories and by prices of competing retailers' products in the same and in different categories. This paper investigates pricing strategies of multi-product retailers taking into account both within- and cross-category pricing effects.

Retail pricing decisions have been extensively studied in the marketing and operations management literature. Various types of pricing models have been proposed (see Shelegia, 2012 for a review). We mainly differentiate between models focused on a single retailer that optimizes pricing of products carried in the store (see Chan et al., 2004 for a review) and models related to pricing for competing retailers. The latter category is relevant to this paper. In particular, pricing models for multi-product competing retailers, such that the pricing decisions of each firm impact the demands of its competitors in the market, is mainly focused on price substitution and complementarity effects between products sold within the same product category at different retail stores (e.g., Fazli & Shulman, 2018; Lus & Muriel, 2009; Guan & Zhao, 2011; Levis & Papageorgiou, 2007; Mizuno, 2003; Garcıa-Gallego & Georgantzıs, 2001; Soon, 2011). This paper extends the existing studies in this field by analyzing pricing decisions for competing multi-product retailers offering products pertaining to different categories. Consumers may view these categories as complementary, substitutable or independent. In our model, the explicit representation of within- and cross-category effects using demand functions derived from consumers' utility allows us to explore competitive strategies by multi-product retailers in a new way.

Literature in both marketing and operations has shown strong empirical evidence that prices in one product category influence consumers' purchases in other product categories (see, e.g., Manchanda et al., 1999; Song & Chintagunta, 2006; Ailawadi et al., 2006; Duvvuri et al., 2007; Leeflang et al., 2008; Leeflang & Parreño-Selva, 2012; Ma et al., 2016; Gelper et al., 2016). For example, Manchanda et al. (1999) explain that "the choice of one category may affect the selection of another category due to the complementary nature (e.g., cake mix and cake frosting) of the two categories. Alternatively, two categories may co-occur in a shopping basket not because they are complementary but because of similar purchase cycles (e.g., beer and diapers) or because of a host of other unobserved factors (e.g., consumer movement through a store)" (p. 96). They find significant cross-category effects after analyzing data for products in the grocery market (e.g., cake frosting and cake mix, detergent and softeners). Using a simulation, they also show that considering complementarity effects can significantly increase the profitability of retailers' price promotions. Similar evidence and impact on profitability is also found by Duvvuri et al. (2007) for different complementary product categories, namely pasta and sauce. While the last two studies focus on complementary cross-category effect, Leeflang & Parreño-Selva (2012) expand the analysis to include data from non-complementary categories, that of aperitif and breakfast, and found that cross-category effects can be related to either complementarity or substitution effects. For example, their study shows that the prices of beer and red wine (substitutable product categories) and the prices of coffee and cookies (complementary product categories) influence each other's sales. However, they do not find a significant relationship between the price of cookies and the demand for beer, meaning that these product categories have null cross-category effects.

This result has also been found by Gelper et al. (2016) who estimate cross-category effects for grocery products pertaining to 15 different categories. They show that cross-category effects are driven by consumption relatedness (substitution or complementarity). For example, they find evidence for substitution effects between soft drinks and frozen juices and complementarity effects between soft drinks and frozen entrees. However, their study shows no cross-category effects between soft drinks and cheeses, which indicates that these categories are unrelated or independent.

The literature about cross-category effects mainly considers one retailer as the decision maker optimizing its profit. Competitors and their strategic interactions are not part of the analysis. A vast literature in marketing, economics and operations management has studied strategic pricing decisions of competing firms (e.g., Rao, 1993; Sigué & Karray, 2007; Shin et al., 2015; Kienzler & Kowalkowski, 2017). This literature can be rooted back to Bertrand's (1883) contribution; interest in pricing strategies has increased from then. These days, a few scientific journals specialize in pricing and, like many other journals, regularly publish papers focusing on strategic pricing interactions between firms. It seems that one missing piece in this literature is strategic pricing in the presence of demand interactions (SPPDI) between product categories. For instance, the Handbook of Pricing Research in Marketing (Rao, 2009) has 26 chapters reporting on and synthesizing hundreds of scientific papers, but, to the best of our knowledge, none of these raise the question of determination of equilibrium prices when each competitor sells products in demand-related categories. By reading Wikipedia,[1] one can learn about 29 different pricing models, but none dealing with SPPDI.

The objective of this paper is to characterize equilibrium strategic pricing in the presence of cross-category interactions. The motivation is straightforward. If retailers (i) compete against each other, and (ii) offer products belonging to demand-related categories, then there is no valid conceptual reason that justifies omitting either of (i) or (ii) when making pricing

[1]https://en.wikipedia.org/wiki/Pricing_strategies, last checked on October 22, 2018.

decisions. More specifically, considering two retailers, each selling two products belonging to two related categories, we wish to answer the following research questions:

1. What are the equilibrium prices?

2. How do the degree of substitutability/complementarity and the degree of competition affect these pricing strategies?

3. Is it in the best interest of one retailer to ignore the demand relationship between the two categories?

4. Are there circumstances under which both retailers benefit from simultaneously ignoring demand relationships between the two categories?

Answering the first two questions will enable us to clearly see the effect of accounting for cross-category interactions on pricing strategies, with and without store competition. One can expect that the nature of the products' relationship (i.e., whether they are complements or substitutes) will play a role at this level.

The other two questions are related to retailers' strategic use of information. More specifically, our objective is to investigate whether intentionally ignoring the presence of cross-category effects can be beneficial to a retailer. While such an objective might be irrelevant in an optimization problem setting, it is not for a game set-up in which decision makers make interdependent decisions. In fact, in an optimization problem, where only one agent is making the decisions, access to more information about the market is usually beneficial to a utility maximizing agent (see Blackwell, 1951, 1953). Indeed, if ignoring this information improves the agent's payoff, then this action would be part of the optimal solution. However, this reasoning does not necessarily carry over to a context where multiple decision makers are considered as in game theory (Hirshleifer, 1971; Kamien et al., 1990a; 1990b; Neyman, 1991; Bassan et al., 2003). In a game setting, ignoring an information item might be the optimal choice for a given player in the game. Of course, this will depend on whether or not the other player is also ignoring the same information. By answering our research questions 2 and 3, we develop a better understanding of the role of information in pricing products belonging to different categories.

The rest of the paper is organized as follows. Section 2 describes the game-theoretic model. Sections 3 and 4 present the equilibrium pricing solutions in the case where retailers do not ignore, and ignore the cross-category effect, respectively. Sections 5 and 6 discuss the results of price and profit comparisons across the two scenarios. Finally, Section 7 concludes.

2 Model

To study pricing strategies for competing retailers in the presence of cross-category effects, we retain the most parsimonious setting, that is, a market formed of two retailers, each offering two products belonging to two different categories. We denote by ic the product sold by retailer i in category c ($i, c = 1, 2$). The two categories under investigation are the same in both outlets,[2] that is, products ic and $(3-i)c$ for $c = 1, 2$ are substitutable (e.g., same or two "close" brands of soda). Each retailer i decides the price of its two products ($p_{ic}, c = 1, 2$). Notations are summarized in Table 1.

i	index of retailers, $i = 1, 2$
c	index of product category, $c = 1, 2$
p_{ic}	price of retailer i in category c, $p_{ic} > 0$
q_{ic}	demand of retailer i in category c, $q_{ic} > 0$
U	consumer utility, $U \geq 0$
t_c	within-category substitution effect in category c, $t_c \in [0, 1]$
s_i	cross-category effect for retailer i, $s_i \in (-1, 1)$
Π_i	profit of retailer i, $\Pi_i > 0$
p^*_{ic}	equilibrium price of retailer i in category c when $t_i, s_i \neq 0$
$\bar{p}^*_{ic}$	equilibrium price of retailer i in category c when $t_1 = t_2, s_1 = s_2$
$\hat{p}^*_{ic}$	equilibrium price of retailer i in category c when $t_1 = t_2 = 0, s_i \neq 0$
$\tilde{p}^*_{ic}$	equilibrium price of retailer i in category c when $t_i \neq 0, s_1 = s_2 = 0$
u_1	superscript used when retailer 1 acts as uninformed, but not retailer 2
Bu	superscript used when both retailers act as uninformed

Table 1: Notations

Following Spence (1976) and Shubik & Levitan (1980), we let the consumer utility from purchasing the four products be given by

$$U(q) = \sum_{i,c=1,2} \left(q_{ic} - \frac{1}{2} q_{ic}^2 \right) - \sum_{i,c=1,2} p_{ic} q_{ic} - \sum_{\substack{c,i,j=1,2 \\ i \neq j}} t_c q_{ic} q_{jc} + \sum_{c,i=1,2} s_i q_{ic} q_{i(3-c)}, \tag{1}$$

where q_{ic} is the demand for the product in category c sold by retailer i and $q = (q_{11}, q_{12}, q_{21}, q_{22})$; $t_c \in [0, 1]$ is a parameter measuring the degree of competition between the two products in category $c = 1, 2$, and $s_i \in (-1, 1)$ represents the cross-category interaction in outlet $i = 1, 2$.

[2] Otherwise, the differences in results would not be solely attributed to differences in pricing strategies.

More specifically, $s_i = 0$ corresponds to two independent product categories (e.g., soda and toothpaste), $s_i < 0$ represents substitutable categories (e.g., soda and frozen juices), and $s_i > 0$ denotes complementary categories (e.g., soda and chips). While the substitutability or complementarity nature of cross-category effects is mainly driven by consumers' purchasing behavior, namely consumption relatedness, retailers can intensify these effects through diverse in-store activities such as close positioning of items on the shelf, in-store demonstrations, or product packaging. Our formulation captures asymmetric cross-category effects by (possibly) having $s_1 \neq s_2$.

The utility function in (1), which has been widely used in economics and marketing (see, e.g., Liu et al., 2014; Choi & Coughlan, 2006; Ingene & Parry, 2007; Lus & Muriel, 2009; Karray et al., 2017), exhibits the classical properties in which the utility of consuming a product decreases when the consumption of the substitute product increases and the consumption of the complementary product decreases. Note that we could add a constant to the right-hand side of (1) to insure that the utility is nonnegative for positive q_{ic}. Instead, we assume that $U(q) \geq 0$ for $q \geq 0$. This assumption does not qualitatively change the results obtained in the paper.

To derive the demand function for each product in each category, we assume that consumers base their purchasing decision on price and do not mind distributing their shopping across stores to take advantage of the best available prices. This assumption is valid for markets where transportation costs are negligible. For example, this can be the case if the stores are located close to each other (physically or virtually) or if consumers are cherry-pickers (Fox & Hoch, 2005)o a behavior typically associated with price-sensitive and promotion-prone consumers. Maximization of consumer's utility function in (1) with respect to quantities yields the following first-order optimality conditions:

$$
\begin{aligned}
1 - p_{11} - q_{11} + 2q_{12}s_1 - 2q_{21}t_1 &= 0, \\
1 - p_{12} - q_{12} + 2q_{11}s_1 - 2q_{22}t_2 &= 0, \\
1 - p_{21} - q_{21} + 2q_{22}s_2 - 2q_{11}t_1 &= 0, \\
1 - p_{22} - q_{22} + 2q_{21}s_2 - 2q_{12}t_2 &= 0.
\end{aligned}
$$

Solving these conditions, we obtain the following demand functions:

$$
q_{ic} = \alpha_{ic} + \beta_{ic}p_{ic} + \gamma_{ic}p_{i(3-c)} + \delta_{ic}p_{(3-i)c} + \theta_{ic}p_{(3-i)(3-c)}, \quad i, c = 1, 2, \tag{2}
$$

where

$$
\begin{aligned}
\alpha_{ic} &= \{(1-2t_{3-c})\left(2s_i(1+2s_{3-i})-4s_{3-i}t_c+(1-2t_c)(1+2t_{3-c})\right) \\
&\quad -4s_{3-i}(s_{3-i}+s_i(1+2s_{3-i})\}/\lambda_{ic}, \\
\beta_{ic} &= -(1-4s_{3-i}^2-4t_{3-c}^2)/\lambda_{ic}, \\
\gamma_{ic} &= -2(s_i-4s_is_{3-i}^2+4s_{3-i}t_ct_{3-c})/\lambda_{ic}, \\
\delta_{ic} &= 2(t_c+4s_is_{3-i}t_{3-c}-4t_ct_{3-c}^2)/\lambda_{ic}, \\
\theta_{ic} &= 4(s_it_{3-c}+s_{3-i}t_c)/\lambda_{ic}, \\
\lambda_{ic} &= (1-4s_i^2)(1-4s_{3-i}^2)+(1-4t_c^2)(1-4t_{3-c}^2)-1-32s_is_{3-i}t_ct_{3-c}.
\end{aligned}
$$

For the above demand system to be economically meaningful, the following restrictions on the parameter values must be satisfied: (i) the market potential is positive (i.e., $\alpha_{ic} > 0$, $i, c = 1, 2$); (ii) own-price effect is negative ($\beta_{ic} < 0, i, c = 1, 2$), and cross-price effect is positive ($\delta_{ic} > 0,\ i, c = 1, 2$); (iii) own-price effect is larger than all other prices' effects (i.e., $|\beta_{ic}| > \max(|\gamma_{ic}|, |\delta_{ic}|, |\theta_{ic}|), i, c = 1, 2$); and (iv) intra-category substitution effects are higher than cross-category effects (i.e., $|\delta_{ic}| > |\theta_{ic}|$ and $|\gamma_{ic}| > |\theta_{ic}|, i, c = 1, 2$) (see Duvvuri et al., 2007; Gelper et al., 2016). We will account for these restrictions throughout the rest of the paper.

We make the following two remarks:

1. We do not make any assumption on the relationship between q_{ic} and the price of the product belonging to the other category, that is, $p_{i(3-c)}, i = 1, 2$. Consequently, γ_{ic} and θ_{ic} can take any sign.

2. Our assumption that each retailer only carries one brand in each category, implies that intra-category competition is synonymous with inter-store competition. One way to rememb the meaning of the coefficients in the demand functions is to think of them as follows:

γ_{ic} : intra-store, inter-category effect. This coefficient is negative (positive) when the two products are complements (substitutes).

δ_{ic} : inter-store, intra-category effect. This coefficient is positive.

θ_{ic} : inter-store, inter-category effect. This coefficient is negative (positive) when both the two products and the two categories are complements (substitutes).

Note that the market potential α_{ic} and all demand price sensitivity parameters ($\beta_{ic}, \gamma_{ic}, \delta_{ic}, \theta_{ic}$ and λ_{ic}) depend on both product substitutability (t_c) and cross category effect (s_i) parameters. Compared to a linear demand function formulation where $\beta_{ic}, \gamma_{ic}, \delta_{ic}, \theta_{ic}$ and λ_{ic} are the main model parameters, as in Chen & Simchi Levi's (2010) formulation of price-dependent demand functions, our consumer utility-derived demand function allows us to capture the effect of product substitutability within and across categories as well as their effects on demand sensitivity to own and other products' prices. See Lus & Muriel (2009) for more discussion on the value of utility-based demand formulation.

To get better insights from the demand equations, we look at some special cases. If the two categories are independent, that is, $s_1 = s_2 = 0$, then all inter-category coefficients would be equal to zero ($\gamma_{ic} = \theta_{ic} = 0$), and the demand function takes the following form:

$$q_{ic} = \frac{1}{1 - 4t_c^2} \left[1 - p_{ic} - 2t_c \left(1 - p_{(3-i)c}\right)\right], \quad i, c = 1, 2.$$

If the two stores do not compete, perhaps because they are far apart physically or perceptually, we would then have $t_1 = t_2 = 0$ and all inter-store (or intra-category) coefficients would be equal to zero ($\delta_{ic} = \theta_{ic} = 0$). The demand function becomes

$$q_{ic} = \frac{1}{1 - 4s_i^2} \left[1 - p_{ic} + 2s_i \left(1 - p_{i(3-c)}\right)\right], \quad i, c = 1, 2.$$

As stated before, the impact of $p_{i(3-c)}$ on q_{ic} depends on the relationship between the two categories; it is negative if the two categories are complements ($s_i > 0$), positive if they are substitutes ($s_i < 0$), and null if they are independent ($s_i = 0$).

Assuming profit-maximization behavior, the optimization problem of each retailer i is as follows:

$$\max_{p_{i1}, p_{i2}} \Pi_i = \sum_{c=1,2} \left(p_{ic} q_{ic}\right), \quad i = 1, 2. \tag{3}$$

In the rest of the paper, we shall use the term uninformed to refer to a retailer who intentionally disregards the information that the two categories are demand-related, that is, who does not account for the cross-category effect.

3 Equilibrium prices with informed retailers

In this scenario, both retailers are informed, meaning that they both account for cross-category pricing effects in their profit maximization problems. Assuming that the retailers make their pricing decisions simultaneously, we seek a Nash equilibrium. The following proposition provides the equilibrium strategies.

Proposition 1 *Assuming an interior solution and* $\beta_{ic} < 0$, $(1 - 4s_{3-i}^2)/\lambda_{ic} > 0$ *for* $i, c = 1, 2$, *the unique equilibrium prices are given by*

$$p_{ic}^{*}=\frac{A_{ic}}{2\left[(1-t_c^2)(1-t_{3-c}^2)-8s_is_{3-i}t_ct_{3-c}-4(s_i^2+s_{3-i}^2-4s_i^2s_{3-i}^2)\right]},\quad i,c=1,2,\qquad (4)$$

where

$$\begin{aligned}A_{ic} &= (1-t_{3-c}^2)\left(1-t_c-2t_c^2\right)-2s_{3-i}(2s_{3-i}+t_c)(1-4s_i^2)-4s_i^2(1-t_c)\\ &- 2s_{3-i}t_ct_{3-c}(1+6s_i)-2s_it_c^2(1+t_{3-c}).\end{aligned}$$

Proof. Retailer i chooses the prices p_{ic} in each category $c=1,2$ aiming to maximize his own profits function Π_i given (3). Therefore, assuming an interior solution, the first-order equilibrium conditions are

$$\frac{\partial \Pi_i}{\partial p_{ic}}=0,\quad i,c=1,2,$$

where

$$\begin{aligned}\frac{\partial \Pi_i}{\partial p_{ic}} &= 2(4s_{3-i}^2+4t_{3-c}^2-1)p_{ic}+4(s_i(4s_{3-i}^2-1)-4s_{3-i}t_ct_{3-c})p_{i(3-c)}\\ &+ 4(s_{3-i}t_c+s_it_{3-c})p_{(3-i)(3-c)}+2(t_c+4s_is_{3-i}t_{3-c}-4t_ct_{3-c}^2)p_{(3-i)c}\\ &- 4s_{3-i}^2+4s_{3-i}t_c(2t_{3-c}-1)-2s_i(1+2s_{3-i})(2s_{3_i}+2t_{3-c}-1)\\ &+ (2t_c-1)(4t_{3-c}^2-1),\quad i,c=1,2.\end{aligned}$$

This expression can be rewritten as:

$$\frac{\partial \Pi_i}{\partial p_{ic}}=\left[\alpha_{ic}-2\beta_{ic}p_{ic}+2\gamma_{ic}p_{i(3-c)}+\theta_{ic}p_{(3-i)(3-c)}+\delta_{ic}p_{(3-i)c}\right]\lambda_{ic},\quad i,c=1,2.$$

From $\frac{\partial \Pi_i}{\partial p_{ic}}=0$, the following reaction functions can be derived:

$$p_{ic}(p_{i(3-c)},p_{(3-i)(3-c)},p_{(3-i)c})=-\frac{1}{2\beta_{ic}}\left[\alpha_{ic}+2\gamma_{ic}p_{i(3-c)}+\theta_{ic}p_{(3-i)(3-c)}+\delta_{ic}p_{(3-i)c}\right],\ i,c=1,2.$$

Straightforward computations yield the equilibrium expressions in (4). The computation of the first and second minors of the Hessian matrix of the profits function Π_i with respect to the decision variables p_{ic}, $c=1,2$, allows us to conclude that the retailers' objective functions (profits) are strictly concave with respect to their decision variables if and only if the following two conditions are satisfied $\beta_{ic} < 0$ and $(1 - 4s_{3-i}^2)/\lambda_{ic} > 0$ for $i, c = 1, 2$.[3] Under these conditions the uniqueness of the equilibrium is guaranteed. ■

The equilibrium prices depend on all the model's parameters in a highly non-linear way, and consequently they are not amenable to a thorough qualitative analysis. However, one can note that equilibrium prices depend on both cross-category (s_i) and within-category effects (t_c) in each category and in each store ($i, c = 1, 2$). This shows that setting multiproduct and multi-category retailers' pricing strategies in a competitive setting is a complex decision that depends on all interactions between products and categories.

To gain some insight into the retailers' pricing strategies, in particular when the equilibrium is indeed interior and unique, we look at some particular cases.

Corollary 1 *Assuming an interior solution, if the game is symmetric, that is, $s_1 = s_2 = s$ and $t_1 = t_2 = t$, then the unique equilibrium prices are given by*

$$\bar{p}_{ic}^* = \frac{1 - 2s - 2t}{2(1 - 2s - t)}, \quad i, c = 1, 2, \tag{5}$$

and the equilibrium profits by

$$\bar{\Pi}_1^* = \bar{\Pi}_2^* = \bar{\Pi}^* = \frac{(1 - 2s)(1 - 2s - 2t)}{2(1 - 2s + 2t)(1 - 2s - t)^2}. \tag{6}$$

Proof. It suffices to set $s_1 = s_2 = s$, and $t_1 = t_2 = t$ in the proof of Proposition 1 to get the result.
The equilibrium prices are positive, and hence the solution is interior, for $t \in (0, 1)$ and either $s \in (-1, 1/2 - t)$ or $s \in (1/2 - t/2, 1)$.

The conditions for strict concavity and hence, for uniqueness now read

$$\begin{aligned} \beta_{ic} &= -\frac{1 - 4s^2 - 4t^2}{\lambda_{ic}} = -\frac{(1 - 4s^2 - 4t^2)}{\left(4(s-t)^2 - 1\right)\left(4(s+t)^2 - 1\right)} < 0, \\ \frac{1 - 4s^2}{\lambda_{ic}} &= \frac{1 - 4s^2}{\left(4(s-t)^2 - 1\right)\left(4(s+t)^2 - 1\right)} > 0. \end{aligned}$$

With the help of Mathematica 11.0 it can be proved $\bar{p}_{ic}^* > 0$ and the strict concavity

[3] Note that the condition $(1 - 4s_{3-i}^2)/\lambda_{ic} > 0$ for $i, c = 1, 2$ is implied by $\beta_{ic} < 0$ when there is no substitution between products in the same category, i.e., for $t_c = 0, c = 1, 2$.

conditions $\beta_{ic} < 0$ and $\frac{1-4s^2}{\lambda_{ic}} > 0$, are satisfied if either of the following conditions holds true:

$$\text{(A)} \quad : \quad t \in (0, 1/2] \text{ and either } \begin{cases} s \in (-1/2 - t, -1/2), \text{ or } s \in (-1/2 + t, 1/2 - t), \\ \text{or } s \in (1/2, 1/2 + t); \end{cases} \tag{7}$$

$$\text{(B)} \quad : \quad t \in (1/2, 1] \text{ and either } s \in (-1, -1/2), \text{ or } s \in (1/2, 1). \tag{8}$$

Note that the equilibrium profits in (6) are strictly positive under the restrictions in (7)-(8). ■

We observe that increasing the degree of cross-category interaction in the symmetric case (s) always leads to lower prices. Indeed, we have

$$\frac{\partial \bar{p}^*_{ic}}{\partial s} = -\frac{t}{(2s + t - 1)^2} < 0, \qquad i, c = 1, 2.$$

Interestingly, the negative sign is independent of the nature of the cross-category interaction, that is, whether the products are complements or substitutes. The intuition is as follows: when the two products are complements, a lower price for one product boosts the demand for both, which is an incentive to lower the other product's price. When they are substitutes, larger s means more competition, which naturally results in lower prices for both competing products.

The impact of varying the intra-category (or inter-store) substitution degree (t) depends on the cross-category interaction as follows:

$$\frac{\partial \bar{p}^*_{ic}}{\partial t} = \frac{2s - 1}{2\,(2s + t - 1)^2} \begin{cases} \leq 0, & \text{if } s \in (-1, 1/2], \\ \geq 0, & \text{if } s \in [1/2, 1). \end{cases}, \quad i, c = 1, 2.$$

When the two products belonging to the two categories are strong complements ($s \in [1/2, 1)$), the direct competitive pressure within each of these categories (higher t) is absorbed by the "captive" nature of the demand for the complement. Otherwise, the within-category substitutability leads to lower prices.

The result concerning the special case of absence of cross-category interaction and inter-store competition is given in the following corollary. Denote by $\tilde{p}^*_{ic}$ the equilibrium price when $s_1 = s_2 = 0$, and by $\hat{p}^*_{ic}$ the equilibrium price when $t_1 = t_2 = 0$.

Corollary 2 *Assuming a unique interior solution, in the absence of cross-category interaction ($s_1 = s_2 = 0$), the unique equilibrium prices are given by*

$$\tilde{p}^*_{ic} = \tilde{p}^*_{c} = \frac{1 - 2t_c}{2(1 - t_c)}.$$

Assuming a unique interior solution, in the absence of within-category (inter-store) competition ($t_1 = t_2 = 0$), the unique equilibrium prices are given by

$$\hat{p}^*_{ic} = \hat{p}^* = \frac{1}{2}.$$

Proof. It suffices to set $s_1 = s_2 = 0$ and $t_1 = t_2 = 0$, respectively in the proof of Proposition 1 to get the results. ■

The above corollary is stated under the assumption of an interior solution. In the absence of cross-category interaction, the equilibrium is interior for $t_c \in (0, 1/2)$. In this case, the price of product i in category c ($\tilde{p}^*_{ic}$) depends only on the substitution rate within this category (t_c) and is independent of the rate in the other category (t_{3-c}). Hence, retailers can decide the prices of their products separately for each category as changes in substitution rates within one category do not affect the prices of the products in the other category offered by the retailers. This is an intuitive result given that consumers' purchases across categories are unrelated in this case.

However, as it can be clearly seen from Proposition 1, when either or both cross-category effects exist, in other words s_1 or/and s_2 is (are) non null, then the price of each retailer's product depends on the level of substitution within the category (t_c) as well as on cross-category effects for the products sold by the retailer and by its competitor.

The second result in the above corollary shows that, in the absence of store competition, the price is independent of the cross-category interaction. This means that when products belonging to the same category are not in competition, for example because they are highly differentiated, then the retailers' pricing strategies will not take into account cross-category effects at equilibrium. This is regardless of the nature of the cross-category interactions that may be present, namely complementarity or substitution. This result supports the findings in the existing literature emphasizing the impact of cross-category effects on pricing strategies (Duvvuri et al, 2007; Ma et al., 2016). While our findings support the view that cross-category effects have an impact on retailers' equilibrium strategies, they show that this impact is only relevant when within-category substitution effects are present.

4 Equilibrium prices with uninformed retailer

In this scenario, at least one of the two retailers is uninformed. To analyze the effects of being uninformed of cross-category effects on equilibrium strategies and outcomes, we shall retain two special cases. In the first case, only one retailer chooses its prices while disregarding cross-category effects. In the second case, both players decide to ignore the cross-category interactions when selecting their prices. We recall that by uninformed retailer, we mean that, although s_i is a market parameter that is different from zero, this retailer intentionally decides to ignore this information and acts *as if* $s_i = 0, i = 1, 2$.

To be sure that all differences are only due to information choice, we shall assume in the rest of the paper that the two retailers are fully symmetric, that is, $t_1 = t_2 = t$, and $s_1 = s_2 = s$. Consequently, the informed equilibrium prices, which will be considered when making some comparisons, are those given in (5). When there is only one uninformed retailer, we suppose that it is retailer 1, and denote its resulting prices by $p_{ic}^{*u_1}, c = 1, 2$. (The superscript u_1 is used to highlight that it is retailer 1 who ignores the information).

Proposition 2 *Suppose that the two products are not independent and that retailer* 1 *ignores the cross-category effects, while retailer* 2 *does not. Assuming an interior solution and*

$$0 \leq t < 1/2 \quad \textit{and} \quad (-1/2-t < s < -1/2) \textit{ or } (-1/2+t < s < 1/2-t) \textit{ or } (1/2 < s < 1/2+t), \tag{9}$$

the unique equilibrium prices are given by

$$\bar{p}_{1c}^{*u_1} = \frac{(1-t)(1-2s) - 2t^2}{2\left(1 - t^2 - 2s\right)}, \quad c = 1, 2, \tag{10}$$

$$\bar{p}_{2c}^{*u_1} = \frac{1 - 2s - t - 2t^2}{2\left(1 - t^2 - 2s\right)}, \; c = 1, 2, \tag{11}$$

and the equilibrium profits by

$$\bar{\Pi}_1^{*u_1} = \frac{\left((1-t)(1-2s) - 2t^2\right)^2}{2\left(1 - t^2 - 2s\right)^2\left(1 - 4t^2\right)}, \tag{12}$$

$$\bar{\Pi}_2^{*u_1} = \frac{(1-2s)\left(1 - 2s - t - 2t^2\right)^2}{2\left(1 - t^2 - 2s\right)^2\left((2s-1)^2 - 4t^2\right)}. \tag{13}$$

Proof. The proof follows the same steps as the proof of Proposition 1. It suffices to set $s_1 = s_2 = 0$ in the objective function of retailer 1 and apply the first-order equilibrium conditions to get the result.

The strict concavity of the retailers' profits with respect to their decision variables ensures that the interior equilibrium is unique. It can be easily checked that the strict concavity conditions are

$$1-4t^2>0, \tag{14}$$

$$\frac{1-4s^2-4t^2}{16s^4+(1-4t^2)^2-8s^2(1+4t^2)}>0, \tag{15}$$

$$\frac{1-4s^2}{16s^4+(1-4t^2)^2-8s^2(1+4t^2)}>0. \tag{16}$$

With the help of Mathematica 11.0, it can be proved that the above three conditions are satisfied for values of $t \in [0,1]$ and $s \in (-1,1)$ such that the conditions in (9) are fulfilled. Under the parameter restrictions in (9), the equilibrium prices $(\bar{p}_{1c}^{*u_1}, \bar{p}_{2c}^{*u_1})$ and the equilibrium profits $(\bar{\Pi}_1^{*u_1}, \bar{\Pi}_2^{*u_1})$ are positive. ■

The equilibrium prices vary as follows with the cross-category effect (s):

$$\begin{aligned}
\frac{\partial \bar{p}_{1c}^{*u_1}}{\partial s} &= -\frac{t^2(t+1)}{(t^2+2s-1)^2}<0,\\
\frac{\partial \bar{p}_{2c}^{*u_1}}{\partial s} &= -\frac{t(t+1)}{(t^2+2s-1)^2}<0.
\end{aligned}$$

This is the same qualitative result that we obtained in the case where both retailers account for products' dependence, and consequently there is no need to repeat the interpretation. Varying the parameter t, we get for $t \in (0,1/2)$

$$\begin{aligned}
\frac{\partial \bar{p}_{1c}^{*u_1}}{\partial t} &= \frac{(2s-1)\left[(t+1)^2-2s\right]}{2(t^2+2s-1)^2},\\
\frac{\partial \bar{p}_{2c}^{*u_1}}{\partial t} &= -\frac{(t+1)^2-2s(1+2t)}{2(t^2+2s-1)^2},
\end{aligned}$$

and

$$\frac{\partial \bar{p}_{1c}^{*u_1}}{\partial t} \quad \text{is} \begin{cases} \leq 0, & \text{for } s \in (-1/2-t,-1/2), \text{ or } s \in (-1/2+t,1/2-t) \\ \geq 0 & \text{for } s \in (1/2,1/2+t) \end{cases},$$

$$\frac{\partial \bar{p}_{2c}^{*u_1}}{\partial t} \quad \text{is} \begin{cases} \leq 0, & \begin{array}{c}\text{for } s \in (-1/2-t,-1/2) \text{ or } s \in (-1/2+t,1/2-t) \\ \text{or } s \in \left(1/2, \frac{(t+1)^2}{2(1+2t)}\right)\end{array} \\ \geq 0 & \text{for } s \in \left(\frac{(t+1)^2}{2(1+2t)}, 1/2+t\right) \end{cases}.$$

Again, given some minor changes in bound values of some of the intervals, the results are very similar to those derived in the previous section. Note, however, that when the cross-category effect is positive and not too high $(1/2<s<\frac{(t+1)^2}{2(1+2t)})$, we have $\frac{\partial \bar{p}_{1c}^{*u_1}}{\partial t} \geq 0$ and

$\frac{\partial \bar{p}_{2c}^{*u_1}}{\partial t} \leq 0$. This means that while retailer 1's price increases with higher values of the product substitution effect (t), the informed competing retailer's price would decrease. Therefore, in the case of low enough cross-category complementarity effects, the uninformed behavior of retailer 1 affects the sensitivity of equilibrium prices to within-category substitution effects.

As a last case, we characterize the equilibrium strategies and outcomes when both retailers disregard cross-category interactions when selecting their prices. In this case, we use the superscript Bu to indicate that both retailers act as uninformed.

Proposition 3 *For $t \in [0, 1/2)$, when both retailers intentionally ignore the cross-category effects, the unique interior equilibrium prices are given by*

$$\bar{p}_{ic}^{*Bu} = \bar{p}^{*Bu} = \frac{1-2t}{2(1-t)}, \quad i, c = 1, 2, \tag{17}$$

and the equilibrium profits by

$$\bar{\Pi}_1^{*Bu} = \bar{\Pi}_2^{*Bu} = \bar{\Pi}^{*Bu} = \frac{1-2t}{2(1-3t^2+2t^3)}. \tag{18}$$

Proof. The proof follows the same steps as the proof of Proposition 1. It suffices to set $s_1 = s_2 = 0$ in both retailers' objective functions and apply the first-order equilibrium conditions to get the result. ■

By definition, the equilibrium is independent of s and, as in the two previous scenarios, we have $\frac{\partial \bar{p}^{*Bu}}{\partial t} = -\frac{1}{2(t-1)^2} < 0$, that is, higher within-category competition leads to lower prices.

5 Effects of ignoring cross-category effects

We now address our third and fourth research questions. Namely: Is it in the best interest of one retailer to ignore the demand relationship between the two categories? Are there circumstances under which both retailers benefit from simultaneously ignoring demand relationships between the two categories?

To assess the impact of retailers' information choice (or use), we compare the pricing strategies and payoffs in the symmetric case ($s_1 = s_2 = s, t_1 = t_2 = t$) obtained in three scenarios. These are: (1) both retailers account for cross-category effects; (2) only one retailer intentionally ignores the cross-category effects; and (3) both retailers intentionally do not account for cross-category effects. For our comparisons to be meaningful, they must be made in a subset defined by the intersection of the three sets that outlines a unique interior equilibrium in the three scenarios. More specifically, we must retain the couples

(s,t) satisfying the conditions in (7)-(8), (9) and $t \in (0, 1/2)$. It is straightforward to observe that the restrictions in (9) define the sought intersection set. To facilitate interpretation of the results, we rewrite these restrictions as follows:

$$0 \le t < 1/2 \quad \text{and} \begin{cases} s \in (-1/2 - t, -1/2), & \text{large substitution effect } (R_1), \\ s \in (-1/2 + t, 0), & \text{small substitution effect } (R_2), \\ s \in (0, 1/2 - t), & \text{small complementarity effect } (R_3), \\ s \in (1/2, 1/2 + t), & \text{large complementarity effect } (R_4). \end{cases}$$

Regions R_1-R_4 are shown in Figure 1, along with the region where there is no feasible solution (UF). In the infeasibility region, at least one price and/or demand is negative. Qualitatively speaking, for a combination of s and t to yield an interior equilibrium, both parameters must be sufficiently high (for s in absolute value) or sufficiently low.

Insert Figure 1

5.1 Price comparisons

In the following three propositions, we compare the different prices in the four regions (R_1-R_4) described above.

Proposition 4 *If the two categories are substitutes ($s < 0$), then the different equilibrium prices compare as follows:*

$$\bar{p}_{ic}^{*Bu} < \bar{p}_{1c}^{*u_1} < \bar{p}_{2c}^{*u_1} < \bar{p}_{ic}^{*}, \; i = 1, 2, \; c = 1, 2.$$

Proof. In turn, assume that the pair (s,t) is in regions R_1 and R_2. Straightforward comparisons of prices in each region given in (5), (10), (11) and (17), yield the results. ■

Proposition 5 *If the two categories exhibit a small complementarity effect, then the different equilibrium prices compare as follows:*

$$\bar{p}_{ic}^{*Bu} > \bar{p}_{1c}^{*u_1} > \bar{p}_{2c}^{*u_1} > \bar{p}_{ic}^{*}, \; i = 1, 2, \; c = 1, 2.$$

Proof. Assume that the pair (s,t) is in region R_3. Straightforward comparison of the prices given in (5), (10), (11) and (17), yields the results. ■

Proposition 6 *If the two categories exhibit a large complementarity effect, then the different equilibrium prices compare as follows:*

$$\bar{p}_{ic}^{*Bu} < \bar{p}_{1c}^{*u_1} < \bar{p}_{ic}^{*} < \bar{p}_{2c}^{*u_1}, \; i = 1, 2, \; c = 1, 2.$$

Proof. Assume that the pair (s, t) is in region R_4. Straightforward comparison of the prices given in (5), (10), (11) and (17), yields the results. ■

The results in Propositions 4-6 show that ranking of the equilibrium prices depends on the nature and intensity of cross-category effects. The most striking result is that price ordering shares many commonalities when the categories are substitutes or close complements (i.e., when they exhibit large complementarity effect). Indeed, in both Propositions 4 and 6, $\bar{p}_{ic}^{*Bu} < \bar{p}_{1c}^{*u_1} < \bar{p}_{ic}^{*}$. Therefore, no matter the intensity and nature of cross-category effects, when both retailers decide to ignore these effects, prices are lower than the price of the uninformed retailer when its competitor accounts for cross-category links. The scenario when both retailers are informed leads to a higher price than $\bar{p}_{ic}^{*Bu}$ and $\bar{p}_{1c}^{*u_1}$. This means that intentionally ignoring cross-category effects leads to lower prices in all cases.

The only difference in Propositions 4 and 6 relates to the price of the informed retailer when the competitor is uninformed ($\bar{p}_{2c}^{*u_1}$). A straightforward conclusion here is that consumers would benefit from lower prices when they buy from the uninformed retailer(s) in cases where the two categories are substitutes (regions R_1 and R_2) or exhibit large complementarity effects (region R_4). Alternatively, the informed retailer(s) charges a lower price in the case of small complementarity effect (region R_3).

To summarize, the results show that when the two categories are substitutes or highly complementary, ignoring cross-category effects leads to lower prices. Otherwise, in the case of low levels of complementarity across categories, prices set by informed retailers are the lowest.

5.2 Profit comparisons

Looking at Nash equilibria in the three scenarios, few differences in profits can be characterized analytically. A first result is the following:

$$\text{sign}\ \left(\bar{\Pi}_2^{*u_1} - \bar{\Pi}_2^{*}\right) \ \text{ is } \begin{cases} > 0, & \text{in regions } R_3 \text{ and } R_4, \\ < 0, & \text{in regions } R_1 \text{ and } R_2. \end{cases}$$

If the product categories are substitutes, an informed retailer achieves a lower profit when the competitor acts as uninformed. Otherwise, it is better not to stick to informed pricing. From Proposition 4, we know that the informed retailer sets its prices at higher levels than the uninformed retailer, and this harms the former's profit. When the two products are complements, the uninformed retailer does not see the opportunity of boosting the demand in one category by reducing the price in the other category, which improves the competitive position of the informed retailer. Consequently, its profit is higher than in the scenario where its competitor is also informed.

A second result is

$$\text{sign}\ \left(\bar{\Pi}^{*Bu}-\bar{\Pi}_1^{*u_1}\right)=\text{sign}\ \left(\bar{p}_{ic}^{*Bu}-\bar{p}_{1c}^{*u_1}\right)=\begin{cases}>0, & \text{in region } R_3,\\ <0, & \text{in regions } R_1,\ R_2 \text{ and } R_4.\end{cases}$$

Clearly, this result is a direct consequence of Propositions 4-6. The only case where it is better for an uninformed retailer to face an uninformed retailer is when the product categories exhibit a small complementarity effect. In this region, the two uninformed retailers simultaneously raise their prices and lessen competition. In all other regions, that is, R_1, R_2 and R_4, it is in the best interest of an uninformed retailer to compete with an informed one. When both retailers are uninformed, the prices are lower and so are the profits.

We could not obtain analytical results in all other cases than the above two. The reason is that the difference in profits is given by a ratio with both the numerator and the denominator being highly non-linear in (s,t). In fact, the numerators in these comparisons are polynomials of degree 7 in (s,t).

To interpret the numerical results to follow, let us suppose that the two retailers play a strategic game where each of them can choose being informed or uninformed. Table 2 summarizes this two-player game in strategic form:

		Retailer 2	
		Uninformed (U)	Informed (I)
Retailer 1	Uninformed (U)	$\left(\bar{\Pi}^{*Bu},\bar{\Pi}^{*Bu}\right)$	$\left(\bar{\Pi}_1^{*u_1},\bar{\Pi}_2^{*u_1}\right)$
	Informed (I)	$\left(\bar{\Pi}_1^{*u_2},\bar{\Pi}_2^{*u_2}\right)$	$\left(\bar{\Pi}^{*},\bar{\Pi}^{*}\right)$

Table 2: Game in strategic form

Figure 2 shows the equilibrium results. It is important to note that the equilibrium is unique for any pair (s,t). However, a different equilibrium is found for different value combinations of s and t in the feasible domain. The method for finding the results shown in Figure 2 consists in the following three steps: (i) choose values for (s,t); (ii) report the values of the different profits in Table 2; and finally, (iii) determine the equilibrium cell. These steps are repeated for all admissible values of (s,t), that is, for all values in regions R_1-R_4. The main takeaways from Figure 2 are:

1. If the product categories are substitutes, or highly complementary, then the unique Nash equilibrium for each feasible pair (s,t) involves at least one uninformed retailer.

2. When the product categories exhibit a small complementarity effect, then the unique Nash equilibrium for each feasible pair (s,t) is (I,I).

Whereas the first statement holds true for any pair of (s,t) in R_1, R_2 or R_4, the second statement is true for *"most"* values in region R_3. Indeed, we see in Figure 2 that for some combinations of values of s and t, the equilibrium is different from (I,I).

To summarize our results, it is safe to state that when not accounting for cross-category effect implies lower prices than when accounting for it, at least one player chooses to be uninformed at equilibrium.

Insert Figure 2

A relevant question is whether ignoring the cross-category effect is part of a prisoner's dilemma type of game. In such a case, players would both be better off using the information that the two categories are related, but they are unable to achieve this Pareto-optimal outcome at equilibrium. For such a situation to occur, we need the following inequalities to hold true:

(i) For each retailer, its profit when informed is larger than its profit when acting as uninformed, in other words;

$$\bar{\Pi}_1^* = \bar{\Pi}_2^* = \bar{\Pi}^* > \bar{\Pi}_1^{*Bu} = \bar{\Pi}_2^{*Bu} = \bar{\Pi}^{*Bu};$$

(ii) Each player gains a higher profit when both retailers are uninformed, than when only one retailer is uninformed, in other words;

$$\bar{\Pi}^{*Bu} > \bar{\Pi}_1^{*u_2}, \quad \bar{\Pi}^{*Bu} > \bar{\Pi}_2^{*u_1};$$

(iii) Finally, each player's profit is higher when it is the sole uninformed player than when both retailers are informed,in other words;

$$\bar{\Pi}_1^{*u_1} > \bar{\Pi}^*, \quad \bar{\Pi}_2^{*u_2} > \bar{\Pi}^*.$$

Again, it is not possible to analytically characterize the region in the feasible (s,t)-space where all above inequalities hold true. However, the task can be performed numerically as we did previously. Figure 3 shows the region where the equilibrium is (U,U), but both players would achieve higher profits if they both play (I,I). As in any prisoner's dilemma game, the Pareto-optimal outcomes cannot be reached at equilibrium.

Insert Figure 3

6 Concluding Remarks

In today's complex environment, retailers must take into account several factors when setting their pricing strategies. In particular, most retailers carry several products in multiple categories. Literature in both marketing and operations has shown strong empirical evidence that prices in one product category influence consumers' purchases in other product categories through a substitution or a complementarity cross-category effect. Further, retailers do not act in silos; they face intense competition from other retailers in the marketplace. They must take into account competitors' reactions when setting their prices.

This paper investigates strategic pricing of multi-product retailers taking into account both within and cross-category pricing effects as well as strategic competitors' reactions. To our knowledge, this is the first attempt to develop a theory that addresses the effects of such diverse pricing interactions on retailers' strategic prices. As many theoretic works in the operations literature focus on pricing strategies in one product category and ignore cross-category effects, this research also explores the implications of retailers intentionally not accounting for cross-category effect.

We developed a game-theoretic model for two competing retailers. Our model takes into account within and cross-category pricing effects. We obtained Nash equilibrium strategies for prices and profits in three scenarios: (i) when both retailers are uninformed; (ii) when both retailers are informed; and (iii) when one retailer is uninformed while the other is not. In each scenario, we performed sensitivity analyses of prices to cross and within-category effects. Our results show that strong complementary cross-category effects can invert the sensitivity of prices to substitution (competition) levels within a category. This competition effect is important when assessing the impact of cross-category effects on prices. While our findings support the view that cross-category effects have an impact on retailers' equilibrium strategies, they show that this impact is only relevant when within-category substitution effects are present (i.e., when products within a category are not highly differentiated).

To analyze the impact of using or not the information on cross-category effect on equilibrium strategies and outcomes, we compared prices and profits across different scenarios and identified the equilibrium for adopting an uninformed behavior. Our results show that the equilibrium, although unique for specific values of cross and within-category effects, depends on the range of these parameters' values. In particular, ignoring the cross-category effect leads to lower prices when the two categories are substitutes or highly complementary, and to higher prices otherwise. These pricing effects then directly impact the equilibrium solution. Specifically, when disregarding cross-category effect implies lower prices than when accounting for it, at least one retailer chooses to behave as uninformed at equilibrium. Finally, we find evidence for prisoner's dilemma situations where both retailers choose to be uninformed at equilibrium, even when acting as informed would yield them higher profits.

This research has some limitations. For example, we do not account for other factors such as advertising, promotions, inventory availability, and so forth that can influence consumer utility when choosing products (Ma et al., 2016). Future research can explore these effects although analytical insights will be hard to obtain given the high complexity of accounting for all these effects.

References

[1] Ailawadi, K. L., B. A. Harlam, J. Cesar and D. Trounce (2006). Promotion profitability for a retailer: The role of promotion, brand, category, and store characteristics. Journal of Marketing Research, 43(4): 518–535.

[2] Bassan, B., O. Gossner, M. Scarsini and S. Zamir (2003). Positive value of information in games. International Journal of Game Theory, 32: 17–31.

[3] Bertrand, J. (1883). Book review of théorie mathématique de la richesse sociale and of recherches sur les principles mathématiques de la théorie des richesses. Journal de Savants, 67: 499-508.

[4] Blackwell, D. (1951). Comparison of experiments, in Proceedings of the Second Berkeley Symposium on Mathematical Statistics and Probability, University of California Press, Berkeley, CA, 93–102.

[5] Blackwell, D. (1953). Equivalent comparison of experiments. Annals of Mathematical Statistics, 24, 265–272.

[6] Chan, L. M. A., Z. J. Shen Max, D. Simchi-Levi and J. Swann (2004). Coordination of pricing and inventory decisions: a survey and classification. In: Simchi-Levi D, Wu SD, Shen Max Z. J. , editors. Handbook of quantitative supply chain analysis: modeling in the E-business Era. Boston: Kluwer.

[7] Choi, C. S. and A. T. Coughlan (2006). Private label positioning: Quality versus feature differentiation from the national brand. Journal of Retailing, 82 (2): 79-93.

[8] Duvvuri, S. D., A. Ansari and S. Gupta (2007). Consumers' price sensitivities across complementary categories. Management Science, 53 (12): 1933-1945.

[9] Guan, G. and X. Zhao (2011). Pricing and inventory management in a system with multiple competing retailers under (r, Q) policies. Computers & Operations Research, 38 (9): 1294-1304.

[10] Fazli, A. and J. D. Shulman (2018). Implications of market spillovers. Management Science, 64 (11): 4996–5013.

[11] Fox, E. J. and S. J. Hoch (2005). Cherry-picking. Journal of Marketing, 69 (1): 46–62.

[12] Garcia-Gallego, A. and N. Georgantzis (2001). Multi-product activity in an experimental differentiated oligopoly. International Journal of Industrial Organization, 19 (3–4): 493-518.

[13] Gelper, S., I. Wilms and C. Croux (2016). Identifying demand effects in a large network of product categories. Journal of Retailing, 92 (1): 25-39.

[14] Hirshleifer J. (1971). The private and social value of information and the reward to inventive activity. American Economic Review, 61: 561–574.

[15] Ingene, C. A. and M. E. Parry (2007). Bilateral monopoly, identical distributors, and game-theoretic analyses of distribution channels. Journal of the Academy of Marketing Science, 35(4): 586-602.

[16] Kamien M. I., Y. Tauman and S. Zamir (1990a). On the value of information in a strategic conflict, Games and Economic Behavior, 2: 129–153.

[17] Kamien M. I., Y. Tauman and S. Zamir (1990b). Information transmission, in Ichiishi T, Neyman A, Tauman Y (eds), Game Theory and Applications, Academic Press, 273–281.

[18] Karray, S., G. Martín-Herrán and S. P. Sigué (2017). Cooperative advertising for competing manufacturers: The impact of long-term promotional effects. International Journal of Production Economics, 184: 21-32.

[19] Kienzler, M. and C. Kowalkowski (2017). Pricing strategy: A review of 22 years of marketing research. Journal of Business Research, 78: 101-110.

[20] Leeflang, P. S. H., J. Parreño-Selva, A. Van Dijk and D. R. Wittink (2008). Decomposing the sales promotion bump accounting for cross-category effects. International Journal of Research in Marketing, 25 (3): 201-214,

[21] Leeflang, P. S. H. and J. Parreño-Selva (2012). Cross-category demand effects of price promotions. Journal of the Academy of Marketing Science, 40 (4): 572-586.

[22] Levis, A. A. and L. G. Papageorgiou (2007). Active demand management for substitute products through price optimization. OR Spectrum, 29: 551-577.

[23] Liu, B., G. Cai and A. Tsay (2014). Advertising in asymmetric competing supply chains. Production and Operations Management, 23 (11): 1845-1858.

[24] Lus, B. and A. Muriel (2009). Measuring the impact of increased product substitution on pricing and capacity decisions under linear demand models. Production and Operations Management, 18 (1): 95-113.

[25] Ma, S., R. Fildes and T. Huang (2016). Demand forecasting with high dimensional data: The case of SKU retail sales forecasting with intra- and inter-category promotional information. European Journal of Operational Research, 249: 245–257.

[26] Manchanda, P., A. Ansari and S. Gupta (1999). The "shopping basket": A model for multicategory purchase incidence decisions. Marketing Science, 18 (2):95-114.

[27] Mizuno, T. (2003). On the existence of a unique price equilibrium for models of product differentiation. International Journal of Industrial Organization, 21: 761-793.

[28] Neyman, A. (1991). The positive value of information. Games and Economic Behavior, 3: 350–355.

[29] Rao, V. R. (1993). Chapter 11 Pricing models in marketing. In Handbooks in Operations Research and Management Science (Vol. 5, pp. 517-552): Elsevier.

[30] Rao, V. R. (2009). Editor. Handbook of Pricing Research in Marketing. Edward Elgar.

[31] Shelegia, S. (2012). Multiproduct pricing in oligopoly. International Journal of Industrial Organization, 30 (2): 231-242.

[32] Shin, H., S. Park, E. Lee and W.C. Benton (2015) A classification of the literature on the planning of substitutable products. European Journal of Operational Research, 246 (3): 686-699.

[33] Shubik, M. and R. Levitan (1980). Market structure and behavior. Harvard University Press.

[34] Sigué, S.P. and S. Karray (2007). Price competition during and after promotions. Canadian Journal of Administrative Sciences, 24 (2): 80-93.

[35] Spence, M. (1976). Product differentiation and welfare. American Economic Review, 66 (2): 407-414.

[36] Song, I. and P. K. Chintagunta (2006). Measuring cross-category price effects with aggregate store data. Management Science, 52 (10): 1594-1609.

[37] Soon, W. (2011). A review of multi-product pricing models. Applied Mathematics and Computation, 217 (21): 8149-8165.