Journal of Process Control 133 (2024) 103142

Contents lists available at ScienceDirect

Journal of Process Control

journal homepage: www.elsevier.com/locate/jprocont

Optimization of crude oil operations scheduling by applying a two-stage stochastic programming approach with risk management

Tomas Garcia Garcia-Verdier [a,b,*], Gloria Gutierrez [a,b], Carlos A. Méndez [c], Carlos G. Palacín [a], Cesar de Prada [a,b]

[a] *Department of Systems Engineering and Automatic Control, Universidad de Valladolid, Dr. Mergelina s/n, Valladolid, 47011, Spain*
[b] *Institute of Sustainable Processes, Dr. Mergelina s/n, Valladolid, 47011, Spain*
[c] *Center for Advanced Process Systems Engineering (CAPSE), INTEC (UNL - CONICET), Industrial Engineering Dpt. (FIQ-UNL), Güemes 3450, Santa Fe, 3000, Argentina*

ARTICLE INFO

Keywords:
Stochastic optimization
Continuous-time representation
Crude oil scheduling
Uncertain oil supply
Conditional Value-at-Risk

ABSTRACT

This paper focuses on the problem of crude oil operations scheduling carried out in a system composed of a refinery and a marine terminal, considering uncertainty in the arrival date of the ships that supply the crudes. To tackle this problem, we develop a two-stage stochastic mixed-integer nonlinear programming (MINLP) model based on continuous-time representation. Furthermore, we extend the proposed model to include risk management by considering the Conditional Value-at-Risk (CVaR) measure as the objective function, and we analyze the solutions obtained for different risk levels. Finally, to evaluate the solution obtained, we calculate the Expected Value of Perfect Information (EVPI) and the Value of the Stochastic Solution (VSS) to assess whether two-stage stochastic programming model offers any advantage over simpler deterministic approaches.

1. Introduction

Oil refineries receive crude oil and transform it into a set of petrol products. Typically, there is a central office on which a group of refineries depend. This central office generates a production plan for a period of time, establishing when crude oil will be received and the type of products that must be produced in that period in each refinery. Then, the operation programming unit of each refinery will develop the detailed scheduling of the operations for that time horizon. Usually, in the schedule development phase, the possibility of unforeseen events is not considered, leading to the potential inapplicability of the generated schedule. One of the main sources of uncertainty that can lead to this situation is the change in the scheduled arrival dates of ships that supply crude to the refineries.

The general structure of a refinery can be divided into three parts based on the processes carried out. The first part concerns the crude oil operations scheduling, which involves crude oil unloading, inventory management, and the feeding schedule for crude distillation units. The second part corresponds to production unit scheduling, which includes both fractionation and reaction processes. Finally, the third part is related to the scheduling, blending, storage, and delivery of final products.

In this paper, we address the optimization problem of crude oil operations scheduling in a refinery that is supplied with crude oil by ships whose arrival dates are subject to uncertainty. This problem involves a complex network of ships, tanks, pipelines, and crude distillation units (CDUs) [1].

We consider a system that consists of a marine terminal and an oil storing and processing section, both connected by a pipeline (see Fig. 1). The terminal is the facility where the unloading operation of the crude oil transported by the ships begins, which means that it plays the role of a link between the ships and the refinery, where the supplied crude is stored in tanks and then processed. Here we analyze the case of a single-dock terminal, so only one ship can be unloaded at a time.

As for the storing and processing section, two areas can be distinguished: the storage tank area and the crude distillation unit area. The first is connected to the marine terminal by a pipeline and, as its name indicates, is composed of tanks that store the crude oil coming from the terminal. Typically, refineries involve two types of tanks: storage and charging tanks. Storage tanks are used for receiving and storing crude oil from ships, while charging tanks are used to create blends that will feed the distillation units, meeting certain quality specifications. Nevertheless, some refineries remove the charging tanks in order to save space and eliminate immobilized capital, implementing the mixing online in the pipelines that feed the CDUs using an appropriate control system. This is our case where only storage tanks are available. Since

* Corresponding author at: Department of Systems Engineering and Automatic Control, Universidad de Valladolid, Dr. Mergelina s/n, Valladolid, 47011, Spain.
E-mail address: tomasjorge.garcia@uva.es (T.G. Garcia-Verdier).

https://doi.org/10.1016/j.jprocont.2023.103142
Received 16 April 2023; Received in revised form 6 November 2023; Accepted 30 November 2023
Available online 13 December 2023

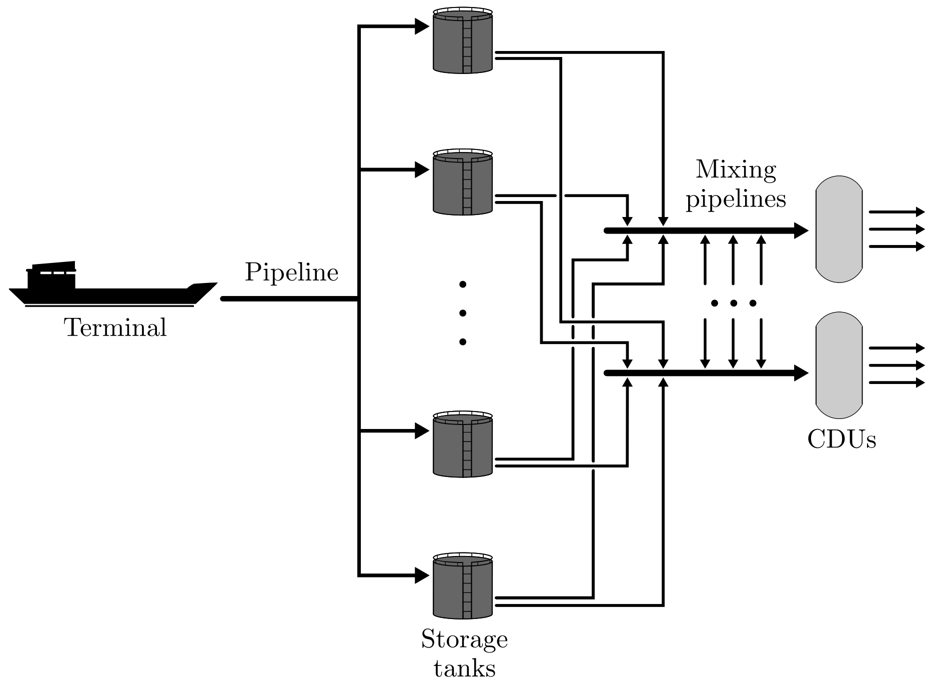

Fig. 1. Schematic of system.

the storage capacity is limited and there are many types of crude oil based on their compositions, the tanks are not exclusively dedicated to a single type of crude; in other words, it is possible to store blends of crude oil.

At the same time, the tanks are connected to the crude distillation unit area through a piping system (mixing pipelines), where the final mixtures of crudes take place in order to achieve the desired flows and properties required by the different crude distillation units (CDUs). All operations are subjected to multiple rules and constraints, among them, the arrival over time of different types and amounts of crude, and the fulfillment of the company production plan.

Regarding crude oil operations scheduling, it means allocating the resources involved (vessels, tanks, CDUs) to operations (e.g., loading a tank, feeding a CDU, etc.) and sequencing them over time so that the constraints and aims of the process are satisfied. Optimal scheduling, in addition, will provide the best use of the resources minimizing a cost function.

One point that is interesting to mention is that analogously to batch process operations scheduling problems, one could think of the volumes of crude oil transferred as batches. However, we should note that in the crude operations scheduling problem, these batches are not defined in advance at the beginning of the scheduling horizon. Therefore, this feature increases the complexity of the problem resolution [2].

2. Literature review

In the last decades, a wide variety of articles have been published in the area of crude oil operations scheduling, covering the development of both deterministic and stochastic models.

2.1. Deterministic models

The paper of Lee et al. [3] involves one of the earlier works to address the optimization of short-term scheduling for crude oil unloading, tank inventory management, and CDU charging. In this article, the authors developed a mixed-integer linear programming (MILP) model that relies on time discretization, in which the bilinear equations arising from mixing operations are replaced with individual component flows to maintain linearity.

In Jia et al. [4], the authors addressed the problem of crude-oil short-term scheduling, which involves optimizing the unloading of crude oil from vessels, its transfer to storage tanks, and the charging of crude distillation units. For this purpose, they developed a novel MILP model based on a continuous-time representation, utilizing the state-task network (STN) representation introduced by Kondili et al. [5].

In the paper Reddy et al. [6], a continuous-time mixed-integer linear programming (MILP) formulation was presented for the short-term scheduling of operations in a refinery that handles crude from very large crude carriers. Moreover, the authors put forward an iterative algorithm to address the crude composition discrepancy. This algorithm entails solving a series of MILPs with gradually reduced size and complexity to achieve a near-optimal solution.

The authors of Furman et al. [7] proposed a mixed-integer nonlinear programming (MINLP) model based on a continuous-time formulation to optimize the scheduling of fluid transfers within tanks and robustly handle the synchronization of time events with material balances. Additionally, they presented a new approach to depict the inflow and outflow from a tank, which holds the potential to reduce the number of time events in continuous-time scheduling formulations. Subsequently, the modeling paradigm was applied to develop charging schedules for refinery crude units.

Mouret et al. [8] developed a continuous-time MINLP formulation to tackle crude-oil scheduling problems. This formulation is based on the representation of a schedule as a sequence of operations and is called the single-operation sequencing (SOS) model. Additionally, they introduced a sequencing rule to address the symmetries that may arise in the model. Finally, a two-step MILP-NLP procedure was implemented to solve the model.

Li et al. [9] developed a novel unit-specific event-based continuous-time MINLP formulation to tackle the crude oil scheduling problem for a marine-access refinery. Furthermore, they introduced a branch-and-bound global optimization algorithm with a piecewise-linear underestimation approach to solve the model.

In Yadav et al. [10], the authors introduced a simplified STN-based formulation to tackle the problem of scheduling crude oil operations using a unit-specific event-based continuous-time representation. The solution strategy proposed by the authors involves relaxing the MINLP model by dropping the nonlinear constraints and solving the resulting MILP model. In case the obtained solution exhibits composition discrepancies, the original MINLP model is also solved to rectify these discrepancies.

Hamisu et al. [11] proposed an enhanced version of the MILP model developed by Lee et al. [3] through the inclusion and modification of a set of constraints that allow for a decrease in operating costs and provide more flexible operation.

The authors of Castro et al. [12] tackled the optimization of scheduling of crude oil blending operations in a refinery. As a solution, they developed an MINLP model based on continuous-time formulation with a single time grid derived from a resource-task network (RTN) superstructure. Subsequently, the MINLP model was solved using a two-step MILP-NLP algorithm, which involves a tight relaxation of the bilinear blending constraints using multiparametric disaggregation.

In Cerdá et al. [13], a MINLP continuous-time approach for scheduling crude oil operations in marine-access refineries was introduced. The developed model is based on global-precedence sequencing variables to establish the order of loading and unloading operations in the storage tanks, and synchronized time slots of variable length are used to model the sequence of feedstock supplied to each CDU.

Zimberg et al. [14] presented a discrete-time MILP model to optimize the reception, blending, and delivery of crude oil from a terminal to a pipeline without considering crude oil processing. The authors proposed replacing the nonlinear equation derived from the blending of crudes in tanks with a set of linear equations that include an adjustment term for composition discrepancies.

The paper of de Assis et al. [15] focused on optimizing operations at a crude oil terminal, specifically, the optimization of crude oil unloading from vessels to storage tanks and transfers from storage tanks to the pipeline that connects the terminal to the refinery. In this work, an MINLP model based on discrete-time formulation was presented, along with an iterative two-step MILP-NLP decomposition algorithm, which involves using piecewise McCormick envelopes to replace bilinear terms and a domain-reduction strategy.

In de Assis et al. [16], the authors tackled the Operational Management of Crude Oil Supply (OMCOS), which involves optimizing the schedule of vessel trips and crude oil operations at a terminal in an integrated manner. As a solution, they proposed a discrete-time MINLP formulation that was solved through an iterative MILP-NLP decomposition approach.

In another paper, de Asis et al. [17] introduced an MILP clustering formulation, whose solution serves as a preliminary step before solving the OMCOS MINLP formulation. Based on the clustering solution, bounds on crude properties inside tanks can be inferred, enabling the linearization of bilinear terms in blending constraints and resulting in an MILP approximation. Subsequently, they applied an MILP-NLP decomposition strategy to achieve a solution for the MINLP model.

2.2. Stochastic programming models

All the works mentioned above tackled the problem through a deterministic approach. However, it is very important to take into account unplanned events for generating practical and useful schedules, so stochastic programming models have also been developed to address the problem of crude oil operations scheduling under uncertainty, among which the following stand out.

In Wang et al. [18], a discrete-time two-stage robust model was proposed to address the crude oil operations scheduling problem considering uncertainty in vessel arrival times and product demand.

Cao et al. [19] presented stochastic chance-constrained mixed-integer nonlinear programming (SCC-MINLP) models based on discrete-time formulation to solve the integrated problem of crude oil short-term scheduling, blending, and storage management under uncertainty in crude distillation unit demands.

In Li et al. [20], the authors addressed the crude oil scheduling problem for a marine-access refinery under demand uncertainty. To achieve this, they utilized the unit-specific event-based continuous-time formulation presented in Li et al. [9] and applied the theory of robust optimization framework to formulate the robust counterpart optimization model.

Oliveira et al. [21] proposed a two-stage stochastic MILP model, based on discrete-time formulation, that simultaneously defines the scheduling of oil pumping through a pipeline and the sequencing of ships berthing at a terminal at the lowest possible cost.

All these papers aimed to minimize the expected value of a certain cost function, but none of them took into account the problem of risk. That is, the minimization of the probability that in some scenarios the cost function may have a very bad value.

At the same time, it is worth noting that most of these authors presented stochastic models using a discrete-time representation, and only one used unit-specific continuous-time formulation. Consequently, we believe that it would be interesting to contribute to the advancement of stochastic models that consider risk management and are based on a predefined set of global time points on an efficient continuous-time formulation to solve the crude oil operations scheduling problem in a refinery with only storage tanks. It is worth noting that global time points refer to instants of time at which the status of the limited resources are evaluated.

3. Contribution

The aim of this paper is to formulate a two-stage stochastic programming model based on continuous-time formulation using global time points to solve the crude oil operations scheduling problem considering uncertainty in the arrival date of supplying ships. Then, a performance analysis using several metrics is conducted to ascertain potential advantages over deterministic models. Moreover, in the paper, the stochastic MINLP model is reformulated to incorporate risk management, and solutions obtained for varying levels of risk aversion are analyzed. Additionally, because of the non-linear terms introduced by the concentrations of the crude mixtures, solving the final MINLP model becomes challenging. Therefore, we present a new approach to address these non-linearities by using a MILP approximation, in which we replace the non-linear equation with two linear constraints. Then, we adopt a two-step MILP-NLP procedure.

This article expands upon the conference paper [22] by evaluating the performance of the stochastic model against deterministic models. This evaluation is based on calculating and interpreting the Expected Value of Perfect Information (EVPI) and the Value of the Stochastic Solution (VSS) measures. Moreover, we carry out a more detailed analysis of solutions obtained using both the risk-neutral approach and the one incorporating risk management. To the best of our knowledge, no other paper considers at once a two-stage stochastic approach, risk management with CVaR, and continuous-time formulation with global time points applied to the optimization of crude oil operations scheduling.

The different points addressed throughout the paper to tackle the crude oil operations scheduling problem under uncertainty in crude oil supply are mentioned next.

In Section 4, we present the crude oil operations scheduling problem in more detail. In Section 5, we develop a two-stage stochastic programming model based on continuous-time formulation with global time points, considering uncertainty in the arrival date of the ships utilizing a discrete set of scenarios with different arrival times. Next, in Section 6, we propose a strategy to cope with the nonlinear constraint resulting from crude oil blending in tanks. Then, the concepts EVPI and VSS are defined in Section 7, which will allow us to assess the solution obtained from the two-stage stochastic programming model against those obtained with deterministic models. In Section 8, we extend the two-stage stochastic programming model, to include risk management, by employing the Conditional Value-at-Risk (CVaR) measure as the objective function. In Section 9, we report an example and the computational results. Here, we compute the EVPI and VSS measures, and we analyze the effect of considering uncertainty in the model. Besides, we compare the results of applying the two-stage stochastic programming risk-neutral approach versus the approach with risk management for different confidence level values. Finally, in Section 10, we draw conclusions and discuss future work.

4. Process operation

The optimization of crude oil operations scheduling can be defined as the process of deciding the best way to operate the system based on the management of four macro-operations in a coordinated way, each of which involves a set of interrelated operations.

- Vessel unloading.
- Tank loading.
- Tank unloading and crude mixing.
- Crude distillation unit loading.

In the following subsections, these macro-operations and the decisions to be made in each of them are explained in more detail.

4.1. Vessel unloading

Initially, the ships arrive at the terminal to unload the crude oil. When two ships arrive at times very close to each other, the order of unloading has to be taken into account to meet the production plan and, at the same time, to minimize vessel demurrage and tardiness costs. Demurrage is defined as the difference between the date on which the unloading of a vessel begins and its arrival date; while tardiness is defined as the difference between the completion of the discharge and the scheduled departure date, being null as long as the vessel finishes on or before the stipulated date.

4.2. Tank loading

Concurrently with the unloading of ships, we must select the receiving tanks. However, it must be taken into account that there is a maximum number of tanks that can be loaded simultaneously, and therefore, their loading must also be sequenced. For example, if we can load a maximum of three tanks at a time and we decide that four tanks will receive the crude, then at least one of the tanks will receive crude oil once the loading of another tank has been completed. In addition to the tank assignment and order, the volumes transferred to each tank must be calculated.

4.3. Tank unloading and crude mixing

In this operation, we must choose the feed from the stored crude oil to each CDU, which consists of selecting and sequencing the participating tanks and calculating the volumes transferred from each of them so that the feed properties in the mixing pipelines are within the established ranges. We must take into account that while the tanks store blends of crude oils, these blends do not necessarily meet the specifications to feed the CDUs. The final blends that meet the specifications are obtained in the mixing pipelines.

As previously mentioned, the tanks can store mixtures of crudes. From this characteristic, we have to consider that the concentration of crude oil in the outlet flow of a tank must be equal to the concentration inside the tank. This behavior is represented by a nonlinear constraint, which is explained in (A.59).

Besides, other constraints must be met, such as the minimum settling time in a tank after receiving a load, in order to allow possible water to settle, and the maximum number of units that can be fed simultaneously from the same tank.

4.4. Crude distillation unit loading

The ‘‘crude distillation unit loading’’ operation is closely related to the ‘‘tank unloading’’ operation, so it is difficult to distinguish which decisions belong to which; in any case, they could be thought of as shared decisions that must be taken considering the constraints and requirements of both.

The distillation units are in charge of processing the blends obtained in the mixing pipelines to meet the demands of the final products. These units must be fed continuously, i.e., their operation cannot be interrupted. Therefore, it complicates the decision-making process and inventory management. As mentioned above, one of the crucial points in the loading of CDUs is compliance with feed quality specifications, i.e., keeping the concentration of blend properties within defined ranges. In addition, there are operating constraints, such as the maximum number of tanks allowed in parallel and the feed flow rates.

One of the main challenges of the crude oil operations scheduling problem lies in coordinating the decisions taken at the terminal and the crude oil section since their objectives differ. While the former seeks to unload the ships as soon as possible to avoid demurrage and departure tardiness costs, the latter aims to have the crude oil available at the most convenient times and in the most convenient qualities and quantities to meet the production plan. One way to address this problem is through the development of a mathematical programming model.

5. Stochastic model formulation

In this work, we make use of mathematical programming to address the problem of crude oil operations scheduling. This problem presents a large number of logical and operational constraints that are not simple to model. Therefore, a challenge lies in developing a model with sufficient complexity to faithfully represent the process but, at the same time, robust and able to be solved in a time according to the users' needs.

Before the development of the model, we must select the time representation to be used and the way to represent the uncertainty. Regarding time representation, we can choose between two basic approaches: discrete-time formulation and continuous-time formulation.

5.1. Time representation

In discrete-time-based models, the programming horizon is divided into a finite number of intervals with a predefined duration, and events – meaning any change in operations – only take place at the beginning or the end of these intervals. This approach facilitates the formulation of the model, in particular regarding the synchronization of events among different resources, but it has the disadvantage that the size of the model, its computational efficiency, and the accuracy of the solutions obtained strongly depend on the number of time intervals defined. The greater the number of intervals, the more precise the solution will be, but the size of the model will increase and, therefore, will require more computational effort for its resolution.

Otherwise, if a smaller number of intervals is defined, the precision of the solution will decrease, and there is a risk of obtaining suboptimal or even infeasible solutions since, for a fixed horizon, a small number of slots would imply that no actions would be taken for an extended period. As a result of this, infeasibility can arise for various reasons. For example, tanks could be emptied (interrupting the feed to units), and certain properties in the blends might not be met, among others. It is also possible that the number of slots may not be sufficient to execute the feasible solution that involves the least possible number of operations over the entire horizon.

One alternative to improve the time precision of the solution and to decrease the number of variables involved is the use of models based on continuous-time formulation, where timing decisions are explicitly

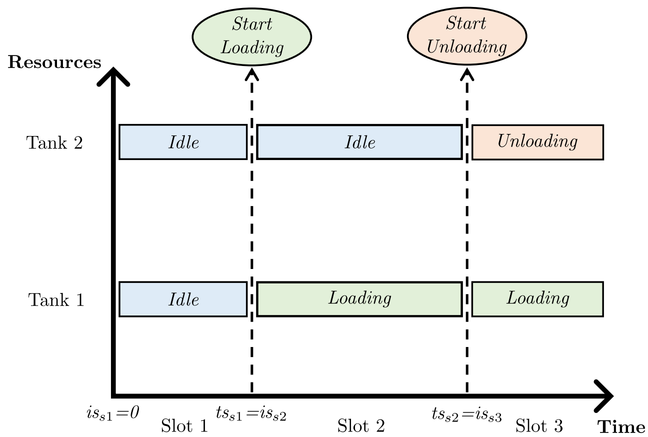

Fig. 2. Continuous-time representation (global time points).

represented as a set of continuous variables defining the exact times at which the events take place [23].

Continuous-time formulations can be represented in different ways, including using global time points or unit-specific time events for network processes, as can be seen in [23]. With global time points, also called MOS-SST in [24], a common time grid is used for all resources. The intervals of time (slots) represent the time between two consecutive events that take place at any resource of the process. In contrast, the unit-specific time events approach (or MOS representation in [24]) defines a unique time grid for each resource, allowing for different operations to start at different times for the same event point.

In our study, we have chosen the option characterized by global time points as a compromise between the ease of synchronization and the complexity of the corresponding continuous-time model. This continuous-time formulation is performed as follows:

- The scheduling horizon is divided into consecutive variable-length slots, synchronized across all the resources (vessels, tanks, and crude distillation units).
- Three mutually exclusive states are defined for the tanks: loading, unloading, and idle.
- A new slot is activated whenever a resource changes its state.
- Even so, a resource can maintain its state during consecutive time slots.

To make it more understandable, Fig. 2 depicts an example involving two tanks and three slots whose start and end times are represented by the variables is_s and ts_s, respectively. Initially, both tanks are idle during slot 1, whose start time is $is_{s1} = 0$. After some time, tank 1 begins loading, and slot 1 ends at that moment, starting slot 2 with $is_{s2} = ts_{s1}$, which is the end time of slot 1. It is important to note that in slot 2, the state of tank 2 ("Idle") has not changed, while the state of tank 1 has changed to "Loading". Then, after some time, tank 2 starts unloading, which ends time slot 2 and begins a new slot 3, whose start time is $is_{s3} = ts_{s2}$. From this moment on, the new state of tank 2 is "Unloading", while the state of tank 1 remains "Loading". Notice that the slot number imposes a precedence of events over time and that, in the proposed formulation, not only the start time is the same for operations belonging to the same slot, but also the duration of the operations.

5.2. Representing the uncertainty

As mentioned earlier, another decision we need to make before developing the mathematical model is whether to formulate a stochastic or a deterministic model.

Deterministic models are very useful and reliable as long as the process to be optimized is not subject to large uncertainties or is not very sensitive to variations in the parameter values. Otherwise, there is a risk that the solution obtained will not be robust enough to adapt to changes in the process environment if the reality is different from the assumption in the model, then the quality of the solution will likely decrease.

When optimizing the scheduling of crude oil operations, we should consider that weather conditions may influence the arrival time of vessels and, thus, the availability of crude oil. In turn, these events affect downstream decisions. Based on this premise, we have considered the arrival dates of ships at the terminal as an important uncertainty that must be considered explicitly. This uncertainty is represented by a set of scenarios that cover possible ship arrival dates with different probabilities that can be obtained with historical data and current conditions.

Therefore, for a more realistic resolution of the scheduling problem in the refinery, it becomes imperative to integrate this uncertainty into the optimization model. Numerous approaches exist for incorporating this kind of uncertainty, including chance-constrained, robust optimization, and two-stage methods, among others.

In chance-constrained, we calculate an optimal policy such that the probability of fulfilling the constraints is greater than a certain user-defined value. The main problem of this approach is the complexity of the numerical solution in a mixed-integer context.

In robust optimization, the decision variable is optimized for the worst case of the uncertain variable so the constraints are satisfied for all values of the uncertainty. The robust solution is guaranteed to remain feasible over the entire range of uncertain parameter realizations [25], but the solution may be too conservative.

Finally, the two-stage stochastic programming approach involves two types of decision variables: the first-stage variables ("here-and-now" variables) which have to be implemented now and influence all future decisions, and the second-stage ones that will be implemented later on when more information about the process is available so that they can be adjusted to the realization of the uncertainty (recourse variables or "wait-and-see" variables). This provides solutions that are less conservative than robust formulations.

In the present work, we propose a model based on two-stage stochastic programming with recourse [26] to tackle the crude oil operations scheduling optimization, considering uncertainty in the arrival date of the vessels, because of the following reasons:

- The uncertainty does not depend on the decisions made and several scenarios can be selected in a sensible way as small variations of the planned arrival dates of ships.
- A discrete probability distribution can be defined for the uncertainty, i.e., for the arrival time of vessels.
- The structure of the problem makes it possible to clearly define which are first-stage decisions and which are second-stage decisions.
- It gives robust solutions without being extremely conservative.

In this problem, the first-stage variables refer to those related to the supply of blends to the CDUs. This involves the allocation of tanks to units, the output flow rates from tanks, and the input flow rates to CDUs. The start time, end time, and length of slots are also considered first-stage decisions.

As for the second-stage decisions, they encompass decisions related to: activities carried out at the marine terminal after a ship has arrived, inventory management (including types of crudes and levels stored in each tank), crude concentration in the output flow from tanks, and therefore crude concentration in the input flows to CDUs. The crude oil concentration indicates the proportion of each type of crude oil in a flow rate. It could also be thought of as the flow rate of each crude oil.

The decision regarding which variables correspond to the first stage is based on the constraint of continuous operation of the CDUs, as well as on the exercise of putting ourselves in the operator's shoes. If we were the decision-makers, then we would have to decide on the feeding of the CDUs at all times, even without knowing the exact arrival dates of the ships. This involves selecting the tanks that will feed the CDUs, the output flow rates from tanks, and the start and end times of these feeding operations. All these decisions determine the flows and composition of the CDU feeds.

On the other hand, each time a ship arrives, we must decide in which tanks the crude oil will be unloaded and the quantity to be transferred. These decisions constitute the recourse variables since they allow us to correct the choices made in the first stage and maintain the solution's feasibility in the scenarios under study.

An important point to consider is related to concentrations. Among the first-stage variables, we have mentioned the flow rate from each tank to each CDU. This flow rate can be defined as the sum of the output flow of each crude oil while respecting their in-tank concentrations. However, the output flow of each type of crude oil is a second-stage variable since the amount of each crude oil in the feeding tank may vary between scenarios. This is equivalent to saying that the composition of each tank is a second-stage variable.

5.3. Scenario definition

As mentioned earlier, the crude oil supply is subject to uncertainty owing to possible deviations in the scheduled arrival time of the ships. This uncertainty is depicted by a discrete set of scenarios that contemplates different arrival times. These scenarios are generated as follows.

For each ship, we assess a most likely arrival date according to the company planning and the potential deviations from it, both in terms of arriving earlier or later. This process generates three possibilities per ship, each accompanied by a corresponding probability of occurrence. Afterward, we build scenarios by considering the combinations of all dates. Here, we assume that the arrivals of the ships are mutually independent events.

5.4. Assumptions and model

The following assumptions have been considered when formulating the mathematical programming model:

1. There is only one pipeline connecting the terminal with the refinery, so only one vessel can unload at any moment.
2. A vessel that has started unloading crude can leave the terminal only once it is completely emptied.
3. Each vessel carries a single type of crude oil, and it is considered that the pipeline has a negligible volume compared to the volume to be unloaded.
4. A tank cannot receive crude from a vessel and feed a CDU at the same time. After receiving crude, a tank should stay idle for some time for brine settling and removal.
5. A maximum number of tanks can be loaded simultaneously, and transfers between tanks are not allowed.
6. There is a maximum number of CDUs that a single tank can feed simultaneously.
7. There is a maximum number of tanks that can feed a CDU at the same time, and the time to change over tanks is negligible.
8. Perfect mixing of crudes occurs in the mixing pipelines.
9. It is not allowed to stop feeding the crude distillation units.

Under these assumptions, a scheduling model of the process operation, which includes balances, constraints, and other relevant factors, was developed. Appendix A includes the nomenclature of sets, parameters, and variables. Additionally, it contains the model's constraints, except for the objective function, which is explained in the next subsection.

5.5. Objective function

The cost associated with each scenario e, including first and second stage terms, is calculated by using (1). The first term, the costs due to the difference between processed volume and required demand, comprises the first-stage cost. The variables op_u and sp_u represent the overproduction and underproduction, respectively, concerning the demand for CDU u. The second term, demurrage and departure tardiness costs, represents the second-stage cost, where the variables $dmg_{b,e}$ and $tdn_{b,e}$ refer to the demurrage and departure tardiness of vessel b under scenario e, respectively. In both cases, the parameters in capital letters correspond to the unit costs related to each variable.

$$
\begin{aligned}
ze_e &= \sum_{u \in U} (COP_u * op_u + CSP_u * sp_u) \\
&+ \sum_{b \in B} (CDMG_b * dmg_{b,e} + CTDN_b * tdn_{b,e}) \quad \forall e \in E
\end{aligned} \tag{1}
$$

The objective function is composed of the first-stage cost and the expected value of the second-stage cost, considering all scenarios e. To calculate it, we sum the costs associated with each scenario (ze_e), weighting them according to their probability of occurrence (π_e), as shown in (2).

$$
MIN \sum_{e \in E} \pi_e * ze_e \tag{2}
$$

5.6. Deterministic equivalent program

There are different ways to solve the two-stage stochastic programming model. In this paper, we use the deterministic equivalent program approach, which consists of solving the first and second-stage variables together, and thus simultaneously obtaining a feasible solution for each scenario.

The optimization model results in a mixed-integer nonlinear programming (MINLP) model as it involves continuous and binary variables. The objective function is given by (2), and the problem is subject to constraints (A.1)–(A.59), and (1), among which only (A.59) is nonlinear. This nonlinearity is caused by the fact that the crude oil concentration in the output flow of a tank must be equal to the concentration inside the tank.

6. MINLP solution procedure

Due to the nonconvex nature of the nonlinear constraint (A.59), it is interesting to develop strategies to address this issue.

For instance, in [8] a two-step procedure has been implemented. In the first step, the nonlinear constraint is ignored, and the resulting MILP model is solved. Subsequently, utilizing the solution from the previous step, the binary variables are fixed, and the MILP solution serves as a starting point for solving the resulting NLP model. This NLP model includes the same constraints as the MILP model in addition to the nonlinear constraint.

The authors of [15] propose an iterative two-step MILP-NLP decomposition algorithm. First, an MILP relaxation is formulated by replacing the bilinear terms with piecewise McCormick envelopes, which establishes a lower bound on the MINLP. Next, the solution of the MILP is utilized as an initial point, and the binary variables are fixed into the MINLP, resulting in a non-linear programming (NLP) problem. The solution of this NLP provides an upper bound when a feasible solution is obtained.

The paper [12] introduces a two-step MILP-NLP algorithm where the bilinear blending constraints are relaxed using multiparametric disaggregation, this technique involves discretizing one of the variables of the bilinear term over a set of powers.

In this paper, we have developed a strategy that also consists of two steps. Initially, a mixed integer linear programming (MILP) model, which is an approximation of the original MINLP, is solved. The approximate MILP formulation is obtained by replacing the nonlinear constraint (A.59), the one which states that the crude oil concentration at the outlet of a tank must be the same as the one inside the tank, with the linear constraints (3) and (4), which determine that a tank maintains the initial crude c concentration until the moment it receives crude oil from a ship.

In more detail, if a tank has not received any loading until slot s inclusive, then the binary variable $xq_{q,s',e}$, indicating if the tank is receiving a load, will be equal to zero for slot s and all previous slots ($s' \leq s$). Thus, from Eqs. (3) and (4), the volume of crude oil type c unloaded ($fcqu_{c,q,u,s,e}$) will be equal to the total volume unloaded ($fqu_{q,u,s}$) during slot s, multiplied by the initial concentration of crude oil c in that tank ($CONC_{q,c}$). In case the tank has previously received a load, then the constraints become idle.

$$fcqu_{c,q,u,s,e} \leq CONC_{q,c} * fqu_{q,u,s} + M3_q * \sum_{s' \in S, s' \leq s} xq_{q,s',e} \quad (3)$$
$$\forall c \in C,\ \forall q \in Q,\ \forall u \in U,\ \forall s \in S,\ \forall e \in E$$

$$fcqu_{c,q,u,s,e} \geq CONC_{q,c} * fqu_{q,u,s} - M3_q * \sum_{s' \in S, s' \leq s} xq_{q,s',e} \quad (4)$$
$$\forall c \in C,\ \forall q \in Q,\ \forall u \in U,\ \forall s \in S,\ \forall e \in E$$

Subsequently, the values of the binary variables in the original MINLP (allocation of vessels to tanks, allocation of tanks to units, et cetera) are fixed according to the solution obtained for the MILP, and the resulting nonlinear programming (NLP) model is solved to get the values of volumes loaded into the tanks, volumes unloaded from the tanks, start-time and duration of operations, among others. In case no feasible solution can be reached, the original MINLP model is solved using an outer approximation solver (DICOPT [27]).

This procedure is summarizes in algorithm 1, and depicted in Fig. 3. In this figure, each circle represents a type of model and its components: the objective function, constraints, and variables. Initially, starting from the MINLP model, we obtain the MILP model (linear approximation) and solve it (MILP solution). Then, based on this solution, we fix the values of the binary variables in the MINLP model (binary variables MILP) and solve the resulting NLP model (solution).

Algorithm 1 MINLP solution strategy

1: Replace equation (A.59) of MINLP model with equations (3) and (4).
2: Solve resulting MILP model.
3: Fix binary variables from MILP solution in MINLP model.
4: Solve resulting NLP model.
5: **if** NLP solution is infeasible **then**
6: Solve MINLP model with DICOPT solver.
7: **end if**

7. Stochastic model performance evaluation

One of the objectives of this paper is to evaluate whether two-stage stochastic programming offers any advantage over simpler deterministic approaches. To this end, Birge and Louveaux [26] proposed the value of the stochastic solution (VSS) and the expected value of perfect information (EVPI) as performance indicators. Before defining both measures, it should be noted that the two-stage stochastic programming model is also known as recourse problem (RP).

7.1. Expected value of perfect information

Suppose that we have perfect information about uncertainty, i.e., we know with complete certainty the arrival date of the ships every time we have to make a decision; in other words, we know what the future scenario will be. Furthermore, let us assume that there is at least one feasible solution for each of the scenarios considered. Thus, we could solve each of them separately obtaining the corresponding optimal solution and the associated value of the objective function. If we were to repeat the procedure of applying the appropriate optimal solution every time we have to make a decision, we would obtain the minimum expected cost in the long run.

Therefore, the minimum expected cost is equal to the sum of the optimal costs associated with each scenario weighted by its probability of occurrence what is known as the wait-and-see solution (WS).

Finally, we obtain the expected value of perfect information (EVPI) as the difference between the RP solution and the WS solution. The EVPI represents how much we would be willing to pay, each time we have to make a decision, to obtain perfect information about the arrival of the ships.

$$EVPI = RP - WS \quad (5)$$

7.2. Value of the stochastic solution

One may ask which is the advantage of using the stochastic solution over the deterministic one. If we do not wish to use the RP, then we can solve a deterministic problem with the expected arrival times of the ships. This approach is known as the expected value problem (EV), and its solution is called the expected value solution.

With this solution, we will apply the first-stage variable values. Then, when a ship arrives, the best we could do is to solve another deterministic problem in which we fix the values of the first-stage variables and optimize the second-stage variables.

In the long run, the cost will correspond to the weighted average of all these solutions, and it is known as the expected result of using the EV solution (EEV).

Finally, the value of the stochastic solution (VSS) compares the EEV solution and the RP solution in order to quantify the reduction of the expected cost when considering the randomness of the uncertainty versus its weighted average.

$$VSS = EEV - RP \quad (6)$$

It should be noted that there is a risk that the solution obtained from the EV model may be infeasible for one or more scenarios. Therefore, when optimizing the second-stage variables, we have incorporated slack variables related to vessel unloading start dates to achieve convergence in those scenarios for which the solution is infeasible.

8. Risk management

The approach described above (A.1)–(A.59), (1), and (2) does not evaluate the risk associated with the objective function, that is, it is risk-neutral and only seeks to minimize the expected cost in the long run without taking into account the probability distribution of the objective function, that is, the probability of having very bad values of the cost function in case some scenarios are realized. However, it is often important to consider this distribution to reduce the risk that the solution obtained could produce high costs in certain scenarios.

For this purpose, there are two popular risk measures: Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR). On the one hand, VaR with confidence level $1-\alpha$ determines the minimum value ω^*, where ω^* is the value of the cost function J such that the probability of obtaining a value of J less than ω^* is $1-\alpha$. On the other hand, CVaR with confidence level $1-\alpha$ represents the average value of the tail of the distribution, above the $\mathrm{VaR}_{1-\alpha}$ value (Fig. 4).

In Fig. 4, the y-axis represents the values of the probability density function. The x-axis represents the values of the random variable: the objective function (J). The symbol ω^* represents the value of J obtained from $\mathrm{VaR}_{1-\alpha}$. The gray area under the curve represents the value of the cumulative distribution function evaluated at ω^*, which is the probability that J is less than ω^*. The blue area under the curve (distribution tail) represents the probability that J is greater than ω^*.

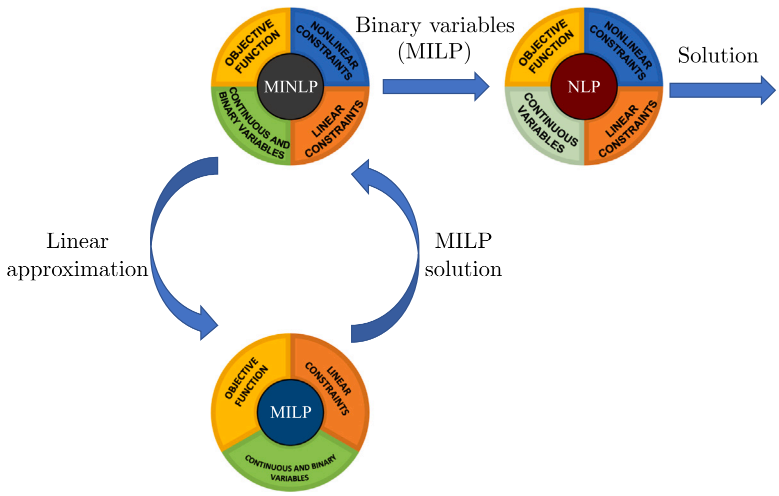

Fig. 3. Solution procedure.

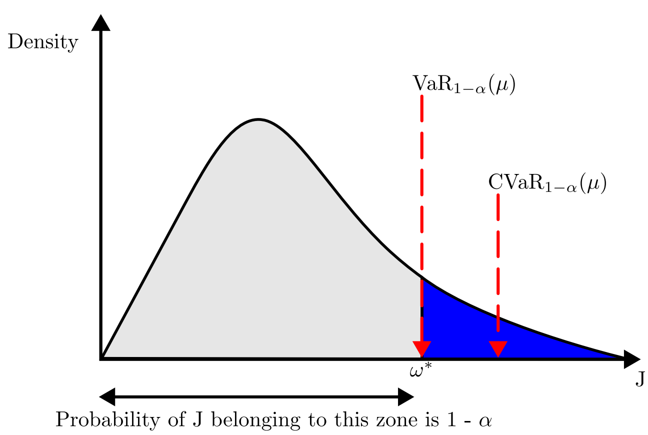

Fig. 4. Graphical representation of the CVaR.

Both risk measures have similar meanings but, even if VaR is easier to interpret, the solution of the optimization problem becomes quite complex, so in this work, we use CVaR because it is simple to calculate and consistent since if the cost function is convex with respect to the decision variables (μ), then the CVaR function is also convex.

The formulation of the two-stage stochastic programming model with risk management maintains the same constraints as the original two-stage model (A.1)–(A.59), and (1) and incorporates the following.

In (7), a non-negative auxiliary variable is defined for each scenario (ϕ_e). It takes a value greater than zero if the cost of the scenario (ze_e) is greater than VaR variable (var); otherwise, it can be made zero. In (8), the value of the CVaR variable ($cvar$) is calculated.

$$ze_e \; - \; var \leq \phi_e \quad \forall e \in E \tag{7}$$

$$cvar \; = \; var \; + \; (1/\alpha) * (\sum_{e \in E} \pi_e * \phi_e) \tag{8}$$

Moreover, the objective function (2) is replaced by (9).

$$MIN \; cvar \tag{9}$$

9. Results

To evaluate each of the presented approaches, and in particular, the two-stage stochastic programming model with risk management developed in this paper, we consider an example that consists of a maritime terminal with a pipeline, five storage tanks, two crude distillation units, and five classes of crude characterized by a single key property. The arrival of two vessels is expected over a 120-hours (5 days) time horizon. The arrival dates and probabilities for each scenario are detailed in Table 1. The expected departure date is 12 hours after the arrival. The demand for CDU 1 is 100 000 m^3 and for CDU 2 is 65 000 m^3 over the scheduling horizon.

Table 1
Arrival times and probabilities.

Scenarios	Probabilities	Arrival time (h)	
		Ship 1	Ship 2
1	0.01	5	35
2	0.03	45	35
3	0.01	85	35
4	0.2	5	65
5	0.5	45	65
6	0.2	85	65
7	0.01	5	95
8	0.03	45	95
9	0.01	85	95

The different formulations mentioned in the paper have been solved and compared in the following sections.

9.1. Risk-neutral approach

In this section, we analyze the results obtained from the two-stage risk-neutral stochastic programming model (A.1)–(A.59), (1), and (2), also known as the recourse problem (RP).

First, in Table 2, we observe that the expected cost of the solution is 20.97 k€. This value corresponds to the sum of the costs of each scenario, weighted according to their probability of occurrence.

Second, the costs associated with each scenario are detailed in Table 3 under the "Risk-neutral (RP)" row. Here, we can see that they are greater than zero in all cases except for scenarios 4 and 6, and the worst case corresponds to scenario 2, which has a cost of 159 k€. It should be mentioned that none of the scenarios incur first-stage costs; in other words, the demand is met exactly, and these costs are exclusively attributed to demurrage and tardiness in the unloading of the ships.

Next, we conduct a detailed analysis of Fig. 5, corresponding to a Gantt chart of ship unloading for the RP solution. On the vertical axis of this diagram, the scenarios are indicated, and on the horizontal axis, the timeline (scheduling horizon). Then, blue and red bars represent the operation of vessels 1 and 2, respectively, and gray bars represent the time intervals during which no vessel is operating. Please note that

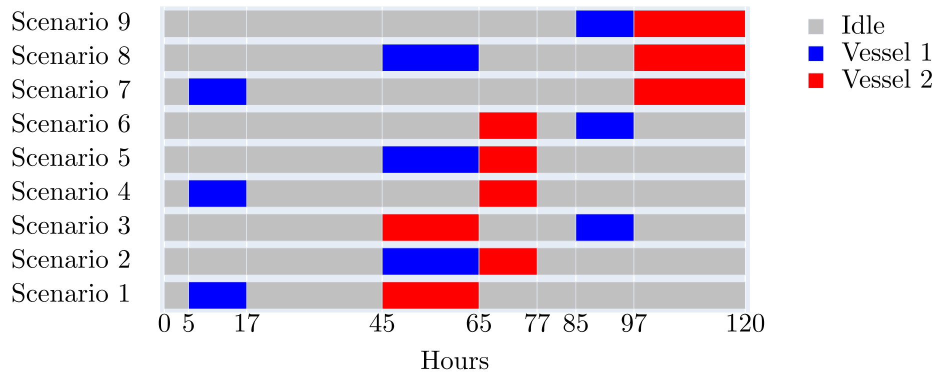

Fig. 5. Gantt chart of risk-neutral solution.

Table 2
Expected costs associated with proposed models, and values of EVPI and VSS.

RP	WS	EEV (×10³ €)	EVPI	VSS
20.97	0.45	1078.13	20.52	1057.16

Table 3
Cost per scenario.

Model	Cost per scenario (×10³ €)								
	e1	e2	e3	e4	e5	e6	e7	e8	e9
$CVaR_{0.99}$	0	30	0	24	54	24	42	72	42
$CVaR_{0.7}$	45	99	45	30	30	30	42	42	42
$CVaR_{0.6}$	45	147	45	24	24	24	42	42	42
Risk-neutral (RP)	69	159	69	0	24	0	42	66	42
EEV	205	123	4123	196	0	4000	3196	3000	7000
WS	0	12	0	0	0	0	0	0	9

slot durations are first-stage variables, and therefore, they are the same for all scenarios.

In scenario 1, vessel 1 unloads the crude oil at the scheduled times. In contrast, vessel 2 experiences demurrage and tardiness, beginning its discharge 10 hours later than the scheduled arrival time and finishing at hour 65 instead of hour 47 (12 hours after arrival). In scenario 2, ship 1 starts unloading at the scheduled time (hour 45) but finishes at hour 65, eight hours later than expected. As for ship 2, it incurs a 30-hour demurrage, and consequently, it also fails to meet the stipulated departure time. In this case, we can observe that the order of ship arrivals is not respected when unloading. Regarding scenario 3, we observe that vessel 2 starts unloading 10 hours later than expected, and its tardiness is 18 hours, as it should have left the terminal by hour 47. In scenarios 4 and 6, both vessels adhere to their scheduled times for starting and finishing unloading, resulting in a cost of zero for these scenarios. In scenario 5, both ships start unloading at the expected time, but ship 1 incurs an eight-hour tardiness. Finally, in scenarios 7, 8, and 9, ship 1 begins unloading at the scheduled time and only finishes later than expected in scenario 8. As for ship 2, in all three scenarios, it starts and finishes unloading at hours 97 and 120, respectively, which means there is a two-hour demurrage and a 13-hour tardiness.

9.2. EVPI and VSS

The cost function values obtained for the recourse problem solution (RP), the expected result of using the EV solution (EEV), and the wait-and-see solution (WS) are shown in Table 2. Moreover, the expected value of perfect information (EVPI) and the value of the stochastic solution (VSS) are presented in the same table to analyze the effect of considering uncertainty. From the value of the EVPI, we can conclude that if we have access to perfect information, the RP solution will improve by up to 20.52 k€, so this is the maximum that we would be willing to pay to obtain that information. Besides, the VSS indicates that it is worth using the two-stage stochastic optimization since the expected cost when using the mean value of the uncertain parameter increases by 1057.16 k€. It is important to mention that this value is very large since there are scenarios for which the first-stage EV solution is infeasible, and this feature is represented by high costs. Therefore, it is preferable to apply the RP solution.

In Table 3, we can see the cost per scenario for the risk-neutral (RP), EEV, and WS models. The solution obtained for the first-stage variables using the EV model is infeasible for scenarios 3, 6, 7, 8, and 9, as indicated by the high associated cost for each of these scenarios. It is worth mentioning that when optimizing the second-stage variables, we introduced slack variables related to vessel unloading start dates to ensure convergence in scenarios where the initial solution is infeasible. These slack variables were incorporated into the objective function with a high cost. As a result, the objective function value is higher than three thousand in these cases, highlighting the extent of infeasibility in the first-stage solution for these scenarios.

We can also observe that the WS solution is zero in several scenarios because, with perfect information, they fulfill all specifications. However, it is non-zero in scenarios 2 and 9. This is because when ships arrive on dates close to each other, the second ship to be unloaded will inevitably incur demurrage and tardiness, even if the first ship unloads at the maximum flow rate.

9.3. CVaR

The results obtained for the two-stage stochastic programming model with risk management (CVaR model) are presented below. Three cases with different confidence levels have been analyzed: 0.99, 0.7, and 0.6.

Table 3 shows the cost associated with each scenario for each of the evaluated cases: CVaR 0.99, CVaR 0.7, and CVaR 0.6. When we compare these three cases, we can observe that only for CVaR 0.99 are there scenarios with zero cost, specifically scenarios 1 and 3. This is due to the fact that, based on the solution obtained for CVaR 0.99, the demand is met exactly, and ships 1 and 2 are unloaded on the scheduled dates in scenarios 1 and 3. For the remaining scenarios, costs are greater than zero due to demurrage or tardiness in the operation of the ships.

Continuing with the analysis of the three solutions, we can observe that the worst-case scenario for CVaR 0.99 is scenario 8. For the CVaR 0.7 and CVaR 0.6 solutions, the worst scenario is number 2. Additionally, when comparing the three cases, we notice that the highest cost is associated with scenario 2 of the CVaR 0.6 solution.

Interestingly, we can observe from Table 3 that for scenarios 7 and 9, the costs associated with the RP and CVaR models are the same. This is because, in all four solutions, vessel 2 starts unloading two hours later than planned and finishes 13 hours later than its expected departure date, resulting in the same demurrage and tardiness costs for these models. It is worth mentioning that in none of the cases are first-stage costs incurred.

Table 4 displays the VaR and CVaR values at confidence levels 0.99, 0.7, and 0.6. In addition, the expected costs for all cases, including the risk-neutral approach, are shown. When we compare the solutions that include risk management, we can observe that as the significance level (α) decreases, the VaR and CVaR values increase. This is because a greater weight is given to the worst-case scenarios, i.e., those scenarios that present a high cost.

Likewise, as α decreases, the expected cost (value of the objective function) increases because more conservative policies are adopted. These policies help obtain solutions that avoid high costs in less probable scenarios but come at the expense of increasing the cost associated with scenarios that have a higher probability of occurring. Furthermore, we can note that the RP solution is better in terms of expected cost, but it may have very high costs in some scenarios, as can be seen in Table 3 for scenario 2.

Figs. 6 to 8 show Gantt charts of vessel operations for the three risk-managed solutions. In these figures, we can observe the arrival and departure times of the two vessels for each of the nine scenarios.

When comparing scenarios within the same case, we observe that the decisions related to the unloading of the vessels vary among them, since they are not first-stage decisions. Moreover, when comparing the three risk-managed solutions, we notice variations in the decisions regarding the start and end times of the time slots.

We also notice that the decisions (unloading dates and operation durations) related to ship 1 for scenarios 1, 3, 4, 6, 7, and 9 remain the same in all cases, including the risk-neutral case. Meanwhile, concerning vessel 2, the decisions related to scenarios 7, 8, and 9 are the same in all cases. Furthermore, only in the case of higher risk aversion ($CVaR_{0.99}$) is the order of arrival of the vessels in scenario 2 respected. This results in the lowest cost case for scenario 2, excluding the WS solution, as can be seen in Table 3.

Fig. 9 illustrates the costs per scenario in each of the three risk-managed solutions and the risk-neutral solution. From this figure, we can see more clearly the effect of the adopted risk level on the obtained solution. As we assume less risk (lower α), the costs of the scenarios tend to be more uniform. Additionally, we can notice that the possibility of having scenarios with very high costs is avoided.

It is worth mentioning that although the total volume of the feed mixture is the same for all scenarios since it is a first-stage variable, the concentration of the property can be different between scenarios if the composition of the mixture changes. This fact is shown in Figs. 10, 11, and 12, which depict the evolution of key property concentration in the feed mixture of CDU 1 for scenarios 3, 5, and 7, corresponding to the $CVaR_{0.99}$ solution.

By analyzing these figures, we can observe that the evolution of the concentration maintains a very similar profile up to hour 65 in the three scenarios, and in none of the cases, the established limits are violated. However, scenarios 3 and 7 are more critical than scenario 5 since, at the end of the horizon, the property's concentration value is at the lower bound or very close to it.

9.4. Time analysis and model statistics

In order to evaluate the complexity of the presented method, an analysis of the computational time has been carried out by varying the number of evaluated scenarios. For this purpose, four cases have been proposed: the first involves 4 scenarios, the second 9 scenarios (the same as those shown in Table 1), the third 16 scenarios, and finally, the fourth case with 25 scenarios. Each of the cases has been solved for the models: RP, $CVaR_{0.7}$, and $CVaR_{0.99}$; and in each of them, eight slots have been used.

Table 4
VaR and CVaR values.

1-α	$VaR_{1-\alpha}$ ($\times 10^3$ €)	$CVaR_{1-\alpha}$ ($\times 10^3$ €)	Expected cost ($\times 10^3$ €)
0.99	72	72	40.5
0.7	30	39.9	32.97
0.6	24	36.53	29.01
Risk-neutral (RP)	–	–	20.97

Table 5
Solution time for different number of scenarios.

Model	Number of scenarios (CPU time in seconds)			
	4	9	16	25
RP	8.9	40.5	1178.9	–
$CVaR_{0.7}$	4.3	19.1	482.7	–
$CVaR_{0.99}$	7.0	38.0	149.1	–

As can be seen from the values in Table 5, as the number of scenarios increases, the solution time increases exponentially, increasing by three orders of magnitude for the RP case. Moreover, it should be noted that it was not possible to obtain solutions in less than 3600 seconds for any of the three proposed models.

From the previous result, we can see that the number of scenarios evaluated constitutes a bottleneck when solving case studies with the proposed model, which is why it is of great interest to the authors to apply and develop decomposition methods in future work.

Lastly, Table 6 summarizes the statistics for each of the instances solved, including EV and WS models. In the EV and WS models the scenarios are solved separately, therefore, the table shows the number of variables and constraints corresponding to a single scenario. However, the Time column shows the total time taken to solve all scenarios. The example has been solved using GAMS 41.3.0 software, Gurobi 9.5.2 for MILPs, and CONOPT 4.29 for NLPs on a computer with Intel Core i9-13900K 3.00 GHz processor and 128 GB RAM.

10. Conclusions

In this paper, we developed a scheduling model to characterize the operation of a system comprising a crude oil section and a maritime terminal of an oil refinery. The model is used to decide the best way of operating the crude section while considering the uncertainty linked with ship arrivals. Alongside formulating a two-stage stochastic scheduling problem, we assessed the advantages of concurrently incorporating a two-stage stochastic approach and a continuous-time formulation for optimizing crude oil operations scheduling compared to deterministic approaches. Furthermore, we examined the impact of integrating risk management into the model and how solutions vary across different aversion levels.

From the value of the stochastic solution, we can conclude that the two-stage formulation offers a more robust solution compared to deterministic approaches, mainly because it allows us to correct the consequences of decisions taken now based on future conditions.

Moreover, introducing CVaR enables the penalization of extreme values that may appear if some scenarios are realized, thereby minimizing risk. While the inclusion of risk management in the two-stage stochastic model increases the expected cost with higher risk aversion, Fig. 9 highlights that risk-aware solutions exhibit greater uniformity compared to risk-neutral ones. Notably, in scenario 2, the cost of the risk-neutral solution exceeds five times that of CVaR at a confidence level of 0.99.

Regarding the solution strategy for the presented MINLP, even though the solution of the approximate MILP model might not be

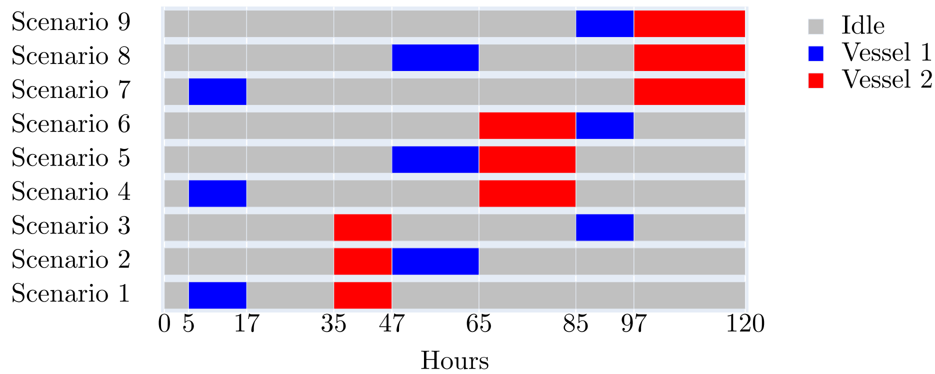

Fig. 6. Gantt chart of $CVaR_{0.99}$ solution.

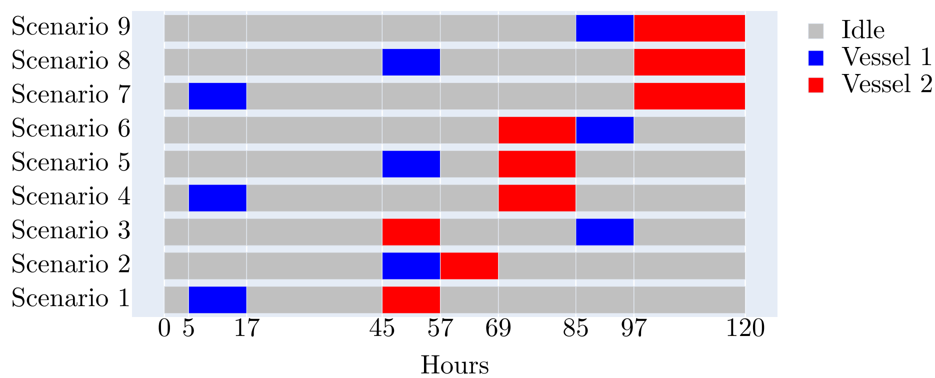

Fig. 7. Gantt chart of $CVaR_{0.7}$ solution.

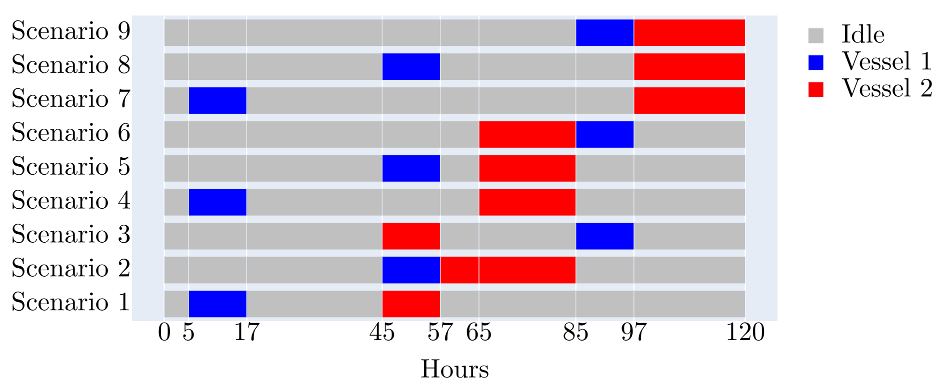

Fig. 8. Gantt chart of $CVaR_{0.6}$ solution.

Table 6
Model statistics.

Model	Continuous variables		Binary variables	Constraints		Time (s)
	MILP	NLP	MILP	MILP	NLP	
$CVaR_{0.99}$	9356		1272	20 016	16 416	38.02
$CVaR_{0.7}$						19.12
$CVaR_{0.6}$						54.39
Risk-neutral	9345			20 006	16 406	40.51
EEV	1273		248	2670	2270	0.77
WS	1257					2.82

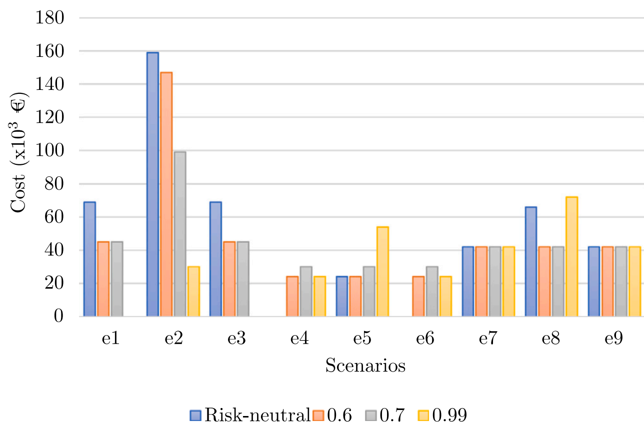

Fig. 9. Cost of scenarios.

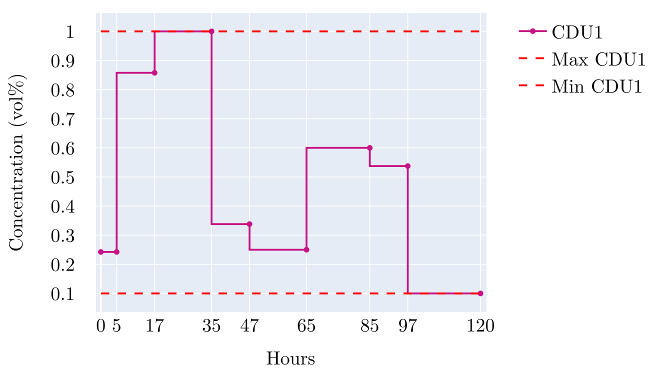

Fig. 10. Evolution of key property concentration in scenario 3 ($CVaR_{0.99}$).

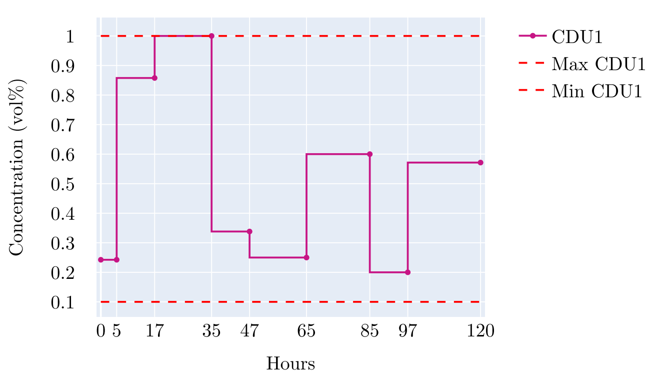

Fig. 11. Evolution of key property concentration in scenario 5 ($CVaR_{0.99}$).

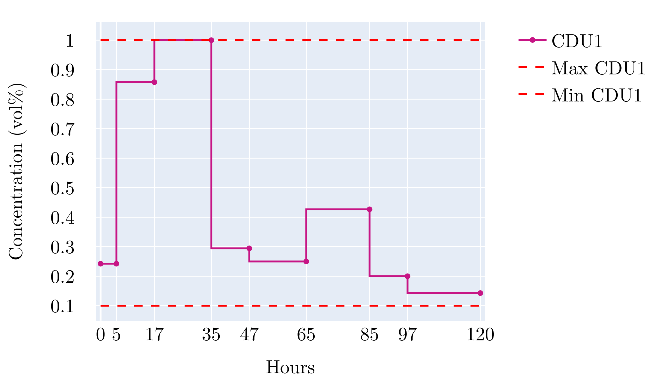

Fig. 12. Evolution of key property concentration in scenario 7 ($CVaR_{0.99}$).

optimal for the MINLP, it provides an efficient way of selecting very good and feasible decisions. However, the authors are aware that the method can be improved. One option would be to define an iterative process in which cutoffs are added to the MILP.

One of the main limitations found in this work corresponds to the number of evaluated scenarios. One way to overcome this difficulty is through the development and application of decomposition methods for stochastic problems. Currently, progress is being made in this direction, and results are expected to be presented soon.

Finally, future work will also focus on extending the scope of the studied system, including downstream processing units, to achieve more comprehensive solutions.

Declaration of competing interest

The authors declare the following financial interests/personal relationships which may be considered as potential competing interests: Tomas Garcia Garcia-Verdier reports financial support was provided by University of Valladolid.

Data availability

Data will be made available on request.

Acknowledgments

Financial support received from the Spanish Government with projects a-CIDiT (PID2021-123654OB-C31) and InCo4In (PGC 2018-099312-B-C31), and from European Social Fund. Furthermore, this work was supported by the Regional Government of Castilla y León and the EU-FEDER (CLU 2017-09, CL-EI-2021-07, UIC 233).

Appendix

This appendix includes the nomenclature and constraints of the model presented. We should mention that certain notation has been adopted from [6]. Besides, one point that is interesting to mention concerns the way precedence is handled. In this work, the precedence between vessels is not subject to the order of the elements of the set “Vessels” as stated in [13], and there is no pre-allocation of time slots for each vessel as in [6]. In the present work, the “pre-defined precedence” concept has been applied, which means that the set of slots is pre-ordered, and the optimization algorithm allocates each vessel to some of these slots [28].

Appendix A. Model constraints

A.1. Notation

A.1.1. Sets

- B = vessels
- BC = vessel–crude pairs. This set indicates which type of crude oil *c* is transported by each vessel *b*, i.e., its elements are pairs (*b*, *c*), where *b* is in B, and *c* is in C.
- C = types of crude oils
- E = scenarios
- K = key components
- Q = tanks
- S = time slots
- U = crude distillation units

A.1.2. Parameters

- $AT_{b,e}$ = arrival time of vessel b under scenario e
- $\underline{CAP}_q$ = minimum inventory of crude mix in a tank
- $\overline{CAP}_q$ = maximum capacity of a tank
- $CDMG_b$ = demurrage or sea waiting cost
- $CONC_{q,c}$ = initial crude c concentration in the tank q
- COP_u = cost due to overproduction concerning the demand of unit u
- CSP_u = cost due to underproduction concerning the demand of unit u
- $CTDN_b$ = departure tardiness cost
- DEM_u = total demand of blended crude for CDU u
- $EDT_{b,e}$ = departure time of vessel b under scenario e
- $\underline{FB}_b$ = minimum rate of crude transfer from vessel b
- $\overline{FB}_b$ = maximum rate of crude transfer from vessel b
- $\underline{FQ}_q$ = minimum rate of crude transfer to tank q
- $\overline{FQ}_q$ = maximum rate of crude transfer to tank q
- $\underline{FU}_u$ = minimum rate of crude transfer to CDU u
- $\overline{FU}_u$ = maximum rate of crude transfer to CDU u
- H = length of the scheduling horizon
- $IIC_{q,c}$ = initial amount of crude c in the tank q
- NR = maximum number of tanks that can feed a CDU
- NT = maximum number of tanks that can be loaded
- NU = maximum number of CDUs that can be loaded simultaneously by a tank
- $\underline{OFQ}_q$ = minimum rate of crude transfer from tank q
- $\overline{OFQ}_q$ = maximum rate of crude transfer from tank q
- $PR_{c,k}$ = volumetric concentration of the key component k in the crude type c
- $\underline{PROP}_{u,k}$ = minimum allowed concentration of key component k in the feedstock of CDU u
- $\overline{PROP}_{u,k}$ = maximum allowed concentration of key component k in the feedstock of CDU u
- ST = time to settle and remove the brine
- $VOL_{b,c}$ = amount of crude c in the vessel b
- α = significance level
- π_e = scenario e probability

A.1.3. Continuous variables

The domain of continuous variables is the set of non-negative real numbers, except for the variables ze_e (cost per scenario) and $cvar$ (Conditional Value-at-Risk), whose domain is the set of all real numbers since they represent the objective functions.

- $cvar$ = Conditional Value-at-Risk
- $dmg_{b,e}$ = demurrage of vessel b under scenario e
- $dmgs_{b,s,e}$ = auxiliary variable to calculate $dmg_{b,e}$
- ds_s = length of slot s
- $fbq_{b,q,s,e}$ = amount of crude transferred from b to q during s
- $fcb_{c,b,s,e}$ = total amount of crude c unloaded from b during s
- $fcbq_{c,b,q,s,e}$ = amount of crude c transferred from b to q during s
- $fcqu_{c,q,u,s,e}$ = amount of crude c transferred from q to u during s under scenario e
- $fqu_{q,u,s}$ = amount of crude mix transferred from q to u during s
- $fu_{u,s}$ = total amount of crude mix transferred to u during s
- $i_{q,s,e}$ = total level in q at the beginning of s
- $ic_{q,c,s,e}$ = amount of c in q at the beginning of s
- $ice_{q,c,s,e}$ = amount of c in q at the end of the horizon
- $ie_{q,s,e}$ = total level in q at the end of the scheduling horizon
- is_s = start-time of slot s
- op_u = overproduction concerning the demand of unit u
- sp_u = underproduction concerning the demand of unit u
- $tdn_{b,e}$ = departure tardiness of vessel b under scenario e
- $tdns_{b,s,e}$ = auxiliary variable to calculate $tdn_{b,e}$
- ts_s = end-time of slot s
- var = Value-at-Risk
- ze_e = cost associated with scenario e
- ϕ_e = auxiliary variable to assess the CVaR

A.1.4. Binary variables

- $xb_{b,s,e}$ = is equal to 1 if vessel b is unloading during s under scenario e; 0 otherwise
- $xfd_{b,s,e}$ = is equal to 1 if vessel b finishes its unloading at the end of s under scenario e; 0 otherwise
- $xid_{b,s,e}$ = is equal to 1 if vessel b starts its unloading at the beginning of s under scenario e; 0 otherwise
- $xq_{q,s,e}$ = is equal to 1 if tank q is receiving crude during s under scenario e
- $y_{q,u,s}$ = is equal to 1 if tank q feeds CDU u during slot s
- $yq_{q,s}$ = is equal to 1 if tank q is delivering crude during slot s
- $zq_{q,s,e}$ = is equal to 1 if tank q is idle or settling during slot s under scenario e

A.2. Constraints

A vessel is unloaded during a slot s ($xb_{b,s,e}$) if it was unloading during the previous slot ($xb_{b,s-1,e}$) and has not finished yet ($xfd_{b,s-1,e}$), or if it starts at the beginning of the current slot ($xid_{b,s,e}$) (A.1). Note that it applies to every scenario as it involves second-stage variables.

$$xb_{b,s,e} = xb_{b,s-1,e} + xid_{b,s,e} - xfd_{b,s-1,e} \quad \forall b \in B,\ \forall s \in S,\ \forall e \in E \tag{A.1}$$

A vessel can finish unloading at the end of a slot as long as it was unloading during that slot (A.2). Note that if the ship is not being unloaded ($xb_{b,s,e} = 0$), then $xfd_{b,s,e}$ will be equal to 0.

$$xb_{b,s,e} \geq xfd_{b,s,e} \quad \forall b \in B,\ \forall s \in S,\ \forall e \in E \tag{A.2}$$

All ships may start and end unloading only once within the scheduling horizon, (A.3) and (A.4) respectively.

$$\sum_{s\in S} xid_{b,s,e} = 1 \quad \forall b \in B,\ \forall e \in E \tag{A.3}$$

$$\sum_{s\in S} xfd_{b,s,e} = 1 \quad \forall b \in B,\ \forall e \in E \tag{A.4}$$

Only one vessel can unload at any moment (assumption 1).

$$\sum_{b\in B} xb_{b,s,e} \leq 1 \quad \forall s \in S,\ \forall e \in E \tag{A.5}$$

A maximum of NT tanks can be loaded simultaneously (assumption 5). That is, the sum of the binary variable indicating that a tank is being loaded ($xq_{q,s,e}$), over all tanks, must be less than or equal to NT.

$$\sum_{q\in Q} xq_{q,s,e} \leq NT \quad \forall s \in S,\ \forall e \in E \tag{A.6}$$

When unloading crude oil from a vessel, it is necessary to have at least one tank being loaded. Note that if no tank is receiving a load, then $\sum_{q\in Q} xq_{q,s,e}$ equals zero.

$$\sum_{q\in Q} xq_{q,s,e} \geq xb_{b,s,e} \quad \forall b \in B,\ \forall s \in S,\ \forall e \in E \tag{A.7}$$

A tank cannot be loaded if there is no ship unloading.

$$xq_{q,s,e} \leq \sum_{b\in B} xb_{b,s,e} \quad \forall q \in Q,\ \forall s \in S,\ \forall e \in E \tag{A.8}$$

A tank may not charge more than NU crude distillation units simultaneously (assumption 6). Therefore, the sum of the variable indicating that a tank is feeding a unit ($y_{q,u,s}$), over all units, must be less than or equal to NU at all times.

$$\sum_{u\in U} y_{q,u,s} \le NU \quad \forall q \in Q,\ \forall s \in S \tag{A.9}$$

At most NR tanks are allowed to concurrently feed a CDU (assumption 7).

$$\sum_{q\in Q} y_{q,u,s} \le NR \quad \forall u \in U,\ \forall s \in S \tag{A.10}$$

Each CDU must continually process feedstock coming from tanks (assumption 9). This means that each CDU must be fed by at least one tank at all times.

$$\sum_{q\in Q} y_{q,u,s} \ge 1 \quad \forall u \in U,\ \forall s \in S \tag{A.11}$$

A tank must be in one of the three states (i.e., loading, unloading, or idle) during a given slot.

$$xq_{q,s,e} + yq_{q,s} + zq_{q,s,e} = 1 \quad \forall q \in Q,\ \forall s \in S,\ \forall e \in E \tag{A.12}$$

A tank must be discharging ($yq_{q,s}$) if it is feeding a CDU ($y_{q,u,s}$).

$$yq_{q,s} \ge y_{q,u,s} \quad \forall q \in Q,\ \forall u \in U,\ \forall s \in S \tag{A.13}$$

A tank must be feeding at least one unit ($\sum_{u\in U} y_{q,u,s}$) if it is being unloaded ($yq_{q,s}$).

$$\sum_{u\in U} y_{q,u,s} \ge yq_{q,s} \quad \forall q \in Q,\ \forall s \in S \tag{A.14}$$

The end-time of a slot is equal to its start-time plus its length.

$$ts_s = is_s + ds_s \quad \forall s \in S \tag{A.15}$$

The start-time of a slot coincides with the end-time of the previous slot. This implies that the durations of operations of all resources are synchronized in each slot.

$$is_s = ts_{s-1} \quad \forall s \in S \tag{A.16}$$

The total length of the time slots must be equal to the length of the scheduling horizon.

$$\sum_{s\in S} ds_s = H \tag{A.17}$$

The big-M method, explained by Winston and Goldberg [29], is applied to compute the amount of crude unloaded to tanks. The M values are determined based on physical limits. For example, M1 takes into account the volume carried by each vessel.

We calculate the upper bound of the volume of crude oil unloaded from a ship to a tank ($fcbq_{c,b,q,s,e}$) as a function of the maximum flow rate that the receiving tank admits ($\overline{FQ}_q$) and the duration of the operation.

$$fcbq_{c,b,q,s,e} \le \overline{FQ}_q * ds_s \quad \forall q \in Q,\ \forall (b,c) \in BC,\ \forall s \in S,\ \forall e \in E \tag{A.18}$$

We calculate the lower bound of the volume of crude oil unloaded from a ship to a tank ($fcbq_{c,b,q,s,e}$) as a function of the minimum flow rate the tank admits ($\underline{FQ}_q$) and the duration of the operation. Note that this constraint is activated only if ship b is unloading ($xb_{b,s,e} = 1$) and tank q is receiving a load ($xq_{q,s,e} = 1$).

$$fcbq_{c,b,q,s,e} \ge \underline{FQ}_q * ds_s - M1_b * (2 - xb_{b,s,e} - xq_{q,s,e}) \quad \forall q \in Q,\ \forall (b,c) \in BC,\ \forall s \in S,\ \forall e \in E \tag{A.19}$$

If vessel b is not being unloaded, then $fcbq_{c,b,q,s,e}$ will be equal to zero.

$$fcbq_{c,b,q,s,e} \le M1_b * xb_{b,s,e} \quad \forall q \in Q,\ \forall (b,c) \in BC,\ \forall s \in S,\ \forall e \in E \tag{A.20}$$

If tank q is not being loaded, then $fcbq_{c,b,q,s,e}$ will be equal to zero.

$$fcbq_{c,b,q,s,e} \le M1_b * xq_{q,s,e} \quad \forall q \in Q,\ \forall (b,c) \in BC,\ \forall s \in S,\ \forall e \in E \tag{A.21}$$

Also, we use the big-M method to calculate the crude volume unloaded from a vessel during a slot s ($fcb_{c,b,s,e}$). The upper and lower bounds are obtained by (A.22) and (A.23), respectively.

$$fcb_{c,b,s,e} \le \overline{FB}_b * ds_s \quad \forall (b,c) \in BC,\ \forall s \in S,\ \forall e \in E \tag{A.22}$$

$$fcb_{c,b,s,e} \ge \underline{FB}_b * ds_s - M1_b * (1 - xb_{b,s,e}) \quad \forall (b,c) \in BC,\ \forall s \in S,\ \forall e \in E \tag{A.23}$$

If ship b is not being unloaded, then $fcb_{c,b,s,e}$ will be equal to zero.

$$fcb_{c,b,s,e} \le M1_b * xb_{b,s,e} \quad \forall (b,c) \in BC,\ \forall s \in S,\ \forall e \in E \tag{A.24}$$

The crude volume unloaded from a vessel during a slot s is equal to the sum of volumes unloaded to each tank.

$$fcb_{c,b,s,e} = \sum_{q\in Q} fcbq_{c,b,q,s,e} \quad \forall (b,c) \in BC,\ \forall s \in S,\ \forall e \in E \tag{A.25}$$

The total volume loaded into a tank during a slot s is calculated by using (A.26).

$$fbq_{b,q,s,e} = \sum_{c\in BC} fcbq_{c,b,q,s,e} \quad \forall b \in B,\ \forall q \in Q,\ \forall s \in S,\ \forall e \in E \tag{A.26}$$

To make each vessel unload fully during the scheduling horizon (assumption 2), we use (A.27).

$$\sum_{s\in S} fcb_{c,b,s,e} = VOL_{b,c} \quad \forall (b,c) \in BC,\ \forall e \in E \tag{A.27}$$

The big-M method is applied to compute the amount of crude unloaded from tanks (A.28)–(A.30). Constraint (A.28) establishes the upper bound of the amount of crude oil discharged from tank q to unit u ($fqu_{q,u,s}$) as a function of the maximum discharge flow rate from the tank ($\overline{OFQ}_q$) and the duration of the operation.

$$fqu_{q,u,s} \le \overline{OFQ}_q * ds_s \quad \forall q \in Q,\ \forall u \in U,\ \forall s \in S \tag{A.28}$$

We compute the lower bound as a function of the minimum discharge flow rate of tank q ($\underline{OFQ}_q$) and the duration of the operation. This constraint will be activated if tank q is feeding unit u ($y_{q,u,s} = 1$).

$$fqu_{q,u,s} \ge \underline{OFQ}_q * ds_s - M2_q * (1 - y_{q,u,s}) \quad \forall q \in Q,\ \forall u \in U,\ \forall s \in S \tag{A.29}$$

If tank q is not feeding unit u, then $fqu_{q,u,s}$ will be equal to zero.

$$fqu_{q,u,s} \le M2_q * y_{q,u,s} \quad \forall q \in Q,\ \forall u \in U,\ \forall s \in S \tag{A.30}$$

The total volume unloaded from a tank during a slot s is calculated by using (A.31). It should be noted that this total volume does not depend on the scenarios as it is a first-stage variable. However, its composition does, as the inventory profile in each tank may be different between scenarios due to receiving crude from ships at different times.

$$fqu_{q,u,s} = \sum_{c\in C} fcqu_{c,q,u,s,e} \quad \forall q \in Q,\ \forall u \in U,\ \forall s \in S,\ \forall e \in E \tag{A.31}$$

The total feed to CDU u during slot s ($fu_{u,s}$) is equal to the sum of the volumes transferred from each tank ($fqu_{q,u,s}$).

$$fu_{u,s} = \sum_{q\in Q} fqu_{q,u,s} \quad \forall u \in U,\ \forall s \in S \tag{A.32}$$

Constraints (A.33) and (A.34) set the upper and lower limits for $fu_{u,s}$, respectively

$$fu_{u,s} \le \overline{FU}_u * ds_s \quad \forall u \in U,\ \forall s \in S \tag{A.33}$$

$$fu_{u,s} \ge \underline{FU}_u * ds_s \quad \forall u \in U,\ \forall s \in S \tag{A.34}$$

The concentration of key components in the feedstock for the CDUs is given by (A.35)–(A.36). Constraint (A.35) sets the upper bound as the multiplication between the maximum allowed concentration of key component k in the feedstock of CDU u ($\overline{PROP}_{u,k}$) and the total volume received by u ($fu_{u,s}$). Similarly, constraint (A.36) establishes the lower bound. Note that, in both cases, the variable $fu_{u,s}$ is on the right-hand side of the inequality to avoid nonlinear constraints.

$$\sum_{q\in Q}\sum_{c\in C} fcqu_{c,q,u,s,e} * PR_{c,k} \leq \overline{PROP}_{u,k} * fu_{u,s} \quad \forall k \in K,\ \forall u \in U,\ \forall s \in S,\ \forall e \in E \tag{A.35}$$

$$\sum_{q\in Q}\sum_{c\in C} fcqu_{c,q,u,s,e} * PR_{c,k} \geq \underline{PROP}_{u,k} * fu_{u,s} \quad \forall k \in K,\ \forall u \in U,\ \forall s \in S,\ \forall e \in E \tag{A.36}$$

The amount of crude c in each tank at the start of slot s ($ic_{q,c,s,e}$) is calculated as the amount of crude oil c at the beginning of the previous slot ($ic_{q,c,s-1,e}$), plus the load of crude oil c received during the previous slot ($fcbq_{c,b,q,s-1,e}$), minus the volume of crude oil c discharged from tank q to the units during the previous slot ($fcqu_{c,q,u,s-1,e}$).

$$ic_{q,c,s,e} = ic_{q,c,s-1,e} + \sum_{b\in BC} fcbq_{c,b,q,s-1,e} - \sum_{u\in U} fcqu_{c,q,u,s-1,e} \quad \forall q \in Q,\ \forall c \in C,\ \forall s \in S\setminus\{s1\},\ \forall e \in E \tag{A.37}$$

The amount of crude c in each tank at the beginning of the horizon is given by (A.38).

$$ic_{q,c,s,e} = IIC_{q,c} \quad \forall q \in Q,\ \forall c \in C,\ s = s1,\ \forall e \in E \tag{A.38}$$

The amount of crude c in each tank at the end of the horizon ($ice_{q,c,s,e}$) is calculated as the amount of crude oil c at the beginning of the last slot of the horizon ($ic_{q,c,s,e}$), plus the crude oil load c received during the final slot ($fcbq_{c,b,q,s,e}$), minus the volume of crude oil c discharged from tank q to the units during that slot ($fcqu_{c,q,u,s,e}$).

$$ice_{q,c,s,e} = ic_{q,c,s,e} + \sum_{b\in BC} fcbq_{c,b,q,s,e} - \sum_{u\in U} fcqu_{c,q,u,s,e} \quad \forall q \in Q,\ \forall c \in C,\ s = |S|,\ \forall e \in E \tag{A.39}$$

The total level in each tank at the start of slot s ($i_{q,s,e}$) and at the end of the horizon ($ie_{q,s,e}$) is given by (A.40)–(A.43).

$$i_{q,s,e} = i_{q,s-1,e} + \sum_{b\in B} fbq_{b,q,s-1,e} - \sum_{u\in U} fqu_{q,u,s-1} \quad \forall q \in Q,\ \forall s \in S\setminus\{s1\},\ \forall e \in E \tag{A.40}$$

$$ie_{q,s,e} = i_{q,s,e} + \sum_{b\in B} fbq_{b,q,s,e} - \sum_{u\in U} fqu_{q,u,s} \quad \forall q \in Q,\ s = |S|,\ \forall e \in E \tag{A.41}$$

Moreover, both at the beginning of each slot (A.42) and at the end of the horizon (A.43), the total level in a tank q is equal to the sum of the volumes of each crude oil s stored in that tank.

$$i_{q,s,e} = \sum_{c\in C} ic_{q,c,s,e} \quad \forall q \in Q,\ \forall s \in S,\ \forall e \in E \tag{A.42}$$

$$ie_{q,s,e} = \sum_{c\in C} ice_{q,c,s,e} \quad \forall q \in Q,\ s = |S|,\ \forall e \in E \tag{A.43}$$

Eqs. (A.44)–(A.47) establish the physical limits for the inventory levels.

$$i_{q,s,e} \leq \overline{CAP}_q \quad \forall q \in Q,\ \forall s \in S,\ \forall e \in E \tag{A.44}$$

$$i_{q,s,e} \geq \underline{CAP}_q \quad \forall q \in Q,\ \forall s \in S,\ \forall e \in E \tag{A.45}$$

$$ie_{q,s,e} \leq \overline{CAP}_q \quad \forall q \in Q,\ s = |S|,\ \forall e \in E \tag{A.46}$$

$$ie_{q,s,e} \geq \underline{CAP}_q \quad \forall q \in Q,\ s = |S|,\ \forall e \in E \tag{A.47}$$

To ensure minimum settling time (assumption 4), we use (A.48). If a tank q receives a charge during slot s ($xq_{q,s,e} = 1$) and is discharged during slot s' ($yq_{q,s'} = 1$), where s' is later than s, then the start time of slot s' must be greater than or equal to the end time of slot s, plus the time required to settle (ST).

$$is_{s'} - ts_s \geq ST * (xq_{q,s,e} + yq_{q,s'} - 1) \quad \forall q \in Q,\ \forall s \in S,\ \forall s' \in S,\ s < s',\ \forall e \in E \tag{A.48}$$

To calculate the difference between processed volume and required demand by each CDU, we use (A.49)–(A.50). Constraint (A.49) sets the overproduction volume at unit u (op_u) as the difference between the volume processed over the horizon and the established demand. Constraint (A.50) computes the underproduction volume at unit u as the difference between the required demand and the volume processed over the horizon. Note that if the demand is not met, then sp_u will be greater than zero, and constraint (A.49) will be idle. Otherwise, if the demand value is exceeded, then op_u will be greater than zero, and constraint (A.50) will be idle.

$$op_u \geq \sum_{s\in S} fu_{u,s} - dem_u \quad \forall u \in U \tag{A.49}$$

$$sp_u \geq dem_u - \sum_{s\in S} fu_{u,s} \quad \forall u \in U \tag{A.50}$$

The discharge of crude oil from vessel b cannot start before its arrival time.

$$is_s \geq AT_{b,e} * xid_{b,s,e} \quad \forall b \in B,\ \forall s \in S,\ \forall e \in E \tag{A.51}$$

The demurrage is calculated as the time elapsed between the arrival of a ship and the start of its unloading.

From constraints (A.52)–(A.54), the auxiliary variable $dmgs_{b,s,e}$ is calculated, which represents how many hours after its arrival a ship b has started unloading in slot s, for scenario e.

In case the ship has not started unloading at the beginning of slot s ($xid_{b,s,e} = 0$), then constraints (A.52) and (A.53) will be inactive, and the variable $dmgs_{b,s,e}$ be zero (A.54). Otherwise, if the ship starts unloading at the beginning of slot s ($xid_{b,s,e} = 1$), then constraints (A.52) and (A.53) are activated, and the value of $dmgs_{b,s,e}$ is computed.

$$dmgs_{b,s,e} \geq is_s - AT_{b,e} * xid_{b,s,e} - H * (1 - xid_{b,s,e}) \quad \forall b \in B,\ \forall s \in S,\ \forall e \in E \tag{A.52}$$

$$dmgs_{b,s,e} \leq is_s - AT_{b,e} * xid_{b,s,e} \quad \forall b \in B,\ \forall s \in S,\ \forall e \in E \tag{A.53}$$

$$dmgs_{b,s,e} \leq H * xid_{b,s,e} \quad \forall b \in B,\ \forall s \in S,\ \forall e \in E \tag{A.54}$$

From constraint (A.55), the demurrage of each ship in each scenario is calculated as the summation of the auxiliary variable $dmgs_{b,s,e}$ over all slots. Note that, at most, a single term of the summation will be greater than zero.

$$dmg_{b,e} = \sum_{s\in S} dmgs_{b,s,e} \quad \forall b \in B,\ \forall e \in E \tag{A.55}$$

If vessel b leaves the terminal after its expected departure time $EDT_{b,e}$, it should pay a penalty that will be proportional to the departure tardiness ($tdn_{b,e}$). Analogously to the calculation of the demurrage, the tardiness of each ship in each scenario ($tdn_{b,e}$) is computed from the auxiliary variable $tdns_{b,s,e}$, using constraints (A.56)–(A.58).

$$tdns_{b,s,e} \geq ts_s - EDT_{b,e} * xfd_{b,s,e} - H * (1 - xfd_{b,s,e}) \quad \forall b \in B,\ \forall s \in S,\ \forall e \in E \tag{A.56}$$

$$tdns_{b,s,e} \leq H * xfd_{b,s,e} \quad \forall b \in B,\ \forall s \in S,\ \forall e \in E \tag{A.57}$$

$$tdn_{b,e} = \sum_{s\in S} tdns_{b,s,e} \quad \forall b \in B,\ \forall e \in E \tag{A.58}$$

If a tank is being discharged, then the crude oil concentration in the outflow must be equal to the concentration inside the tank. In other words, this principle states that the proportion of each crude in the volume transferred ($fcqu_{c,q,u,s,e}/fqu_{q,u,s}$) must be the same as the proportion of each crude in the volume stored ($ic_{q,c,s,e}/i_{q,s,e}$). This rule is satisfied by (A.59). It should be noted that this equation yields two bilinear terms which are non-convex.

$$i_{q,s,e} * fcqu_{c,q,u,s,e} = ic_{q,c,s,e} * fqu_{q,u,s} \quad \forall c \in C,\ \forall q \in Q,\ \forall u \in U,\ \forall s \in S,\ \forall e \in E \tag{A.59}$$

Appendix B. List of abbreviations and symbols

- CDU: crude oil distillation unit
- CVaR: conditional value-at-risk
- EEV: expected result of using the EV solution
- EV: expected value problem
- EVPI: expected value of perfect information
- M1: M-value applied to constraints of calculation of volume loaded to tanks
- M2: M-value applied to constraints of calculation of volume discharged from tanks
- M3: M-value applied to linear approximation constraints
- MILP: mixed-integer linear programming
- MINLP: mixed-integer nonlinear programming
- NLP: nonlinear programming
- RP: recourse problem
- VaR: value-at-risk
- VSS: value of the stochastic solution
- WS: wait-and-see solution

References

[1] H. Yang, D.E. Bernal, R.E. Franzoi, F.G. Engineer, K. Kwon, S. Lee, I.E. Grossmann, Integration of crude-oil scheduling and refinery planning by Lagrangean decomposition, Comput. Chem. Eng. 138 (2020) http://dx.doi.org/10.1016/j.compchemeng.2020.106812.

[2] N.Q. Wu, L.P. Bai, M.C. Zhou, An efficient scheduling method for crude oil operations in refinery with crude oil type mixing requirements, IEEE Trans. Syst. Man Cybern. 46 (2016) 413–426, http://dx.doi.org/10.1109/TSMC.2014.2332138.

[3] H. Lee, J.M. Pinto, I.E. Grossmann, S. Park, Mixed-integer linear programming model for refinery short-term scheduling of crude oil unloading with inventory management, Ind. Eng. Chem. Res. 35 (1996) http://dx.doi.org/10.1021/ie950519h.

[4] Z. Jia, M. Ierapetritou, J.D. Kelly, Refinery short-term scheduling using continuous time formulation: Crude-oil operations, Ind. Eng. Chem. Res. 42 (2003) http://dx.doi.org/10.1021/ie020124f.

[5] E. Kondili, C.C. Pantelides, R.W.H. Sargent, A general algorithm for short-term scheduling of batch operations—i. milp formulation, Comput. Chem. Eng. 17 (1993) 211–227, http://dx.doi.org/10.1016/0098-1354(93)80015-F.

[6] P.C.P. Reddy, I.A. Karimi, R. Srinivasan, A new continuous-time formulation for scheduling crude oil operations, Chem. Eng. Sci. 59 (2004) http://dx.doi.org/10.1016/j.ces.2004.01.009.

[7] K.C. Furman, Z. Jia, M.G. Ierapetritou, A robust event-based continuous time formulation for tank transfer scheduling, Ind. Eng. Chem. Res. 46 (2007) http://dx.doi.org/10.1021/ie061516f.

[8] S. Mouret, I.E. Grossmann, P. Pestiaux, A novel priority-slot based continuous-time formulation for crude-oil scheduling problems, Ind. Eng. Chem. Res. 48 (2009) 8515–8528, http://dx.doi.org/10.1021/ie8019592.

[9] J. Li, R. Misener, C.A. Floudas, Continuous-time modeling and global optimization approach for scheduling of crude oil operations, AIChE J. 58 (2012) 205–226, http://dx.doi.org/10.1002/aic.12623.

[10] S. Yadav, M.A. Shaik, Short-term scheduling of refinery crude oil operations, Ind. Eng. Chem. Res. 51 (2012) 9287–9299, http://dx.doi.org/10.1021/ie300046g.

[11] A.A. Hamisu, S. Kabantiok, M. Wang, Refinery scheduling of crude oil unloading with tank inventory management, Comput. Chem. Eng. 55 (2013) 134–147, http://dx.doi.org/10.1016/j.compchemeng.2013.04.003.

[12] P.M. Castro, I.E. Grossmann, Global optimal scheduling of crude oil blending operations with RTN continuous-time and multiparametric disaggregation, Ind. Eng. Chem. Res. 53 (2014) 15127–15145, http://dx.doi.org/10.1021/ie503002k.

[13] J. Cerdá, P.C. Pautasso, D.C. Cafaro, Efficient approach for scheduling crude oil operations in marine- access refineries, Ind. Eng. Chem. Res. 54 (2015) http://dx.doi.org/10.1021/acs.iecr.5b01461.

[14] B. Zimberg, E. Camponogara, E. Ferreira, Reception, mixture, and transfer in a crude oil terminal, Comput. Chem. Eng. 82 (2015) 293–302, http://dx.doi.org/10.1016/j.compchemeng.2015.07.012.

[15] L.S. de Assis, E. Camponogara, B. Zimberg, E. Ferreira, I.E. Grossmann, A piecewise McCormick relaxation-based strategy for scheduling operations in a crude oil terminal, Comput. Chem. Eng. 106 (2017) 309–321, http://dx.doi.org/10.1016/j.compchemeng.2017.06.012.

[16] L.S. Assis, E. Camponogara, B.C. Menezes, I.E. Grossmann, An MINLP formulation for integrating the operational management of crude oil supply, Comput. Chem. Eng. 123 (2019) 110–125, http://dx.doi.org/10.1016/j.compchemeng.2018.12.014.

[17] L.S. Assis, E. Camponogara, I.E. Grossmann, A MILP-based clustering strategy for integrating the operational management of crude oil supply, Comput. Chem. Eng. 145 (2021) http://dx.doi.org/10.1016/j.compchemeng.2020.107161.

[18] J. Wang, G. Rong, Robust optimization model for crude oil scheduling under uncertainty, Ind. Eng. Chem. Res. 49 (2010) 1737–1748, http://dx.doi.org/10.1021/ie900358z.

[19] C. Cao, X. Gu, Z. Xin, Stochastic chance constrained mixed-integer nonlinear programming models and the solution approaches for refinery short-term crude oil scheduling problem, Appl. Math. Model. 34 (2010) 3231–3243, http://dx.doi.org/10.1016/j.apm.2010.02.015.

[20] J. Li, R. Misener, C.A. Floudas, Scheduling of crude oil operations under demand uncertainty: A robust optimization framework coupled with global optimization, AIChE J. 58 (2012) 2373–2396, http://dx.doi.org/10.1002/aic.12772.

[21] F. Oliveira, P.M. Nunes, R. Blajberg, S. Hamacher, A framework for crude oil scheduling in an integrated terminal-refinery system under supply uncertainty, European J. Oper. Res. 252 (2016) 635–645, http://dx.doi.org/10.1016/j.ejor.2016.01.034.

[22] T.G. García-Verdier, G. Gutiérrez, C. Méndez, C.G. Palacín, C. de Prada, Minimizing risk in the scheduling of crudes in an oil refinery, IFAC-PapersOnLine 55 (2022) 809–814, http://dx.doi.org/10.1016/J.IFACOL.2022.07.544, URL https://linkinghub.elsevier.com/retrieve/pii/S2405896322009508.

[23] C.A. Méndez, J. Cerdá, I.E. Grossmann, I. Harjunkoski, M. Fahl, State-of-the-art review of optimization methods for short-term scheduling of batch processes, Comput. Chem. Eng. 30 (2006) 913–946, http://dx.doi.org/10.1016/j.compchemeng.2006.02.008.

[24] S. Mouret, I.E. Grossmann, P. Pestiaux, Time representations and mathematical models for process scheduling problems, Comput. Chem. Eng. 35 (2011) 1038–1063, http://dx.doi.org/10.1016/j.compchemeng.2010.07.007.

[25] M. Tejeda-Iglesias, N.H. Lappas, C.E. Gounaris, L. Ricardez-Sandoval, Explicit model predictive controller under uncertainty: An adjustable robust optimization approach, J. Process Control 84 (2019) 115–132, http://dx.doi.org/10.1016/j.jprocont.2019.09.002.

[26] J. Birge, F. Louveaux, Introduction to Stochastic Programming, second ed., in: Springer Series in Operations Research and Financial Engineering, Springer New York, NY, 2011, pp. 1–469.

[27] I. Grossmann, J. Viswanathan, A. Vecchietti, R. Raman, E. Kalvelagen, GAMS/DICOPT: a discrete continuous optimization package, 11, 2002.

[28] C.G. Palacín, Efficient Scheduling of Batch Processes in Continuous Processing Lines, Universidad de Valladolid, 2020, http://dx.doi.org/10.35376/10324/43323, URL http://uvadoc.uva.es/handle/10324/43323.

[29] W.L. Winston, J.B. Goldberg, Operations Research: Applications and Algorithms, fourth ed., in: Mathematics in Science and Engineering, Thomson Brooks/Cole, 2004.