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On the first order dynamics of subphonon lifetime transient stimulated Brillouin scattering

P. Chamorro-Posada [a,*], J. Bengoechea de la Llera [b]

[a] Dpto. de Teor de la Senal y Comunicaciones, Ingenieria Telematica, Universidad de Valladolid, Paseo Belen 15, Valladolid, 47011, Spain
[b] Lumiker Aplicaciones Tecnologicas S.L, Parque tecnologico de Vizcaya, Astondo Bidea 612, Derio, Vizcaya 48160, Spain

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ABSTRACT

In this work, we address the phonon dynamics in the stimulated Brillouin scattering process. There exists a wide consensus on the fact that the slowly varying approximation for the acoustic field cannot be invoked in the transient subphonon regime. We present an analysis where we set the precise limits of validity of this approximation. Our study shows that the resonant behavior of the Stokes interaction permits, in general, a first order treatment of this process in the theoretical analysis of current practical applications. An improved-accuracy first order model is also put forward. Numerical calculations are used to support our conclusions.

1. Introduction

Stimulated Brillouin scattering (SBS) finds many applications [1]. For instance, in strain and temperature optical fiber sensors [2–6], for optical pulse compression [7–11], for correcting wavefront aberrations through phase conjugation [12], for fast and slow light systems [13–20] or optical amplification [21,22]. The acoustic field dynamics dictate the main characteristics of SBS in its practical usages because the phonon lifetime determines the width of the Brillouin gain spectrum. Therefore, the SBS threshold can be significantly increased if the pump coherence or modulation bandwidths become comparable to the inverse of the phonon lifetime [23]. Optical pulse compression is a purely dynamic effect and its accurate modeling requires to take into account the evolution of the acoustic field in the medium [8,9,7,10,11]. Similarly, in Brillouin fiber sensors, either a phase modulated or a pulsed Stokes and/or pump optical signal is employed and the theoretical modeling requires an adequate treatment of the phonon dynamics in the optical fiber [2–4].

When the signals participating in the stimulated Brillouin scattering process bear smooth variations, as it typically happens, for example, when optical pulses with duration larger than the phonon lifetime are employed, the dynamics of the acoustic field can be reduced to a first order model by the introduction of the slowly varying approximation (SVA). In the transient subphonon lifetime regime, abrupt pulse transitions of the order of the phonon oscillation period can be found and the SVA, in principle, could be under suspect. The customary treatment of the problem is, then, to use the full second order equation to rigorously analyze the scattering dynamics [1,4,7,18–22,24–28]. In spite of the existence of a broad consensus on this regard, some works have provided both numerical [3] and approximate analytical [2] results in the subphonon lifetime regime obtained invoking the SVA that fit remarkably well to the experimental results. This work aims to illuminate this apparent contradiction.

We present a detailed study of the phonon dynamics in the SBS process. We show that the Navier-Stokes equation that describes the evolution of the acoustic field can in fact be simplified to a first order equation even in the subphonon lifetime regime. Both approximate and exact first order models for the phonon dynamics are studied. These are obtained, respectively, by simply neglecting the second order term and by modifying the first order dynamics to accurately reproduce the second order response. In all cases, numerical simulations performed with subphonon lifetime optical pulses show no significant difference with those obtained solving the full second order model. Finally, the precise limits for the application of the SVA are presented.

2. Model equations

The evolution equations of the pump, Stokes, and acoustic phonon fields were established some decades ago by Boyd from the Navier-Stokes equation [29]. Here, we rewrite them with the particular notation used in [4],

* Corresponding author.
E-mail address: pedcha@tel.uva.es (P. Chamorro-Posada).

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$$\frac{\partial A_P}{\partial t} + v\frac{\partial A_P}{\partial z} = -jvQA_S \quad (1)$$

$$\frac{\partial A_S}{\partial t} - v\frac{\partial A_S}{\partial z} = -jvQ^* A_P \quad (2)$$

$$\frac{\partial^2 Q}{\partial t^2} + 2(\Gamma - j\Omega)\frac{\partial Q}{\partial t} + (\Omega_B^2 - \Omega^2 - 2j\Gamma\Omega)Q = -\Gamma\Omega g_B A_P A_S^*, \quad (3)$$

where the optical powers are related to the complex field amplitudes as $P_{P,S} = |A_{P,S}|^2 A_{eff}$. Ω_B is the Brillouin resonance frequency, which corresponds to the hypersonic acoustic field frequency, A_{eff} is the effective area of the optical field, $v = c/n$, g_B is the Brillouin gain at resonance, and $\Gamma = 1/\tau_{ph}$, with τ_{ph} the phonon lifetime. To obtain (3), the spatial derivatives are neglected in the Navier-Stokes equation because the speed of the acoustical waves is much smaller than the velocity of light in the medium, and the interaction is assumed to be local [4,7].

3. Analysis

Eq. (3) describes the response of the phonon field driven by the optical signals that appear in the right hand side of the equation. For sufficiently long and smooth pulses, the second order derivative can be safely neglected according to the SVA. This condition implies that $|\partial^2 Q/\partial t^2| \ll 2|(\Gamma - j\Omega)||\partial Q/\partial t|$. The period of the acoustic vibration is shorter than 100 ps and, therefore, the leading term determining the validity of the SVA in the coefficient of $\partial Q/\partial t$ is the magnitude of Ω.

In [7], the precise argument used to keep the full second order Eq. (3) for the subphonon lifetime transient regime is that the spectral width of the optical pulses is usually not less than one order of magnitude smaller than Ω. The actual bandwidth of a pulsed signal does not depend solely on its duration, but also on the particular pulse waveform. For pulsed optical signals, the magnitudes of the time derivatives will depend on the steepness of their transitions and the specific shaping of the rising and falling edges of the pulses. When the duration of the optical pulses becomes of the order of the phonon lifetime [4,7], the signals involved in the SBS process are typically switched on and off in a very short time in such a way that an aprioristic estimation of $|\partial^2 Q/\partial t^2|$ based on the bandwidth of the optical signals could suggest that this term is comparable to $2|(\Gamma - j\Omega)||\partial Q/\partial t|$ and the SVA would, in principle, be questionable. The common practice of retaining the second order term of Eq. (3) would then seem to be well-justified. We address all these issues in detail in the following sections, where we show that, in fact, this is not the case.

3.1. Resonant and non-resonant contributions to the phonon dynamics

The impulse response of Eq. (3) is [7]

$$h(t) = \frac{-v\Gamma g_B \Omega}{2\sqrt{\Omega_B^2 - \Gamma^2}} \exp(-\Gamma t)\exp(j\Omega t) \times \sin\left(\sqrt{\Omega_B^2 - \Gamma^2}\, t\right) u(t), \quad (4)$$

where $u(t)$ is the Heaviside step function, and the solution of the phonon dynamics (3) can be written as a convolution integral [7]

$$Q(z,t) = \int_{-\infty}^{t} h(t-\tau) A_P(z,\tau) A_S^*(z,\tau) d\tau. \quad (5)$$

The corresponding transfer function is

$$H(s) = \frac{-\Gamma\Omega g_B}{s^2 + 2(\Gamma - j\Omega) + (\Omega_B^2 - \Omega^2 - 2j\Gamma\Omega)} \quad (6)$$

which has poles

$$p_{1,2} = -\Gamma + j\left(\Omega \mp \sqrt{\Omega_B^2 - \Gamma^2}\right), \quad (7)$$

where the minus and plus sign correspond, respectively, to subscript 1 and 2.

The transfer function can be expanded as a sum of contributions from each pole

$$H(s) = \frac{j\Gamma\Omega g_B}{2\sqrt{\Omega^2 - \Gamma^2}} \left[\frac{1}{s - \left(-\Gamma + j\left(\Omega + \sqrt{\Omega_B^2 - \Gamma^2}\right)\right)} - \frac{1}{s - \left(-\Gamma + j\left(\Omega - \sqrt{\Omega_B^2 - \Gamma^2}\right)\right)} \right]. \quad (8)$$

Since $\Gamma \ll \Omega_B$, we can approximate the resonance frequencies of the two poles as

$$\Omega_1 = \Omega - \sqrt{\Omega_B^2 - \Gamma^2} \simeq \Omega - \Omega_B + \frac{\Gamma^2}{2\Omega_B}, \quad (9)$$

and

$$\Omega_2 = \Omega + \sqrt{\Omega_B^2 - \Gamma^2} \simeq \Omega + \Omega_B + \frac{\Gamma^2}{2\Omega_B}. \quad (10)$$

If we expand the sine in the impulse response (4), we obtain, at a given point within the Brillouin medium, two contributions to the convolution integral (5), namely,

$$Q_1(t) = C_1 \exp(-\Gamma t)\exp(j\Omega_1 t) \times \int_{-\infty}^{t} A_P(\tau) A_S(\tau)^* \exp(\Gamma\tau)\exp(-j\Omega_1\tau) d\tau \quad (11)$$

and

$$Q_2(t) = C_2 \exp(-\Gamma t)\exp(j\Omega_2 t) \times \int_{-\infty}^{t} A_P(\tau) A_S(\tau)^* \exp(\Gamma\tau)\exp(-j\Omega_2\tau) d\tau \quad (12)$$

with $C_l = \frac{j(-1)^{(l+1)} v\Gamma g_B \Omega}{4\sqrt{\Omega_B^2 - \Gamma^2}}$, $l = 1, 2$, for the responses from poles p_1 and p_2, respectively.

A_P and A_S are the baseband complex envelopes of the pump and Stokes waves, respectively. For an efficient interaction, we have $\Omega \sim \Omega_B$. Thus, from (9), we have $\Omega_1 \sim 0$, and the integral in (11) gives a resonant interaction contribution. On the other hand, the same considerations imply that $\Omega_2 \sim 2\Omega_B$ in (10). So, the off-resonant interaction described by (12), with a rapidly varying integrand, will produce a vanishing first order response [31]. For an abrupt pulsed response, there exists also a higher order contribution to the integral (12) produced by the pulse edges [31], but this term is much smaller than Q_1 and will not affect the system dynamics. When the duration of the pulses becomes comparable to $2\pi/\Omega_2$, the argument of the integral in Eq. (12) is no longer rapidly varying, and the contributions from Q_1 and Q_2 become comparable. Notwithstanding, in this extreme regime, both contributions are very small, and this limit is not likely to be of interest for most applications of SBS.

Consequently, for SBS with optical pulses of duration T, the contribution from (11) clearly dominates over Q_2 provided that $T \gg \pi/\Omega_B$ and, under this condition, the effective acoustic response is of first order even for signals in the subphonon lifetime regime that might bear very fast variations. This is analogous to the rotating wave approximation used in the analysis of the interaction of an atomic medium and a classical field [30]. The first resonance at Ω_1 corresponds to the Stokes interaction, which is implicitly assumed in the model equations. The second order dynamics of the phonon field brings into (3) a second, non-resonant, contribution at Ω_2, associated to the anti-Stokes interaction. Nevertheless, this will have a negligible practical effect in the solution of Eqs. (1)–(3).

3.2. Improved accuracy first order dynamics

From the analysis of the previous section, it can be concluded that the phonon dynamics are essentially of first order under rather broad conditions, running deep in the sub-phonon lifetime regime. In fact, we can construct a first order evolution equation which is almost as accurate as the second order (3) if we keep only the exact contribution from pole p_1 in (8) as

$$H_2(s) = \frac{j\Gamma\Omega g_B}{2\sqrt{\Omega^2-\Gamma^2}}\frac{1}{s-\left(-\Gamma+j\left(\Omega+\sqrt{\Omega_B^2-\Gamma^2}\right)\right)}. \quad (13)$$

The corresponding first-order evolution equation reads

$$\frac{\partial Q}{\partial t}+(-\Gamma+j\sqrt{\Omega_B^2-\Gamma^2})Q = \frac{j\Gamma\Omega g_B}{2\sqrt{\Omega^2-\Gamma^2}}A_P A_S^* \quad (14)$$

and it can be used as a replacement of (3).

It is interesting to compare our model Eq. (14) with the result obtained by directly neglecting the second order derivative in (3). This latter approach results in the equation

$$2(\Gamma-j\Omega)\frac{\partial Q}{\partial t}+(\Omega_B^2-\Omega^2-2j\Gamma\Omega)Q = -\Gamma\Omega g_B A_P A_S^* \quad (15)$$

which has a pole at

$$p = -\frac{\Gamma\left(\Omega_B^2+\Omega^2\right)}{2(\Gamma^2+\Omega^2)}+j\frac{\Omega\left[\left(\Omega^2-\Omega_B^2\right)-2\Gamma^2\right]}{2(\Gamma^2+\Omega^2)}. \quad (16)$$

Again, assuming $\Gamma \ll \Omega_B \sim \Omega$, the pole can be approximated as

$$p \simeq -\Gamma-j\left(\Omega-\Omega_B+\frac{\Gamma^2}{\Omega_B}\right), \quad (17)$$

which corresponds to an error of $\sim \frac{\Gamma^2}{2\Omega_B}$ in the resonance frequency when compared with the exact dynamics (3) and the proposed model (14). The value of the decay constant of model (15), on the other hand, is a good approximation to the exact value of (14).

3.3. Pulsed interactions

As discussed in Section 3.1, the worst case regarding the validity of SVA in (3) is found when abruptly pulsed optical signals are involved. In this section we address this particular case, showing that infinitely steep transitions in the optical field are consistent with the SVA. For this purpose, we derive an approximate analytical expression for $Q(t)$ that will be also valuable in the subsequent analyses of the numerical results presented in Section 4.

We assume, for definiteness, a continuous wave (CW) pump $A_P = A_{P,0}$ and a Stokes signal with the shape of a pulse on top of a CW background $A_S = A_{S,0} + A(t)$. However, the conclusions derived from our analysis are of a broad character and they can be extrapolated to any situation where pulsed optical signals are involved. For simplicity, we also consider an Stokes excitation at resonance, with $\Omega_1 = 0$ in (11), obtaining

$$Q_1(t) = \frac{C_1 A_{P,0}A_{S,0}^*}{\Gamma}+C_1 A_{P,0}\exp(-\Gamma t)\int_{-\infty}^{t} A(\tau)^*\exp(\Gamma\tau)d\tau. \quad (18)$$

We also assume a Stokes pulse $A(t)$ of duration T, with $1/(2\Omega_1) \ll T \ll \tau_{ph}$. Under the condition $1/(2\Omega_1) \ll T$, $Q_2(t)$ is much smaller than $Q_1(t)$ and it can be neglected, as discussed in the previous section. Since the Stokes pulse $A(t)$ is of duration $T \ll \tau_{ph}$, we have $\exp(-\Gamma t) \simeq 1$ in the range $0 \leqslant t \leqslant T$. We also consider ideally sharp transitions in the stokes pulse $A(t) = A_0(u(t)-u(t-T))$, therefore maximizing its possible bandwidth. This permits, for $t > T$, to approximate

$$Q(t) \simeq \frac{C_1 A_{P,0}A_{S,0}^*}{\Gamma}+C_1 A_{P,0}A_0^*\frac{exp(\Gamma T)-1}{\Gamma}\exp(-\Gamma t) \quad (19)$$

$$\frac{dQ(t)}{dt} \simeq -C_1 A_{P,0}A_0^*[\exp(\Gamma T)-1]\exp(-\Gamma t), \quad (20)$$

$$\frac{d^2Q(t)}{dt^2} \simeq \Gamma C_1 A_{P,0}A_0^*[\exp(\Gamma T)-1]\exp(-\Gamma t), \quad (21)$$

thus obtaining the result $\left|d^2Q(t)/dt^2\right| / |dQ(t)/dt| = \Gamma \ll \Omega_1$, which is accordant with the validity of the SVA even for highly steep pulses.

When the duration of the pulse $A(t)$ is increased, while in the sub-phonon transient regime, $T \lesssim \tau_{ph}$, the variation of $\exp(-\Gamma t)$ in $0 \leqslant t \leqslant T$ is still small, but it will have some effect on the calculations. Nevertheless, the previous considerations regarding the relative values of the derivatives of $Q(t)$, $\left|d^2Q(t)/dt^2\right| / |dQ(t)/dt| \ll \Omega_1$, will hold in general for these longer pulses.

The approximate analytical results presented in this section permit to reaffirm the consistency of the SVA for pulsed optical signals of short duration despite the presence of very abrupt transitions even when considerations based on estimates of the bandwidth of the optical pulses might suggest the opposite [7]. Moreover, these approximate solutions depict in a very clear manner the flaw in the widespread reasoning that initiates this study. The rationale behind the use of the full second order dynamics for the phonon field when broad spectrum optical pulses are used does not take into account the fact that the bandwidth of the acoustic field, as illustrated by Eq. (20), has little dependence on the spectral width of the optical signals and, on the contrary, is largely determined by the phonon decay rate, as shown by the time-dependence $\exp(-\Gamma t)$ in Eq. (19).

4. Numerical results

In order to assess our analysis, we have performed numerical simulations comparing the results obtained with and without using the SVA for the phonon dynamics. A split-step method [7] has been employed for the calculations. The time-step in the discretization has been taken sufficiently small such that any effect due to the pole at Ω_2 can be adequately captured in the simulations.

We base our numerical survey on the scenario described in Ref. [4] that corresponds to a simple Brillouin optical time-domain analysis (BOTDA) system. Because of its simplicity, this configuration is particularly adequate to address the validity of our analysis while it does not imply any loss of generality. In the setup, the CW pump and Stokes signals are first set at the two ends of the measurement fiber. We input the pump and Stokes continuous wave (CW) signals at $z = 0$ and $z = L$, respectively, where L is the length of the fiber. Once the steady-state (SS) has been reached, a short Stokes pulse is injected into the fiber. This produces a disturbance on the output pump signal that depends on the detuning from the Brillouin resonance which, in turn, is a function of the measurement variable. We consider an optical fiber with a length of $L = 10$ m and homogeneous conditions along its length. Typical parameters of a single-mode fiber working at $\lambda_0 = 1.3$ μm are assumed [4]: $\tau_{ph} = 10$ ns, $F_B = \Omega_B/(2\pi) = 12.8$ GHz, $g_B = 5\times10^{-11}$ m/W, $A_{eff} = 50$ μm^2, and $v = 0.2$ m/ns.

The Stokes measurement pulse $A(t)$ of duration T is assumed to have the same shape as in [4]

$$A(t) = \left[\frac{1}{2}\left(\tanh\left(\frac{t+T/2}{a}\right)-\tanh\left(\frac{t-T/2}{a}\right)\right)\right]^{\frac{1}{2}} \quad (22)$$

and it is superimposed to the CW Stokes background. In all the cases analyzed, the value of a is chosen for setting the rise (and fall) time of the pulse to 100 ps, which is of the order of magnitude of the acoustic field oscillation period, the pump power is $P_p = 5$ mW, the Stokes pulse peak power is $P_s = 10$ mW and the extinction ration between the Stokes pulse peak power and the CW background is $ER = 15$ dB.

The space-time evolution of the pump, Stokes and acoustic field intensities for a $T = 3$ ns Stokes pulse are depicted in Fig. 1 at the left, middle and right columns, respectively. The top row shows the results obtained from the integration of the full second order model (Eq. 3). The middle row displays the results obtained under the SVA and the bottom row shows the relative errors of the results of the middle row in relation with the results from the top row. The CW pump and Stokes signals are injected in the fiber at $z = 0$ and $z = L$, respectively. When a steady-state (SS) condition is established, a $T = 3$ ns Stokes pulse is launched at $z = L$ on top of the CW Stokes signal. The propagation of the Stokes pulse is accompanied by a disturbance of the phonon field intensity and a depletion of the pump. After the Stokes pulse has traveled along the fiber, the SS situation is recovered after a transient in the pump signal that has a maximal duration of the order $2T_r$, where $T_r = L/v$ is the propagation delay in the fiber, at $z = L$. The basis of the BOTDA measurement is the read-out of this transient response in the pump signal at the fiber output.

As regards the error due to the SVA, the analysis presented in Section 3.1 predicts a very small error due to the abrupt pulse edges with a magnitude that depends on the pump and Stokes CW and pulse intensities and the fiber parameters and that is independent from the pulse duration.

By analyzing the relative error in the acoustic field signal under the SVA, we find that it is indeed of a very small value, as predicted by the theoretical analysis and, certainly, it is localized close to the edges of the Stokes optical pulse. Furthermore, the impact of this small error in the acoustic field intensity on the pump power results in a relative error that is still three orders of magnitude smaller in the read-out signal. It is noteworthy that the acoustic field intensity resulting from the propagation of the Stokes pulse superimposed to the pump and Stokes CW signals is very accurately described, for $t > T$, by the analytical expression given in Eq. (19).

Figs. 2 and 3 show the corresponding results for Stokes pulses with $T = 0.5$ ns and $T = 0.2$ ns, respectively, while all the other parameters take the same values in all the simulations. We can observe that the intensity of the resulting acoustic field diminishes with the decrease of the Stokes pulse energy as it is shortened. Nevertheless, the amplitude of the relative error in the acoustic field intensity is kept always at the same order of magnitude, as expected from the analysis presented in Section 3.1, and localized close to the edges of the Stokes pulse. Correspondingly, the order of magnitude of its impact on the relative errors found in P_p and P_s are, consistently, very similar for all pulse durations.

Furthermore, in conformity with our previous analysis, the results of the simulations show that the peak values of the phonon field intensity decrease with the energy in the Stokes pulse, but they display a space-time waveform that does not vary significantly with the change of the width of the optical pulse and it is determined by the value of Γ therefore confirming the validity of the SVA for very short optical pulses.

Even though the duration of the Stokes pulse is smaller than the phonon life-time, the variation of the pump power at the output permits to determine the Brillouin gain spectrum [24,3,4]. This is due to the fact that the transient generated by this short pulse produces a mensurable

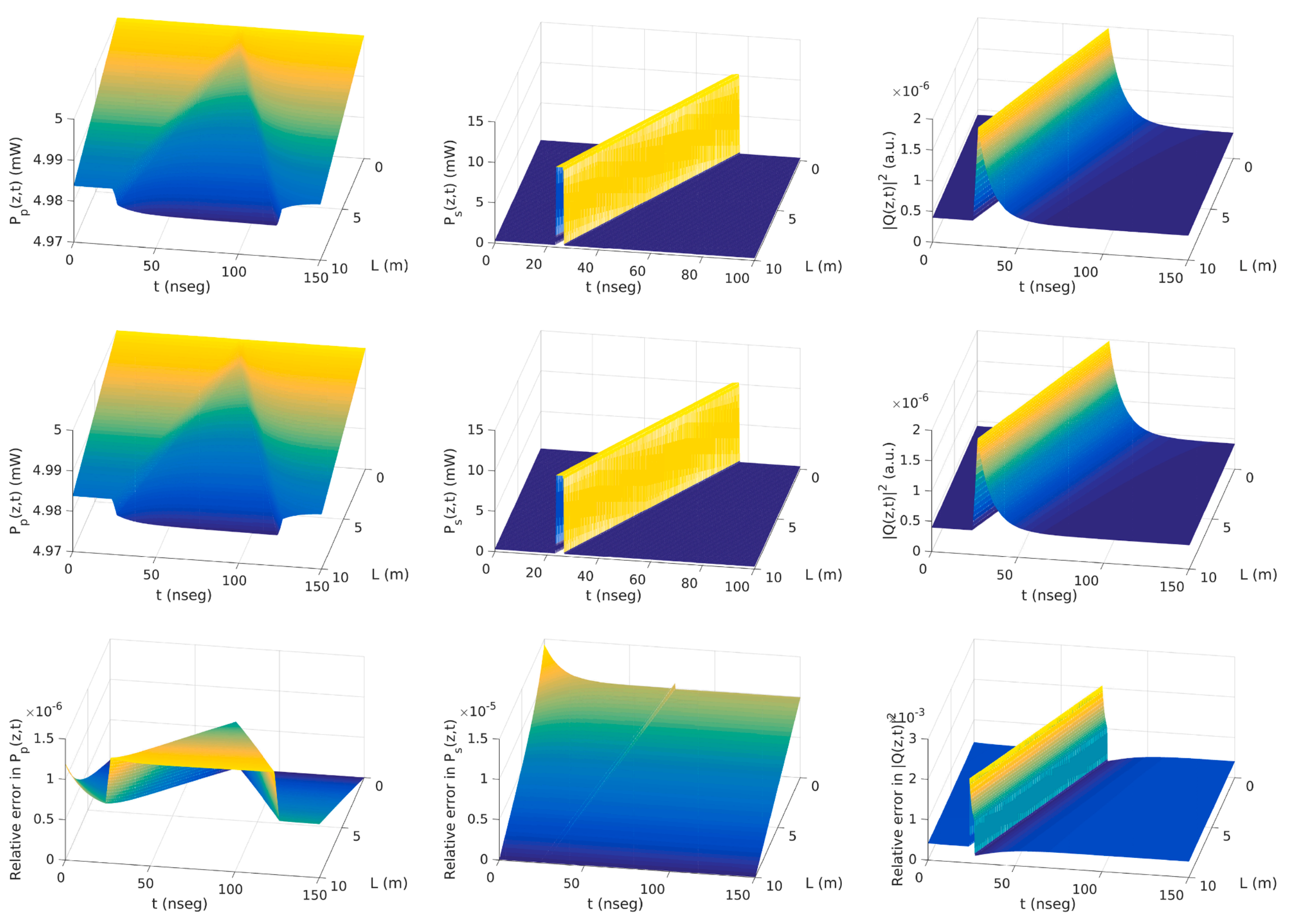

Fig. 1. Numerical result for a 3 ns pulse. The results displayed at the left, center and right column correspond, respectively, to the pump, Stokes, and phonon signals. The top row displays the results obtained using Eq. (3), the middle row those using Eq. (15), and the bottom row the relative errors between the two.

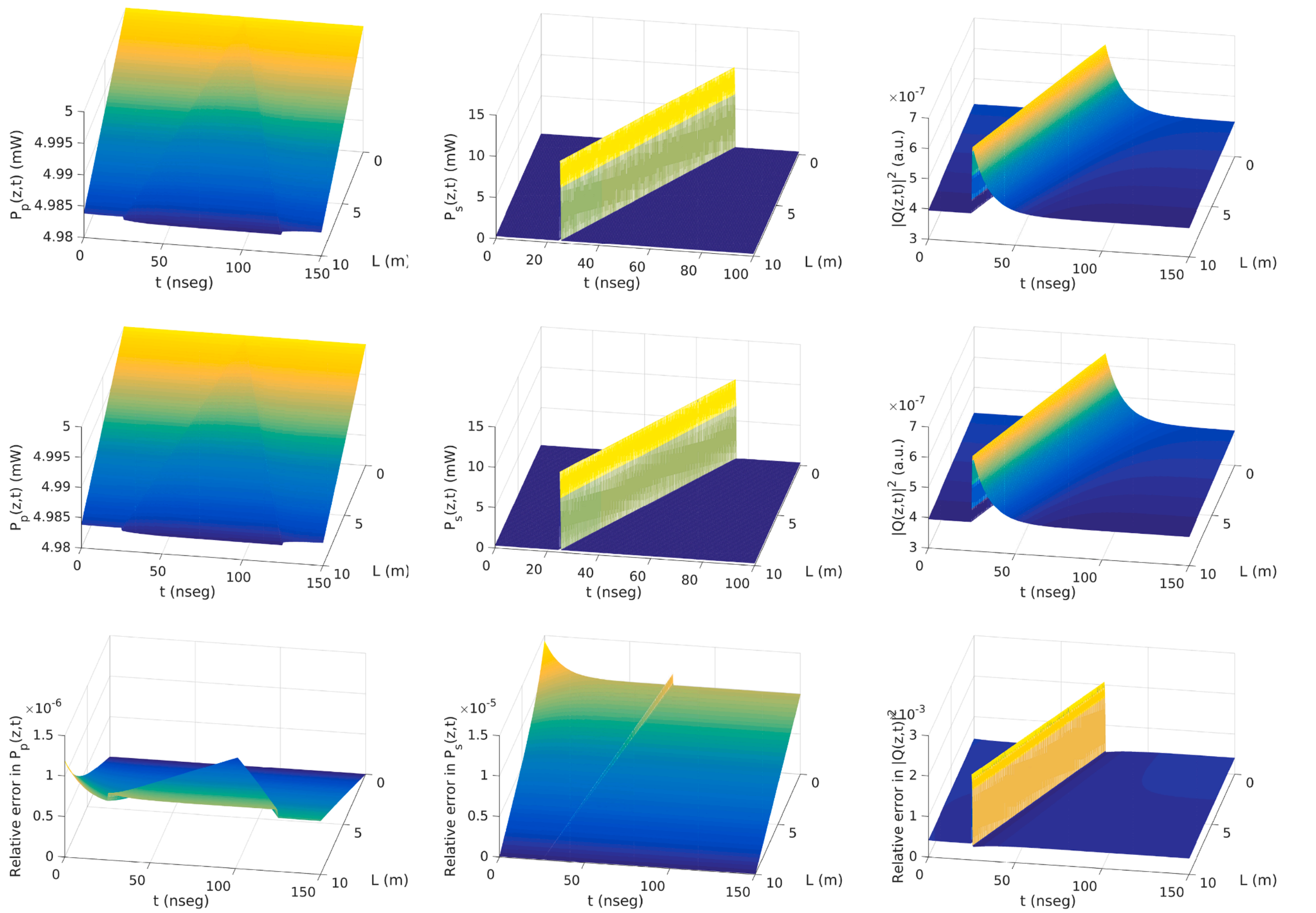

Fig. 2. Numerical result for a 0.5 ns pulse. The results displayed at the left, center and right column correspond, respectively, to the pump, Stokes, and phonon signals. The top row displays the results obtained using Eq. (3), the middle row those using Eq. (15), and the bottom row the relative errors between the two.

effect on the previous steady-state distribution [24,3,4]. Both the DC and AC pump fluctuations, $\alpha_{DC} = P_p - P_p(z = L, t)$ and $\alpha_{AC} = P_p(z = L, t = 0) - P_p(z = L, t)$, respectively, can be used for the measurement, even though higher contrast is provided by α_{AC} [4].

Fig. 4 shows the AC pump fluctuations at $z = L, \alpha_{AC} = P_p(z = L, t = 0) - P_p(z = L, t)$ $\alpha_{AC}(t)$, as a function of the frequency detuning Δf of the Stokes field from the Brillouin resonance obtained by scanning the full Brillouin response. The results for Stokes pulse widths of 3 ns, 0.5 ns and 0.2 ns are displayed. As expected, the shapes of the measured responses remain very similar under the variation of the duration of the Stokes pulses whereas the amplitude of the measured response diminishes as the energy in the Stokes pulse is reduced. Also, the results of the calculations performed with the second order model (top row) are almost identical to those obtained under the SVA (bottom row), in accordance with the predictions from our analysis.

For the assessment of the results derived from our analysis, we have chosen an implementation of a simple BOTDA system that is well-document in the literature [4], relevant to the discussion presented in this work, and convenient because of its simplicity. There exist alternative state-of-the-art Brillouin measurement systems that offer a performance largely improved in relation to that of this simple scheme. Nevertheless, the aim of this numerical survey is to clearly illustrate that the SVA can be applied even in the sub-phonon lifetime transient SBS as predicted by our analyses. The results presented are relevant for more elaborate and efficient Brillouin sensing systems [5], pulse compression [7–11] and amplification[21,22], and fast and slow light systems [18–20], whenever short optical pulses are involved.

5. Conclusion

We have presented an analysis of the dynamical effects of the acoustic fluctuations in stimulated Brillouin scattering in the subphonon lifetime regime. Our results show that the phonon dynamics are essentially of first order in this regime unless the widths of the optical pulses are comparable with half the period of the acoustic oscillations, that is, of a few tenths of ps for the parameters used in our simulations. Such ultra-short optical pulses are unlikely to be of interest in SBS applications except, possibly, for very high energy implementations.

This main result arises due to the fact that only one of the poles of the full second order evolution equation depicts a resonant interaction with the optical fields, whereas the existence of a second pole has a negligible effect on the system dynamics, except for a low-order contribution arising from the edges of optical pulses with very abrupt transitions.

The theoretical analyses are supported by detailed numerical calculations that have been performed for a fiber optic setup involving a Stokes pulse with durations shorter than the phonon lifetime ranging from a few ns to a few hundreds of ps.

The results obtained from the numerical simulations are consistent with those derived from our theoretical analyses, confirming the existence of a negligible error in the acoustic field solution that is localized at the edges of the optical pulse used in the simulations. As expected, the magnitude of this error is independent of the pulse width when this is the only parameter that is changed in the calculations. The impact of this very small relative error on the measured optical signals is far lesser: the magnitude of the relative error in the optical signals resulting from that of the acoustic field intensity is three orders of magnitude smaller.

The pitfall that leads to the inclusion of the second order term in the

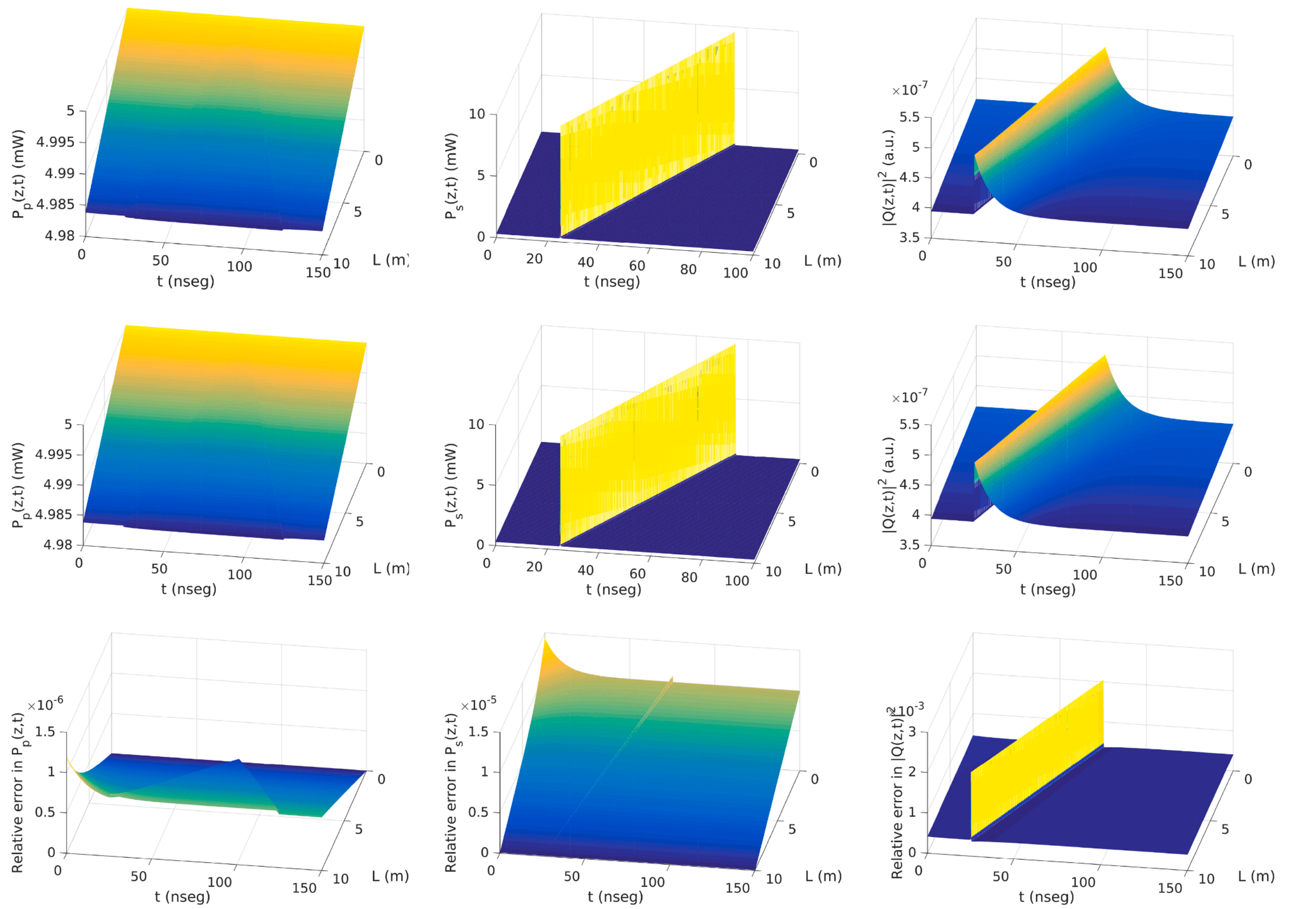

Fig. 3. Numerical result for a 0.2 ns pulse. The results displayed at the left, center and right column correspond, respectively, to the pump, Stokes, and phonon signals. The top row displays the results obtained using Eq. (3), the middle row those using Eq. (15), and the bottom row the relative errors between the two.

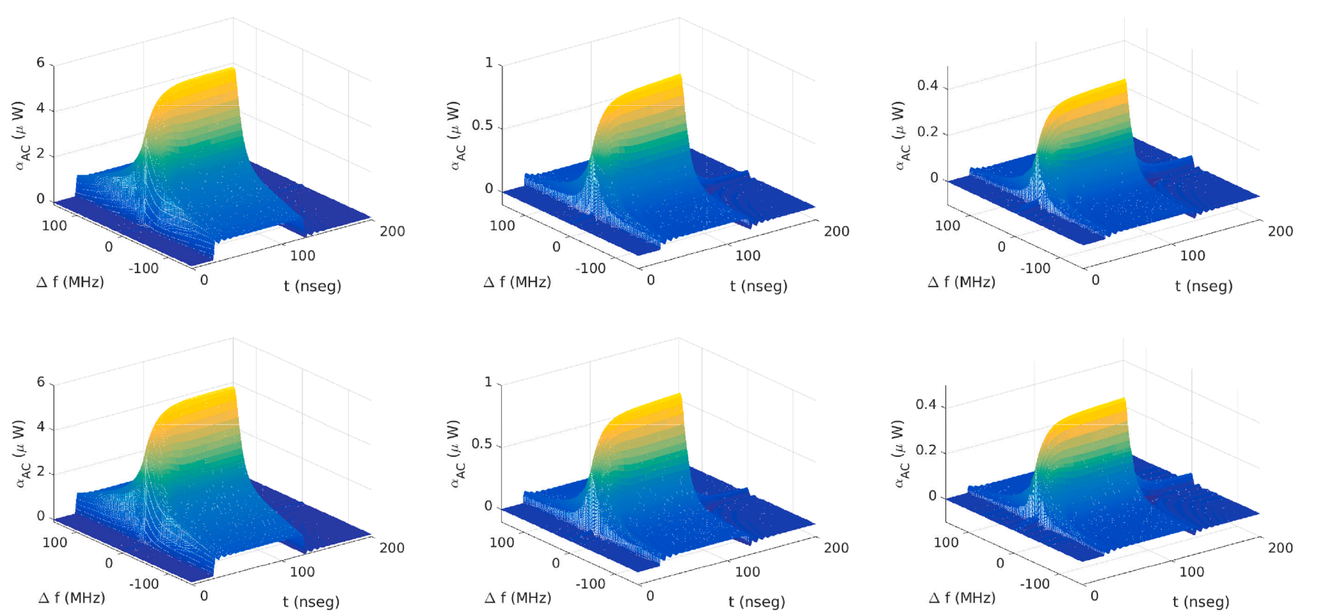

Fig. 4. Numerical results obtained for α_{AC} measurements with a 3 ns pulse (left column), a 0.5 ns pulse (center column), and a 0.2 ns pulse (right column) without and with the use of the SVA (top and bottom row, respectively).

acoustic field dynamics has it origin in a reasoning based on the bandwidth of the optical pulses involved in the SBS process, but our analysis shows that under very broad conditions the waveform of the phonon field is independent of the bandwidth of the optical signals and it is predominantly determined by the phonon lifetime Γ, with the result that the range of applicability of the SVA is far wider than expected. This is illustrated in a particularly insightful manner by a complementary analysis based on the approximate solutions for SBS using short optical pulses.

CRediT authorship contribution statement

P. Chamorro-Posada: Conceptualization, Investigation, Writing - original draft. **J. Bengoechea de la Llera:** Investigation.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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